Chapter 1

Introduction

Many years of experimental discoveries and simultaneous theoretical attempts to understand the particle physics phenomena reflect human's keen interest and curiosity towards the comprehensive knowledge of fundamental constituents of matter and their interactions. Most of the mass of the visible matter around us comes from composite objects called "Hadrons", mainly the nucleons. In the standard model of particle physics [1], hadrons are classified into two families: "mesons- bound states of a quark and an antiquark" and "baryons- bound states of three quarks". Quarks and gluons together called the fundamental constituents of hadrons. Gluon, the spin-1 boson is a fundamental quanta of strong interaction like the photon is of electromagnetic interaction but gluon carries color charge while photon is charge neutral. The area of hadronic physics studies strongly interacting matter in all its manifestations in connection with the underlying gauge theory, Quantum Chromodynamics (QCD). The field owns a long and exciting history starting from the phenomenology of hadron- hadron interaction to the present day ideas on the quark-gluon structure of hadrons [2].

Strong interaction physics poses many basic questions on our present-day understanding of the structure of matter which is of utmost significance. Our knowledge about the structure of the nucleons, the primary building blocks of atomic nuclei and hadronic matter in the universe, is still far from complete. Experimental data from polarised DIS experiments show that the quark spins account for only about 30% of the total nucleon's spin which is in conflict with the constituent quark model [3]. What exactly accounts for remaining 70% of the spin is one of the most important objectives of high energy spin research [4]. Despite the availability of enormous information on the partonic substructure of hadronic matter, understanding of exactly how quarks and gluons evolve into a hadron via dynamics of confinement is also an open key issue. The theory which describes strong interaction is not yet under quantitative control analytically due to associated non-perturbative nature [5]. Precise calculations are essential in understanding a wide range of hadronic phenomena.

In this intricate sphere of hadrons, studies involving pseudoscalar mesons, mainly η and η' mesons grab large attention of the scientific community due to following reasons:

- Understanding of the low-energy dynamics of the pseudoscalar mesons is a persistent problem in the hadronic world.
- Light pseudoscalar mesons, at a fundamental level, are so-called Goldstone bosons of spontaneously broken chiral symmetry. Questions like how the transition from partons to Goldstone mode occurs, what is their partonic substructure are important objectives.
- Despite having similar quark content, the masses of η and η' mesons differ so much.
- $SU(3)_f$ breaking effects induce mixing of $\eta - \eta'$. There is also a small mixing with π^0 and η_c.
- While pions and kaons fit well into their status of Goldstone bosons of broken chiral symmetry, η and mainly η' are too massive to be pure Goldstone bosons.

- η' is associated with $U(1)$ axial anomaly, hence it has a strong affinity towards glue.

- Due to association with non-perturbative gluon dynamics, the interactions of these mesons with other hadrons are characterized by OZI violation. Their coupling constants with nucleons are vulnerable to OZI-violation.

- Although η' is predominantly flavor-singlet and η is largely octet, η bound states in nuclei are sensitive to singlet component of η and hence to non-perturbative glue and axial U(1) dynamics.

- QCD axial anomaly plays a major role in the "proton spin puzzle" by flavor-singlet Goldberger-Treiman relation.

- Rigorous studies on exclusive processes involving η and η' are important tools to test the QCD approaches. They also give important information about the quark-gluon structure of these mesons.

In this chapter, we highlight some important aspects of the physics of η and η' mesons.

