FUNDAMENTAL CONCEPTS AND INTRODUCTION

1.1 Fluid, Types of Fluid

A fluid is that type of matter which needs some support to recollect its size and shape, which in turn means that fluid, is a substance which flows or is capable of flowing. The most common examples of fluid are water and air.

1.2 Newtonian and non-Newtonian fluid

A Newtonian fluid is a fluid in which relation between the stress tensor and rate of strain tensor is linear function (see Figure 1.1) which is established by Isaac Newton in 1687. Air, water and glycerin are few examples of Newtonian fluids.

A fluid is said to be non-Newtonian if relation between the stress tensor and rate of strain tensor is not a linear function (see Figure 1.2). Therefore, non-Newtonian fluid is represented by curve. Blood, grease and sugar are some of non-Newtonian fluid.

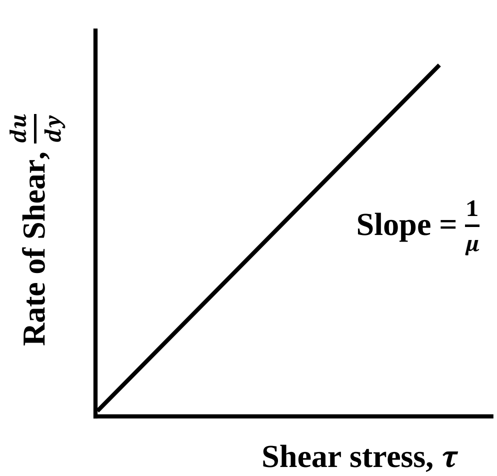

Figure 1.1: Newtonian Fluid

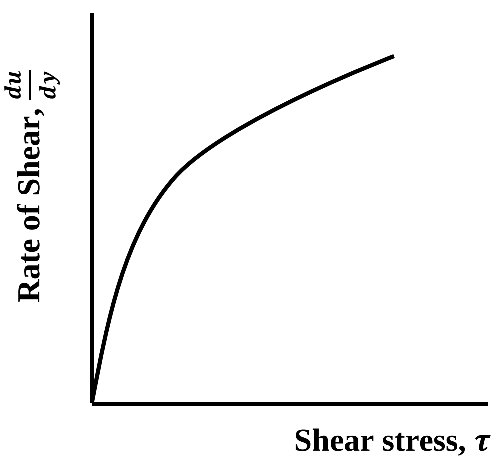

Figure 1.2: non-Newtonian Fluid

1.3 Classification of non-Newtonian fluid

The Non-Newtonian fluids are divided in to three categories.

- Time Independent Fluids
- Time dependent Fluids
- Elastic viscous Fluids

1.3.1 Time Independent Fluids

Time independent fluids are those fluids in which the values of shear rate at a point within the fluid is determined only by the recent values of shear stress. Viscosity of this type of fluid is dependent on temperature and shear rate. This type of fluid also known as, purely viscous, inelastic or generalized Newtonian fluids

The characterization of flow behavior depending on how viscosity changes with the shear rate.

- Shear thinning: Viscosity of the fluid reduces with increased shear rate. These fluids are also called Pseudo-plastic fluids. Shampoo, slurries and ketchup are common examples of shear thinning fluids.
- Shear thickening: Viscosity of the fluid raises with raised shear rate. These fluids are known as dilatant fluids. Wet sand and concentrated starch suspensions are examples of shear thickening fluids.
- Plastic: Shear stress must be applied before flow occurs. Tomato paste, tooth paste, hand cream and grease are common examples of these fluids

1.3.2 Time dependent Fluids

They are those fluids in which the relation between shear rate and rate of stress shows further dependence. In this type of Non-Newtonian fluid, viscosity of the fluid changes with temperature, shear rate and time.

The characterization of flow behavior depending on how viscosity changes with the time.

- Thixotropic (time thinning): Viscosity of the fluid reduces with time. Some examples of such fluid are yoghurt and paint.
- Rheopectic (time thickening): Viscosity of the fluid increase with time. Gypsum paste is time thickening fluids.

1.3.3 Visco-elastic fluids

They are those fluids which display a mixture of viscous fluid behavior and elastic behavior which are predominantly viscous but show partial elastic recovery after deformation, are termed as elastic viscous fluids. The term viscoelastic fluid is also used for such fluids. Plastics, polymer, colloidal solutions and foodstuffs, etc. are types of elastic viscous.

1.4 Micro-polar fluid

Micro-polar fluids are fluids with microstructure. They belong to a class of fluids with non-symmetric stress tensor which can be considered as polar fluids. Physically, micro-polar fluids may suggest fluids, including rigid, randomly oriented particles suspended in a viscous medium, where the deformation of fluid particles are ignored. Recently, the study of micro polar fluid has attracted many scholars as Navier–Stokes equations of Newtonian fluids cannot describe effectively the characteristics of fluid with suspended particles. Eringen [1] introduced the simple micro-polar fluids theory. According to his theory, simple micro-polar fluids is a fluent medium whose properties and behavior are affected by the local motion of the fluid fundamentals. Keeping this in mind, Eringen [2] simplified micro-polar fluids theory and obtained a subclass of these fluids called micro-polar fluids. This theory is one of the best established theories of fluids with microstructure. This theory is useful in explaining the features of certain fluids such as liquid crystals, suspensions and animal blood.

