APPENDIX

Chapter 3, Section-I

$d_1 = a_1$ $\quad$ $d_2 = a_3 M + \frac{a_1}{k}$ $\quad$ $d_3 = Gr\ a_2$

$d_4 = Gm\ a_5$ $\quad$ $d_5 = \frac{d_1}{a_4} - 1$ $\quad$ $d_6 = \frac{d_2}{d_5}$

$d_7 = \frac{d_3}{d_5}$ $\quad$ $d_8 = d_1 Sc - 1$ $\quad$ $d_9 = \frac{d_2}{d_8}$

$d_{10} = \frac{d_4}{d_8}$ $\quad$ $d_{11} = -\frac{d_7}{d_6}$ $\quad$ $d_{12} = \frac{d_7}{{d_6}^2}$

$d_{13} = \frac{d_7}{(1-d_6)} - d_{11} - \frac{d_{12}}{(1-d_6)}$ $\quad$ $d_{14} = -\frac{d_{10}}{d_9}$ $\quad$ $d_{15} = \frac{d_{10}}{{d_9}^2}$

$d_{16} = \frac{d_{10}}{(1-d_9)} - d_{14} - \frac{d_{15}}{(1-d_9)}$ $\quad$ $d_{17} = d_{16} + d_{13}$ $\quad$ $d_{18} = d_{11} + d_{14}$

$I_1 = \frac{df_1}{dy}\Big|_{y=0}$ $\quad$ $I_2 = \frac{df_2}{dy}\Big|_{y=0}$ $\quad$ $I_3 = \frac{df_3}{dy}\Big|_{y=0}$

$I_4 = \frac{df_4}{dy}\Big|_{y=0}$ $\quad$ $I_5 = \frac{df_5}{dy}\Big|_{y=0}$ $\quad$ $I_6 = \frac{df_6}{dy}\Big|_{y=0}$

$I_7 = \frac{df_7}{dy}\Big|_{y=0}$ $\quad$ $I_8 = \frac{df_8}{dy}\Big|_{y=0}$ $\quad$ $I_9 = \frac{df_9}{dy}\Big|_{y=0}$

$I_{10} = \frac{df_{10}}{dy}\Big|_{y=0}$ $\quad$ $I_{11} = \frac{df_1}{dy}\Big|_{y=0}$ $\quad$ $I_{12} = \frac{df_2}{dy}\Big|_{y=0}$

$I_{13} = \frac{dg_1}{dy}\Big|_{y=0}$ $\quad$ $I_{14} = \frac{dg_2}{dy}\Big|_{y=0}$ $\quad$ $I_{15} = \frac{dg_3}{dy}\Big|_{y=0}$

$I_{16} = \frac{dg_4}{dy}\Big|_{y=0}$ $\quad$ $I_{17} = \frac{dg_5}{dy}\Big|_{y=0}$ $\quad$ $h_{18} = \frac{dg_6}{dy}\Big|_{y=0}$

$I_{19} = \frac{dg_7}{dy}\Big|_{y=0}$ $\quad$ $I_{20} = \frac{dh_1}{dy}\Big|_{y=0}$

Chapter 3, Section-II

$$g_1(y,s) = d_{13}f_3(y,s) + d_{11}f_4(y,s) + d_{12}f_5(y,s)$$

$$g_2(y,s) = d_{16}f_3(y,s) + d_{14}f_4(y,s) + d_{15}f_6(y,s)$$

$$g_3(y,s) = d_{13}f_7(y,s) + d_{11}f_8(y,s) + d_{12}f_9(y,s),$$

$$g_4(y,s) = d_{16}f_{10}(y,s) + d_{14}f_{11}(y,s) + d_{15}f_{12}(y,s)$$

$$f_1(y,t) = \frac{e^{-i\omega t}}{2}\left[e^{-y\sqrt{\frac{1}{d_1}(d_2-i\omega)}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} - \sqrt{(d_2-i\omega)t}\right) + e^{y\sqrt{\frac{1}{d_1}(d_2-i\omega)}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} + \sqrt{(d_2-i\omega)t}\right)\right]$$

$$f_2(y,t) = \frac{e^{i\omega t}}{2}\left[e^{-y\sqrt{\frac{1}{d_1}(d_2+i\omega)}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} - \sqrt{(d_2+i\omega)t}\right) + e^{y\sqrt{\frac{1}{d_1}(d_2+i\omega)}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} + \sqrt{(d_2+i\omega)t}\right)\right]$$

