CHAPTER - 2

Theoretical Models

2.1 Introduction

For years, scientist has suspected that the atomic nucleus need not be lightly bound, compact or appear as a spherical object. The exact nature and dynamics of this remains an open question. The idea of the nucleus having cluster structures dates back to 1931 when the theory of α-decay was proposed by Gamow [1]. In fact, the existing nuclear models at the time assumed that the nuclei were composed of α-particles and/or alpha particles plus protons and electrons.

For example, nuclei such as ^{8}Be, ^{12}C, ^{16}O, considered as $n\alpha$-nuclei where n is the number of alpha particles, are thought to be formed from alpha clusters. So one would say that ^{16}O is composed of four (4) α clusters (see Fig. 2.1). However, despite the satisfactory explanation of the alpha decay phenomena and a possible explanation of nuclear character, it was not until the discovery of neutrons in 1932 that it was possible to give a realistic description of the nuclei. In light of this development, several realistic models have been used for describing the nuclei, over the years with varying degrees of success. Given that no nuclear model gives a complete description of a nucleus, the cluster model has also been used to give a simultaneous description of nuclei, and has thus been improved to a level that it can give comparable predictions with so-called realistic models [1].

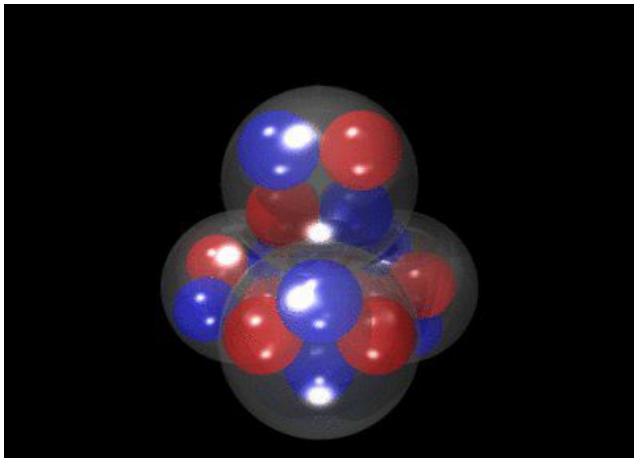

Figure 2.1 The arrangement of four alpha particle clusters in the nucleus ^{16}O [2].

Understanding the structure of a nucleus as a system of particles has been a major research area since the nucleus was discovered. Several methods have been proposed to describe

some of the experimentally observed properties of the nucleus. Since it was understood that the nucleus consists of nucleons, a model predicts that these nucleus were always moving independently of each other under a binding force. The nucleons were later found to be protons and neutrons. Although some of the suggested models were found to be successful in reproducing some of the properties of the nucleus, others such as the shell model were not able to reproduce all of the experimentally observed characteristics of all the possible nuclei.

As time evolved, more models were put forward until the cluster models were proposed, at the time of the α-decay from a nucleus. The cluster model describes the nucleus as a binary system where the correlated nucleons could be pictured as a cluster of a few nucleons orbiting a core containing the remaining nucleons. This model was used by several scientists to predict not just the structure, but other properties of some light nuclei, e.g. ^{6}Li and ^{16}O. Some of these nuclei were described as consisting of α-particle clusters. Since it was quite successful in describing a large number of light nuclei, scientists decided to investigate their ability to reproduce properties of heavy nuclei. Hence, to date, the model is still being used to investigate properties of heavy nuclei such as ^{212}Po, etc. [3].

2.2 Nuclear models

A nucleus is a system of a number of protons and neutrons interacting with strong nuclear forces. Because of the complexity of the nuclear force, the nucleus is usually described using a number of models. These are the nuclear shell model, the collective model, interacting Boson model and the cluster model etc., most of these are phenomenological. Apart from these models there are many microscopic models, like: Hartree-Fock, Hartree-Fock Bogolyubov, Tamm-Dankov, Random phase approximation, etc. But these kinds of models are very difficult to include in Coupled Channel (CC) calculations due to their complexity.

The shell model is considered as a microscopic model because, in its simplest form, it considers a nucleus as a system of nucleons moving independently in an average effective self-consistent potential. However, it proves difficult to extrapolate to the heavier nuclei. An alternative approach is to use the cluster model which provides a simple method for describing the properties of heavy nuclei. The cluster model seeks to describe the nucleus in term of a strong correlated group of nucleons moving collectively in an average potential with respect to

the remaining nucleons. This group of nucleons is generally referred to as the cluster, and the remaining nucleons form the core nucleons [3].