1.1 η and η' Mesons: discovery, basic properties

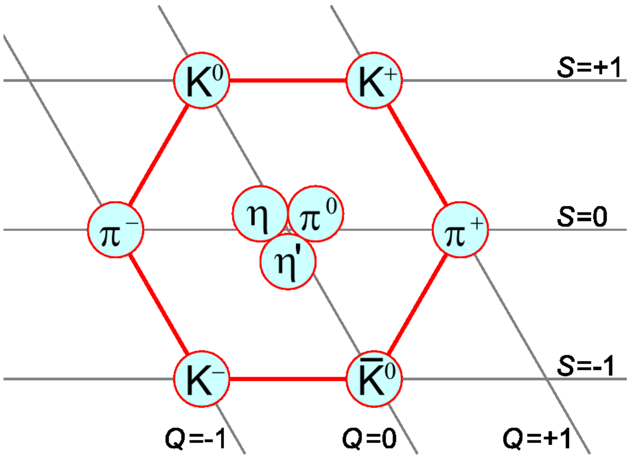
After the discovery of new "resonances" in the late '50s in pion-nucleon scattering, efficient theoretical models were needed in order to organize and understand the plenty of new data. A model proposed by Sakata [6] considered nucleons and Λ as the basic building blocks of matter. In this scheme, mesons with zero isospin were needed to accompany pions and kaons. That was the earliest hint towards the existence of the pseudoscalar η and η' mesons. η meson was predicted in a famous "Eight-fold way" model given by Gell-Mann and Ne'emann independently in 1961-62 [7, 8] which considers $SU(3)_f$ symmetry as a basic underlying symmetry for organizing mesons and baryons. In 1962, η was discovered in pion-nucleon collisions at the Bevatron [9]. A More fundamental theoretical model, "the Quark Model " was given by Gell-Mann and Zweig in 1964 [3]. The fundamental particles were called "Quarks" and up, down and strange quarks together form a three-dimensional representation of the symmetry group SU(3) in the model. Any higher dimensional representation can be built by combining the fundamental representation 3 and its conjugate $\overline{3}$. The direct sum decomposition of the SU(3) symmetry among u, d, s quarks result into, $3 \times \overline{3} = 1 \oplus 8$. The ninth resonance in the pseudoscalar multiplet was predicted and later discovered independently by two groups in 1964 [10, 11] and is known as η'.

Table 1.1: Properties of η and η' mesons

Light unflavored mesons $S = C = B = 0$	J^{PC}	Mass in MeV	Full width Γ	main decay modes
η-meson	0^{-+}	547.862 ± 0.017	1.31 ± 0.05 KeV	$\eta \to 2\gamma, \eta \to 3\pi$
η'-meson	0^{-+}	957.78 ± 0.06	0.198 ± 0.009 MeV	$\eta' \to 2\gamma$,$\eta' \to \pi^+\pi^-\eta$, $\eta' \to \rho^0\gamma$

In the case of light mesons, made entirely out of (u, d, s) quarks, nine pseudoscalar mesons can be obtained by combining quarks and antiquarks in all possible combinations. Fig. (1.1) shows the pseudoscalar meson nonet. Particles in the same horizontal line have the same strangeness S, and on the same diagonal line share the same Charge Q. The pseudoscalar mesons (π's, η, K's, $\overline{K}$) are members of SU(3) octet and η' is mainly SU(3) singlet. η and η' mesons have I=0 (iso-singlet). In the quark model, hadrons are labeled with two kinds of quantum numbers:one set comes from poincarè symmetry-J^{PC} where J,P,C stand for total angular momentum, p-symmetry (parity), c-symmetry (charge conjugation) respectively. Another is flavor quantum numbers. This arrangement of hadrons was one of the major breakthroughs in physics. Basic properties of pseudoscalar mesons η and η' are given in Table. 1.1 [12].

Figure 1.1: The pseudoscalar meson nonet in the quark model



1.2 Symmetries and pseudoscalar η and η'

1.2.1 $SU(3)_f$ symmetry and its breaking

The quark part of the QCD Lagrangian is written as [13],

$$\mathcal{L}_\psi = \overline{\psi}_a(i\gamma_\mu(\partial^\mu + igA^\mu) - m)\psi^a, a = 1.....N_f. \tag{1.1}$$

When all N_f quarks have same mass m, the above Lagrangian is invariant under $SU(N_f)$ transformations. The infinitesimal transformations of fields are:

$$\begin{aligned} \delta\psi^a &= -i\delta\alpha^A(T^A)^a_b\psi^b, \\ \delta\overline{\psi}^a &= i\delta\alpha^A\overline{\psi}_b(T^A)^b_a, \end{aligned}$$

$$\delta A_\mu = 0.$$

The generators T^A satisfy

$$[T^A, T^B] = if_{ABC}T^C, A, B, C = 1......N_f^2 - 1 \tag{1.2}$$

and there are $N_f^2 - 1$ conserved currents.

$$
\begin{aligned}
j_\mu^A(x) &= -i\frac{\partial \mathcal{L}}{\partial(\partial^\mu \psi^a)}(T^A)^a_b \psi^b, \\
j_\mu^A(x) &= \overline{\psi}_a \gamma_\mu (T^A)^a_b \psi^b, \\
\partial^\mu j_\mu^A &= 0.
\end{aligned}
$$