$$f_3(y,t) = \frac{1}{2}\left[e^{-y\sqrt{\frac{d_2}{d_1}}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} - \sqrt{d_2 t}\right) + e^{y\sqrt{\frac{d_2}{d_1}}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} + \sqrt{d_2 t}\right)\right]$$

$$f_4(y,t) = \frac{1}{2}\left[\left(t - \frac{y}{2\sqrt{d_2}d_1}\right)e^{-y\sqrt{\frac{d_2}{d_1}}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} - \sqrt{d_2 t}\right) + \left(t + \frac{y}{2\sqrt{d_2}d_1}\right)e^{y\sqrt{\frac{d_2}{d_1}}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} + \sqrt{d_2 t}\right)\right]$$

$$f_5(y,t) = \frac{e^{d_6 t}}{2}\left[e^{-y\sqrt{\frac{1}{d_1}(d_6+d_2)}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} - \sqrt{(d_6+d_2)t}\right) + e^{-y\sqrt{\frac{1}{d_1}(d_6+d_2)}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} + \sqrt{(d_6+d_2)t}\right)\right]$$

$$f_6(y,t) = \frac{e^{d_9 t}}{2}\left[e^{-y\sqrt{\frac{1}{d_1}(d_9+d_2)}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} - \sqrt{(d_9+d_2)t}\right) + e^{-y\sqrt{\frac{1}{d_1}(d_9+d_2)}}\ erfc\left(\frac{y}{2\sqrt{d_1 t}} + \sqrt{(d_9+d_2)t}\right)\right]$$

$$f_7(y,t) = erfc\left(\frac{y}{2\sqrt{a_4 t}}\right),$$

$$f_8(y,t) = \left(\frac{y^2}{2\,a_4} + t\right) erfc\,\left(\frac{y}{2\sqrt{a_4\,t}}\right) - \frac{y\sqrt{t}}{2\sqrt{a_4\,\pi}} e^{-\frac{y^2}{4ta_4}},$$

$$f_9(y,t) = \frac{e^{d_6 t}}{2}\left[e^{-y\sqrt{\frac{d_6}{a_4}}}\, erfc\,\left(\frac{y}{2\sqrt{a_4 t}} - \sqrt{d_6 t}\right) + e^{y\sqrt{\frac{d_6}{a_4}}}\, erfc\,\left(\frac{y}{2\sqrt{a_4 t}} + \sqrt{d_6 t}\right)\right]$$

$$f_{10}(y,t) = erfc\,\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)$$

$$f_{11}(y,t) = \left(\frac{y^2 Sc}{2} + t\right) erfc\,\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right) - \frac{y\sqrt{\mathrm{Sc}\,t}}{2\sqrt{\pi}} e^{-\frac{y^2 Sc}{4t}}$$

$$f_{12}(y,t) = \frac{e^{d_9 t}}{2}\left[e^{-y\sqrt{Sc\,d_9}}\, erfc\,\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{d_9 t}\right) + e^{y\sqrt{Sc\,d_9}}\, erfc\,\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{d_9 t}\right)\right]$$

$$h_1(t) = e^{-i\omega t}\sqrt{\frac{d_2 - i\omega}{d_1}}\,\mathrm{erf}\left(\sqrt{(d_2 - i\omega)t}\right) + \frac{e^{-d_2 t}}{\sqrt{\pi d_1 t}}$$

$$h_2(t) = e^{i\omega t}\sqrt{\frac{d_2 + i\omega}{d_1}}\,\mathrm{erf}\left(\sqrt{(d_2 + i\omega)t}\right) + \frac{e^{-d_2 t}}{\sqrt{\pi d_1 t}}$$

$$h_3(t) = -\sqrt{\frac{d_2}{d_1}}\,\mathrm{erf}\left(\sqrt{d_2 t}\right) + \frac{e^{-d_2 t}}{\sqrt{\pi d_1 t}}$$

$$h_4(t) = -\frac{1}{\sqrt{4 d_2 d_1}}\,\mathrm{erf}\left(\sqrt{d_2 t}\right) - t\sqrt{\frac{d_2}{d_1}}\,\mathrm{erf}\left(\sqrt{d_2 t}\right) + \sqrt{\frac{t}{\pi d_1}}\, e^{-d_2 t}$$