In micro-polar fluid theory, two new variables of the velocity are added which were not existing in the Navier-Stokes model. These are micro-rotation variables which represent rotation and micro-inertia tensors which define the distribution of particles and molecules classified the atomic fluid particles. The details of micro-polar fluid theory can be found in the book by Eringen [3] and Lukaszewicz [4].

1.5 Second grade fluid

A Second grade fluid is a fluid; whose stress tensor is the sum of all tensors that can be designed from the velocity field with couple of derivatives. The constitutive equation of a Second grade fluid is a linear relation between the stress and the first Rivlin-Ericksen tensor, square of the first Rivlin-Ericksen tensor and second Rivlin-Ericksen tensor [5]. This constitutive equation has three coefficients and used for fluids of the visco-elastic type. The governing differential equations of a Second grade fluid are of higher order than the Navier-Stokes equations and thus, one needs conditions in addition to the usual adherence boundary condition in general. For a detailed discussion and some interesting examples, one can refer to Rajagopal [6].

1.6 Casson fluid

Casson fluid is a shear thinning fluid which is supposed to have an immeasurable viscosity at zero rate of shear, a yield stress below which no movement happens, and a zero viscosity at an infinite rate of shear. Casson fluid is classified as the most popular non-Newtonian fluids which has several applications in food processing, metallurgy, drilling operations and bio-engineering operations. Casson fluid model was introduced by Casson [7] for the prediction of the flow behavior of pigment-oil suspensions. Examples of Casson fluid are jelly, tomato sauce, honey, soup, concentrated fruit juices and Human blood.

1.7 Types of Fluid flow

1.7.1 Steady and unsteady fluid flow

Flow, in which properties and conditions do not change with time. If $\emptyset$ represents velocity, density, temperature, pressure etc. of the fluid then $\frac{\partial \emptyset}{\partial t} = 0$ for steady state flow. Thus in steady flow, the various field quantities become functions of the space co-ordinates only. On the other hand, the flow is said to be unsteady if $\frac{\partial \emptyset}{\partial t} \neq 0$. So, in unsteady flow, fluid properties and conditions are time dependent.

1.7.2 Compressible or incompressible fluid flow

If density changes with pressure, fluids are regarded as compressible. On other hand, the flow is incompressible, if density is constant w.r.t pressure. Liquids are usually nearly impossible to compress, whereas gases (also considered a fluid) are compressible.

1.7.3 Rotational or Irrotational flow

To test whether a flow has rotational component, one can put a small object in the flow and let the flow carry it. If the small object turns, flow is rotational otherwise flow is irrotational.

1.8 Magnetohydrodynamics flow

Fluid dynamics is an important science used to solve many natural phenomena such as flying of birds, swimming of fishes and the development of weather conditions. The study of the laws that govern the conversion of energy from one form to another, the direction in which the heat will flow, and the availability of energy to do work is the subject of the Thermodynamics. The study of charge particle in motion, the forces created by electric and magnetic field, and the relationship between them give rise to the subject Electrodynamics. The collective effects of these three significant branches of science namely, Fluid dynamics, Thermodynamics and Electrodynamics give rise to the topic Magneto-fluid dynamics (MFD) which in the form of definition read as the science of motion, electrically conducting fluid in the presence of a magnetic field. It has two subtopics: Magnetohydrodynamics (MHD) and Magnetogasdynamics (MGD). MHD deals with electrically conducting liquids whereas MGD deals with ionized compressible gases.

Magnetohydrodynamics (often referred to as MHD) deals with the dynamics of fluids having non-negligible electrical conductivity which interact with a magnetic field. As a result, motion of an electrically conducting fluid in the presence of a magnetic field, electric current is induced in the fluid. An electrically conducting fluid moving in presence of a magnetic field (transverse) which generate a force called the Lorentz force. This force has a tendency to modify the initial motion of the conducting fluid. Moreover, the induced currents generate their own magnetic field, which is added to the primitive magnetic field. Thus there is an interlocking between the motion of the conductor and the electromagnetic field. MHD has several applications, namely, fusion research, MHD accelerator, power generator and in causing delay in the transition from laminar to turbulent flow. First theory of laminar flow of an electrically conductive liquid in a homogenous magnetic field was introduced by Hartman [8], whereas the finding of Alfven waves completed the formation of MHD as an individual science by Alfven [9]. Alfven received Nobel Prize in 1942 for his study. Huang et al. [10] discussed MHD waves and instabilities in the heat-conducting solar wind plasma. Wang et al. [11] studied energy of Alfven waves generated during magnetic reconnection.

1.9 Heat transfer

The study of heat transfer contains thermal energy which is transfer as a result of temperature variance. In studying the heat transfer, understanding of the temperature distribution in a system is necessary. Heat flow takes place whenever there is a temperature gradient in a system. Once the temperature distribution is known, the heat flux, which is the amount of heat transfer per unit area, per unit time is obtained from the rule connecting the heat flows to the temperature gradient. The basic categories of heat transfer are

(i) Conduction (ii) Convection (iii) Radiation and (iv) Advection

i. Conduction

Conduction can be termed exactly like heat is distributed by conduction when adjacent particles wobble against one another. The most important way of heat transfer is conduction when solid objects can be in thermal contact. Fluids, especially gases, are comparatively less conductive.

ii. Convection

Convection, which arises in a fluid by the mixing of one portion of the fluid with another portion due to gross moments of the mass of fluid.

