CHAPTER

1

FUNDAMENTAL CONCEPTS AND INTRODUCTION

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1.1 Fluid

A fluid is a material that continuously deforms under an applied shear stress. Fluid has no accurate shape and continuously deforms until a shear stress acts upon it. This shear stress may be less or more through which fluid changes its shape.

1.2 Fluid mechanics

Study of behavior of fluids subjected to a system of forces is known as fluid mechanics. It can be divided in following fields:

1.2.1 Statics

It deals with fluid elements which are at rest relative to each other.

1.2.2 Dynamics

It is study of the motion of liquids, gases and plasma from one place to another. Fluid dynamics has applications like computing forces and moments on aircraft, mass flow rate of petroleum passing through pipelines, prediction of weather, etc.

1.3 Viscosity

This is the internal property of a fluid by virtue of which it offers resistance to the flow. Mathematically it is defined as:

$$Viscosity = \mu = \frac{shear\ stress}{rate\ of\ shear\ strain}. \quad (1.1)$$

Water is thin having low viscosity and honey is thick having higher viscosity.

1.3.1 Dynamic viscosity (Absolute viscosity)

Dynamic viscosity measures the internal resistance of fluid. This resistance arises from attractive forces between molecules of the fluid. Mathematically, it is

$$Viscosity = \mu = \frac{shear\ stress}{rate\ of\ shear\ strain}. \tag{1.2}$$

1.3.2 Kinematic viscosity

The kinematic viscosity represents the ratio of dynamic viscosity μ to the density of the fluid ρ, it is represented by ν. Mathematically it is written as

$$\nu = \frac{\mu}{\rho}. \tag{1.3}$$

1.4 Nanofluid

Performance and compactness of engineering equipment such as heat exchangers, nuclear reactors and electronic devices can be upgraded, if thermal conductivity of conventional fluids such as oil, water and ethylene glycol mixture is improved. Pioneering technique to improve thermal conductivity of conventional fluids, is uniform and stable suspension of nanoparticles in the fluid. Suspension of copper or alumina nanoparticles in water are examples of nanofluids. Other metallic, nonmetallic, and polymeric particles can also be added into fluids to form nanofluids. Suspension of small amount of nanoparticles in conventional fluids, increases thermal conductivity significantly.

1.5 Physical properties of nanofluid [105]

Due to low concentration and extremely small size of suspended nanoparticles, the particles are supposed to move with same velocity as of fluid. Also, by assuming local thermal equilibrium, solid particle – liquid mixture may behave as a conventional single-phase fluid. Some of the physical properties of nanofluid for single-phase model are:

1.5.1 Density

Nanofluid density based on nanoparticle volume fraction is

$$\rho_{nf} = (1 - \emptyset)\rho_f + \emptyset\rho_s . \tag{1.4}$$

1.5.2 Specific heat capacity

Effective specific heat, based on heat capacity concept is given by

$$(\rho c_p)_{nf} = (1-\emptyset)(\rho c_p)_f + \emptyset(\rho c_p)_s \,. \tag{1.5}$$

1.5.3 Thermal expansion coefficient

Expression of thermal expansion coefficient of nanofluid is

$$(\rho\beta)_{nf} = (1-\emptyset)(\rho\beta)_f + \emptyset(\rho\beta)_s \,. \tag{1.6}$$

1.5.4 Electrical conductivity

The effective electrical conductivity of nanofluid is given by

$$\sigma_{nf} = \sigma_f\left[1+\frac{3(\sigma-1)\emptyset}{(\sigma+2)-(\sigma-1)\emptyset}\right],\ \sigma = \frac{\sigma_s}{\sigma_f}\,. \tag{1.7}$$

1.5.5 Dynamic viscosity

There are many models for viscosity of a nanofluid mixture which consider percentage of nanoparticles suspended in the base fluid. Most popular model that relates nanofluid viscosity, base fluid viscosity and nanoparticle concentration is as follows

$$\mu_{nf} = \frac{\mu_f}{(1-\emptyset)^{2.5}}\,. \tag{1.8}$$

1.5.6 Thermal conductivity

Nanofluid model for thermal conductivity adopted in studies of natural convection is

$$k_{nf} = k_f\left[1-3\,\frac{\emptyset(k_f-k_s)}{2k_f+k_s+\emptyset(k_f-k_s)}\right]. \tag{1.9}$$

Table 1.1 demonstrates values of thermo-physical properties for water as base fluid and different materials used as suspended particles. Various thermal conductivity and viscosity models used in nanofluid flow studies are presented in Table 1.2 and Table 1.3 respectively.

Table 1.1: Thermo-physical properties of water and nanoparticles.

Physical properties	Fluid phase (water)	Copper	Alumina	Silver
$C_p(J/(KgK))$	4179	385	765	235
$\rho(Kg/m^3)$	997.1	8933	3970	10500
$k(W/(mK))$	0.613	401	40	429
$\beta \times 10^{-5}(K^{-1})$	21	1.67	0.85	1.89
$\sigma((\Omega m)^{-1})$	0.05	$5.96*10^{7}$	10^{-10}	$3.60*10^{7}$

