Synopsis of the Thesis entitled

A Study of Anisotropic Matter Distributions in General Relativity

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Submitted by

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1 Introduction

General Theory of Relativity is a geometric theory of gravitation. Among the four forces of interaction, gravitation is the only long range force. General Theory of Relativity is based on the following fundamental principles.

1. The principle of equivalence: In the neighbourhood of any point, one cannot distinguish between the gravitational field produced by the attraction of masses and the field produced by accelerating frame of reference.

2. Principle of covariance: The laws of physics must take the same form in all coordinate system.

3. The physical events are described in four dimensional spacetime manifold with metric $ds^2 = g_{ij}dx^i dx^j$.

4. The spacetime curvature is created by stress energy within the spacetime. In the presence of matter, this can be described by Einstein's field equations
$$\Re_{ij} - \frac{1}{2}\Re g_{ij} = -\frac{8\pi G}{c^2}T_{ij}, \tag{1}$$
where T_{ij}'s are components of energy-momentum tensor which contains all information about the physical content of the spacetime.

The Einstein's field equations comprise a set of ten second order partial differential equations connecting the metric and physical variables. However, for a spherically symmetric static metric, they reduce to the following set of three equations

$$\frac{8\pi G}{c^4}T_0^0 = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \tag{2}$$

$$\frac{8\pi G}{c^4}T_1^1 = e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}, \tag{3}$$

$$\frac{8\pi G}{c^4}T_2^2 = e^{-\lambda}\left(\frac{\nu''}{2} + \frac{\nu'\lambda'}{4} + \frac{\nu'^2}{4} - \frac{\lambda' - \nu'}{2r}\right). \tag{4}$$

For anisotropic fluid distributions the energy-momentum tensor can be expressed in the form [Maharaj and Maartens (1989) ([8]),

$$T_{ij} = (\rho + p)\, u_i u_j - p g_{ij} + \pi_{ij}, \tag{5}$$

where, ρ and p denote the energy-density and isotropic pressure of the fluid, respectively and u_i is the 4-velocity of the fluid. The anisotropic stress-tensor π_{ij} has the form

$$\pi_{ij} = \sqrt{3}S\left[C_i C_j - \frac{1}{3}(u_i u_j - g_{ij})\right], \tag{6}$$

where, $C^i = (0, -e^{-\lambda/2}, 0, 0)$. For a spherically symmetric anisotropic distribution, $S(r)$ denotes the magnitude of the anisotropic stress. The non-vanishing components of the energy-momentum tensor are the following:

$$T_0^0 = \rho, \quad T_1^1 = -\left(p + \frac{2S}{\sqrt{3}}\right), \quad T_2^2 = T_3^3 = -\left(p - \frac{S}{\sqrt{3}}\right). \tag{7}$$

Consequently, radial and transverse pressures of the distribution are given by

$$p_r = -T_1^1 = \left(p + \frac{2S}{\sqrt{3}}\right), \tag{8}$$

$$p_\perp = -T_2^2 = \left(p - \frac{S}{\sqrt{3}}\right), \tag{9}$$

so that the anisotropy takes the form

$$S = \frac{p_r - p_\perp}{\sqrt{3}}. \tag{10}$$

Theoretical investigations by Ruderman (1972)([13]) and Canuto (1974) ([2]) suggest that when the density of matter distribution exceeds that of nuclear density, it is likely that the distribution is anisotropic. Bowers and Liang (1974) ([1]) have shown that anisotropy may have non-negligible effect on equilibrium mass and surface redshift. Since then number of researchers worked on anisotropic superdense stars incorporating charge as well as devoid of charge. Recently a lot of interest has been developed among researchers in developing models of charged as well as uncharged anisotropic fluid distributions compatible with observational data.

We have studied, in the Thesis, models of anisotropic charged as well as uncharged superdense distributions of matter compatible with observational data on the background of spacetimes possessing definite geometry, viz., pseudospheroidal and paraboloidal spacetimes. The Thesis is divided into six chapters.

2 Layout of The Thesis

Chapter 1 contains introduction to general theory of relativity and the theoretical background needed for the problems studied in the subsequent chapters. It also contains the summary of each chapter of the thesis .

In **Chapter 2** we study the solution of Einstein's field equations (EFEs) for a static spherically symmetric anisotropic distribution by generalizing the ansatz of Finch and Skea [Class. Quantum Grav. **6** 467, 1989] described by $g_{rr} = \left(1 + \frac{r^2}{R^2}\right)^n$. By using the physical acceptability and regularity conditions we have obtained the bounds on the model parameter p_0 in terms of the dimensionless parameter n which lies in the interval $\left(1, \frac{4}{\sqrt{3}}\right)$. The model so developed is in good agreement with the observational data of pulsars , viz., 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela X-1, PSR J1614-2230, SAX J1808.4-3658 and Her X-1 (referred in Gangopadhyay *et al* [5]).