The convective mode of heat transfer is divided into two types:

- Free or Natural convection in which the fluid moves as a result simply of changes in density of portions at different temperatures which results to buoyancy forces in streams and currents.
- Forced convection in which the motion is maintained by external ways such as stirrers, fans, and pumps. i.e., Circulation sets through means of some mechanism.

iii. Radiation

The transfer of energy from a body by means of the release or absorption of electromagnetic radiation is referred as radiation. Thermal radiation increases without the existence of matter through the vacuum of space. In contrast to conduction and convection, thermal radiation can be collected in a small spot with the help of reflecting mirrors, which is utilized in concentrating solar power generation.

iv. Advection

Thermal energy is move by the physical transfer of a hot or cold object from one place to other.

1.10 Mass Transfer

Mass transfer is obtained as the transfer of matter by feature of species concentration variance in a system. The variance in concentration delivers a driving force for the transfer of mass. Mass transfer always occurs in the direction of reducing concentration gradient. The phenomena of mass transfer are very corporate in the concept of stellar structure and visible effects are measureable at least on the solar surface.

The participation and application of mass transfer process spread to greater extent in several fields of Engineering, Science and Technology. In the fields of Electric Engineering, Civil Engineering, Environmental Engineering, Air conditioning, Aeronautics, Metallurgy, Refrigeration, industrial and biological process and mass transfer process in general.

In biological functions or process like respiratory mechanisms, oxygenation or purification of blood, kidney function, osmosis and assimilation of food and drugs, these processes are involved. Some of the mass transfer phenomena found in nature are smoke formation, evaporation of clouds, dispersion of fog, grooves of fruit trees, damage of crops due to freezing and pollution of the environment and distribution temperature and moisture over agricultural fields.

1.11 Porous media

The perfect porous medium, appropriate perfectly the definition required, may be most clearly understood by imagining a body of ordinary unconsolidated sand. It is present in such a medium immeasurable holes of varying sizes and shapes including “pore space” or gaps between the separate solid particles of sand. Moreover, each pore is connected by confined channels to other pores, the whole making a totally interconnected network of openings which form the channels through which the contained fluid may flow. It is the whole several interconnected of the minute openings that characterizes the ideal “porous media” assumed in this work and definitely differentiates this subject from that of the usual hydrodynamics or hydraulics by Muskat [12].

A porous medium is a solid containing invalid spaces (pores), connected or unconnected, distributed within it in either a regular or random manner. These so called pores may contain a variety of fluids such as air, water, oil etc. If pores represent a certain portion of the bulk volume, a complex network can be formed which is able to carry fluids. Various examples can be named where porous media play an important role or where the technology requires them as a tool.

The porosity of porous media is determined as the ratio of the volume of the pores to the total bulk volume of the media. By definition, the porosity can only range from 0 to 1, or as a percentage between 0–100 percent.

The porous medium is also described by its supreme valuable fluid flow property called permeability which is a measure of the flow conductivity in the porous medium. The permeability is a statistical average of the fluid conductivity of all the flow channels in the medium. The average conductivity takes into account the variation in size, shape, direction and interconnection of all the flow channels.

The permeability (k) can be defined as,

$$k = \frac{-Q\,\mu}{A\,\rho g}\left(\frac{\partial h}{\partial s}\right)^{-1} \tag{1.1}$$

Where A is cross sectional area of the fluid, Q is the total discharge of the fluid, μ is the viscosity of the fluid, g is the acceleration due to gravity, ρ is the density of the fluid and $\frac{\partial h}{\partial s}$ is the hydraulic gradient.

Darcy's law is a simple proportional relationship between the instantaneous discharge rate through a porous medium. The viscosity of the fluid and the pressure difference over a given distance. In modern notations this is expressed, in refined form by:

$$q = \frac{-k}{\mu}\frac{\partial P}{\partial x} \tag{1.2}$$

where q is the flux, $\frac{\partial P}{\partial x}$ is the pressure gradient, in the flow direction and μ is the dynamic viscosity of the fluid. The coefficient k is independent of the nature of the fluid but it depends on the geometry of the medium.

1.12 Chemical reaction effects

In convective heat and mass transfer developments, distribution rates can be changed by chemical reaction. The effect of chemical reaction depends on whether the reaction is mixed or identical. In detailed, a reaction is called of first order, if the rate of chemical reaction is directly related to the concentration. In nature, the existence of pure water or air is not possible. Some external mass either may be existing naturally or mixed with air or water. The study of chemical reaction procedures is beneficial for improving a number of chemical technologies, such as food processing, polymer production and manufacturing of ceramics or glassware. In chemical reaction problems, Kr is chemical reaction parameter. If Kr > 0, then it is a destructive chemical reaction (means endothermic, i.e., heat is absorbed) if Kr < 0, then it characterizes a propagative chemical reaction (means exothermic, i.e., heat is generated). In many chemical reaction processes, there is chemical reaction between external mass and the fluid in which the plate moves.

1.13 Soret effect

In the simultaneous existence of heat and mass transfer in a moving fluid, it founds connection between the fluxes and the driving potentials. It is noticed that mass fluxes can be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect, named after the Swiss scientist Charles Soret [13], who investigated thermal diffusion in solutions in 1879–1881.

