SYNOPSIS

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ANALYSIS OF MAGNETIC FIELD EFFECT ON NANO FLUID FLOW

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MHD is the study of the motion of the electrically conducting fluid in the presence of magnetic field. Interaction between the electrically conducting fluid and a magnetic field is used as a control mechanism in material manufacturing industry, as the convection currents are suppressed by Lorentz force which is produced by the magnetic field. The study of magnetohydrodynamic (MHD) flow has essential applications in physics, chemistry and engineering. Industrial equipment, such as magnetohydrodynamic (MHD) generators, pumps and bearings are affected by the interaction between the electrically conducting fluid and a magnetic field.

Research institutes like **THE HELMHOLTZ-ZENTRUM DRESDEN-ROSSENDORF** (member of the Helmholtz Association of German Research Centres), **MHD RESEARCH INSTITUTE** based at the University of Latvia, **THE MAX PLANCK INSTITUTE**, Göttingen are dedicated to research related to MHD.

First theory of laminar flow of an electrically conductive liquid in a homogenous magnetic field was introduced in 1937 by Hartman [1]. Huang et al. [2] discussed MHD waves and instabilities in the heat-conducting solar wind plasma. Wang et al. [3] studied energy of Alfven waves generated during magnetic reconnection. The set of equations which describe MHD are combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism.

The term ‘‘nanofluid” (the term was introduced by Choi and Eastman [4]) refers to a liquid containing a suspension of metallic or non-metallic nanometer-sized solid particles or fibers. The study of nanofluid flow is highly significant due to application of such fluids in heat transfer devices. Due to the higher thermal conductivity and convective heat transfer rates, nanofluids are used in a wide variety of engineering applications. The suspension of nanoparticles enhances the thermal conductivity and the convective heat transfer coefficients of several fluids such as oil, water and ethylene glycol mixture.

In nanoparticles, due to the increase of surface area to the volume, some physical properties such as thermal, electrical, mechanical, optical and magnetic property of the materials can be changed significantly. The most important point is that nano structured materials exhibit different and unique properties as compared to the materials with the same compositions. Experimental studies conducted by Eastman et al. [5] have displayed that with 1%–5% volume of metallic oxide particles, the effective thermal conductivity of the resulting mixture can be increased by 20% compared to that of the base fluid. Numerical simulation of nanofluid flow in presence of magnetic field was carried out by Sheikholeslami et al. [6].

In this thesis, effect of magnetic field on one, two and three dimensional unsteady free convective nanofluid flow with Heat and Mass transfer is discussed. This thesis consists of eight chapters.

Chapter 1 is taken in order to build up a stronger structure in logical manner to provide knowledge of fundamentals of MHD flow, basic concepts of nanofluid, heat and mass transfer effects, radiation effects, heat generation effects and Soret effects. A brief history of the development of the subject is also given. Relevant literature has been surveyed. Further, Laplace transform technique for solving system of linear partial differential equations and Homotopy analysis method for solving system of non-linear equations are discussed.

The gravity-driven convection heat transfer is a vital phenomenon in the cooling mechanism of many engineering systems like in electronics industry, solar collectors and cooling systems for nuclear reactors, because of its minimum cost, low noise, smaller size and reliability. There has been increasing interest in studying the problem of MHD with convection boundary layer flow and heat transfer characteristics over a vertical plate [7]. **Aim of Chapter 2 is the study of gravity-driven convective boundary layer flow of nanofluids past an oscillating vertical plate in the presence of a uniform transverse magnetic field and thermal radiation.** The fluid flow is assumed to be induced due to the motion of the plate. Water based nanofluids containing nanoparticles of copper (Cu) and Silver (Ag) have been considered in the present work.