Table 1.2: Different models for thermal conductivity of nanofluids

Model	Equation
Wasp	$\frac{k_{nf}}{k_f} = \frac{k_p+2k_f-2\phi(k_f-k_p)}{k_p+2k_f+\phi(k_f-k_p)}$
Jang and Choi	$\frac{k_{nf}}{k_f} = (1-\phi) + Bk_p\phi + 18\times 10^6 \frac{3d_f}{d_p} k_f Re_{dp}^2 Pr\phi$
Bruggenman	$k_{nf} = 0.25k_f(3\phi-1)k_p/k_f + [3(1-\phi)-1] + \sqrt{\Delta_B}$
	$\Delta_B = \left[(3\phi-1)k_p/k_f + (3(1-\phi)-1)\right]^2 + 8k_p/k_f$
Chon	$k_{nf} = k_f[1 + 64.7\phi^{0.7640}\left(\frac{d_f}{d_p}\right)^{0.396}\left(\frac{k_f}{k_p}\right)^{0.7476} Pr_T^{0.9955} Re^{1.2321}$
Koo and Kleinstreuer	$\frac{k_{nf}}{k_f} = \frac{k_p+2k_f-2\phi(k_f-k_p)}{k_p+2k_f+\phi(k_f-k_p)} + 5\times 10^4 \beta\phi\rho_f(C_p)_f\sqrt{\frac{k_B T}{d_p\rho_f}} f(T,\phi)$
Charuyakorn	$\frac{k_{nf}}{k_f} = \left[\frac{k_p+2k_f-2\phi(k_f-k_p)}{k_p+2k_f+\phi(k_f-k_p)}\right](1 + b\phi Pe_p^m)$
Stationary	$k_{nf} = k_f[1 + k_p\phi d_f/(k_f(1-\phi)d_p)]$
Yu and Choi	$\frac{k_{nf}}{k_f} = \left[\frac{k_p+2k_f-2\phi(k_f-k_p)(1+\eta)^3}{k_p+2k_f+\phi(k_f-k_p)(1+\eta)^3}\right]$
Patel	$k_{nf} = 1 + \frac{k_p d_f\phi}{k_f d_p(1-\phi)}\left[1 + c\frac{2k_B T d_p}{\pi\alpha_f\mu_f d_p^2}\right]$
Mintsa	$k_{nf} = k_f[1.72\phi + 1.0]$

Table 1.3: Different models for viscosity of nanofluids

Model	**Equation**
Brinkman	$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$
Einstein	$\mu_{nf} = (2.5\phi + 1)\mu_f, \quad \phi < 0.05$
Pak and Cho	$\mu_{nf} = \mu_f(1 + 39.11\phi + 533.9\phi^2)$
Nguyen	$\mu_{CuO} = -0.6967 + \frac{15.937}{T} + 1.238\phi + \frac{1356.14}{T^2} - 0.259\phi^2 - \frac{30.88\phi}{T} - \frac{19652.74}{T^3} + 0.01593\phi^3 + \frac{4.38206\phi^2}{T} + \frac{147.573\phi}{T^2}$ $\mu_{Al_2O_3} = \exp(3.003 - 0.04203T - 0.5445\phi + 0.0002553T^2 + 0.0524\phi^2 - 1.622\phi^{-1})$
Jang	$\mu_{nf} = (2.5\phi + 1)\mu_f\left[1 + \eta\left(\frac{d_p}{H}\right)^{-2\varepsilon}\phi^{\frac{2}{3}}(\varepsilon + 1)\right]$
Koo and Kleinstreuer	$\mu_{nf} = 5 \times 10^4\beta\phi\rho_f\sqrt{\frac{k_BT}{d_p\rho_p}}f(T,\phi), \quad \begin{cases}\beta = 0.0137(100\phi)^{-0.8229} & for\ \phi < 1\% \\ \beta = 0.0011(100\phi)^{-0.7272} & for\ \phi > 1\%\end{cases}$
Maiga	$\mu_{nf} = \mu_f(1 + 7.3\phi + 123\ \phi^2)$
Brownian	$\mu_{nf} = \mu_f(1 + 2.5\phi + 6.17\phi^2)$
Nguyen	$\mu_{nf} = \mu_f(1 + 0.025\phi + 0.015\phi^2)$
Masoumi	$\mu_{nf} = \mu_f + \rho_p V_B d_p^2/(72C\delta)$
Gherasim	$\mu_{nf} = \mu_f 0.904e^{14.8\phi}$

1.6 Casson nanofluid

Casson nanofluid is a shear thinning fluid which is supposed to have an immeasurable viscosity at zero rate of shear, a yield stress below which no movement happens, and a zero viscosity at an infinite rate of shear. Casson nanofluid has numerous applications in metallurgy, food processing, drilling operations and bio-engineering operations. Example of Casson nanofluid is human blood.

1.7 Types of fluid flow

Classification of fluid flow is as follows:

1.7.1 Uniform and non-uniform flows

The flow is said to be uniform if magnitude and direction of flow velocity is same at each point of the flow. In case of non-uniform flow, velocity is not same at each point of the flow at any given instant.

1.7.2 Steady and unsteady flow

A flow is said to be steady flow in which fluid properties do not change with time at a specific point. A flow is said to be unsteady, if fluid properties change with time.

1.7.3 Laminar and turbulent flow

When a fluid flows in parallel layers, without disruption (disturbance) between the layers, it is called streamline or laminar flow. It is the flow of a fluid when each particle of the fluid follows a smooth pattern, the pattern which never interfere with one another. In streamline flow, velocity of the fluid is constant at any point. In contrast, turbulent flow is the flow in which fluid undergoes irregular fluctuations. In turbulent flow, speed of fluid at a point continuously changes in both magnitude and direction.

1.7.4 Compressible and incompressible flow

The flow type in which density is constant within the fluid, is called an incompressible flow. The mathematical equation for an incompressible flow is given by

$$\frac{\partial \rho}{\partial t} = 0. \tag{1.10}$$

1.8 Magnetohydrodynamics flow

The magnetohydrodynamics is a combination of three words. Magneto means magnetic field, hydro stands for liquid and dynamics for movement. Study of motion of electrically conducting fluids in which current is induced by magnetic field, is known as magnetohydrodynamics (MHD). Examples of such fluids are plasma, electrolytes, salt water and liquid metals. Motion of a conducting fluid across magnetic field generates electric currents, which modify magnetic field; and at the same time electric currents react with magnetic field to produce a body force, which in turn modifies the motion (Figure 1.1). This study was primarily inspired by geophysical and astrophysical problems and by problems associated with fusion reactor.