1.14 Hall current effects

It is observed that in an ionized fluid where the density is low and/or if a very strong magnetic field is present so that the cyclotron frequency beats collision frequency, charged particles can rotate round the lines of force numerous times before suffering crashes with other particles. This results in a drift of charged particles in a direction perpendicular to the directions of electric and magnetic fields. Thus if an electric field is applied at right angle to the magnetic field, total current will not flow along with electric field. This tendency of electric current to flow across an electric field in the presence of a magnetic field is called Hall effect and resulting current is known as Hall current. Hall effect was discovered by Hall [14] in 1879.

1.15 Fundamental Equations

The basic equations governing the motion of an incompressible viscous and electrically conducting fluid in the presence of a magnetic field

- Equation of continuity
- Equation of motion
- Energy equation
- Mass transfer equation
- Maxwell equation

1.15.1 Equation of continuity

The equation of continuity is a differential equation that describes the transport of conserved mass. A conserved mass means such mass that cannot increase or decrease, it can only move from one place to another place. The conservation of mass gives the equation of continuity which may be written as

$$\frac{D\rho}{Dt} + \rho\,\mathrm{div}(\vec{u}) = 0 \qquad (1.3)$$

1.15.2 Equation of motion

The equations of motion are derived from the Newton's second law of motion, which states that

Rate of change of linear momentum = Total force

Therefore, the equations of motion is

$$\rho\left[\frac{D\vec{u}}{Dt}\right] = \rho\vec{F} - \nabla P + \rho_e\vec{E} + J \times B + div\,(\vec{\tau}). \tag{1.4}$$

Where, $\vec{\tau} = \tau_{ij} = 2\mu\, e_{ij} + \left(\xi - \frac{2}{3}\mu\right) e_{kk}\delta_{ij}$ & $e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ (1.5)

1.15.3 Energy equation

The law of conservation of heat transfer is an empirical law of Physics. It said that the full amount of heat transfer in an isolated system remains constant over time. The energy equation in this case includes both viscous and Joulean dissipations functions then reduces to

$$\rho C_p\left[\frac{\partial T}{\partial t} + (\vec{u}.\nabla)T\right] = k_4\nabla^2 T + \phi + \frac{\mathbf{J}^2}{\sigma} + Q. \tag{1.6}$$

1.15.4 Mass transfer equation

Though mass diffusion is concerned with the conductivity of the medium, the magnetic field has minor effect on this process. So, ordinary diffusion equation can be applied for MHD problems under the suitable assumptions is

$$\frac{\partial C}{\partial t} + (\vec{u}.\nabla)C = D\nabla^2 C \tag{1.7}$$

1.15.5 Maxwell equations

Maxwell's equations are a set of four partial differential equations that relate the electric and magnetic fields to their sources, charge density and current density. Individually, the equations are known as Gauss's law, Gauss's law for magnetism, Faraday's law of induction, and Ampère's law with Maxwell's correction. The set of equations is named after James Clerk Maxwell which can be expressed as

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{1.8}$$

$$\nabla \times \boldsymbol{H_1} = \mathbf{J} \tag{1.9}$$

$$\nabla.\mathbf{B} = 0 \tag{1.10}$$

$$\nabla.\mathbf{J} = 0 \tag{1.11}$$

1.16 Constitutive equations of micro-polar fluid

The constitutive equations for micro-polar fluids with stress tensor τ_{ij} and couple stress tensor C_{ij} are given as Lukaszewicz [4]

$$\tau_{ij} = \left(-P + \lambda \nu_{k,k}\right)\delta_{ij} + \mu\left(\nu_{i,j} + \nu_{j,i}\right) + \mu_r\left(\nu_{j,i} - \nu_{i,j}\right) - 2\mu_r \varepsilon_{mij} N_m \tag{1.12}$$

and

$$C_{ij} = C_0 N_{k,k}\delta_{ij} + C_d\left(N_{i,j} + N_{j,i}\right) + C_a\left(N_{j,i} - N_{i,j}\right) - 2\mu_r \varepsilon_{mij} N_m \tag{1.13}$$

where λ and μ are the usual viscosity coefficients, μ_r is the dynamic micro rotation viscosity, *p* is the pressure, *N* is the angular velocity of rotation of particles of the fluid and $C_0, C_a \; and \; C_d$ are constants, called coefficients of angular viscosities.

The symmetric part of the stress tensor τ_{ij}in equation (1.12) is given by

$$\tau_{ij}{}^{(s)} = \left(-P + \lambda \nu_{k,k}\right)\delta_{ij} + \mu\left(\nu_{i,j} + \nu_{j,i}\right) \tag{1.14}$$

Local conservation laws of mass, linear and angular momentum and energy for polar fluids were obtained by using stress tensor given by (1.12) or (1.14) and couple stress (1.13) along with one additional equation known as Angular momentum equation, which is given by

$$\rho I \frac{DN}{Dt} = \vec{\nabla}.\overrightarrow{C_{ij}} + \varepsilon_{ijk}\tau_{jk} \tag{1.15}$$

Where, *I* - micro-inertia coefficient

1.17 Constitutive equations of Second grade fluid

The constitutive equation for viscoelastic fluid of Second grade model is given by

$$\tau = \mu\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial \mathrm{u}}{\partial \mathrm{y}} \tag{1.16}$$

Where, $t, \tau, \frac{\partial \mathrm{u}}{\partial \mathrm{y}}$ and λ_1 is the time, shear stress, rate of strain rate and material constants respectively and μ is viscosity. This model reduces to classical Navier Stokes model by substituting $\lambda_1 = 0$.

