Existence and Uniqueness of Solution of Fractional Order Differential Equations.

Abstract

In this work, qualitative property like existence and uniqueness of the solution of fractional order differential equations studied. Fractional order differential equation with and without impulses governing physical phenomena is also considered. Thesis consist of Six chapters:

Through out the all chapters ${}^cD^\beta$ denotes Caputo fractional order differential operator.

Chapter one

The impulsive fractional quasiliner integro-differential equation

$$\begin{aligned} {}^cD^\beta x(t) &= A(t,x)x(t) + f(t, x(t), Tx(t), Sx(t)) \quad t \neq t_k,\ k = 1, 2, \cdots, p \\ \Delta x(t_k) &= I_k(x(t_k)), \quad t = t_k,\ k = 1, 2, \cdots, p \end{aligned}$$

Where, $A(t,x)$ is bounded quasi-linear operator on X and $f : [0, T_0] \times X \times X \times X \to X$, $T, S : X \to X$ are defined by $Tx(t) = \int_0^t h(t, s, x(s))ds$ and $Sx(t) = \int_0^{T_0} k(t, s, x(s))ds$, $h : D_0 \times X \to X,\ D_0 = \{(t, s); 0 \leq s \leq t \leq T_0\}$ and $k : D_1 \times X \to X,\ D_1 = \{(t, s); 0 \leq t, s \leq T_0\}$ are continuous, with local condition $x(0) = x_0$ and nonlocal condition $x(0) = x_0 - g(x)$ over the interval $[0, T_0]$ in a Banach space X. Existence and uniqueness of mild solution of the problem obtained. Inclusion of Fredholm integral operator S in the equation is more relavent in modelling of many physical phenomena arises in the field of viscoelasticity. An example is added to illustrate the efficacy of the method.

Chapter Two

The work discussed in 1st chapter is extended in 2nd chapter by adding a delay condition on it and derived sufficient conditions for existence and uniqueness of mild solution of the

integro-differential equations:

$$\begin{aligned} {}^cD^\beta x(t) &= A(t,x)x(t) + f(t, x(\phi(t)), Tx(t), Sx(t)) \quad t \neq t_k,\ k = 1, 2, \cdots, p \\ \Delta x(t_k) &= I_k(x(t_k)), \quad t = t_k,\ k = 1, 2, \cdots, p \\ x(0) &= x_0 - g(x), \end{aligned}$$

over the interval $[0, T_0]$ in a Banach space X. Where, $A(t,x)$ is bounded quasi linear operator on X and $f : [0, T_0] \times X \times X \times X \to X$, $T, S : X \to X$ are defined by $Tx(t) = \int_0^t h(t, s, x(\psi(s)))ds$ and $Sx(t) = \int_0^{T_0} k(t, s, x(\xi(s)))ds$, where $h : D_0 \times X \to X$, $D_0 = \{(t,s); 0 \leq s \leq t \leq T_0\}$ and $k : D_1 \times X \to X$, $D_1 = \{(t,s); 0 \leq t, s \leq T_0\}$ are the operators satisfying condition of the hypotheses.

Chapter Three

The sufficient conditions for existence and uniqueness of mild and classical solution of fractional order impulsive integro-differential equations of the following form is established in this chapter. And also derived conditions in which mild and classical solution are congruence.

$$\begin{aligned} {}^cD^\beta x(t) &= Ax(t) + f(t, x(t), Tx(t), Sx(t)) \quad t \neq t_k,\ k = 1, 2, \cdots, p \\ \Delta x(t_k) &= I_k(x(t_k)), \quad t = t_k,\ k = 1, 2, \cdots, p \\ x(0) &= x_0 \end{aligned}$$

over the interval $[0, T_0]$ in a Banach space X. Here, A is bounded linear operator on X and $f : [0, T_0] \times X \times X \times X \to X$,$T, S : X \to X$ are defined by $Tx(t) = \int_0^t h(t, s, x(s))ds$ and $Sx(t) = \int_0^{T_0} k(t, s, x(s))ds$. Where $h : D_0 \times X \to X$, $D_0 = \{(t,s); 0 \leq s \leq t \leq T_0\}$ and $k : D_1 \times X \to X$, $D_1 = \{(t,s); 0 \leq t, s \leq T_0\}$ are the operators satisfying condition of the hypotheses.

Chapter Four

This chapter, includes existence and uniqueness of mild and classical solution of generalized fractional order impulsive evolution equations

$$\begin{aligned} {}^cD^\beta x(t) &= Ax(t) + g_k(t, x(t)) \quad t \in (t_{k-1}, t_k)\ k = 1, 2, \cdots, p \\ \Delta x(t_k) &= I_k(x(t_k)), \quad t = t_k,\ k = 1, 2, \cdots, p \\ x(0) &= x_0 \end{aligned}$$

in which perturbing force is different after every impulse over the interval $[0, T_0]$ on a Banach space X. A is bounded linear operator on X and $I_k : [0, T_0] \times X \to X$ for $k = 1, 2, \cdots, p+1$. Also established conditions in which mild and classical solutions are congruent.

Chapter Five

Existence of mild solution for not-instantaneous impulses fractional order integro-differential equations with local and nonlocal conditions in Banach space is established in this paper.

$$\begin{aligned} {}^cD^\beta x(t) &= Ax(t) + f\left(t, x(t), \int_0^t a(t,s,x(s))ds\right), \; t \in [s_k, t_{k+1}), \; k = 1, 2, \cdots, p \\ x(t) &= I_k(k, x(t)), \; t \in [t_k, s_k) \end{aligned}$$

with local condition $x(0) = x_0$ and nonlocal condition $x(0) = x_0 + h(x)$ over the interval $[0, T_0]$ in a Banach space X. Here $A : X \to X$ is linear operator, $Kx = \int_0^t a(t,s,x(s))ds$ is nonlinear Volterra integral oprator on X, $f : [0, T_0] \times X \times X \to X$ is nonlinear function and $I_k : [0, T_0] \times X$ are set of nonlinear functions applied in the interval $[t_k, s_k)$ for all $i = 1, 2, \cdots, p$.

Chapter Six

This chapter established sufficient conditions for the mild solution of the fractional hybrid equations:

$$\begin{aligned} {}^cD^\beta\big[x(t) - f(t, x(t))\big] &= g(t, x(t)) \\ x(0) &= h(x) \end{aligned}$$

with nonlocal condition over the interval $[0, T_0]$ on partially ordered Banach space X. The nonlocal condition in this equation is more relevant in many physical phenomena in physics.