In the mathematical formulation, flow is confined to y > 0, where y is the coordinate measured in the normal direction to the plate. The fluid is assumed to be electrically conducting with a uniform magnetic field of strength B_0, applied in a direction perpendicular to the plate. At time t = 0, the plate is at rest with the constant ambient temperature T_0. At time t > 0, the plate begins to oscillate in its own plane according to $u_0 \sin \omega t$, where u_0 is amplitude of the plate oscillations and the temperature of the plate is raised or lowered to T_w. As we have optically thick nanofluid, we can use Rosseland approximation [8] for radiative flux. Under the above assumptions, the momentum and energy equations in the presence of thermal radiation and magnetic field past an oscillating vertical plate can be expressed as

$$\rho_{nf}\frac{\partial u}{\partial t} = \mu_{nf}\frac{\partial^2 u}{\partial y^2} + g(\rho\beta)_{nf}(T - T_0) - \sigma_{nf}B^2 u \qquad (1)$$

$$\left(\rho c_p\right)_{nf}\frac{\partial T}{\partial t} = k_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \qquad (2)$$

Where

$$\rho_{nf} = (1-\emptyset)\rho_f + \emptyset\rho_s \tag{3}$$

$$\mu_{nf} = \frac{\mu_f}{(1-\emptyset)^{2.5}} \tag{4}$$

$$\sigma_{nf} = \sigma_f[1 + \frac{3(\sigma-1)\emptyset}{(\sigma+2)-(\sigma-1)\emptyset}],\ \sigma = \frac{\sigma_s}{\sigma_f} \tag{5}$$

$$(\rho\beta)_{nf} = (1-\emptyset)(\rho\beta)_f + \emptyset(\rho\beta)_s \tag{6}$$

$$k_{nf} = k_f[1 - 3\frac{\emptyset(k_f-k_s)}{2k_f+k_s+\emptyset(k_f-k_s)}] \tag{7}$$

$$\left(\rho c_p\right)_{nf} = (1-\emptyset)(\rho c_p)_f + \emptyset(\rho c_p)_s \tag{8}$$

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{9}$$

The momentum and energy conservation equations are made dimensionless and analytic solution is obtained using the Laplace transform. The results for velocity and temperature are obtained and plotted graphically. It is found that the velocity of the nanofluid increases with radiation parameter Nr, Grashof number Gr and time while decreases with increase in magnetic field and Prandtl number Pr. Temperature of nano-fluids increases with time while decreases with increase in Nr and Pr. This result of **Chapter 2** is published in **Journal of the Nigerian Mathematical Society (Elsevier) [9].**

The convective heat transfer phenomena in nature are often attended by mass transfer. Convective mass transfer process creates the support of various procedures in the chemical engineering. This appears like sufficient purpose to contain mass transfer in heat convection as well. Heat and Mass transfer problems, involving porous media have many engineering applications such as ground water pollution, geothermal energy recovery, flow through filtering media, thermal energy storage and crude oil extraction. Vadasz [10] explained heat and mass transfer in porous media. Investigation of MHD nanofluid flow in porous channel has been carried out by Sheikholeslami [11]. **Section 1 of chapter 3 deals with the mathematical modelling of flow, heat and mass transfer in the unsteady natural convection MHD flow of electrically conducting nanofluid, past over an oscillating vertical plate.** Plate with different boundary conditions through porous medium is studied. In this case, governing equations can be expressed as

$$\rho_{nf}\frac{\partial u}{\partial t} = \mu_{nf}\frac{\partial^2 u}{\partial y^2} - \sigma_{nf}B^2u - \frac{\mu_{nf}\varphi}{k_1}u + g(\rho\beta)_{nf}(T-T_0) + g(\rho\beta_C)_{nf}(C-C_0) \tag{10}$$

$$\left(\rho c_p\right)_{nf}\frac{\partial T}{\partial t} = k_{nf}\frac{\partial^2 T}{\partial y^2} \tag{11}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} \tag{12}$$

$$u = 0, \quad T = T_0, \ \ C = C_0; \ as \ \ y \geq 0 \ and \ \ t < 0 \tag{13}$$