1.18 Constitutive equations of Casson fluid

The constitutive equation for the Casson fluid can be written as

$$\tau_{ij} = \begin{cases} 2\left(\mu B + \frac{Py}{\sqrt{2\pi}}\right) e_{ij} & \pi > \pi_c \\ 2\left(\mu B + \frac{Py}{\sqrt{2\pi_c}}\right) e_{ij} & \pi < \pi_c \end{cases} \tag{1.17}$$

Where $\pi = e_{ij}e_{ij}$ and e_{ij} is the $(i,j)^{th}$ component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μB is plastic dynamic viscosity of the non-Newtonian fluid and P_y is yield stress of fluid.

$$P_y = \frac{\mu B\sqrt{2\pi}}{\gamma} \tag{1.18}$$

From the definition of viscosity given by Batchelor [158], ratio of sheer stress τ^* to viscosity μ is constant in case of Newtonian.

$$\tau^* = \mu\frac{\partial u}{\partial y} \tag{1.19}$$

Some fluids require a gradually increasing shear stress to maintain a constant strain rate, In the case of Casson fluid (Non Newtonian) flow where $\pi > \pi_c$

$$\mu = \mu B + \frac{P_y}{\sqrt{2\pi}} \tag{1.20}$$

Substituting equation (1.18) in equation (1.20), the Kinematics viscosity can be expressed as

$$\nu = \frac{\mu B\left(1+\frac{1}{\gamma}\right)}{\rho} \tag{1.21}$$

1.19 Laplace Transform Technique (LTT)

The branches of science and engineering in which Laplace transform technique are used for solving linear system of partial differential equations with constant coefficients and ordinary differential equations in which the coefficients are the variables or simultaneous ordinary differential equations. Laplace transform technique can also be applied in mechanics (dynamics and statics), electrical circuits, to analysis the characteristic of beam and several partial differential equations subject to initial and boundary conditions etc. Thus, it can be understood that the Laplace transform has its remarkable applications in many branches of pure and applied mathematics.

The Laplace transform is named after mathematician and astronomer Pierre-Simon Laplace, who used a similar transform (now called z transform) in his work on probability theory. The Laplace transform of $f(t), t \geq 0$ denoted by $F(s)$ $or\ L\{f(t)\},$ is an integral transform given by the Laplace integral:

$$L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\ dt \tag{1.22}$$

Provided this improper integral exist, i.e. the integral is convergent.

The Laplace transform is an operation which transformed a function of time domain t, defined on [0, ∞), to a function of frequency domain s.

1.19.1 Advantages of this methods

Laplace transform technique gives analytical results which is more accurate than numerical results. Laplace transform technique can be the best suitable for time dependent problems. Some elementary functions, such as a constant, the exponential, the sine, and the cosine functions do not have Fourier transforms (one of the most popular analytical methods) because they are not integral on R. The easiest (suitable) way to overcome these sever limitations are the use of 1aplace transform technique. In particular, one can apply it to deal with problems in which one of the variables is time.

1.19.2 Laplace transforms technique in MHD

The physical aspects of any fluid flow is expressed in terms of system of partial differential equation with initial and boundary condition, in which Laplace transform technique can be used properly; as it is the art of substituting the governing equations of fluid flow with numbers and proceeding these numbers in space and/ or time into an ordinary differential equation, which can be solved by established rules and, then Inverse Laplace transforms techniques are useful to get the required results. This method is the perfectly fitted for unsteady free convective MHD problems through porous medium. The Laplace transform and inverse Laplace transform of special function and many other function which is practically growth in MHD problems using Laplace transform technique [167-168].

1.19.3 Solving partial differential equation using Laplace transform technique

Given a function $u(x,t)$ well-defined for all $t > 0$ and supposed to be bounded one can apply the Laplace transform in t considering x as a parameter

$$L\big(u(x,t)\big) = \int_0^\infty e^{-st} u(x,t)\, dt = U(x,s) \qquad (1.26)$$

For solving PDEs, the Laplace transform of following partial derivative is required.

$$L\big(u_t(x,t)\big) = \int_0^\infty e^{-st} u_t(x,t)\, dt = [e^{-st} u(x,t)\,]_0^\infty + s\int_0^\infty e^{-st} u(x,t)\, dt = sU(x,s) - u(x,0) \qquad (1.27)$$

So, the outcome is

$$L\big(u_t(x,t)\big) = sU(x,s) - u(x,0) \qquad (1.28)$$

Similarly, it obtained

$$L\big(u_{tt}(x,t)\big) = s^2 U(x,s) - su(x,0) - u_t(x,0) \qquad (1.29)$$

It required the Laplace transform of x derivative.

$$L\big(u_x(x,t)\big) = \int_0^\infty e^{-st} u_x(x,t)\, dt = U_x(x,s) \qquad (1.30)$$

$$L\big(u_{xx}(x,t)\big) = \int_0^\infty e^{-st} u_{xx}(x,t)\, dt = U_{xx}(x,s) \qquad (1.31)$$

1.20 Review of Relevant Literature

Analytical methods as well as Numerical methods have been successfully used by many scientists to solve several types of problems of magnetic field effects on unsteady free convective flow of different types of non-Newtonian fluid like, micro-polar fluid, Second grade fluid and Casson fluid with heat and mass transfer through porous medium. Some of them (related to the thesis) are briefly reviewed here.