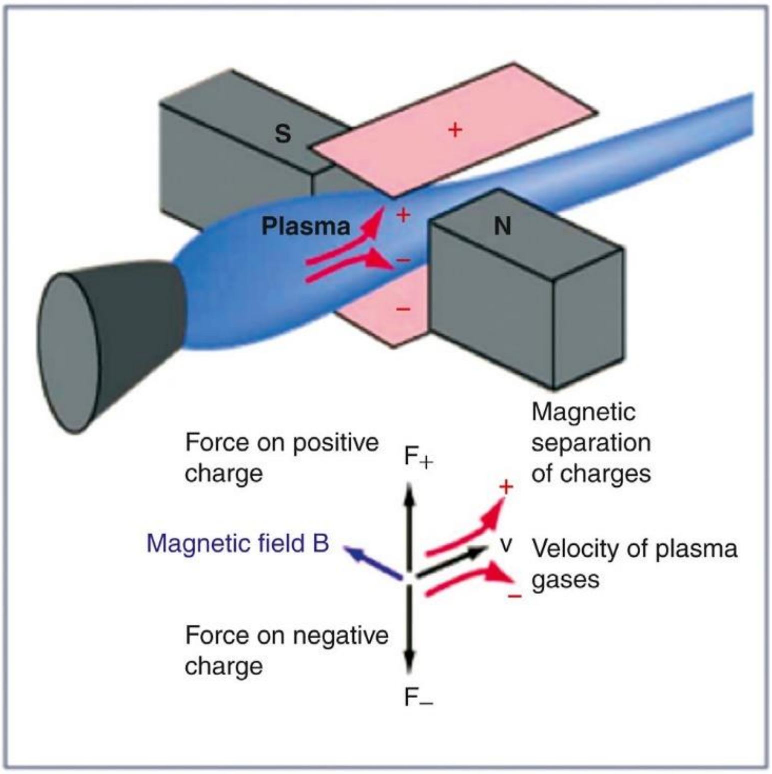

Figure 1.1: Induced current in a moving conductive fluid in the presence of a magnetic field

Technological problems like controlled thermonuclear fusion, thrust production for propulsive devices, pumping of liquid metals, high temperature resistant coating, re-entry problems of ballistic missiles, liquid metal lubrication, power conversion (i.e extraction of electrical energy directly from a hot plasma) etc. need the study of the flow of an electrically conducting fluid. In magnetohydronamics heat transfer problems, the additional body force term, viz, the Lorentz force term comes into play in the momentum equation and the term corresponding to Joule heating appears in the energy equation. In a forced convective system, the energy equation remains uncoupled from Maxwell`s equation and Navier-Stokes equations.

1.9 Heat transfer

Study of heat transfer includes thermal energy, where transfer is a result of temperature variance. In studying heat transfer, understanding of temperature distribution in a system is necessary. Heat flow takes place whenever there is a temperature gradient in a system. Once temperature distribution is known, heat flux, which is amount of heat transfer per unit area per unit time is obtained from rule connecting heat flows to temperature gradient. There are three fundamental modes of transfer of heat, which are conduction, convection and radiation.

1.9.1 Conduction

Conduction is a process by which heat energy is transmitted through collisions between neighboring molecules. In other words, conduction is way through which energy is transferred by movement of electrons or ions. Ironing clothes, melting piece of ice in hand, metal spoon in boiling water and a cup with hot coffee, are few examples of conduction.

1.9.2 Convection

In fluid dynamics, convection is energy transfer due to bulk fluid motion. Convective heat transfer arises between a fluid in motion and a bounding surface. If there is a difference in temperature of fluid and bounding surface, then thermal boundary layer is created. Fluid particles which interact with the surface, attain equilibrium at surface temperature and transfer energy in next layer and so on. Through this mode, temperature gradients are produced in fluid. Area of fluid containing these temperature gradients identified as thermal boundary layer.

A mathematical expression for convection phenomenon is

$$q = hA(T_w - T_0), \qquad (1.11)$$

where h and A denote heat transfer coefficient and area respectively.

Since convective heat transfer is by both random molecular motion and bulk motion of fluid; molecular motion is more adjacent to surface where fluid velocity is less. Convective heat transfer depends upon nature of flow. Therefore, convection has three forms: Natural (free) convection, Forced convection and Mixed convection.

1.9.2.1 Natural convection

Natural convection is a heat transport process, in which fluid motion is not developed by any external source but, only by density difference in fluid taking place due to temperature gradients. It happens due to temperature difference which affects density of the fluid. It is also known as free convection.

1.9.2.2 Forced convection

Forced convection is a process in which fluid motion is produced by an external source. It is a special type of heat transfer in which heat transfer is caused by dependent source like a fan and pump etc.

1.9.2.3 Mixed convection

It is a combination of both forced convection and natural convection.

1.9.3 Radiation

Radiation is the energy transfer due to release of photons or electromagnetic waves from a surface volume. Radiation doesn't require any medium to transfer heat. The energy produced by radiation is transformed by electromagnetic waves.

1.10 Mass transfer

Mass transfer is obtained as transfer of matter, as a result of concentration variance in a system. Mass transfer always occurs in direction of reducing concentration gradient. Phenomenon of mass transfer is evident in concept of stellar structure and on solar surface.

Application of mass transfer process spread to greater extent in several fields of engineering, science and technology. Mass transfer is involved in biological functions or processes like respiratory mechanisms, oxygenation or purification of blood, kidney function, osmosis and assimilation of food and drugs. This phenomenon is found in natural processes like smoke formation, evaporation of clouds, dispersion of fog, grooves of fruit trees, damage of crops due to freezing, pollution of environment, distribution of temperature and moisture over agricultural fields.

1.11 Porous media

Porous media have received increased attention over the past three decade because of their use in existing and emerging technologies, which directly impact the advancement of known grand challenges in areas such as bioengineering, electronics, energy and water filtration.

Permeability is most important physical property of the porous medium. Permeability is measure of ability of a medium to transmit fluid. It describes how well the pore spaces are connected. Value of permeability is determined by structure of the porous material, it is roughly a measure of mean square pore diameter in the material. Effective permeability measures the ability of a single fluid to flow through a rock when another fluid is also present in the pore spaces.

1.12 Soret effect

In simultaneous existence of heat and mass transfer in a moving fluid, flux and driving potential are related to each other. It is noticed that mass flux can be created by temperature gradients, and this embodies thermal diffusion (Soret) effect, named after Swiss scientist Charles Soret.

1.13 Chemical reaction effect

In convective heat and mass transfer, distribution rates can be changed by chemical reaction. Effect of chemical reaction depends on whether reaction is mixed or identical. A reaction is of first order, if rate of chemical reaction is directly proportional to concentration. In nature, existence of pure water or air is not possible. Study of chemical reaction procedures is beneficial for improving number of chemical technologies, such as food processing, polymer production and manufacturing of ceramics or glassware.