$$u = \sin(wt) \ or \ \cos(wt) \ ,$$

$$T = \begin{cases} T_0 + (T_w - T_0)\, {}^{t}/_{t_0} \ if \ \ 0 < t < t_0 \\ \quad T_w \qquad\qquad\qquad\quad if \quad t \geq t_0 \end{cases},$$

$$C = \begin{cases} C_0 + (C_w - C_0)\, {}^{t}/_{t_0} \ if \ \ 0 < t < t_0 \\ \quad C_w \qquad\qquad\qquad\quad if \quad t \geq t_0 \end{cases}, \ \ as \ \ t \geq 0 \ and \ y = 0 \tag{14}$$

$$u \to 0, T \to T_0, \ \ C \to C_0; \ as \ \ y \to \infty \ and \ t \geq 0 \tag{15}$$

The governing non-dimensional system of linear partial differential equations (10) to (12) with initial and boundary conditions (13) to (15) are solved analytically using Laplace transform. This result is published in **Applied Thermal Engineering (Elsevier) [12]** **. In Section 2 of Chapter 3, work of Kataria and Mittal [12] is extended considering radiation effects on unsteady MHD flow of non-Newtonian electrically conducting Casson nanofluids near an infinite vertical plate with ramped wall temperature in a porous medium, published in Mathematics Today [13].**

The effect of heat generation is very important in industrial processes. A thorough observation of the literature shows that, study of the effects of internal heat generation is limited. Thus, **Chapter 4 analyzes the effects of thermal diffusion and heat generation on the unsteady natural convection flow of radiating and electrically conducting nanofluid past over an oscillating vertical plate embedded in porous medium.** Thus equations (11) and (12) are replaced by equations (16) and (17).

$$\frac{\partial T}{\partial t} = \frac{k_{nf}}{(\rho c_p)_{nf}} \frac{\partial^2 T}{\partial y^2} + \frac{Q(T - T_0)}{(\rho c_p)_{nf}} \tag{16}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 T}{\partial y^2} - k' \, (C - C_0) \tag{17}$$

The governing dimensionless equations (10), (16) and (17) with initial and boundary conditions (13) to (15) are solved analytically using Laplace transform technique. This result included in Chapter 4**,** is published in **Journal of Molecular Liquids (Elsevier) [14].** Features of the fluid flow, heat and mass transfer characteristics are analyzed by plotting graphs and the physical aspects are discussed in detail. Skin friction, Nusselt number and Sherwood number are derived and represented through tabular form.

Two dimensional MHD flow problems are of more importance and realistic compared to one dimensional problems. Due to this reason, **Chapter 5 is dedicated to the study of effects of magnetic field on two dimensional nanofluid flow with heat transfer.** Fluid under consideration is Al_2O_3-Water nanofluid. Squeezing flow is assumed to be between two horizontal parallel plates, $l(t) = L(1-at)^{1/2}$ units apart (Here L is the initial position of the plate, $a > 0$ signifies that plates are squeezed until they touch each other at $t = \frac{1}{a}$ and plates move in opposite direction for $a < 0$). A uniform magnetic flux with density $B(t) = B_0/\sqrt{1-at}$ (B_0 is the initial value) is applied. Under these assumptions, governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (18)$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma_{nf}B^2(t)u \quad (19)$$

$$\rho_{nf}\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu_{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad (20)$$

$$(\rho c_p)_{nf}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_{eff}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu_{nf}\left(2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right) - \frac{\partial q_r}{\partial y} + \frac{(\rho c_p)_{nf}Q(T-T_0)}{\rho_{nf}} \quad (21)$$

Effects of thermal interfacial resistance, nanoparticle volume fraction, Brownian motion, nanoparticle size and base fluid on thermal conductivity [15] are considered. Also micro mixing in suspensions is taken into account while calculating viscosity.