Recently, the study of micro polar fluid has attracted many scholars as Navier–Stokes equations of Newtonian fluids cannot effectively describe the characteristics of fluid with suspended particles. Eringen [2] first introduced the theory of micro-polar fluid and this theory is useful in explaining the characteristics of certain fluids such as liquid crystals, suspensions and animal blood. Consequently, Ariman et al. [15] wrote outstanding reviews on applications of micro-polar fluids. Further, El-Hakiem [16] studied natural convection of micro-polar fluid by using perturbation technique. Chamkha et al. [17] and Abdulaziz and Hashim [18] investigated the fully developed free convection of a micro-polar fluid in vertical channel. Ikbal et al. [19] studied two-layered micro-polar fluid flow whereas, Xin-yi et al. [20] considered flow of micro-polar fluid through porous channel with expanding or contracting walls. Alloui et al. [21] discussed double-diffusive and Soret-induced convection of a micro-polar fluid in a vertical channel. Mohamed [22] worked on the solution of mixed convection flow of a micro-polar fluid with viscous dissipation. Jena et al. [23] considered diffusive convection of micro-polar fluids in a square. Sudhakara et al. [24] studied steady heat transfer in a thin film flow of a micro-polar fluid over an inclined permeable bed. Asma et al. [25] considered heat and mass in unsteady flow of a micro-polar fluid with wall couple stress. Due to increasing significance, application of non-Newtonian fluids is required in engineering. It is due to many applications in their several areas, such as the plastic manufacture, performance of lubricants, food processing, or movement of biological fluids. Second grade fluids can model many fluids such as dilute polymer solutions, slurry flows and industrial oils. Tan and Masuoka [26] examined the Stokes' first problem for a Second grade fluid and Rashidi et al. [27] investigated unsteady squeezing flow of a second-grade fluid, whereas Hayat et al. [28] considered with unsteady stagnation point flow of Second grade fluid with variable free stream. Hameed et al. [29] obtained the solution of magnetic and heat transfer on fractional Second grade fluid in a vertical tube. Samiulhaq et al. [30] defined exact solutions for unsteady free convection flow of a Second grade fluid past a vertical flat plate.

In modern engineering, many characteristics of flow are not understandable with the Newtonian fluid model. Hence non-Newtonian fluids theory has become useful. Non-Newtonian fluid exerts non-linear relationships between the shear stress and rate of shear strain. It has an extensive variety of applications in engineering and industry, especially in the extraction of crude oil from petroleum products. Casson fluid is one of such fluids. Casson fluid is classified as the most popular non-Newtonian fluid which has several applications in food processing, metallurgy, drilling operations and bio-engineering operations. Casson fluid model was introduced by Casson [7] for the prediction of the flow behavior of pigment-oil suspensions. Pramanik [31] solved the problem based on Casson fluid flow past an exponentially porous stretching surface in presence of thermal radiation as well as Mahanta and shaw [32] discussed the 3D Casson fluid flow past linearly stretching sheet. Dash et al. [33] considered Casson fluid flow in a pipe filled with a homogeneous porous medium. Akbar and Butt [34] studied physiological transportation of Casson Fluid in a Plumb Duct whereas, Bhattacharyya et al. [35] and Abbas et al. [36] obtained solution of Casson fluid flow over permeable shrinking sheet with thermal radiation. Recently, Sandeep et al. [37] considered 3D-Casson fluid flow within boundary layer formed on a surface at absolute zero and Sulochana et al. [38] discussed heat source or sink effect on 3D-Casson fluid in the presence of Soret and thermal radiation.

Magnetohydrodynamics is the study of the motion of the fluid in the presence of magnetic field. The study of MHD flows has great importance, as these flows are quite prevalent in nature. The application of MHD to natural events received delayed motivation when astrophysicists came to realize that how it is established throughout the universe conducting, ionized gases and significantly strong magnetic fields. First introduced theory of laminar flow of an electrically conductive liquid in a homogenous magnetic field by Hartman [8]. Huang et al. [10] discussed on MHD waves and instabilities in the heat-conducting solar wind plasma. Lee [39] deals with two and three dimensional MHD simulations whereas, Kumar et al. [40] studied coronal heating by MHD waves.

There are many applications for the parabolic motion such as solar cookers, solar concentrators and parabolic through solar collector. Murty et al. [41] deals with evaluation of thermal performance of heat exchanger unit for parabolic solar cooker and Raja et al. [42] considered design and manufacturing of parabolic through solar collector system. Muthucumaraswamy and Geetha [43] discussed effects of parabolic motion on isothermal vertical plate with constant mass flux. Singh and Singh [44] studied MHD free convective flow past a semi-infinite vertical permeable wall. Many researchers like, Eldabe and Salwa [45], Nadeem et al. [46], Akbar and Khan [47], Akbar [48] discussed influence of magnetic field on unsteady free convective flow of Casson fluid whereas, Nadeem et al. [49-50] considered magnetic field effects on unsteady free convective flow of Casson nano fluid. Akbar et al. [51-52] studied MHD stagnation point flow of Carreau fluid/ Powell fluid toward a permeable shrinking sheet. The work based on induced magnetic field effects on unsteady free convective flow, which is done by [53-55]. Furthermore, some other work on MHD flow of different types of nanofluid are available in Sheikholeslami et al. [56], Hatami et al. [57], Sheikholeslami and Ellahi [58], Freidoonimehr et al. [59], Shehzad et al. [60], Abbasi et al. [61], Hayat et al. [62] and Khan et al. [63]. Borrelli et al. [64] discussed MHD flow of micro-polar fluid in a vertical channel.