A good example of heavy nuclei which can be described in terms of a stable core and cluster system is the ^{212}Po. The shell model has successfully been used to describe the known excited state of ^{212}Po. It, however, fails to describe the alpha decay widths, the ground state decay widths can only be described by assuming that both the shell model and cluster model exist in this nucleus [4].

2.3 Optical Model

The existence of giant resonance is usually explained on the basis of optical model. This model of the nucleus is capable of explaining the behaviour of reaction cross sections at both low (in the resonance region this is not true, this is part of the low energy region) as well as high energies. At low energies the optical model deals with the energy average of the reaction cross section. If the incident neutron energy is not sharply defined, a number of resonances may be covered in the energy spread. One can then average the cross section and look for its energy and mass number dependence. For such purposes, we often invoke the analogy of an optical wave through a “cloudy” crystal ball.

In a nuclear reaction, the scattered wave may be divided into two categories: (i) elastic scattering in which only the direction of wave propagation is changed and (ii) inelastic scattering in which the particle is scattered into an exit channel different from the incident one. The former corresponds to a refraction of the optical waves and the latter corresponds to absorption due to the fact that the crystal ball is cloudy [5].

In this model, the interaction between the incident particle and the target nucleus is treated in analogy to the transmission of light through a partially absorbing medium. The real part of such a potential gives rise to scattering, while the imaginary part is responsible for the absorption, producing reactions.

The assumption of a single particle potential is in consonance with similar assumption in the nuclear shell model. In the latter model, which uses a real potential not only the ground state properties, but also those of the excited bound states, can be derived, based on certain additional assumptions. When the energy rises above the separation energy of the particle, an additional

effect becomes important. The entering nucleon may initiate a nuclear reaction, so that the description of the particle in a potential as a simple state is not adequate and the absorption of the latter has to be taken into account by introducing a complex part in the potential.

Further examining the optical analogy, light interacts with the atoms or molecules in a semi-transparent medium, which results in the absorption of a part of the incident wave and final re-emission of the partially attenuated wave with a phase change. The process is usually described by introducing a complex refractive index. For the nucleons there is a change in the de Broglie wavelength due to the potential inside the nucleus. This is analogous to the fact that a part of the incident light energy may also be re-emitted with changed frequency. Thus the parallelism between the motion of the nucleon in the complex potential and that of the light wave in a semi-transparent medium becomes evident. The complex potential actually provides a complex refractive index for the nuclear matter [6].

The aim of the optical model is to find a potential to describe smooth variations of the scattering cross section as a function of energy E and target nucleon number A. The general situation of a scattering may be quite complex; however, great simplification may be obtained if we are interested only in the average properties, away from resonances and states strongly excited by direct reactions.

In the optical model elastic scattering is described by solving the Schrödinger equation with an average potential resulting from the interaction of the nucleons of the projectile and target. This potential has a real part responsible for the attractive refraction, i.e., the gap sustained by the projectile as it passes through the target, and an imaginary part which describes the absorption, i.e., a decrease in the incident flow and which simulates the existence of other channels reaction than the elastic scattering e.g., such as fusion, transfer and inelastic scattering. The term optical is obvious analogy with the passage of a ray of light by a glass ball with a given opacity. The average nuclear potential resulting from the interactions between nucleons forming the target and the projectile is:

$$V_N = V_R(r) + V_I(r) \tag{2.1}$$

Now the two most commonly used potentials which we have used in our work has been described below.

2.3.1 Wood – Saxon potential

The shape factor of the optical potential commonly used is the Wood-Saxon [7]:

$$V_N(r) = -\frac{V_R}{1+e^{(r-R_r)/a_r}} - \frac{iV_I}{1+e^{(r-R_I)/a_i}} \tag{2.2}$$

where:

V_R and V_I represent the depths of the real and imaginary parts, respectively;

a_r and a_i are the real and imaginary diffusivities.

R_R and R_I are the rays real and imaginary, which obey the relation:

$$R_r = r_r\left(A_A^{1/3} + A_a^{1/3}\right) \text{ and } R_i = r_i\left(A_A^{1/3} + A_a^{1/3}\right) \tag{2.3}$$

where:

A_A and A_a represent the mass of the target and the projectile, respectively.

