Appendix

A Density of State

A central property of electronic materials is the Density of States (DOS) $N(E)$, which informs us of the density of mobile electrons or holes present in the solid at a given temperature. Formally, in two dimensions, the total number of states available between energy E and an interval $\mathrm{d}E$ is given by the differential area in k-space $\mathrm{d}A$ divided by the area of one k-state. Mathematically, this is equivalent to

$$N(E) = \mathrm{g}_S\, \mathrm{g}_V\, \frac{\mathrm{d}A}{(2\pi)^2/\Omega}, \tag{A.1}$$

where $\mathrm{g}_S = 2$ is the spin degeneracy, g_V is the zone degeneracy, and Ω is the area of the lattice. There are six equivalent K-points, and each K-point is shared by three hexagons; therefore, $\mathrm{g}_V = 2$ for graphene. To determine $\mathrm{d}A$, let us consider a circle of constant energy in k-space. The perimeter of the circle is $2\pi k$ and the differential area obtained by an incremental increase of the radius by $\mathrm{d}k$ is $2\pi k\mathrm{d}k$. Therefore, the DOS, which is always a positive value or zero, is

$$N(E_k) = \frac{2}{\pi}\left|k\left(\frac{\mathrm{d}E}{\mathrm{d}k}\right)^{-1}\right|, \tag{A.2}$$

where $N(E)$ has been normalized to the Ω. To obtain the DOS for Graphene Systems, we first use the energy dispersion relations calculated within the Dirac approximation in Eq. (A.2);

$$E_k^{\mathrm{MLG}} = \gamma k \quad \Rightarrow \frac{\mathrm{d}E}{\mathrm{d}k} = \gamma \quad \& \quad N^{\mathrm{MLG}}(E_k) = \frac{2k}{\pi\gamma} \tag{A.3}$$

$$E_k^{\mathrm{BLG}} = \frac{\hbar^2k^2}{2m} \quad \Rightarrow \frac{\mathrm{d}E}{\mathrm{d}k} = \frac{\hbar^2k}{m} \quad \& \quad N^{\mathrm{BLG}}(E_k) = \frac{2m}{\pi\hbar^2} \tag{A.4}$$

$$E_k^{\mathrm{MLGG}} = \sqrt{(\gamma k)^2 + \Delta^2} \quad \Rightarrow \frac{\mathrm{d}E}{\mathrm{d}k} = \frac{\gamma^2 k}{\sqrt{(\gamma k)^2+\Delta^2}} \quad \& \quad N^{\mathrm{MLGG}}(E_k) = \frac{\sqrt{(\gamma k)^2+\Delta^2}}{\pi\gamma^2} \tag{A.5}$$

B Carrier Density

The electron carrier density is simply the number of states that are occupied per unit area at a given temperature. The occupation probability for electrons at finite temperatures is given by the Fermi–Dirac distribution, $f(E) = \left(1 + e^{(E-\mu_c)/k_BT}\right)^{-1}$. The net carrier density $|n_e - n_p| = n_c$, where n_e stands for the filled states for $E > 0$, and n_p for the empty states with $E < 0$, is calculated as the convolution of the density of states with the Fermi Dirac distribution in the energy space:

$$|n_e - n_p| = n_c = \int_0^\infty dE\, N(E) f(E) - \int_{-\infty}^0 dE\, N(E)(1 - f(E)), \tag{B.1}$$

where $N(E)$ is the density of state for Graphene Systems.

[I] Monolayer Graphene (MLG)

The net Carrier density for MLG can be evaluated from Eqs. (A.3) & (B.1);

$$n_c^{\mathrm{MLG}} = \frac{2}{\pi\gamma^2}\left(\int_0^\infty dE\, \frac{E^{\mathrm{MLG}}}{1+e^{(E-\mu_c)/k_BT}} - \int_{-\infty}^0 dE\, \frac{E^{\mathrm{MLG}}\, e^{\left(E^{\mathrm{MLG}}-\mu_c\right)/k_BT}}{1+e^{(E-\mu_c)/k_BT}}\right) \tag{B.2}$$

$$n_c^{\mathrm{MLG}} = \frac{2}{\pi\gamma^2}\left(\int_0^\infty dE\, \frac{E^{\mathrm{MLG}}}{1+e^{(E-\mu_c)/k_BT}} - \int_0^\infty dE\, \frac{E^{\mathrm{MLG}}}{1+e^{(E+\mu_c)/k_BT}}\right) \tag{B.3}$$