$$h_5(t) = e^{d_6 t}\sqrt{\frac{d_2 + d_6}{d_1}}\,\mathrm{erf}\left(\sqrt{(d_2 + d_6)t}\right) + \frac{e^{-d_2 t}}{\sqrt{\pi d_1 t}}$$

$$h_6(t) = e^{d_9 t}\sqrt{\frac{d_2 + d_9}{d_1}}\,\mathrm{erf}\left(\sqrt{(d_2 + d_9)t}\right) + \frac{e^{-d_2 t}}{\sqrt{\pi d_1 t}}$$

$$h_7(t) = \sqrt{\frac{1}{\pi a_4 t}}$$

$$h_8(t) = \frac{1}{2}\sqrt{\frac{t}{\pi a_4}}$$

$$h_9(t) = -e^{d_6 t}\sqrt{\frac{d_6}{a_4}}\,\mathrm{erf}\left(\sqrt{d_6 t}\right) + \sqrt{\frac{1}{\pi a_4 t}}$$

$$h_{10}(t) = \sqrt{\frac{Sc}{\pi t}}$$

$$h_{11}(t) = \frac{1}{2}\sqrt{\frac{tSc}{\pi}}$$

$$h_{12}(t) = -e^{d_9 t}\sqrt{Sc\ d_9}\,\mathrm{erf}\left(\sqrt{d_9 t}\right) + \sqrt{\frac{Sc}{\pi t}}$$

$$h_{13}(t) = d_{13}h_3(t) + d_{11}h_4(t) + d_{12}h_5(t)$$

$$h_{14}(t) = d_{16}h_3(t) + d_{14}h_4(t) + d_{15}h_6(t)$$

$$h_{15}(t) = d_{13}h_7(t) + d_{11}h_8(t) + d_{12}h_9(t)$$

$$h_{16}(t) = d_{16}h_{10}(t) + d_{14}h_{11}(t) + d_{15}h_{12}(t)$$

Chapter 4

$a = a_1$ $\quad b = a_3 M + \frac{a_1}{k}$ $\quad \mathrm{c} = Gr$

$\mathrm{d} = Gm$ $\quad a_6 = a_4$ $\quad a_7 = H$

$a_8 = \frac{1}{Sc}$ $\quad a_9 = Sr$ $\quad a_{10} = Kr$

$a_{11} = \frac{a_8}{a_6}$ $\quad a_{12} = \frac{a_7}{a_6}$ $\quad a_{13} = a_{10} - a_{12}$

$b_{14} = a_{11} - 1$ $\quad a_{14} = \frac{a_9}{a_6}$ $\quad a_{15} = \frac{a_{13}}{b_{14}}$

$a_{16} = \frac{a_{14}}{b_{14}}$ $\quad a_{17} = \frac{-a_{16}\,a_7}{a_{15}}$ $\quad a_{18} = \frac{-a_{16}\,(a_{15}+a_7)}{{a_{15}}^2}$

$a_{20} = \frac{a}{a_6}$ $\quad a_{21} = \frac{a.\ a_7}{a_6}$ $\quad a_{19} = \frac{a_{16}\,(1-a_7)}{1+a_{15}} - a_{17} - \frac{a_{18}}{1+a_{15}}$

$b_{21} = a_{20} - 1$ $\quad a_{22} = b + a_{21}$ $\quad a_{23} = \frac{a_{22}}{b_{21}}$

$a_{24} = \frac{c}{b_{21}}$ $\quad b_{24} = \frac{a}{a_8} - 1$ $\quad a_{25} = \frac{a\,a_{10}}{a_8} + b$

$a_{26} = \frac{a_{25}}{b_{24}}$ $\quad a_{27} = \frac{d}{b_{24}}$ $\quad a_{28} = \frac{d\,a_{16}}{b_{24}}$

$a_{29} = \frac{d\,a_{16}}{b_{21}}$ $\quad a_{30} = \frac{-a_{24}}{a_{23}}$ $\quad a_{31} = \frac{a_{24}}{{a_{23}}^2}$