$$k_{eff} = k_{static} + k_{Brownian} = k_f\left[1 - 3\frac{\emptyset(k_f - k_{s,R})}{2k_f + k_{s,R} + \emptyset(k_f - k_{s,R})}\right] + 5\times 10^4 \emptyset \rho_f c_{pf}\sqrt{\frac{\kappa_b T}{\rho_s d_s}}F(T,\emptyset,d_s), \quad (22)$$

Subject to

$$u = 0;\ v = \frac{dl}{dt};\ T = T_L\ at\ y = l(t) \quad (23)$$

$$u = 0;\ v = 0;\ \frac{\partial T}{\partial y} = 0\ at\ y = 0 \quad (24)$$

The governing dimensionless system obtained from Equations (18) to (22) subject to boundary conditions (23) and (24) are solved using Homotopy analysis method (HAM), derived by Liao [16]. In this chapter, the solutions are obtained using appropriate codes in Mathematica. From graphical presentation, we conclude that Skin friction can be minimized by decreasing nanoparticle volume fraction $\emptyset$, whereas Nusselt number surges with $\emptyset$.

Above work is further extended in Chapter 6, considering mass transfer.

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu_{nf}}{(\rho c_p)_{nf}}\left(2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right)$$

$$+\frac{(\rho c_p)_s}{(\rho c_p)_f}\left(\frac{D_T}{T_w}\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right)+D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)\right)-\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y}+\frac{(\rho c_p)_{nf}Q(T-T_0)}{\rho_{nf}} \tag{25}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)+\frac{D_T}{T_w}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \tag{26}$$

The governing equations will be (18) - (20), (22), (25) - (26) subject to boundary conditions (23) and (24). Solution is obtained using HAM and discussed in detail. These results are communicated for publication.

Chapter 7 deals with three dimensional nanofluid flow in the presence of magnetic field through porous medium. System under consideration is rotating. Effects of thermal interfacial resistance, nanoparticle volume fraction, Brownian motion and nanoparticle size on thermal conductivity are considered. Also micro mixing in suspensions is taken into account while calculating viscosity. In this case, flow is assumed to be between two horizontal parallel plates, L units apart, through a porous medium. A coordinate system (x, y, z) is chosen such that origin is positioned at the lower plate. The lower plate is subject to stretching by two equal forces in opposite directions. The plates along with the fluid rotate about y axis, with angular velocity Ω. A uniform magnetic flux with density B is applied along y-axis. Under these assumptions, governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{27}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+2\Omega w\right)=\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma_{nf}B^2u-\frac{\mu_{nf}\varphi}{k_1}u \tag{28}$$

$$\rho_{nf}\left(v\frac{\partial v}{\partial y}\right)=\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{29}$$

$$\rho_{nf}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}-2\Omega w\right)=\mu_{nf}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\sigma_{nf}B^2w-\frac{\mu_{nf}\varphi}{k_1}w \tag{30}$$

$$(\rho c_p)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=k_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\mu_{nf}\left(2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right]+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2+\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)^2\right)-\frac{\partial q_r}{\partial y} \tag{31}$$

Subject to

$$u=ax;\ v=0;\ w=0;\ T=T_w\ at\ y=0 \tag{32}$$

$$u=0;\ v=0;\ w=0;\ T=T_L\ at\ y=L \tag{33}$$

HAM is applied to solve the linearized system. This result is published in **Chinese Journal of Physics (Elsevier) [17].**

We have extended this work in Chapter 8, taking mass transfer into account. Mathematical modelling of the problem give rise to system of the equations (27) - (30) along with equations

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{(\rho c_p)_s}{(\rho c_p)_f}\left(\frac{D_T}{T_w}\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right)+ D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right)\right)-\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y} \quad (34)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)+\frac{D_T}{T_w}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \quad (35)$$

subject to the conditions (32) - (33). HAM is employed to solve the simplified system and obtained solutions are explained through graphs. Results are communicated.

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