1.14 Mathematical model

Equations of MHD define the motion of a conducting fluid such as liquid metal or plasma in a magnetic field. There are two fundamental physical effects in MHD. The first effect is induced electric current due to movement of conductor into a magnetic field. Electric current induced in the conductor, creates its own magnetic field. This induced magnetic field tends to nullify the original, externally supported field. Conversely, when the magnetic field enters the conductor and the conductor is moved out of the field, the induced field strengthens the applied field.

Consequence of this phenomenon is that, lines of force seem to be dragged along with the conductor. Second crucial effect is dynamical. When currents are induced by flow of a conducting fluid through a magnetic field, Lorentz force act on the fluid and alter its motion. In MHD, the motion varies the field, and the field in turn reacts back and alters the motion. This makes the theory highly nonlinear.

1.14.1 Lorentz force

Lorentz force is the force experienced by charged particles moving in electromagnetic field. This force is given by

$$\vec{F} = q(\vec{u} \times \vec{B}), \tag{1.12}$$

where $\vec{F}$ is the force acting on charged particles, q is charge of particle, $\vec{u}$ is velocity of particle, and $\vec{B}$ is magnetic induction.

1.14.2 Faraday's law

When a charged particle moves in a magnetic field, it experiences the retarding force as well as produces voltage.

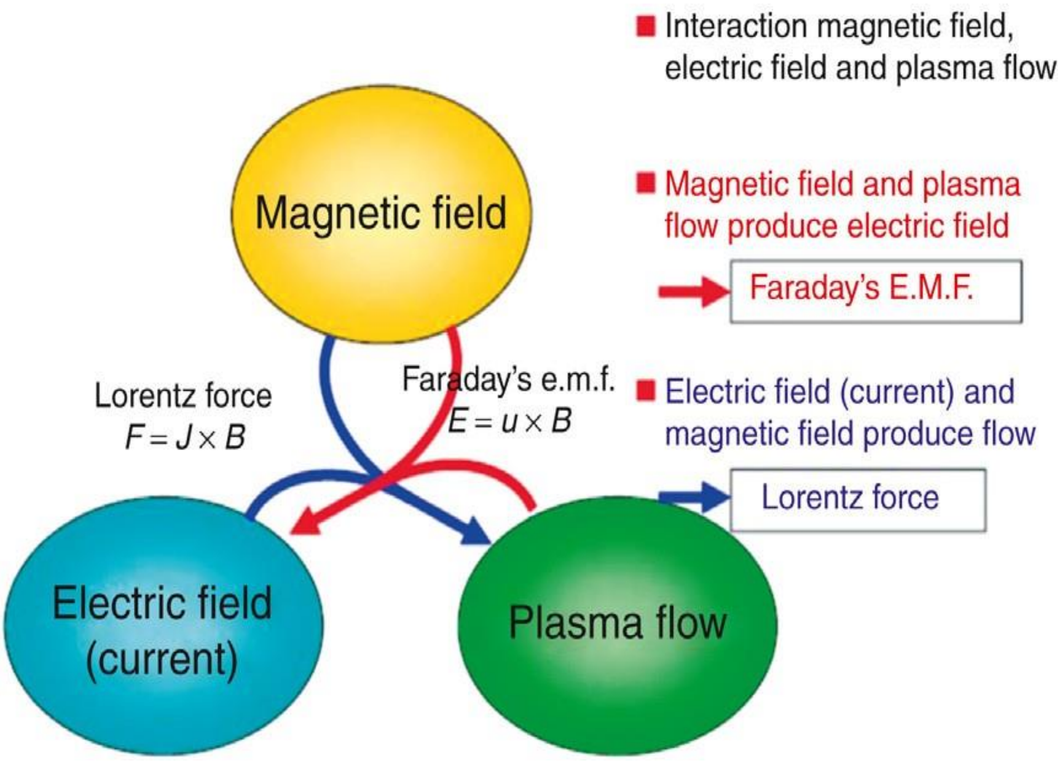

Figure 1.2: Relation between Faraday's Law and Lorentz Force.

1.14.3 Ohm's law

Ohm's law states that total electric current in a conductor is proportional to total electric field $\overrightarrow{E}$. Additionally, a fluid moving with velocity u in presence of a magnetic field $\vec{B}$ is subject to an additional electric field $\vec{u} \times \vec{B}$. Mathematically

$$\vec{j} = \sigma(\vec{E} + \vec{u} \times \vec{B}). \tag{1.13}$$

1.14.4 Maxwell equations

Maxwell's equations are a set of four partial differential equations that relate electric and magnetic fields to their sources, charge density and current density. Individually, equations are known as Gauss's law, Gauss's law for magnetism, Faraday's law for induction and Ampère's law with Maxwell's correction. Set of equations is named after James Clerk Maxwell, which can be expressed as

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{1.14}$$

$$\nabla \times \boldsymbol{H_1} = \mathbf{J}, \tag{1.15}$$

$$\nabla . \mathbf{B} = 0, \tag{1.16}$$

$$\nabla . \mathbf{J} = 0. \tag{1.17}$$

1.14.5 Equation of continuity

The equation of continuity is a differential equation that describes transport of conserved mass. A conserved mass means such mass that cannot increase or decrease, it can only move from one place to another place. Consider a small control volume. Let ρ be mass density of the fluid at a particular instant and u, v and w are component of velocity of flow entering the three faces of the parallelepiped

For $x - direction$

Fluid influx $= \rho \times velocity(x - direction) \times area\ of\ the\ face = \rho udydz$ (1.18)

Fluid efflux $= \rho\, u\, dydz + \frac{\partial}{\partial x}(\rho\, u\, dydz)dx$ (1.19)

Gain in mass per unit time in $x -$ direction $= Fluid\ influx - Fluid\ efflux$

$$= \rho\, u\, dydz - (\rho\, u\, dydz + \frac{\partial}{\partial x}(\rho\, u\, dx)dydz) \tag{1.20}$$

Similarly,

Gain in mass per unit time in $y -$ direction $= \rho\, v\, dxdz - (\rho\, v\, dxdz + \frac{\partial}{\partial y}(\rho\, v\, dy)dxdz)$ (1.21)