Chapter 3 deals with a new class of solutions of Einstein's field equations representing a static spherically symmetric anisotropic matter distribution on the background of pseudo-spheroidal spacetime characterized by the metric potential $g_{rr} = \frac{1+K\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}$, where K and R represent geometric parameters. The field equations are integrated by assuming a particular, physically acceptable form for the radial pressure p_r given by the expression $p_r = \frac{p_0}{R^2}\frac{\left(1-\frac{r^2}{R^2}\right)\left(1+\frac{r^2}{R^2}\right)}{\left(1+K\frac{r^2}{R^2}\right)^2}$. We have obtained suitable bounds of model parameters K and p_0 on the basis of the physical acceptability conditions viz., regularity, stability and energy conditions. It is found that the model is compatible with the wide range of compact stars viz., 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela

X-1, PSR J1614-2230, SMC X-4 and Cen X-3.

In order to validate the model for physical acceptability , we have studied in detail the regularity, energy and stability conditions using numerical and graphical methods for the pulsar 4U 1820-30 by taking the mass of the pulsar as $1.58M_{\odot}$ and radius is $9.1km$ The values of the parameters in this case are $p_0 = 1.08$ and $K = 3.1$.

In the second part of this chapter, we have taken a different form for radial pressure, viz., $p_r = \frac{K-1}{R^2}\frac{\left(1-\frac{r^2}{R^2}\right)}{\left(1+K\frac{r^2}{R^2}\right)^2}$. The bound for the geometric parameter K is obtained as $2.4641 \leq K \leq 4.1231$ using the physical acceptability conditions. For validating the present model , we have studied in detail the regularity, stability and energy conditions for the pulsar candidate PSR J1614-2230 having mass equal to $1.97M_{\odot}$ and radius $9.69km$ corresponding to $K = 3.997$.

In **Chapter 4**, we have studied anisotropic charged fluid distributions on pseudo-spheroidal spacetime. By choosing suitable expressions for radial pressure $p_r = \frac{p_0}{R^2}\frac{\left(1-\frac{r^2}{R^2}\right)\left(1+\frac{r^2}{R^2}\right)}{\left(1+K\frac{r^2}{R^2}\right)^2}$ and electric field intensity $E = \sqrt{\alpha}\frac{r}{R^2}$, where $\alpha \geq 0$ is a constant, the field equations are integrated. The parameters K, R and α are determined by imposing the physical acceptability conditions. The present model is in good agreement with the observational data of various compact stars like 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela X-1, SMC X-4, Cen X-3 given by Gangopadhyay *et al.* ([5]). When $\alpha = 0$, the model reduces to the uncharged anisotropic distribution described as first model in chapter 3. In order to examine the nature of physical quantities throughout the distribution, we have considered a particular pulsar 4U 1820-30, whose tabulated mass and radius are $M = 1.58M_{\odot}$, and $R = 9.1(km)$, respectively, for $K = 2.718$ and $\alpha = 0.05$. It is found that all physical variables behave well for this particular pulsar.

We have studied a second model in this chapter by assuming a different form for radial pressure p_r and electric field intensity E, namely, $p_r = \frac{K-1}{R^2}\frac{\left(1-\frac{r^2}{R^2}\right)}{\left(1+K\frac{r^2}{R^2}\right)^2}$ and $E^2 = \frac{\alpha(K-1)}{R^2}\frac{\frac{r^2}{R^2}}{\left(1+K\frac{r^2}{R^2}\right)^2}$. The bounds of geometric parameter K and the parameter α appearing in the expression for E^2 are obtained by imposing the requirements for a physically acceptable model. It is found that the model is in good agreement with the observational data of number of compact stars like 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela X-1, PSR J1614-2230, Cen X-3 given by Gangopadhyay *et al.* ([5]). When $\alpha = 0$, the model reduces to the uncharged anisotropic distribution discussed as a second model in chapter 3.

Chapter 5 provides new exact solutions of Einstein's field equations (EFEs) by assuming a linear equation of state, $p_r = \alpha(\rho - \rho_R)$ where p_r is the radial pressure and ρ_R is the surface density. The background spacetime metric is a paraboloidal spacetime metric characterized by the metric potential $g_{rr} = 1 + \frac{r^2}{R^2}$. By assuming estimated mass and radius of strange star candidate 4U 1820-30, various physical and energy conditions are used for estimating the range of parameter α. The suitability of the model for describing pulsars like PSR J1903+327, Vela X-1, Her X-1 and SAX J1808.4-3658 has been explored and respective ranges of α, for which all physical and energy conditions are satisfied throughout the distribution, are obtained.

In **Chapter 6** we have obtained an exact solutions of Einstein's field equations on the background of paraboloidal spacetime using Karmarkar condition, namely, $R_{1414}R_{2323} = R_{1212}R_{3434} + R_{1224}R_{1334}$. For a spherically symmetric static paraboloidal spacetime this condition is equivalent to $\frac{2\nu''}{\nu'} + \nu' = \frac{2}{r}$, where the metric potential $g_{tt} = e^{\nu}$. The physical acceptability conditions of the model are investigated and found that the model is compatible with a number of compact star candidates like Her X-1, LMC X-4, EXO 1785-248, PSR J1903+327, Vela X-1 and PSR J1614-2230. A noteworthy feature of the model is that it is geometrically significant and simple in form.

Some of the important references used in the thesis are cited below:

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