Heat transfer which is defined as the transmission of thermal energy and heat transfer between physical systems. The transfer of heat is usually from a region of higher temperature to lower temperature zone. In heat transfer problems, temperature represents the amount of thermal energy available while heat flow represents the movement of thermal energy from one point to another. Thus, the study of heat transfer has a principal role in numerous commercial and engineering applications aircraft engineering, water heaters, lubrication of bearings, cooling compressors of engine cylinders and nuclear technology. New growth concerning laminar and turbulent flows along with the boundary layer theory has managed to the study of heat transfer progression to a huge amount in consideration of above research developments. Heat transfer characteristics of unsteady free convective MHD flow are described by several researchers like, Abbas et al. [65], Ahmed et al. [66] and Sheikholeslami et al. [67]. Mahmoud and Waheed [68] discussed heat transfer effects on MHD micro-polar fluid flow over a stretching surface with heat generation. Recently, Kataria et al. [69] studied heat transfer effects on MHD micro-polar fluid flow between two vertical walls. Research work on heat transfer effects on MHD flow of nanofluid presented by Sheikholeslami et al. [70], Sheikholeslami and Chamkha [71], Sheikholeslami and Bhatti [72], Ibrahim [73], Sheikholeslami et al. [74], Hatami and Safari [75] and Hatami et al. [76]. Abbasi and Shehzad [77] studied heat transfer effects on three-dimensional flow of Maxwell fluid with temperature dependent thermal conductivity.

The convective heat transfer phenomena in nature are often attended by mass transfer. Convective mass transfer process creates the support of various procedures in the chemical engineering. This appears like sufficient purpose to contain mass transfer in heat convection as well. An analogy happens between convective mass transfer and convective heat transfer. This analogy is educationally actual significant because it provides a chance to organize the understanding of heat transfer and to learn mass transfer with the least memorization. Muthucumaraswamy et al. [78] considered mass transfer effects on exponentially accelerated isothermal vertical plate. Haque et al. [79] studied MHD micro-polar fluid flow with constant heat and mass fluxes whereas, Das et al. [80] discussed heat and mass transfer of a Second grade MHD fluid over a convectively heated stretching sheet. Nadeem and Akbar [81] considered MHD on the peristaltic flow in an annulus with heat and mass transfer. Concurrently, unsteady free convective MHD flow with heat and mass transfers from numerous geometries in porous media has several engineering and geophysical applications such as-underground energy transport drying of porous solids, thermal insulation and geothermal reservoirs. Hayat et al. [82] studied MHD Second grade fluid flow in porous channel. Rassoulinejad-Mousavi et al. [83-84] discussed forced convection in a circular tube filled with a Darcy–Brinkman–Forchheimer porous medium. Ali et al. [85-86] studied MHD flow with heat and mass transfer past a vertical plate embedded in porous medium. Hussnan et al. [87] defined the solution of unsteady MHD free convection flow in a porous medium with constant mass diffusion.

Nadeem et al. [88] and Khalid et al. [89] discussed unsteady free convective MHD Casson fluid flow with heat and mass transfer through porous medium. Das et al. [90] studied Magneto-nanofluid flow past an impulsively started porous plate.

The Stefan-Boltzmann rule states that, “the emissive power of a black body is directly proportional to fourth power of its absolute temperature”. If surface experiences different temperature, then the heat flux is equivalent to the net radiation flow. Rosseland [91] point out radiative heat in fluid flow. Hayat et al. [92] considered thermal radiation effects on MHD Second grade fluid flow whereas, Prakash and Muthtamilselvan [93] studied effect of radiation on MHD flow of micro-polar fluid between porous vertical channels. Makanda et al. [94] discussed radiation effects on MHD Casson fluid flow from in non-Darcy porous medium with viscous dissipation. Several researchers like, Rashidi et al. [95], Kataria and Mittal [96-97], Akbar et al. [98] and Sheikholeslami [99-100] considered thermal radiation effects on unsteady free convective MHD nanofluid flow. Rashidi et al. [101] and Narayana [102] studied mixed convective MHD fluid flow with thermal radiation. Rashidi et al. [103] and Ahmed et al. [104] discussed free convective heat and mass transfer MHD fluid flow in the presence of the radiation through porous medium. Ahmed et al. [105] considered MHD double diffusive convection in the stagnation region of an in the presence of thermal radiation effect. Sheikholeslami and Shehzad [106] studied thermal radiation of ferrofluid in existence of Lorentz forces.

The study of MHD flow with heat and mass transfer in presence of chemical reaction has many applications in hydrometallurgical and chemical industries. Chemical reaction can be classified into heterogeneous and homogeneous processes. Animasaun [107] and Makanda et al. [108] studied chemical reaction and magnetic field effects on unsteady free convective flow of Casson fluid. Research work of thermal radiation and chemical reaction effects on MHD flow is important in engineering and technology which is done by, Seth et al. [109], Kataria and patel [110], Reddy [111], Nayak et al. [112], Narayana and Babu [113].

Heat generation/absorption played important role in MHD flow with heat and mass transfer. Recently, Hayat et al. [114] and Shehzad et al. [115-116] studied heat generation effects on unsteady three-dimensional MHD flow with different fluid. Fetecau et al. [117] considered slip effects on the unsteady radiative MHD free convection flow over a moving plate with heat source whereas, Abbasi et al. [118] discussed mixed convection flow of Maxwell nanofluid with heat generation/absorption.