Gain in mass per unit time in $z -$ direction $= \rho\, w\, dxdy - (\rho\, w\, dxdy + \frac{\partial}{\partial z}(\rho\, w\, dz)dxdy)$ (1.22)

Net mass gain in fluid along the three axes $= -\left[\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z}\right] dxdydz$ (1.23)

The total mass gain must equal to the rate of mass decrease within the control volume, which is given as

$$\frac{\partial\rho}{\partial t}dxdydz \tag{1.24}$$

that is, equation (1.23) must be equal to equation (1.24), this yields

$$-\left[\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}+\frac{\partial\rho w}{\partial z}\right]dxdydz=\frac{\partial\rho}{\partial t}dxdydz \tag{1.25}$$

Simplifying, we have

$$\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}+\frac{\partial\rho w}{\partial z}+\frac{\partial\rho}{\partial t}=0 \tag{1.26}$$

The equation above is a three dimensional continuity equation for both compressible and incompressible fluid. If the density is constant, that is $\frac{\partial\rho}{\partial t}=0$, we obtain

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1.27}$$

which is the continuity equation for incompressible fluids.

For 2-dimensional continuity equation ($x\ and\ y\ directions$ where w $=$ 0), we obtain

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1.28}$$

For one-dimensional continuity equation ($x\ direction, v = w = 0$), we have

$$\frac{\partial u}{\partial x}=0 \tag{1.29}$$

1.14.6 The Navier Stokes equation

The equation governing the flow of a fluid is the Navier–Stokes (or momentum) equation:

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}\right)=-\vec{\nabla}p+\rho\upsilon\nabla^2\vec{u}+\text{other forces} \tag{1.30}$$

where $\vec{u}$ is its velocity and p is the pressure. When the fluid contains electrical charge ρ_c per unit volume, then there is a force per unit volume of $\rho_c\,\vec{E}$.

When an electric current density $\vec{J}$ flows through the fluid, there is a force per unit volume of $\vec{J}\times\vec{B}$.

Then the Navier–Stokes equation becomes

$$\rho\left(\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}\right)=-\vec{\nabla}p+\rho\upsilon\nabla^2\vec{u}+\rho_c\,\vec{E}+\vec{J}\times\vec{B}+\text{ other forces} \tag{1.31}$$

In the applications of our interest, speeds are very small compared with the speed of light. In this case, electric force $\rho_c \vec{E}$ is negligible compared to $\vec{j} \times \vec{B}$ in (1.31), giving

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u}\right) = -\vec{\nabla} p + \rho \upsilon \nabla^2 \vec{u} + \vec{j} \times \vec{B} + \text{ other forces} \quad (1.32)$$

1.14.7 Energy equation

The law of conservation of heat transfer is an empirical law of Physics. It said that full amount of heat transfer in an isolated system remains constant over time. Energy equation in this case includes both viscous and Joul's dissipations functions then reduces to

$$\rho C_p \left[\frac{\partial T}{\partial t} + (\vec{u}.\nabla)T\right] = k_4 \nabla^2 T + \phi + \frac{J^2}{\sigma} + Q. \quad (1.33)$$

1.14.8 Mass transfer equation

Though mass diffusion is concerned with conductivity of medium, magnetic field has minor effect on this process. So, ordinary diffusion equation can be applied for MHD problems under suitable assumptions is

$$\frac{\partial C}{\partial t} + (\vec{u}.\nabla)C = D\nabla^2 C \quad (1.34)$$

1.15 Constitutive equations of Casson nanofluid

The constitutive equation for Casson fluid can be written as

$$\tau_{ij} = \begin{cases} 2\left(\mu B + \frac{Py}{\sqrt{2\pi}}\right) e_{ij} & \pi > \pi_c \\ 2\left(\mu B + \frac{Py}{\sqrt{2\pi_c}}\right) e_{ij} & \pi < \pi_c \end{cases} \quad (1.35)$$

Where $\pi = e_{ij} e_{ij}$ and e_{ij} is $(i, j)^{th}$ component of deformation rate, π is product of component of deformation rate with itself, π_c is a critical value of this product based on non- Newtonian model, μB is plastic dynamic viscosity of non-Newtonian fluid and P_y is yield stress of fluid.

$$P_y = \frac{\mu B \sqrt{2\pi}}{\gamma} \quad (1.36)$$

From definition of viscosity, ratio of sheer stress τ^* to viscosity μ is constant in case of Newtonian.

$$\tau^* = \mu \frac{\partial u}{\partial y} \tag{1.37}$$

Some fluids require a gradually increasing shear stress to maintain a constant strain rate, In case of Casson fluid (Non Newtonian) flow where $\pi > \pi_c$

$$\mu = \mu B + \frac{P_y}{\sqrt{2\pi}} \tag{1.38}$$

Substituting equation (1.18) in equation (1.20), Kinematics viscosity can be expressed as

$$\nu = \frac{\mu B\left(1+\frac{1}{\gamma}\right)}{\rho} \tag{1.39}$$

1.16 Laplace transform technique (LTT)

Many ideas of classical analysis needed their sources in study of physical problems important to boundary value problems. Study for a solution of this initial boundary values problems leads to discovery of new mathematical tool - tools that are currently of huge practice in pure and applied mathematics, and other engineering branches, is Laplace transform.

The branches of science and engineering in which Laplace transform technique are used for solving linear system of partial differential equations with constant coefficients and ordinary differential equations in which coefficients are variables or simultaneous ordinary differential equations. Laplace transform technique can also be applied in mechanics (dynamics and statics), electrical circuits, to analysis characteristic of beam and several partial differential equations subject to initial and boundary conditions etc. Thus, it can be understood that Laplace transform has its remarkable applications in many branches of pure and applied mathematics.

1.16.1 Laplace transforms technique in MHD

The physical aspects of any fluid flow is expressed in terms of system of partial differential equation with initial and boundary condition, in which Laplace transform technique can be used properly; as it is art of substituting governing equations of fluid flow with numbers and proceeding these numbers in space and / or time into an ordinary differential equation, which can be solved by established rules and, then Inverse Laplace transforms techniques are useful to get required results. This method is perfectly fitted for unsteady free convective MHD problems through porous medium.