At zero temperature, chemical potential set to Fermi energy, i.e. $\mu_c = E_f^{\mathrm{MLG}} = \gamma k_f$. Now we can easy to extract from relation (B.3) by making the following substitution,

$$\left.\frac{1}{1+e^{\left(E^{\mathrm{MLG}}-E_f^{\mathrm{MLG}}\right)/k_BT}}\right|_{T=0} = 1 - step\left(E^{\mathrm{MLG}} - E_f^{\mathrm{MLG}}\right), \tag{B.4}$$

which will lead to the following

$$n_c^{\mathrm{MLG}} = \frac{2}{\pi\gamma^2}\int_0^\infty dE^{\mathrm{MLG}}\, E^{\mathrm{MLG}}\left[\left(1 - step\left(E^{\mathrm{MLG}} - E_f^{\mathrm{MLG}}\right)\right) - \left(1 - step\left(E^{\mathrm{MLG}} + E_f^{\mathrm{MLG}}\right)\right)\right] \tag{B.5}$$

Finally, the net carrier density in MLG is define as

$$n_c^{\mathrm{MLG}} = \frac{\left(E_f^{\mathrm{MLG}}\right)^2}{\pi\gamma^2}. \tag{B.6}$$

[II] Bilayer Graphene (BLG)

The net carrier density for BLG can be evaluated from Eqs. (A.4) & (B.1);

$$n_c^{\mathrm{BLG}} = \int_0^{\infty} dE^{\mathrm{BLG}}\, N^{\mathrm{BLG}}(E) f(E) - \int_{-\infty}^{0} dE^{\mathrm{BLG}}\, N^{\mathrm{BLG}}(E)(1 - f(E)) \tag{B.7}$$

$$n_c^{\mathrm{BLG}} = \frac{2m}{\pi\hbar^2}\left(\int_0^{\infty} dE^{\mathrm{BLG}}\, \frac{1}{1+e^{(E^{\mathrm{BLG}}-\mu_c)/k_BT}} - \int_{-\infty}^{0} dE^{\mathrm{BLG}}\, \frac{e^{(E^{\mathrm{BLG}}-\mu_c)/k_BT}}{1+e^{(E^{\mathrm{BLG}}-\mu_c)/k_BT}}\right) \tag{B.8}$$

$$n_c^{\mathrm{BLG}} = \frac{2m}{\pi\hbar^2}\left(\int_0^{\infty} dE^{\mathrm{BLG}}\, \frac{1}{1+e^{(E^{\mathrm{BLG}}-\mu_c)/k_BT}} - \int_0^{\infty} dE^{\mathrm{BLG}}\, \frac{1}{1+e^{(E^{\mathrm{BLG}}+\mu_c)/k_BT}}\right) \tag{B.9}$$

At zero temperature, chemical potential set to Fermi energy, i.e. $\mu_c = E_f^{\mathrm{BLG}} = \hbar^2 k_f^2/2m$. Now we can easy to extract from Eqs. (B.9) and (B.3) which will lead to the following

$$n_c^{\mathrm{BLG}} = \frac{2m}{\pi\hbar^2}\int_0^{\infty} dE^{\mathrm{BLG}}\, \left[\left(1 - step\left(E^{\mathrm{BLG}} - E_f^{\mathrm{BLG}}\right)\right) - \left(1 - step\left(E^{\mathrm{BLG}} + E_f^{\mathrm{BLG}}\right)\right)\right]. \tag{B.10}$$

Finally, the net carrier density in BLG is define as

$$n_c^{\mathrm{BLG}} = \frac{2mE_f^{\mathrm{BLG}}}{\pi\hbar^2}. \tag{B.11}$$

[III] Monolayer Gapped Graphene (MLGG)

The net carrier density for MLLG can be calculated similarly to that in MLG except the lower integration limit in Eq. (B.1) should be depend on Δ since the density of states is proportional to energy at $|E| > |\Delta|$ and is zero at $|E| < |\Delta|$. Now rewrite Eq. (B.1) with modification as

$$n_c^{\mathrm{MLGG}} = \int_{\Delta}^{\infty} dE\, N^{\mathrm{MLGG}}(E) f(E) - \int_{-\infty}^{-\Delta} dE\, N^{\mathrm{MLGG}}(E)(1 - f(E)) \tag{B.12}$$

$$n_c^{\mathrm{MLGG}} = \frac{2}{\pi\gamma^2}\left(\int_{\Delta}^{\infty} dE\, \frac{\sqrt{E^2+\Delta^2}}{1+e^{(E-\mu_c)/k_BT}} - \int_{-\infty}^{-\Delta} dE\, \frac{\sqrt{E^2+\Delta^2}\, e^{(E-\mu_c)/k_BT}}{1+e^{(E-\mu_c)/k_BT}}\right) \tag{B.13}$$

At zero temperature, chemical potential set to Fermi energy, i.e. $\mu_c = \mu_f = \sqrt{\left(\gamma k_f\right)^2 + \Delta^2}$.