The generators also called the "charges",

$$
Q^A = \int d^3x j_0^A(x) \tag{1.3}
$$

commute with the Hamiltonian,

$$
[Q^A, H] = 0. \tag{1.4}
$$

This implies the existence of multiplet with equal mass. This mode is called the Wigner-Weyl mode of realization of symmetry. In reality, quarks have different masses. In such case, the Lagrangian in Eq. (1.1) cannot remain invariant under $SU(N_f)$ transformations hence the associated Noether current is not conserved. This cause explicit breaking of the flavor symmetry and hadrons with degenerate masses in $SU(N_f)$ multiplets are not observed. In the case of light quarks u,d and s, only approximate symmetry can be expected as masses of u and d quarks are of the order of $5 \sim 10$ MeV and that of s-quark is ~ 100 MeV which are relatively smaller than the typical hadronic scale $\sim 1GeV$. In the case of pseudoscalar mesons, π, K, η, η' mass differences are much larger than that of vector mesons as the dynamics of chiral symmetry breaking plays the role. Using approximate flavor symmetry, Gell-Mann-Okubo mass formula [14] gives a relation for mass differences in a given multiplet.

1.2.2 $SU(3)_{f5}$ symmetry and its breaking

Chiral symmetry is an important symmetry of massless QCD. In the QCD Lagrangian, the handedness of massless quarks is not affected by strong interactions. If $m_u = m_d = m_s = 0$, the Lagrangian given in Eq.(1.1) is invariant under below infinitesimal $SU(3)_{f5}$ transformation of fields:

$$\delta\psi^a = -i\delta\alpha^A (T^A)^a_b \gamma_5 \psi^b. \tag{1.5}$$

and

$$\delta\overline{\psi}_a = -i\delta\alpha^A \overline{\psi}_b \gamma_5 (T^A)^b_a. \tag{1.6}$$

with $\delta A_\mu = 0$. The invariance of Lagrangian under the interchange of left and right-handed quarks leads to chiral symmetry. The associated Noether current is conserved.

$$\partial^\mu j^A_{5\mu} = 0. \tag{1.7}$$

Because quark masses are not zero, in reality, the chiral symmetry breaks under axial flavor transformations and associated Noether current is not conserved. In the case of light quarks (u, d, s), if only approximate $SU(3)_{f5}$ symmetry prevails then as a consequence there must be the presence of the parity doublets of approximately equal mass hadrons (axial transformations change the parity of the field). But, no such parity doublets are observed. This indicates that the Wigner-Weyl mode of realization of chiral symmetry in which ground state of the theory respects the symmetry is not valid [15]. The ground state is not invariant under the symmetry group transformation and the symmetry is said to be spontaneously broken. The generators of the group do not annihilate the vacuum:

$$Q^A_5 \mid 0\rangle \neq 0. \tag{1.8}$$

This is the Nambu-Goldstone mode of realization of chiral symmetry [15]. The immediate consequence is the existence of eight massless pseudoscalar particles which are Nambu-Goldstone bosons. However, the octet of pseudoscalar mesons is not massless. This is because quarks are not massless in the real world and the symmetry which is spontaneously broken is not exact, but only approximate. These Nambu-Goldstone bosons acquire small masses compared to massive hadrons and in the chiral limit, they are massless. In the pseudoscalar nonet, πs, Ks fit well in this picture but the case of η and η' is much different as they suffer from an axial $U_{(1)}$ anomaly.

1.3 $\eta - \eta'$ mixing

The basic SU(3) symmetry theory of quarks which considers only strong force, predicts following particles

$$\eta_8 = \frac{1}{\sqrt{6}}(u\overline{u} + d\overline{d} - 2s\overline{s}) \tag{1.9}$$

$$\eta_1 = \frac{1}{\sqrt{3}}(u\overline{u} + d\overline{d} + s\overline{s}). \tag{1.10}$$

η_1 belongs to singlet and η_8 is purely octet. However, significant mixing of these eigenstates occur and the actual quark composition of these mesons are linear combination of η_8 and η_1.

$$U(\theta_p)\begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} = \begin{pmatrix} \eta \\ \eta' \end{pmatrix}, \tag{1.11}$$

where θ_p is a mixing angle and

$$U(\theta_p) = \begin{pmatrix} \cos\theta_p & -\sin\theta_p \\ \sin\theta_p & \cos\theta_p \end{pmatrix} \tag{1.12}$$