1.17 Homotopy analysis method (HAM)

Two continuous functions from one topological space to another are called homotopic if one can be continuously deformed into the other, such a deformation is called a homotopy between the two functions.

The basic idea of HAM method [40] is to produce a succession of approximate solutions that tend to exact solution of the problem. Presence of auxiliary parameters and functions in approximate solution, results in production of a family of approximate solutions, rather than a single solution produced by traditional perturbation methods.

The general approach used by HAM is to solve non-linear equation,

$$\mathcal{N}\big(u(t)\big) = 0, \quad t > 0, \tag{1.40}$$

where $\mathcal{N}$ is a nonlinear operator and $u(t)$ is unknown function of independent variable t.

1.17.1 Zero-order deformation equation

Let $u_0(t)$ denote an initial guess of exact solution of Equation (1.40), $\hbar \neq 0$ an auxiliary parameter, $H(t) \neq 0$ auxiliary function and $\mathcal{L}$ an auxiliary linear operator with property,

$$\mathcal{L}\big(f(t)\big) = 0 \ \text{ when } f(t) = 0. \tag{1.41}$$

The auxiliary parameter $\hbar$, auxiliary function $H(t)$, and auxiliary linear operator $\mathcal{L}$ play important roles within HAM to adjust and control convergence region of solution series. Liao [40] constructs, using $q \in [0, 1]$ as an embedding parameter, so - called zero-order deformation equation,

$$(1-q)\mathcal{L}[\Phi(t;q) - u_0(t)] = q\hbar H(t)\mathcal{N}[\Phi(t;q)], \tag{1.42}$$

where $\Phi(t;q)$ is solution which depends on $\hbar, H(t), \mathcal{L}, u_0(t)$ and q. When $q = 0$, zero-order deformation Equation (1.42) becomes,

$$\Phi(t;0) = u_0(t), \tag{1.43}$$

when $q = 1$, since $\hbar \neq 0$ and $H(t) \neq 0$, then Equation (1.42) reduces to,

$$\mathcal{N}[\Phi(t;1)] = 0. \tag{1.44}$$

So, $\Phi(t;1)$ is exactly solution of nonlinear Equation (1.40). Expanding $\Phi(t;q)$ in Taylor's series with respect to q, we have

$$\Phi(t;q) = u_0(t) + \sum_{m=1}^{\infty} q^m u_m(t), \tag{1.45}$$

where,

$$u_m(t) = \frac{1}{m!}\frac{\partial^m \Phi(t;q)}{\partial q^m}\Big|_{q=0.} \tag{1.46}$$

If power series (1.45) of $\Phi(t;q)$ converges at $q = 1$, then we get following series solution,

$$u(t) = u_0(t) + \sum_{m=1}^{\infty} u_m(t), \tag{1.47}$$

where terms $u_m(t)$ can be determined by so-called high-order deformation equations which are described below.

1.17.2 High-order deformation equation

Define vector

$$\overrightarrow{u_n} = \{u_0(t), u_1(t), u_2(t), \ldots, u_n(t)\}. \tag{1.48}$$

Differentiating Equation (1.42) m times with respect to embedding parameter q, then setting $q = 0$ and dividing them by $m!$, we have so-called m^{th}-order deformation equation,

$$\mathcal{L}[u_m(t) - \chi_m u_{m-1}(t)] = \hbar H(t) R_m(\vec{u}_m, t), \tag{1.49}$$

where

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & otherwise \end{cases} \tag{1.50}$$

$$R_m(\vec{u}_{m-1}, t) = \frac{1}{(m-1)!} \frac{\partial^{m-1} \mathcal{N}[\Phi(t;q)]}{\partial q^{m-1}} |_{q=0.} \tag{1.51}$$

For any given nonlinear operator $\mathcal{N}$, term $R_m(\vec{u}_{m-1}, t)$ can be easily expressed by Equation (1.51). Thus, we can gain $u_1(t), u_2(t), \ldots$ by means of solving linear high-order deformation Equation (1.49) one after other in order. m^{th}-order approximation of $u(t)$ is given by,

$$u(t) = \sum_{k=0}^{m} u_k(t). \tag{1.52}$$

Liao [40] points out that so-called generalized Taylor's series provides a way to control and adjust convergence region through an auxiliary parameter $\hbar$ such that homotopy analysis method is particularly suitable for problems with strong nonlinearity. Abbasbandy [1] gives meaning of auxiliary parameter $\hbar$, and hence uncovers essence of generalized Taylor's expansion as kernel of homotopy analysis method.

1.17.3 Convergence analysis

One of chief aims of HAM method is to produce solutions that will converge in a much larger region than solutions obtained with traditional perturbation methods. Solutions obtained using this method depend on our choice of linear operator $\mathcal{L}$, auxiliary function $H(t)$, initial approximation $u_0(t)$ and value of auxiliary parameter $\hbar$.

Choice of base functions influence convergence of solution series significantly. For example, solution may be expressed as a polynomial or as a sum of exponential functions. It is expected that, base functions that more closely mimic behavior of actual solution should provide much better results than base functions whose behavior differs greatly from behavior of actual solution. Choice of a linear operator, auxiliary function, and initial approximation often determines base functions present in solution. Having selected a linear operator, auxiliary function, and an initial approximation, deformation equations can be developed and solved in series solution. Solution obtained in this way, still contains auxiliary parameter $\hbar$. This solution should be valid for a range of values of $\hbar$. In order to determine optimum value of $\hbar$, $\hbar$ curves of solution are plotted. These curves are obtained by plotting partial sums $u_m(t)$ or their first few derivatives evaluated at a particular value of t against parameter $\hbar$. As long as equation (1.40) with given initial or boundary conditions has a unique solution, partial sums and their derivatives will converge to correct solution for all values of $\hbar$ for which solution converges. Which means that $\hbar$ curves will be essentially horizontal over range of $\hbar$ for which solution converges. As long as, $\hbar$ is chosen in this horizontal region, solution must converge to actual solution of equation (1.40).

