CHAPTER - 5

AN INVENTORY MODEL FOR WEIBULL DETERIORATING ITEMS WITH PRESERVATION TECHNOLOGY INVESTMENT

5.1 Introduction

Deterioration is a basic characteristic of most of the items. All perishable products deteriorate with time. Agricultural products, animal husbandry products, dairy products, pharmaceutical products, and many other products deteriorate at more or less rate. A higher rate of deterioration significantly affects the total profit. So, it is necessary to control the deterioration rate to avoid losses. This is possible by investing in preservation technologies like cooling, freezing, drying, vacuuming, irradiation, high pressure, bio-preservation, etc. to control deterioration rate.

The majority of the studies on deteriorating inventory modeling did not consider the controllable deterioration situation and ignored the possibility of preservation technology investment. In this chapter, we developed an inventory model for Weibull deteriorating items allowing preservation technology investment. Also, both instantaneous and non-instantaneous cases are taken care.

5.2 Assumptions

- The demand is constant.
- The lifetime (t) of the product follows three-parameter Weibull distribution $f(t) = \alpha\beta(t - T_d)^{\beta-1}e^{-\alpha(t-T_d)^\beta}$, where T_d (≥ 0) (deterioration free life) is the location parameter, α (> 0) is the scale parameter, β (> 0) is the shape parameter and. The cumulative distribution function is $F(t) = \int_{T_d}^{t} f(t)dt = 1 - e^{-\alpha(t-T_d)^\beta}$, hence the deterioration rate is $\frac{f(t)}{1-F(t)} = \alpha\beta(t - T_d)^{\beta-1}$.

- The deterioration rate can be reduced through investing in preservation technology. The proportion of the reduced deterioration rate is $m(\xi) = 1 - e^{-\eta\times\xi}$, where, $\eta(\geq 0)$ is the simulation coefficient representing the percentage increase in $m(\xi)$ per dollar increase in ξ. When $\xi = 0$, the reduced deterioration rate $m(\xi) = 0$, and for $\xi \to \infty$, $\lim_{\xi\to\infty} m(\xi) = 1$. But we set constraint $0 \leq \xi \leq \xi'$, where, ξ' is the maximum PT investment allowed.
- Shortages are permitted with partially backlogging. The portion of the unsatisfied demand that backlogged is $D(A, P)e^{-\delta(T-t)}$ where backlogging parameter δ is a positive constant and $(T - t)$ is the waiting time.
- Instant and infinite replenishment rate.
- The inventory system involves a single item.
- There is no salvage value or resale for the deteriorated items.

5.3 Model Development

5.3.1 Case-1: Instantaneous deterioration:

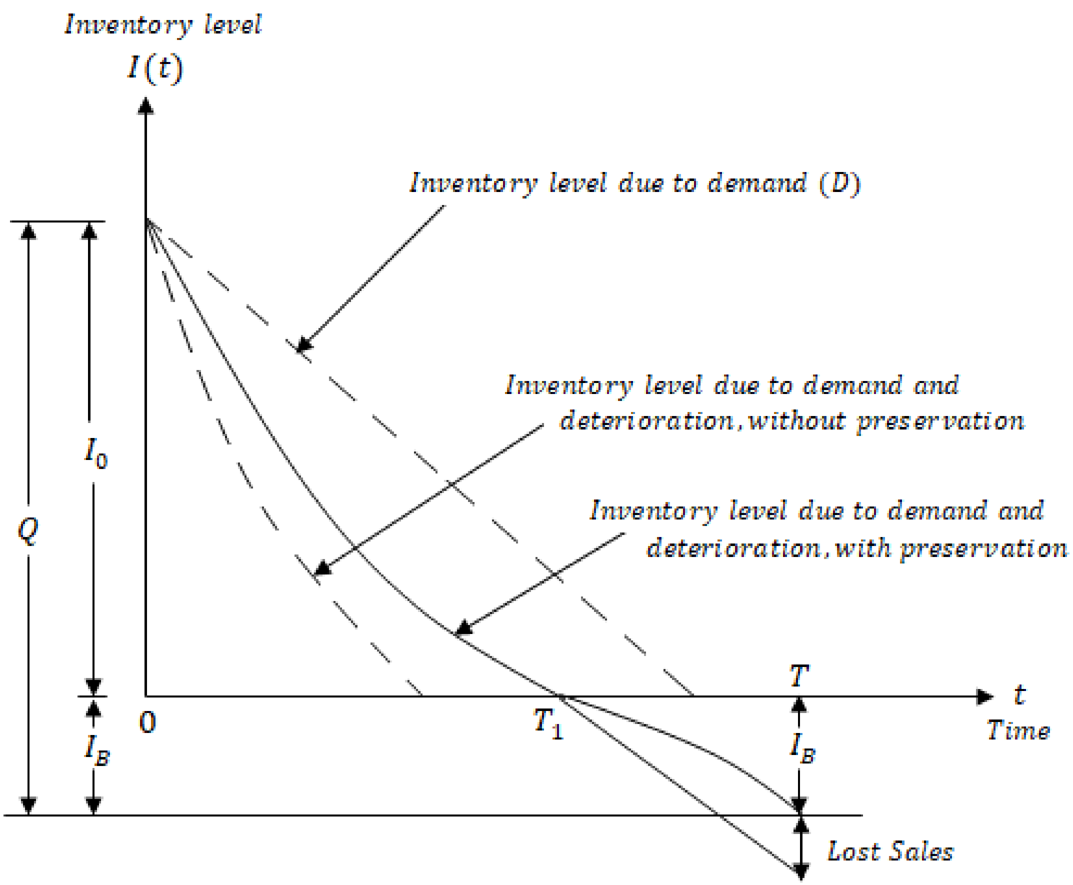
As shown in figure 5.3.1, the inventory level will continuously decrease during the interval [0, T_1] due to demand and deterioration with preservation technology. During the interval $[T_1, T]$ shortages are allowed with partial backlogging.