The η is considered to be close to η_8 and η' to η_1. Eq.(1.11) describes the mixing in the singlet-octet basis [16–18]. The mixing of $\eta - \eta'$ in the quark flavor basis can be written as,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = U(\phi)\begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \tag{1.13}$$

where $U(\phi)$ is the unitary matrix given as

$$U(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}, \tag{1.14}$$

and $\mid \eta_q\rangle = \frac{1}{\sqrt{2}} \mid u\overline{u} + d\overline{d}\rangle$, $\mid \eta_s\rangle = \mid s\overline{s}\rangle$. One can transform one orthogonal basis into another basis. $\eta - \eta'$ mixing is a subject of huge interest and it has been studied in phenomenology widely. Definition of meson decay constants and consistent extraction of mixing parameters from experimental data involves the aspect of $\eta - \eta'$

mixing. The decay constants of mesons η and η' are defined as [18, 19],

$$\langle 0 \mid J^i_{\mu 5} \mid P(p)\rangle = i f^i_P p_\mu (i = 1, 8, q, s, P = \eta, \eta'), \tag{1.15}$$

where

$$\frac{1}{\sqrt{2}}(\overline{u}(0)\gamma_\mu\gamma_5 u(0) + \overline{d}(0)\gamma_\mu\gamma_5 d(0)) = j^q_{\mu 5}, \tag{1.16}$$

$$\overline{s}(0)\gamma_\mu\gamma_5 s(0) = j^s_{\mu 5}. \tag{1.17}$$

In Eq. (1.16), the current is an isoscalar combination of u-and d-quark currents while in Eq.(1.17), it is the s-quark current. In the singlet-octet basis, the axial quark-currents are written as,

$$J^8_{\mu 5} = \frac{1}{\sqrt{6}}(\overline{u}\gamma_\mu\gamma_5 u + \overline{d}\gamma_\mu\gamma_5 d - 2\overline{s}\gamma_\mu\gamma_5 s), \tag{1.18}$$

$$J^1_{\mu 5} = \frac{1}{\sqrt{3}}(\overline{u}\gamma_\mu\gamma_5 u + \overline{d}\gamma_\mu\gamma_5 d + \overline{s}\gamma_\mu\gamma_5 s). \tag{1.19}$$

The general parametrization of mixing of octet and singlet decay constants gives [18, 19],

$$\begin{pmatrix} f^8_\eta & f^1_\eta \\ f^8_{\eta'} & f^1_{\eta'} \end{pmatrix} = \begin{pmatrix} \cos\theta_8 & -\sin\theta_1 \\ \sin\theta_8 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} f_8 \\ f_1 \end{pmatrix}. \tag{1.20}$$

The angles θ_8 and θ_1 are significantly different due to $SU(3)_f$ breaking effects. In the quark-flavor basis [17],

$$\begin{pmatrix} f^q_\eta & f^s_\eta \\ f^q_{\eta'} & f^s_{\eta'} \end{pmatrix} = \begin{pmatrix} \cos\phi_q & -\sin\phi_s \\ \sin\phi_q & \cos\phi_s \end{pmatrix} \begin{pmatrix} f_q \\ f_s \end{pmatrix}. \tag{1.21}$$

Although the relations between decay constants appear analogous in the both basis, it is found that in the quark-flavor basis [18],

$$\langle 0 \mid J^s_{\mu 5} \mid \eta_q\rangle \simeq 0, \langle 0 \mid J^q_{\mu 5} \mid \eta_s\rangle \simeq 0. \tag{1.22}$$

Above relation approximately holds, but similar relation for octet-singlet basis does not hold. The phenomenological analysis [20] gives $\phi_q \simeq 39.4$ and $\phi_s \simeq 38.5$. The negligible difference between ϕ_q and ϕ_s leads to the relation $\phi_q = \phi_s = \phi$. The well-known $U_A(1)$ anomaly (discussed later in this chapter) plays a crucial role in understanding the $\eta - \eta'$ mixing. The anomaly mediates $\eta_q \leftrightarrow \eta_s$ transitions and leads to the mixing. In the quark model, $\eta - \eta'$ mixing is viewed as a mixture of $SU(3)_f$ singlet and octet components in their wavefunctions. However, QCD introduces one more dynamical degree of freedom, the gluon and $SU(3)_f$ singlet can be made up of pure glue configurations like gg. The wavefunctions of η and η' contain the possibility of gluonic admixture in it. η, being mainly octet have lower possibility of glue configurations than η' which is mainly singlet. Understanding of $\eta - \eta'$ mixing phenomena is an essential element in understanding non-perturbative features of QCD.