Raju et al. [119] obtained analytical and numerical solution of MHD flow over an exponentially moving vertical plate with Heat Absorption.

The diffusion of material is irregularly heated which is by the temperature gradient in the system. The effect is named after the Swiss Scientist Soret [13], who was the first to study thermos-diffusion. Nadeem et al. [120], Sengupta and Ahmed [121] studied thermo-diffusion effects on MHD oblique stagnation-point flow of a viscoelastic fluid whereas, Khan et al. [122] considered thermo-diffusion effects on MHD stagnation point flow towards a stretching sheet in a nanofluid. Hayat et al. [123-125] and Anantha et al. [126] discussed soret and dufour effects on unsteady free convective MHD flow with heat and mass transfer. Kataria and Patel [127] obtained analytic solution of MHD Casson fluid flow with soret and heat generation. Olanrewaju and Abbas [128] discussed hydromagnetic flow of a Second grade fluid in the presence of thermal radiation and thermal diffusion. Kumaran and Sandeep [129] obtained thermophoresis and Brownian moment effects on parabolic flow of MHD Casson and Williamson fluids with cross diffusion.

Hall effect was discovered by Hall [14]. It has a dynamic role in defining flow structures of fluid flow problems. Hall effects is likely to be significant in MHD power generation, nuclear power reactors, underground energy storage system, Hall current accelerator, magnetometers, Hall effect sensors, spacecraft propulsion etc. and in several areas of astrophysics and geophysics. Narayana et al. [130], Oahimire and Olajuwon [131] and Olajuwon et al. [132] studied effects of Hall current and radiation on MHD micro-polar fluid flow with heat and mass transfer. Seth et al. [133-134] and Hussain et al. [135] discussed Hall current effect on unsteady free convective MHD flow with heat and mass transfer past a moving vertical plate in rotating system. Raptis and Singh [136] studied rotation effects on MHD free-convection flow past an accelerated vertical plate. Rashidi et al. [137] investigated of entropy generation in MHD and slip flow over a rotating porous disk whereas, Rashidi and Erfani [138] defined the analytical solution for steady MHD flow rotating disk with viscous dissipation and ohmic Heating. Nadeem and Saleem [139] obtained exact solution of unsteady mixed convection MHD flow on a rotating cone in a rotating frame. However, in all the investigations carried out by researchers considered ramped temperature profiles, it is to be noted that interval for ramped profile varies from material to material depending upon the specific heat capacity of the material. Variable ramped temperature profiles appear in real world situation in building air-conditioning systems, fabrication of thin-film photovoltaic devices, and phase transition in processing of materials, turbine blade heat transfer, heat exchangers etc. Lee and Yovanovich

[140] discussed laminar natural convection from a vertical plate with a step change in wall temperature. Seth et al. [141-150], Nandkeolyar et al. [151-152] and Khan et al. [153] considered unsteady free convective MHD flow with ramped wall temperature. Khalid et al. [154] obtained exact solutions of Nano fluids with ramped wall temperature, whereas Samiulhaq et al. [155], Kataria and Patel [156] considered unsteady MHD Second grade fluid flow with ramped wall temperature in a Porous Medium. Recently, Kataria and Patel [157] studied effects of thermo-diffusion and parabolic motion on unsteady free convective MHD Second grade fluid flow with ramped wall temperature and ramped surface concentration.

1.21 Dimensionless Parameters

Dimensionless parameters help us to understand the physical importance of a particular phenomenon attendant with the problem. The basic equations are made dimensionless using certain dependent and independent characteristic values. Some of the dimensionless parameters used in thesis are clarified below

(i) Thermal Grashof number (*Gr*)

The ratio of the buoyancy to viscous force acting on a fluid. It often arises in the study of situations involving free convection

$$Gr = \frac{g\beta_T L^3(T_w - T_\infty)}{\nu^2}$$

(ii) Mass Grashof number (*Gm*)

The ratio of the mass buoyancy force to hydrodynamics viscous force acting on a fluid. It often arises in the study of situations involving free convection

$$Gm = \frac{g\beta_C L^3(C_w - C_\infty)}{\nu^2}$$

(iii)Prandtl number (*Pr*)

It is defined as ratio of momentum and thermal diffusivity.

$$Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

(iv)Schmidt number (*SC*)

It is defined as ratio of momentum and mass diffusivity.

$$Sc = \frac{\nu}{D}$$

(v) Hartmann number (M^2)

It is defined as the ratio of electromagnetic force to the viscous force. It is a dimensionless number which gives a measure of the relative importance of drag forces resulting from magnetic induction and viscous forces in flow, and determines the velocity profile for such flow.

$$M^2 = \frac{\sigma {B_0}^2 L^2}{\nu \, \rho}$$

(vi) Soret Number (Sr)

It is noticed that, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. Soret number corresponds to concentration equation. Soret number is represented by

$$Sr = \frac{D_T}{D_m}$$

(vii) Nusselt Number

Nusselt number represents the dimensionless temperature gradient at the surface.

$$Nu = \frac{h \, L}{k}$$

(viii) Sherwood Number

Sherwood number represents dimensionless concentration gradient at the surface.

$$Sh = \frac{h_m L}{D}$$