$a_{33} = \frac{-a_{27}}{a_{26}}$ $\quad a_{34} = \frac{a_{27}}{{a_{26}}^2}$ $\quad a_{32} = \frac{a_{24}}{1-a_{23}} - a_{30} - \frac{a_{31}}{1-a_{23}}$

$$a_{36} = \frac{a_{28}\, a_7}{a_{15}\, a_{26}} \qquad a_{37} = \frac{a_{28}\,(a_7 + a_{15})}{{a_{15}}^2 (a_{26} + a_{15})} \qquad a_{35} = \frac{a_{27}}{1 - a_{26}} - a_{33} - \frac{a_{34}}{1 - a_{26}}$$

$$a_{38} = \frac{a_{28}\,(a_{26} - a_7)}{{a_{26}}^2 (a_{26} + a_{15})} \qquad a_{40} = \frac{a_{29}\, a_7}{a_{15}\, a_{23}} \qquad a_{39} = \frac{a_{28}(1 - a_7)}{(1 - a_{26})(1 + a_{15})} - a_{36} - \frac{a_{37}}{1 + a_{15}} - \frac{a_{38}}{1 - a_{26}}$$

$$a_{41} = \frac{a_{29}\,(a_7 + a_{15})}{{a_{15}}^2 (a_{23} + a_{15})} \qquad a_{42} = \frac{a_{29}\,(a_{23} - a_7)}{{a_{23}}^2 (a_{23} + a_{15})} \qquad a_{43} = \frac{a_{29}(1 - a_7)}{(1 - a_{23})(1 + a_{15})} - a_{40} - \frac{a_{41}}{1 + a_{15}} - \frac{a_{42}}{1 - a_{23}}$$

$$a_{46} = a_{31} - a_{42} \qquad a_{47} = a_{35} + a_{38} \qquad a_{44} = a_{32} + a_{35} + a_{39} - a_{43}$$

$$a_{48} = a_{37} - a_{41} \qquad a_{49} = a_{43} - a_{32} \qquad a_{45} = a_{30} + a_{33} + a_{36} - a_{40}$$

$$a_{50} = a_{40} - a_{30} \qquad a_{51} = a_{42} - a_{31} \qquad a_{52} = -a_{35} - a_{39}$$

$$a_{53} = -a_{33} - a_{36} \qquad a_{54} = -a_{34} - a_{38} \qquad a_{55} = \frac{-a_{28}\,(a_{15} + a_7)}{a_{15}(a_{26} + a_{15})}$$

$$a_{56} = \frac{a_{28}\,(a_{26} - a_7)}{a_{26}(a_{26} + a_{15})} \qquad a_{57} = \frac{-a_{29}\,(a_{15} + a_7)}{a_{15}(a_{23} + a_{15})} \qquad a_{58} = \frac{a_{29}\,(a_{23} - a_7)}{a_{23}(a_{23} + a_{15})}$$

$$a_{59} = a_{30} + a_{36} - a_{40} \qquad a_{60} = -a_{30} - a_{58} \qquad a_{61} = a_{55} - a_{57}$$

$$a_{62} = a_{30} + a_{58} \qquad a_{63} = \frac{a_{16}\,(a_{15} + a_7)}{a_{15}} \qquad a_{64} = a_{33} - a_{56}$$

$$F_1(y,s) = \frac{e^{-y\sqrt{\frac{s+b}{a}}}}{s + i\omega} \qquad F_2(y,s) = \frac{e^{-y\sqrt{\frac{s+b}{a}}}}{s - i\omega} \qquad F_3(y,s) = \frac{e^{-y\sqrt{\frac{s+b}{a}}}}{s}$$

$$F_4(y,s) = \frac{e^{-y\sqrt{\frac{s+b}{a}}}}{s^2} \qquad F_5(y,s) = \frac{e^{-y\sqrt{\frac{s+b}{a}}}}{s - a_{23}} \qquad F_6(y,s) = \frac{e^{-y\sqrt{\frac{s+b}{a}}}}{(s - a_{26})}$$

$$F_7(y,s) = \frac{e^{-y\sqrt{\frac{s+b}{a}}}}{(s + a_{15})} \qquad F_8(y,s) = \frac{e^{-y\sqrt{\frac{s-a_7}{a_6}}}}{s} \qquad F_9(y,s) = \frac{e^{-y\sqrt{\frac{s-a_7}{a_6}}}}{s^2}$$

$$F_{10}(y,s) = \frac{e^{-y\sqrt{\frac{s-a_7}{a_6}}}}{(s - a_{23})} \qquad F_{11}(y,s) = \frac{e^{-y\sqrt{\frac{s-a_7}{a_6}}}}{(s + a_{15})} \qquad F_{12}(y,s) = \frac{1}{s} e^{-y\sqrt{\frac{s-a_{10}}{a_8}}}$$

$$F_{13}(y,s) = \frac{1}{s^2} e^{-y\sqrt{\frac{s-a_{10}}{a_8}}} \qquad F_{14}(y,s) = \frac{e^{-y\sqrt{\frac{s-a_{10}}{a_8}}}}{(s - a_{26})} \qquad F_{15}(y,s) = \frac{e^{-y\sqrt{\frac{s-a_{10}}{a_8}}}}{(s + a_{15})}$$