1.18 Dimensionless parameters

Dimensionless parameters help us to understand physical importance of a particular phenomenon. Basic equations are made dimensionless using certain dependent or independent characteristic values. Some of dimensionless parameters used in thesis are clarified below.

1.18.1 Thermal Grashof number (Gr)

Thermal Grashof number (Or Grashof number) is the ratio of buoyancy to viscous force acting on a fluid. It often arises in study of situations involving free convection. Its expression is

$$Gr = \frac{g\beta_T L^3 (T_W - T_0)}{\nu^2} \tag{1.53}$$

1.18.2 Mass Grashof number (Gm)

The ratio of mass buoyancy force to hydrodynamics viscous force acting on a fluid is known as Mass Grashof number. It often arises in study of situations involving free convection and it is expressed by

$$Gm = \frac{g\beta_C L^3 (C_W - C_0)}{\nu^2} \tag{1.54}$$

1.18.3 Prandtl number (Pr)

It is defined as ratio of momentum and thermal diffusivity.

$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \tag{1.55}$$

1.18.4 Schmidt number (Sc)

It is defined as ratio of momentum and mass diffusivity.

$$Sc = \frac{\nu}{D} \tag{1.56}$$

1.18.5 Magnetic parameter or Hartmann number (M)

It is defined as ratio of electromagnetic force to viscous force. It measures relative importance of drag forces resulting from magnetic induction and viscous forces in flow.

$$M = \frac{\sigma B^2 L^2}{\nu\,\rho} \tag{1.57}$$

1.18.6 Soret Number (Sr)

It is noticed that, mass fluxes can also be created by temperature gradients and this embodies thermal diffusion (Soret) effect. Soret number is represented by

$$Sr = \frac{D_T}{D_m} \tag{1.58}$$

1.18.7 Reynolds number (Re)

It is used to identify different flow behaviors like laminar or turbulent flow. It measure ratio of inertial force and viscous force. Mathematically,

$$Re = \frac{\frac{\rho u^2}{L}}{\frac{\mu u}{L^2}} \Longrightarrow Re = Lu/\nu \tag{1.59}$$

At low Reynolds number, laminar flow arises, where viscous forces are dominant whereas at high Reynolds number, turbulent flow arises, where inertial forces are dominant.

1.18.8 Eckert Number (Ec)

It expresses relationship between kinetic energy dissipated in flow and thermal energy conducted into or away from fluid.

$$Ec = \frac{u^2}{c_p(T_w - T_0)} \tag{1.60}$$

1.18.9 Brownian diffusion coefficient (D_B)

Brownian diffusion occurs due to continuous collision between molecules and nanoparticles of fluid. Brownian diffusion coefficient D_B is given by

$$D_B = \frac{K_B T C_c}{3\Pi \mu dp} \tag{1.61}$$

where K_B and C_c represent Boltzmann constant and correction factor respectively.

1.18.10 Thermophoresis diffusion coefficient (D_T)

Thermophoresis diffusion occurs when particles diffuse due to effect of temperature gradient. It is given by

$$D_T = \frac{-\upsilon_{th} T}{\nu \nabla T} \tag{1.62}$$

where υ_{th} and ∇T denote thermophoretic velocity and temperature gradient respectively.

1.18.11 Skin friction coefficient (C_f)

It occurs between solid and fluid surface through which motion of fluid becomes slow. Skin friction coefficient can be defined as,

$$C_f = -\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{1.63}$$

1.18.12 Nusselt number (Nu)

It is temperature gradient at surface. Its expression is

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \tag{1.64}$$

1.18.13 Sherwood number (Sh)

Sherwood number represents concentration gradient at surface.

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{1.65}$$

1.19 Review of relevant literature

Analytical methods as well as Numerical methods have been successfully used by many scientists to solve several types of problems of magnetic field effects on unsteady free convective one dimensional, two dimensional and three dimensional flow of different types of fluids with heat and mass transfer through porous medium.

All physical mechanisms have a critical scale below which properties of a material changes totally. Modern nanotechnology offers physical and chemical routes to prepare nanometer sized particles or nanostructured materials engineered on atomic or molecular scales with enhanced thermo-physical properties compared to their respective bulk forms. These nanoparticle-fluid suspensions are termed nanofluids, obtained by dispersing nanometer sized particles in a conventional base fluids like water, oil, ethylene glycol etc. Nanoparticles of materials such as metallic oxides (Al_2O_3, CuO), nitride ceramics (AlN, SiN), carbide ceramics

(SiC, TiC), metals (Cu, Ag, Au), semiconductors (TiO_2, SiC), single, double or multi walled carbon nanotubes (SWCNT, DWCNT, MWCNT) etc. have been used for preparation of nanofluids. In last decade, significant amount of experimental as well as theoretical research were done to investigate thermophysical behavior of nanofluids. All these studies reveal fact that micro structural characteristics of nanofluids have a significant role in deciding effective thermal conductivity. Some recent studies concerning nanofluid flow can be found in works of Koo and Kleinstreuer [37], Makinde [45], Sheikholeslami et. al [91-92], Nadeem et al. [49]. Acharya et al. [2] considered squeezing flow of Cu-water and Cu-kerosene nanofluids between two parallel plates. Azmi et al. [7] studied effective thermal conductivity and effective dynamic viscosity of nanofluids.

In modern engineering, many characteristics of flow are not understandable with Newtonian fluid model. Hence, non-Newtonian fluids theory has become useful. Non-Newtonian fluid exerts non-linear relationship between shear stress and rate of shear strain. It has an extensive variety of applications in engineering and industry; especially in extraction of crude oil from petroleum products. Casson fluid is one of such fluids. Casson fluid is classified as most popular non-Newtonian fluid which has several applications in food processing, metallurgy, drilling operations and bio-engineering operations. Casson fluid model was introduced by Casson [9] for prediction of flow behavior of pigment-oil suspensions. Pramanik [57] solved problem based on Casson fluid flow past an exponentially porous stretching surface. Mahanta and shaw [42] discussed Casson fluid flow past linearly stretching sheet. Akbar and Butt [3] studied physiological transportation of Casson fluid. Recently, Usman et al. [113] obtained result of Casson nanofluid flow over an inclined permeable stretching cylinder via collocation method.