The Coulomb potential is usually represented by the potential of the uniform charged sphere:

$$\begin{aligned} V_C(r) &= \frac{Z_A Z_a e^2}{2R_C}\left(3 - \frac{r^2}{R_C^2}\right) \qquad r \le R_C \\ V_C(r) &= \frac{Z_A Z_a e^2}{r} \qquad\qquad\qquad r > R_C \end{aligned} \tag{2.4}$$

where:

Z_A and Z_a are the atomic numbers of projectile and target;

R_c Coulomb radius is given by: $R_C = r_C A_A^{1/3} + r_C A_a^{1/3}$

With the full potential we can write the radial Schrödinger equation:

$$\frac{d^2 f_l(r)}{dr^2} + \left[\frac{2\mu}{\hbar^2}\left(E - V_N - V_C - \frac{l(l+1)}{r^2}\right)\right] f_l(r) = 0 \qquad (2.5)$$

where:

r is the distance between the nuclei;

l is the orbital angular momentum;

E is the incident energy;

$f_l(r)$ is the radial wave function.

In the case of spinless particles the total wave function is:

$$\chi_l(r,\theta) = \frac{f_l(r) P_l(\cos\theta)}{r} \qquad (2.6)$$

The radial wave function $f_l(r)$ is obtained by integrating eq. 2.5 to a limiting value called matching radius (Rm), from which the effect of the nuclear potential is negligible. The radial wave function $f_l(r)$ is from this point to be treated as a superposition of Coulomb incident and scattered waves.

$$f_l(r) = F_l(r) + iG_l(r) + S_l[F_l(r) - iG_l(r)] \qquad (2.7)$$

where:

$F_l(r)$ and $G_l(r)$ are the Coulomb wave functions;

S_l is the scattering matrix, which is a very important quantity in, that is, it contains all information about the effect that produces the target wave function spread (and thus the observable). So one can write all the scattering observables in terms of the S-matrix elements.

The scattering amplitude is given by:

$$f(\theta) = f_C(\theta) + \frac{1}{2ik}\sum (2l+1)(S_l - 1)i^{i\sigma_l} P_l(\cos\theta) \qquad (2.8)$$

where:

$f_C(\theta)$ is the Coulomb scattering amplitude;

σ_l is the Coulomb gap.

And the elastic differential cross section is obtained from:

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \qquad (2.9)$$

In the optical model calculations, the potential parameters used are varied systematically to obtain a good fit for the experimental elastic differential cross section.

2.3.2 São – Paulo potential

The São Paulo potential (SPP), which is a kind of folding model potential that takes into account the Pauli principle due to exchange of nucleons between the projectile and target, is another theoretical model for the heavy-ion nuclear interaction [8]. In principle, the bare (or nuclear) potential between two heavy ions can be associated with the fundamental nucleon-nucleon interaction folded into a product of the nucleon densities of the nuclei [9].

The São Paulo potential has been successful in describing the elastic scattering and peripheral reaction channels for a large number of heavy-ion systems in a very wide energy region, from sub-Coulomb to 200 MeV/nucleon [10–24]. It has also described the total reaction and fusion cross sections for hundreds of systems [25–27]. Within this model, the nuclear interaction is connected with the folding potential through [28]:

$$V_N(R,E) = V_F(R)\, e^{-4\mathrm{V}^2/c^2} \tag{2.10}$$

where c is the speed of light, V is the local relative velocity between the two nuclei,

$$\mathrm{V}^2(R,E) = \frac{2}{\mu}\,[E - V_C(R) - V_N(R,E)] \tag{2.11}$$

and V_C is the Coulomb potential.

The velocity-dependence of the potential arises from the effects of the Pauli non-locality [10,29]. The SP potential is obtained numerically by solving Eqs. (2.10) and (2.11) by an iterative process. The folding potential depends on the matter densities of the nuclei involved in the collision:

$$V_F(R) = \int \rho_1(\vec{r}_1)\,\rho_2(\vec{r}_2)\,V_0\,\delta(\vec{R} - \vec{r}_1 + \vec{r}_2)\,d\vec{r}_1\,d\vec{r}_2 \tag{2.12}$$