1.4 Nature of Quantum Chromodynamics

The results of deep inelastic scattering experiments revealed many aspects of quarks. Numerous information on their spin, charge, interactions and on the structure of the proton inspired to build a foundation of a theory which can describe the behavior of quarks inside a hadron. QCD was proposed in 1973 by David Gross, Frank Wilczek, and David Politzer as a gauge theory of strong interactions. It forms the SU(3) part of $SU(3) \times SU(2) \times U(1)$ standard model of particle physics. The Lagrangian of QCD is written as [13],

$$\mathcal{L}_{QCD} = \sum_{f=1,2,3}^{n} \overline{\psi}_f^{\,i} (i\not{D} - m_f)_{ij} \psi_f^j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} - \frac{1}{2\lambda} (\partial^\mu A^a_\mu)^2 + \mathcal{L}_{ghost}. \tag{1.23}$$

First term in the above equation represents non-interacting quarks with mass m_f.

$$D_{\mu,ij} = \partial_\mu \mathbb{I}_{ij} + i g_s (t.A_\mu)_{ij} \tag{1.24}$$

is the covariant derivative which makes Lagrangian invariant under local gauge transformations. A^a_μ are coloured vector fields. The second term in the Lagrangian is a kinetic term for A^a_μ field. $G^a_{\mu\nu}$ is non-abelian field strength tensor and written as

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\nu \tag{1.25}$$

In Quantum Electrodynamics (QED), the field strength tensor is given by,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1.26}$$

There lies an important difference between these two field-strength tensors. $G^a_{\mu\nu}$ contains one extra term which is needed in order to make $\mathcal{L}_{kinetic}$ invariant under local gauge transformation. This extra term signifies interesting physics of strong interactions. it tells that gluons not only mediate the strong interaction but also are self-interacting particles. This leads to "asymptotic freedom" at high energies or short-distances and "confinement" at low energies or large-distances. The non-linear interactions between gluons in QCD has "anti-screening" effect which leads to the weaker coupling constant at shorter distances. In other words, in the limit of very high energy, quarks are quasi-free and this "asymptotic freedom" makes perturbative calculations possible in QCD in that region. "Confinement" makes inter-quark coupling larger at low energies and it becomes impossible to detach individual quarks from hadrons. In QCD, the vacuum polarization effects are extremely strong, unlike QED. This makes QCD vacuum extremely complex, full of spontaneously appearing, disappearing and interacting virtual gluons and virtual quark-antiquark pairs. Due to these peculiarities, the main challenge in QCD thus becomes to establish a connection between hadrons (which are observed) and perturbatively calculable quarks and gluons degrees of freedom that appear in the Lagrangian (which are never observed in isolation).

1.5 Axial U(1) anomaly and η' meson

As a result of spontaneously broken axial SU(2) symmetry, pions become the corresponding Goldstone bosons. If the third flavor is taken into account, all the mesons in pseudoscalar octet become Goldstone bosons of spontaneously broken $SU(3)_{f5}$ symmetry. Under the axial U(1) group, quark fields transform as [15],

$$\psi(x) \rightarrow e^{i\beta\gamma_5}\psi(x). \tag{1.27}$$

In the case of massless quarks, a Goldstone boson is expected due to spontaneously broken axial U(1) symmetry. Quark mass term in the Lagrangian breaks the symmetry explicitly and corresponding Goldstone boson will not be massless. η', which is singlet under $SU(3)_f$, have similar quark content as η and have right quantum numbers to be a Goldstone boson of spontaneously broken axial U(1) symmetry but is heavy, having mass ~ 958 MeV, almost twice the mass of the η. Hence, it cannot be associated with this symmetry. Axial U(1) symmetry is not realized in the real world. Either it does not exist or it is more badly broken than $SU(3)_{f5}$. This is axial U(1) problem [21]. It is an established fact that this symmetry suffers an anomaly which is called Adler-Bell-Jackiw anomaly [22, 23]. An anomaly in the theory arises when certain conservation law, which is valid in classical theory, gets violated upon quantization. Noether current associated with the symmetry of Lagrangian in classical field theory is not conserved when quantum corrections are taken into account. In QCD, the anomalous divergence of singlet axial current is represented as [18, 24],

$$\partial^\mu(\overline{\psi}\gamma_\mu\gamma_5\psi) = 2im_\psi\overline{\psi}\gamma_5\psi - \frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}, (\psi = u, d, s), \tag{1.28}$$