$f_i(y,t) = L^{-1}F_i(y,s)\,, \quad i = 1\ to\ 15$

$h_i(y,t) = L^{-1}H_i(y,s)\,,\ i = 1\ to\ 3$

$g_i(y,t) = L^{-1}G_i(y,s)\,, i = 1\ to\ 15$

$h_1 = g_2 + g_3 + g_4$

$h_2 = f_{13} + g_{10} + g_{11}$

$h_3 = g_1 + g_7 + g_{14} + g_{15}$

$g_1 = \frac{i}{2}f_1 - \frac{i}{2}f_2$

$g_6 = a_{35}f_3 + a_{33}f_4 + a_{34}f_6$

$g_2 = a_{44}f_3 + a_{45}f_4 + a_{46}f_5 + a_{47}f_6 + a_{48}f_7$

$g_{11} = a_{19}f_8 + a_{17}f_9 + a_8 f_{11}$

$g_5 = a_{59}f_3 + a_{60}f_5 + a_{61}f_7 + a_{56}f_6$

$g_3 = a_{49}f_8 + a_{50}f_9 + a_{51}f_{10} + a_{41}f_{11}$

$g_{12} = a_{17}f_{12} + a_{63}f_{15}$

$g_7 = a_{50}f_8 + a_{62}f_{10} + a_{57}f_{11}$

$g_4 = a_{52}f_{12} + a_{53}f_{13} + a_{54}f_{14} - a_{37}f_{15}$

$g_{13} = a_{17}f_8 + a_{63}f_{11}$

$g_8 = a_{35}f_{12} + a_{33}f_{13} + a_{34}f_{14}$

$g_9 = a_{36}f_{12} + a_{55}f_{15} + a_{56}f_{14}$

$g_{15} = a_{53}f_{12} - a_{55}f_{15} - a_{64}f_{14}$

$g_{10} = a_{19}f_{12} + a_{17}f_{13} + a_8 f_{15}$

$g_{14} = a_{45}f_3 + a_{60}f_5 + a_{64}f_6 + a_{61}f_7$

$J_1 = \left.\frac{df_1}{dy}\right|_{y=0}$ $\ldots\ldots J_{15} = \left.\frac{df_{13}}{dy}\right|_{y=0}$

$J_{16} = \left.\frac{dg_1}{dy}\right|_{y=0} \ldots J_{30} = \left.\frac{dg_{15}}{dy}\right|_{y=0}$

$J_{31} = \left.\frac{dh_1}{dy}\right|_{y=0} \ldots\ldots J_{33} = \left.\frac{dh_3}{dy}\right|_{y=0}$

$$L^{-1}\left(\frac{e^{-y\sqrt{s+b}}}{s}\right) = \frac{1}{2}\left[e^{-y\sqrt{b}}\ erfc\ \left(\frac{y}{2\sqrt{t}} - \sqrt{bt}\right) + e^{y\sqrt{b}}\ erfc\ \left(\frac{y}{2\sqrt{t}} + \sqrt{bt}\right)\right]$$

$$L^{-1}\left(\frac{e^{-y\sqrt{s+b}}}{s^2}\right) = \frac{1}{2}\left[\left(t - \frac{y}{2\sqrt{b}}\right)e^{-y\sqrt{b}}\ erfc\ \left(\frac{y}{2\sqrt{t}} - \sqrt{bt}\right) + \left(t + \frac{y}{2\sqrt{b}}\right)e^{y\sqrt{b}}\ erfc\ \left(\frac{y}{2\sqrt{t}} + \sqrt{bt}\right)\right]$$

$$L^{-1}\left(\frac{e^{-y\sqrt{s+b}}}{(s+a)}\right) = \frac{e^{-at}}{2}\left[e^{-y\sqrt{\frac{1}{a}(b-a)}}\ erfc\ \left(\frac{y}{2\sqrt{t}} - \sqrt{(b-a)t}\right) + e^{y\sqrt{\frac{1}{a}(b-a)}}\ erfc\ \left(\frac{y}{2\sqrt{t}} + \sqrt{(b-a)t}\right)\right]$$