The rate of change in the inventory level can be described by the differential equations as given below.

$$\frac{dI_1(t)}{dt} + \alpha\beta t^{\beta-1}\big(1 - m(\xi)\big)\, I(t) = -D, \qquad 0 \le t \le T_1 \tag{5.3.1.1}$$

$$\frac{dI_2(t)}{dt} = -De^{-\delta(T-t)}, \qquad T_1 \le t \le T \tag{5.3.1.2}$$

Figure 5.3.1: Graphical representation of Instantaneous deterioration inventory system



The solution of equation (5.3.1.1) with the boundary condition $I_1(T_1) = 0$ is

$$I_1(t) = D\left[(T_1 - t) + \frac{\alpha\big(1-m(\xi)\big)}{\beta+1}\left({T_1}^{\beta+1} - t^{\beta+1}\right) - \alpha\big(1-m(\xi)\big)\left(T_1 t^{\beta} - t^{\beta+1}\right) - \frac{\alpha^2\big(1-m(\xi)\big)^2}{(\beta+1)}\left({T_1}^{\beta+1}t^{\beta} - t^{2\beta+1}\right)\right] \tag{5.3.1.3}$$

Initial inventory at $t = 0$ is

$$I_0 = D\left[T_1 + \frac{\alpha\big(1-m(\xi)\big)}{\beta+1}{T_1}^{\beta+1}\right] \tag{5.3.1.4}$$

Deterioration cost:

$$DC = C_d\left[I_0 - \int_0^{T_1} D\,dt\right] = C_d D\left[\frac{\alpha\big(1-m(\xi)\big)}{\beta+1}{T_1}^{\beta+1}\right] \tag{5.3.1.5}$$

Holding cost:

$$HC = C_h\int_0^{T_1} I_1(t)\,dt \tag{5.3.1.6}$$

$$HC = C_h D\left[\frac{{T_1}^2}{2} + \frac{\alpha\beta\big(1-m(\xi)\big)}{(\beta+1)(\beta+2)}{T_1}^{\beta+2} - \frac{\alpha^2\big(1-m(\xi)\big)^2}{2(\beta+1)^2}{T_1}^{2(\beta+1)}\right] \tag{5.3.1.7}$$

The solution of equation 5.3.1.2 with boundary condition $\mathrm{I}_2(T_1) = 0$ is

$$I_2(t) = \frac{-D}{\delta}\left[e^{-\delta(T-t)} - e^{-\delta(T-T_1)}\right] \tag{5.3.1.8}$$

The maximum amount of demand that can be backlogged per cycle is, obtained by putting $t = T$.

$$I_B = \frac{D}{\delta}[1 - e^{-\delta(T-T_1)}] \tag{5.3.1.9}$$

Order quantity per cycle:

$$Q = I_0 + I_B = \mathrm{D}\left[T_1 + \frac{\alpha\big(1-\mathrm{m}(\xi)\big)}{\beta+1}{T_1}^{\beta+1} + \frac{1}{\delta}[1 - \mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}]\right] \tag{5.3.1.10}$$

Lost sale cost:

$$LSC = C_s\int_{T_1}^{T}\left(D - De^{-\delta(T-T_1)}\right)dt \tag{5.3.1.11}$$

$$= C_sD\left[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T-T_1)}}{\delta}\right] \tag{5.3.1.12}$$

Purchase cost:

$$PC = CQ = C\mathrm{D}\left[T_1 + \frac{\alpha\big(1-\mathrm{m}(\xi)\big)}{\beta+1}{T_1}^{\beta+1} + \frac{1}{\delta}[1 - \mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}]\right] \tag{5.3.1.13}$$