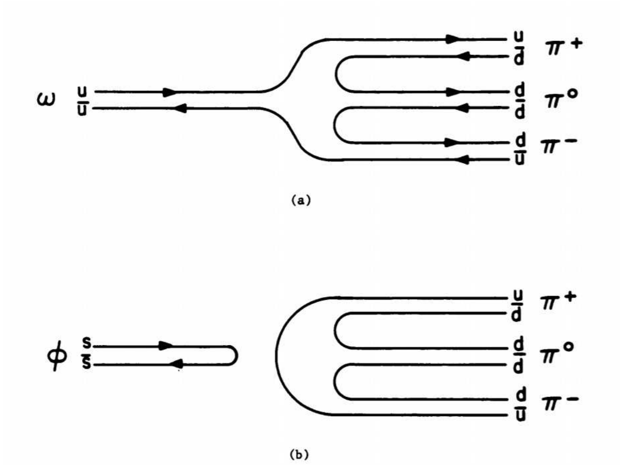
$$\tilde{G}^{a\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}, \epsilon^{0123} = +1. \tag{1.29}$$

Here $G^a_{\mu\nu}$ is the non-abelian field strength tensor and $\tilde{G}^{a\mu\nu}$ is its dual. Thus, gluons enter in the η' physics by an anomaly equation. In the limit $m_\psi \to 0$, η' remains massive due to the occurrence of this anomaly. In the above equation, R.H.S term shows total divergence and integration over Euclidean space must vanish as gauge fields vanish at infinity. Then, how does this divergence have an effect? This dilemma was solved by t'Hooft by instanton configurations [25, 26]. Thus, a large mass of η' is associated with axial U(1) anomaly and non-perturbative gluon dynamics.

1.6 OZI rule and η,η' mesons

OZI rule was independently proposed by Susumu Okubo, George Zweig, and Jugoro Iizuka [27–29]. It explains why certain decay modes are suppressed though it is kinematically favored. For example, the decay $\phi \to \pi^+ + \pi^- + \pi^0$ is suppressed over $\phi \rightarrow K^+ + K^-$ which has much lower Q values. Decay of $\phi \rightarrow \pi^+ + \pi^- + \pi^0$ is suppressed but the decay of $\omega \rightarrow \pi^+ + \pi^- + \pi^0$ is allowed. The occurrence of this phenomena is related to the nature of QCD. Due to asymptotic freedom, the coupling constant decreases with increasing energy and for suppressed channels, gluons must have high energy thus coupling becomes weak. In the case of η and η' mesons, the non-perturbative glue through $U(1)_A$ dynamics plays an important role resulting in OZI-violating interactions of these mesons.

Figure 1.2: Quark line diagrams for (a) OZI allowed decay $\omega \rightarrow 3\pi$ and (b) OZI forbidden decay $\phi \rightarrow 3\pi$.



1.7 QCD Sum Rules

Due to confinement phenomena, quarks and gluons are bound inside hadrons and this hinders direct experimental measurements of the fundamental QCD parameters. Lagrangian of QCD as shown in Eq. (1.23), can be used analytically only in the region where perturbation theory works. Understanding of QCD dynamics at distances $\sim \frac{1}{\Lambda_{QCD}}$ is essential in order to extract all the properties of hadrons in terms of their fundamental constituents. QCD sum rule formalism, invented by Shiftman, Vainshtein and Zakharov [30] is an analytical approach which depends upon the firm relation between QCD Green function, specifically their Operator Product Expansion (OPE) beyond perturbation theory and their hadronic correlators. The underlying idea of this technique is to approach non-perturbative regime of QCD from the perturbatively calculable side i.e. to begin with short distances and move to large distances where confinement effects are significant. OPE beyond perturbation theory [31] and quark-hadron duality [32] are two main pillars of QCD sum rule formalism. Operator Product Expansion was proposed by K. Wilson. It introduces the dynamics of quark-gluon confinement. The two-point current correlator in QCD can be written as [33],

$$\prod(q^2) = i\int d^4x \exp^{iqx}\langle 0 \mid T\{J(x)J(0)\} \mid 0\rangle, \tag{1.30}$$

where $J(x)$ is the local current constructed from the quark and gluon fields accommodated in QCD Lagrangian. Analogously, this current can also be written in the form of the hadronic fields with identical quantum numbers. A relation between these two descriptions follows from the Cauchy's theorem in complex energy plane (quark-hadron duality). The above correlator contains perturbative as well as non-perturbative information manifesting quark-gluon confinement. In the OPE [33],

$$\prod(q^2) \mid_{QCD} = C_0\hat{I} + \sum_{N=0} C_{2N+2}(q^2,\mu^2)\langle 0 \mid \hat{O}_{2N+2}(\mu^2) \mid 0\rangle, \tag{1.31}$$