For V_C a double sharp-cutoff Coulomb potential was used. To obtain a global parameter-free description of the nuclear interaction, a systematization of nuclear densities was developed. This systematic was based on an extensive study involving charge distributions extracted from electron scattering data and theoretical densities calculated through the Dirac-Hartree-Bogoliubov model and adopt the two parameter Fermi (2pF) distribution to describe the nuclear densities. Within the derived systematization, the radii of the 2pF distributions of a nucleus with A nucleons are well described by

$$\mathrm{R_0} = (1.31\mathrm{A}^{1/3} - 0.84)\ \mathrm{fm} \tag{2.13}$$

and the nuclear matter densities have an average diffuseness of a = 0.56 fm. The imaginary part of the interaction is assumed to have the same shape of the real part [Eq. (2.10)], with one single adjustable parameter N_I related to its strength, such as

$$W(\mathrm{R}, \mathrm{E}) = N_I \, V_N(\mathrm{R}, \mathrm{E}) \qquad (2.14)$$

For more than 40 systems, all the elastic scattering angular distributions, over wide energy ranges, were simultaneously well fitted with only one free parameter, the average value of N_I , which was derived to be $N_I = 0.78$, for energies above the Coulomb barrier.

We have used the code ECIS [30] for our optical model calculations. The name “ECIS” is made of the first letters of “Equations Couplées en Itérations Séquentielles", by reference to the method of solution used in the codes, although the usual method of solution is also present and has been written with as much care. The ECIS method is designed to solve sets of coupled differential equations when the coupling terms are not too strong. The iteration technique searches for the one required solution among the many which are mathematically possible.

The method supposes some ordering of the channels: first the ground state, then the state most strongly coupled to it. All channels must be coupled to some preceding one. The result of each iteration depends on this chosen order. If there is more than one equation related to the ground state the whole calculation must be repeated. The efficiency of the method is proportional to the ratio of the total number of equations to the number of those related to the ground state. The usual methods can also be used, but the iteration method is compulsory for spin-orbit deformation and Dirac formalism.

There are many ECIS codes starting from ECIS68 to the recent ECIS06. The code which we have used for our calculations is ECIS95. ECIS-95 is a generalization of ECIS-79: for the rotational model vibrational bands are included. An option for solving the Dirac equation has been added. It also contains the statistical model including width fluctuation corrections as formulated by Peter Moldauer. Besides the use of Bessel expansion for form factors, the use of deformation lengths and the use of 'symmetrized' Woods-Saxon potentials, it includes (i) two bound state transitions for particle-mode excitations with the possibility of the particle in the continuum (ii) expression of cross sections in terms of Legendre polynomials (iii) possibility of angular distribution for uncoupled states without giving explicitly all the reduced nuclear matrix elements (iv) for Coulomb excitation, use of the magnetic multipole.

2.4 Continuum Discretized Coupled Channel (CDCC) formalism

The method of Continuum Discretized Coupled Channel, abbreviated as CDCC, is a method of analyzing direct nuclear reactions which involve break-up of loosely bound particles, such as deuteron, ^{6,7}Li, ^{9}Be etc. The process of nuclear breakdown may occur in a collision between two nuclei depending on the collision energy and binding energy of the nucleus. In the case of stable nuclei with high binding energies, this process is unlikely at low energies and starts to be important from 10 MeV / u. In the case of exotic light nuclei binding energies are much smaller. Due to the low binding energies of exotic nuclei and due to the halo nucleons, it is expected that the probability of nuclear breakdown is greatly increased for these nuclei even at low collision energies. The fact that binding energies are small means that effects due to coupling with the continuum may be important in collision with exotic nuclei. These effects are usually not considered when it comes to stable nuclei and only the bound states are coupled. In the case of exotic nuclei close to the continuum has an important effect and consequences on other channels such as elastic scattering.

In CDCC calculation, the breakdown process (break-up) describes an inelastic scattering of the projectile excited states to the continuum of the projectile. Thus the wave functions represent the excited states of the projectile in the continuum. One needs a model to describe these functions. In the case of ^{6}He three bodies are involved because it breaks into an alpha particle and two neutrons that are not linked. Therefore the wave functions of three bodies would be more appropriate. However this makes calculating the CDCC extremely bulky and few groups in the world are able to achieve these calculations currently [31].

The phenomenological optical model essentially provides a highly adaptable technique for describing elastic scattering and it has been used for complex particles as well as for nucleons. The model gives no detail about the inelastic processes. The optical model potential is the potential acting on an unbound nucleon and is partly absorptive. The absorption represents the fact that such a nucleon has enough energy to collide with a nucleon in the nucleus, and thus be absorbed from the incident beam. The optical model is essentially a generalization of the shell model which applies to nucleons of any energy-not just to nucleons of energy such that they are bound in a nucleus.

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