In heat transfer problems, temperature represents amount of thermal energy available, while heat flow represents movement of thermal energy from one point to another. Thus, study of heat transfer has a principal role in numerous commercial and engineering applications such as aircraft engineering, water heaters, lubrication of bearings, cooling compressors of engine cylinders and nuclear technology. Few studies relevant to nanofluid flow with heat transfer can be seen in Hayat et al. [21], Hatami et al. [17–18], Sheikholeslami [63], Sheikholeslami and Bhatti [65]. Convective heat transfer phenomena in nature are often attended by mass transfer. Convective mass transfer process is significant in chemical engineering. Thus, it is important to include mass transfer in heat convection problems. Sheikholeslami and Ganji [73] discussed three dimensional nanofluid flow with heat and mass transfer in a rotating system.

The study of magnetohydrodynamics (MHD) flow has essential applications in physics, chemistry and engineering. Industrial equipment, such as MHD generators, pumps, bearings and boundary layer control are affected by interaction between electrically conducting fluid and a magnetic field. One of the basic and important problems in this area is hydromagnetic behavior of boundary layers along fixed or moving surfaces in presence of a transverse magnetic field. MHD boundary layers are observed in various technical systems employing liquid metal and plasma flow in presence of transverse magnetic fields. Rashidi and Erfani [59], Nadeem et al. [50], Kataria et al. [34] and Sheikholeslami et al. [82, 101] have contributed significantly in this area.

It is more important to mention vigorous and predictable applications of MHD nanofluid flow and heat transfer including wound treatment, sterilized devices, gastric medications and prodigious significance in developments such as targeted drug release, asthma treatment, cancer therapy, synergistic effects in immunology, magnetic cell separation, magnetic resonance imaging, elimination of tumors with hyperthermia etc. Such valuable applications invited many researchers to study in related areas. Akbar et al. [4] investigated MHD flow of Cu-H_2O nanofluids with magnetic induction. Hatami et al. [19] discussed analytical solution of MHD nanofluid flow in non-parallel walls. Hayat et al. [20] considered three dimensional

MHD nanofluid flow with heat and mass transfer. Freidoonimehr et al. [14] studied MHD nanofluid flow over stretching vertical surface. Ganji and Malvandi [15] investigated magnetic field effects on natural convection of nanofluid flow. Many researchers like, Hatami et al. [16], Sheikholeslami and Chamkha [67], Sheikholeslami and Seyendnezhad [109] and Sheikholeslami et al. [108] considered variable magnetic field effects on MHD flow of nanofluid with heat transfer. Hayat et al. [24] and Nadeem et al. [51] obtained numerical simulation of MHD flow of different types of nanofluid. Sheikholeslami and Ellahi [69] studied three dimensional mesoscopic simulation of magnetic field effect on natural convection of nanofluid. Considering specific applications of MHD nanofluid flow in presence of different types of physical conditions, investigations [66], [70], [72], [77-78], [80-81], [85], [88-90], [93-94], [96] and [100] are performed. Moreover, Sheikholeslami and Ganji [103] investigated FHD and MHD effects on ferrofluid flow and convective heat transfer. Kamran et al. [27] defined numerical solution of MHD flow of Casson nanofluid combined with Joule heating and slip boundary conditions.

Transport in porous medium has broad applications in biological, physical and chemical parts; for example, mass of liquid, seepage, energy and momentum in single or multiphase flow in porous media realm. Another remarkable characteristic of porous matrix is that, it reduces flow instability, if any, appeared in flow domain. Significance of porous medium in MHD flow problems are derived by Das et al. [11], Ellahi et al. [12], Kataria and Patel [36], Vafai [115]. Sheikholeslami [87] considered forced convection MHD nanofluid in a porous lid whereas, Sheikholeslami and Shehzad [98] and Sheikholeslami and Zeeshan [102] studied magnetic field effects on nanofluid convective flow in a porous enclosure.

Radiation is evident in engineering, physics, aerodynamics, rockets, solar power technology, gas cooled nuclear reactors, counting combustion, furnace design, nuclear reactor protection and photo chemical reactors etc. Few representative examinations interconnected with phenomenon of thermal radiation effects on MHD flow problems can be found in articles [58, 95, and 99]. Abbasbandy [1] discussed homotopy analysis method for solving system with radiation. Mahapatra et al. [43] discussed fluid flow through Forchheimer porous medium in presence of thermal radiation. Further investigations on effects of thermal radiation on MHD flow of nanofluid are carried out by Sheikholeslami et al. [86, 107]. Oyelakin et al. [55] analyzed thermal radiation effects on unsteady Casson nanofluid flow over a stretching sheet.

Study of MHD flow with heat and mass transfer in presence of chemical reaction has many applications in hydrometallurgical and chemical industries. Chemical reaction can be classified into heterogeneous and homogeneous reactions. Mustafa et al. [47] and Nayak et al. [53] considered influence of chemical reaction on MHD flow with heat and mass transfer. It is known fact that study of thermal radiation and chemical reaction effects has conventional superior consideration because of its abundant applications in manufacturing and modern technologies. Relevant literature is discussed in articles [33, 38, 52, 60 and 62].

Phenomenon of heat generation/absorption plays a significant role in adjusting heat transfer rate. Hayat et al. [25] studied MHD flow with heat source and power law heat flux.

Mass transfer produced by temperature gradient is called Soret or thermal diffusion effect. Kataria and Patel [32, 35] considered Soret effects on free convective MHD Casson fluid flow in porous medium. Waqas et al. [116] discussed double diffusion impact for temperature dependent conductivity of Powell–Eyring liquid.

Thermophoresis and Brownian motion are important phenomena in heat and mass transfer fluid flow problems. Sheikholeslami and Ganji [104] considered effects of Brownian motion on MHD nanofluid flow. Animasaun [6] considered thermophoresis effects on free convective heat and mass transfer of non-Darcian MHD dissipative Casson fluid flow with chemical reaction. Combined effects of thermophoresis and Brownian motion on MHD nanofluid flow problems are considered by Babu et al. [8], Hayat et al. [22], Kumaran and Sandeep [39], Sheikholeslami et al. [68, 79], Sulochana et al. [110] and Ullah et al. [111].