here, μ^2 is the renormalization scale. The Wilson coefficients $C_{2N+2}(q^2,\mu^2)$ carry the information on short-distance effects [34], calculable in pQCD and vacuum condensates encapsulate non-perturbative long-distance effects. Thus, factorization of long- and short-distance physics is achieved in the OPE. Non-vanishing vacuum expectation values of quark and gluon fields, the condensates $\langle 0 \mid \overline{q}q \mid 0\rangle, \langle 0 \mid G^a_{\mu\nu}G^a_{\mu\nu} \mid 0\rangle$, etc. cannot be obtained from first principles analytical calculations. Values of some condensates are estimated from the current algebra studies such as Partially Conserved Axial-vector Current (PCAC). The first term in the Eq. (1.31) is the lowest dimension unit operator and it is purely perturbative. The operator of the next lowest dimension (three) is $\overline{q}q$. The value of $\overline{q}q$ is determined using PCAC relation,

$$(m_u + m_d)\langle \overline{u}u + \overline{d}d\rangle = -m_\pi^2 f_\pi, \tag{1.32}$$

where m_π is the pion mass and f_π is the pion decay constant. The value of gluon condensate is closely related to the semiclassical QCD vacuum structure instanton effects [25]. Apart from quark and gluon condensates there prevail a plethora of higher order condensates. Dispersion relation connects the two-point function with the physical states [24].

$$\prod^{phys}(q^2) = \frac{1}{\pi}\int_0^\infty \frac{Im\prod(s)}{s-q^2}ds. \tag{1.33}$$

An enormous wealth of data coming from the world of low-energy hadronic physics are understood in QCD via dispersion relations. Borel transform in QCD sum rules were also suggested by the inventors of QCDSRs [30]. The operator $\hat{B}$ is written as,

$$\hat{B} = \lim_{n\to\infty Q^2\to\infty} \frac{(Q^2)^{n+1}}{n!}(-\frac{\partial}{\partial Q^2})^n, \tag{1.34}$$

with $M^2 = \frac{Q^2}{n}$ fixed and $q^2 = -Q^2$. Upon Borel transform,

$$\prod(M^2) \equiv \hat{B}\prod(Q^2) = \frac{1}{\pi M^2}\int_0^\infty dsIm\prod(s)e^{-s/M^2}. \tag{1.35}$$

with

$$\hat{B}(\frac{1}{Q^2})^n = \frac{1}{(n-1)!}(\frac{1}{M^2})^{n-1}, \tag{1.36}$$

$$\hat{B}(\frac{1}{s+Q^2}) = e^{-s/M^2}, \tag{1.37}$$

. Borel transformation technique gives several advantages.

- It improves factorially the convergence of the power series. Due to this, prediction of lowest-lying states' properties is more reliable.

- The exponential weight function in the Eq.(1.35) makes the integral over imaginary part well-convergent and enhances the lowest-lying state's contribution while suppresses the contributions coming from higher order resonances exponentially.

Thus the procedure of QCD sum rule formalism has following important steps.

- Hadrons are represented in terms of the interpolating quark currents.
- Correlation function is calculated by an Operator Product Expansion (OPE), an approach which separates short-and long-distance parts.
- Short-distance part is calculated by using perturbative technique.
- Long-distance part is parameterized in terms of universal vacuum condensates or light-cone distribution amplitudes depending upon the problem.
- Result of QCD calculation is matched with the result obtained by summing over hadronic states by the Borel transformed dispersion relation.

QCD sum rules come in wide variety [35]. There are Laplace sum rules, finite energy sum rules, the Gaussian transform sum rules, etc. Eq.(1.35) represents a Laplace sum rule which is suitable for the prediction of the properties of lowest-lying mesons or baryons. Thus, the method of QCDSR is very useful in hadronic physics as it can determine properties of lowest hadronic states from close to the first principles [36].

1.8 Exclusive processes in QCD

Exclusive processes are a type of scattering processes in which kinematics of all initial and final state particles are specified [37]. These processes allow one to decide one well-defined physical process. Exclusive processes in QCD are of utmost significance as they carry all the complexity of perturbative as well as non-perturbative QCD and give a deeper insight into the structure of hadrons. In the hard processes with large momentum transfers, the properties of the hadronic amplitudes are controlled by both the short- and the long-distance physics. Due to non-perturbative interactions, hadrons are made up of quarks and gluons. The properties of exclusive processes are closely related with the features of hadronic wave functions. Exclusive reactions cover a wide range of processes from space-like and time-like form factors measured in electron-hadron scattering and electron-positron annihilation to hadron scattering reactions. The meson-photon Transition Form Factor (TFF) is a simplest type of exclusive reaction in which only one hadron is involved. The process is, $\gamma^*(q_1) + \gamma(q_2) \to P(p)$. The TFF can be defined by an invariant amplitude [38](see Eq.(1.39)),

$$\Gamma^\mu = ie^2 F_{P\gamma}(Q^2)\epsilon^{\mu\nu\alpha\beta}\epsilon_\nu(q_2)q_{1\alpha}q_{2\beta}. \tag{1.38}$$