The net carrier density in MLGG can be calculated similarly to the above and it is define as

$$n_c^{\mathrm{MLGG}} = \frac{{E_f}^2}{\pi\gamma^2}. \tag{B.14}$$

C Chemical Potential

The finite-temperature chemical potential μ_c is determined by the conservation of the total electron charge density as

$$n_c = \int_0^\infty dE\ \frac{N(E)}{1+e^{(E-\mu_c)/k_BT}} - \int_0^\infty dE\ \frac{N(E)}{1+e^{(E+\mu_c)/k_BT}}. \tag{B.15}$$

[I] Monolayer Graphene (MLG)

Finite temperature chemical potential $\left(\mu_c^{\mathrm{MLG}}(T)\right)$ can be evaluated from Eqs. (B.6) & (B.15);

$$\frac{\left(E_f^{\mathrm{MLG}}\right)^2}{\pi\gamma^2} = \frac{2}{\pi\gamma^2}\left(\int_0^\infty dE^{\mathrm{MLG}}\ \frac{E^{\mathrm{MLG}}}{1+e^{\left(E^{\mathrm{MLG}}-\mu_c^{\mathrm{MLG}}\right)/k_BT}} - \int_0^\infty dE^{\mathrm{MLG}}\ \frac{E^{\mathrm{MLG}}}{1+e^{\left(E^{\mathrm{MLG}}+\mu_c^{\mathrm{MLG}}\right)/k_BT}}\right). \tag{B.16}$$

Using the scaled parameter; $\frac{E^{\mathrm{MLG}}}{E_f^{\mathrm{MLG}}} = x$, $\frac{\mu_c}{E_f^{\mathrm{MLG}}} = s$ and $\frac{T}{T_f} = y$, Eq.(B.16) can be written as

$$\frac{1}{2}\left(\frac{1}{y}\right)^2 = \int_0^\infty dx\ \frac{x}{1+e^{(x-s)/y}} - \int_0^\infty dE\ \frac{x}{1+e^{(x+s)/y}}\,. \tag{B.17}$$

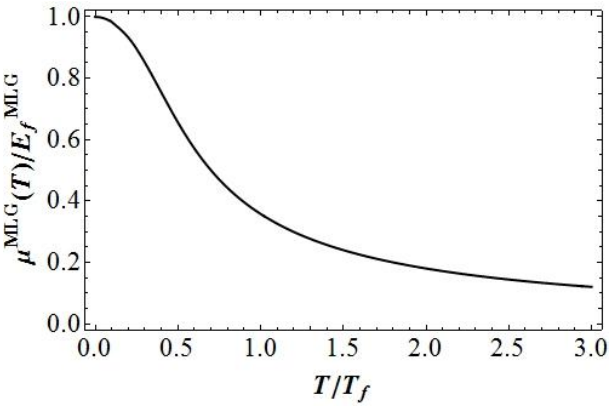

Figure A.1: The chemical potential of MLG as a function of temperature.

Equation (B.17) is plotted in Figure A.1 which shows that variation of $\mu_c^{\mathrm{MLG}}(T)$ as a function of temperature.

[II] Bilayer Graphene (BLG)

Finite temperature chemical potential $\left(\mu_c^{\mathrm{BLG}}(T)\right)$ can be evaluated from Eqs. (B.11) & (B.15);

$$\frac{2mE_f^{\mathrm{BLG}}}{\pi\hbar^2} = \frac{2m}{\pi\hbar^2}\left(\int_0^\infty dE\ \frac{1}{1+e^{\left(E^{\mathrm{BLG}}-\mu_c^{\mathrm{BLG}}\right)/k_BT}} - \int_0^\infty dE\ \frac{1}{1+e^{\left(E^{\mathrm{BLG}}+\mu_c^{\mathrm{BLG}}\right)/k_BT}}\right). \tag{B.18}$$

Using the scaled parameter; $\frac{E^{\mathrm{BLG}}}{E_f^{\mathrm{BLG}}} = x$, $\frac{\mu_c^{\mathrm{BLG}}}{E_f^{\mathrm{BLG}}} = s$ and $\frac{T}{T_f} = y$, Eq.(B.18) can be written as

$$1 = \int_0^\infty dx\ \frac{1}{1+e^{(x-s)/y}} - \int_0^\infty dx\ \frac{1}{1+e^{(x+s)/y}} \tag{B.19}$$

$$s = 1 \Rightarrow \mu = E_f \tag{B.20}$$

Equation (B.20) shows that the chemical potential of BLG is temperature independent and very different from that of the MLG.

[III] Monolayer Gapped Graphene (MLLG)

The expression of chemical potential for MLGG can be defined same as that of MLG by introducing Eq. (B.14) into Eq. (B.15).