Here $\epsilon_\nu(q_2)$ is the polarization vector of the real photon and $Q^2 = -q_1^2$. At large momentum transfer, collinear factorization is a well-known approach [39] by which the transition form factor is calculated. Form factor can be written as a convolution of "Hard-Scattering Amplitude" (HSA) and the "Distribution Amplitude" (DA). HSA is calculable perturbatively and the soft part "the DA" requires non-perturbative input.

Among the meson-photon transition reactions, the process $\gamma^*\gamma \to \pi^0$ is the simplest hard exclusive process in which the behaviour of the TFF at large Q^2 is determined by the product of two electromagnetic currents near the light cone [40]. The form factor $F_{\gamma^*\gamma\to\pi^0}(q_1^2, q_2^2)$ can be defined by the product of two electromagnetic currents as,

$$\int d^4y \exp^{iq_1y}\langle\pi^0(p) \mid T\{j_\mu^{em}(y)j_\nu^{em}(0)\} \mid 0\rangle = ie^2\epsilon^{\mu\nu\alpha\beta}q_{1\alpha}q_{2\beta}F_{\gamma^*\gamma\to\pi^0}(q_1^2, q_2^2) \tag{1.39}$$

Above equation involves a time-ordered product of two electromagnetic currents at small light cone separations. Hence, it can be studied using Wilson's operator product expansion [34]. With this expansion, the transition form factor can be written as [41],

$$F_{\gamma^*\gamma\to\pi^0}(Q) = \sum_n C_n(Q)M_n. \tag{1.40}$$

The coefficients C_n are universal and M_n's are matrix elements of local operators between π^0 and the vacuum. The leading contribution to the form factor corresponds to the contribution of the leading twist-two operators and can be written in the factorized form,

$$F_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{\sqrt{2}f_\pi}{3}\int_0^1 dx T_H(x, Q^2, \mu, \alpha_s(\mu))\phi_\pi(x,\mu), \tag{1.41}$$

where $\phi_\pi(x,\mu)$ is the pion distribution amplitude at the scale μ. The twist of an operator appearing in the matrix element is defined as $\tau = d - j$ [13], where d and j are the dimension and spin of the operators. The operators with lowest twist dominate in the light cone expansion.

1.9 Objectives of the present study and organisation of thesis

This thesis focuses on the studies of coupling constants of η and η' mesons with nucleons and electromagnetic transition form factors of η, η'. From the previous sections, it is clear that the phenomenology of η and η' mesons carry interesting physics. The studies on aspects involving production and interactions of these mesons have applications both in nuclear and particle physics.

In chapter 2, the coupling constants of η and η' mesons with nucleon are calculated. These calculations are done by using a well-known QCD sum rules approach. Using quark-flavor basis, coupling constants are calculated at physical points by linear extrapolation of results calculated at non-physical points. Light-cone expansion of a quark propagator contains anomalous gluons which couple to η, η' mesons. By explicitly including this contribution, the effects are studied and analyzed. The reliable determination of $g_{\eta NN}$ and $g_{\eta' NN}$ have implications in understanding $U(1)_A$ dynamics of QCD, in the "proton spin problem", in construction of realistic NN potential and in estimates of electric dipole moment of neutron.

In chapter 3, The Transition Form Factors of η and η' mesons are studied and the sub-leading power corrections to these TFFs from twist-six contributions are calculated. These corrections are calculated in the standard collinear factorization approach. Meson mass and quark mass corrections which give rise to $SU(3)_f$ breaking effects are also taken into account. Obtained results are superimposed on the results of these TFFs up to twist-four available in the literature. TFFs act as an important tool to test some approaches of QCD. In the case of η and η', these TFFs have a special role in determining their quark-gluon structure and fixing their distribution amplitude and these are essential inputs for the study of various exclusive processes.

In chapter 4, the summary of the present thesis is given. It highlights the main findings of the research and also inspires further study by stating hits and misses of the present study.