Total sales revenue:

$$SR = P\left[\int_0^{T_1} D\ dt + \int_{T_1}^{T} De^{-\delta(T-t)}\ dt\right]$$

$$= PD\left[T_1 + \frac{1}{\delta}\left(1 - e^{-\delta(T-T_1)}\right)\right] \tag{5.3.1.14}$$

Ordering cost

$$OC = C_O \tag{5.3.1.15}$$

Preservation technology investment:

$$PTI = T_1\xi \tag{5.3.1.16}$$

Total profit per unit time:

$$TP_1(T_1, T, \xi) = \frac{1}{T}[SR - PC - DC - LSC\ - HC - OC - PTI]$$

$$TP_1(T_1, T, \xi) = \frac{PD}{T}\left[T_1 + \frac{1}{\delta}\left(1 - e^{-\delta(T-T_1)}\right)\right]$$

$$-\frac{CD}{T}\left[T_1 + \frac{\alpha\left(1 - m(\xi)\right)}{\beta + 1}{T_1}^{\beta+1} + \frac{1}{\delta}\left(1 - e^{-\delta(T-T_1)}\right)\right]$$

$$-\frac{C_d D}{T}\left[\frac{\alpha\left(1 - m(\xi)\right)}{\beta + 1}{T_1}^{\beta+1}\right]$$

$$-\ \frac{C_s D}{T}\left[T - T_1 - \frac{1}{\delta} + \frac{e^{-\delta(T-T_1)}}{\delta}\right]$$

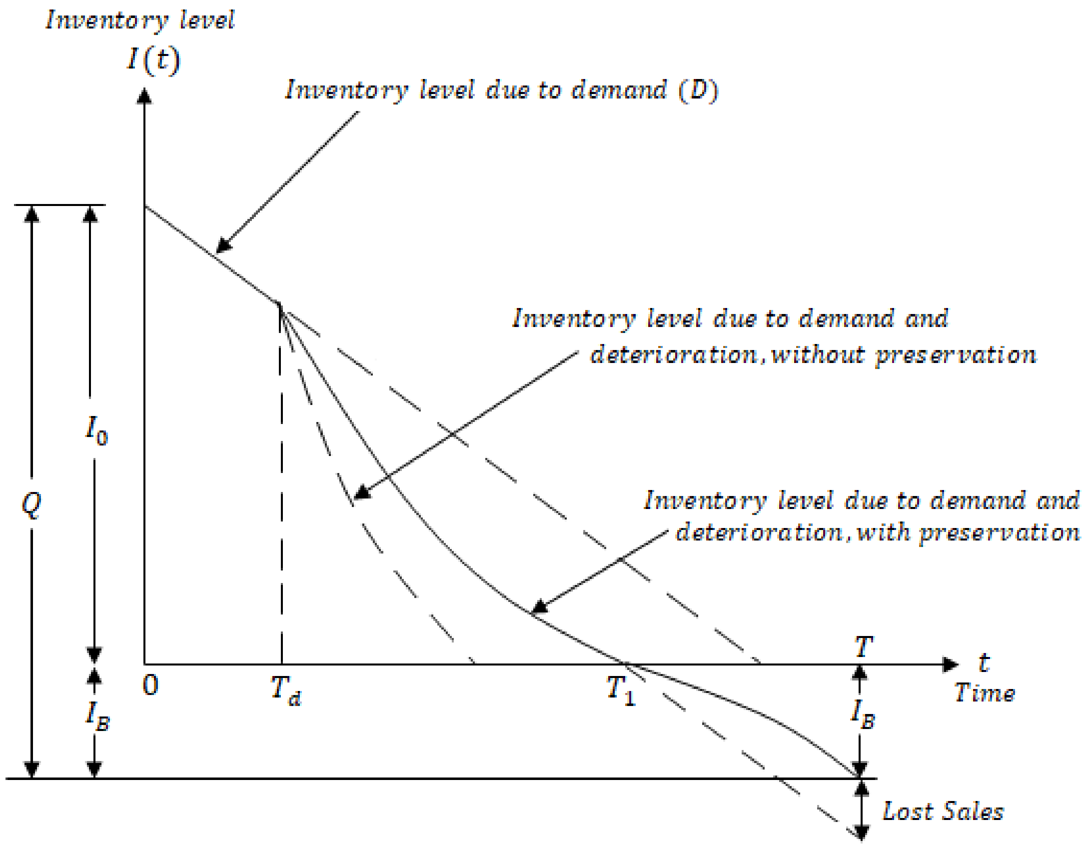
$$-\frac{C_h D}{T}\left[\frac{{T_1}^2}{2} + \frac{\alpha\beta\left(1 - m(\xi)\right)}{(\beta + 1)(\beta + 2)}{T_1}^{\beta+2} - \frac{\alpha^2\left(1 - m(\xi)\right)^2}{(\beta + 1)(2\beta + 2)}{T_1}^{2(\beta+1)}\right]$$

$$-\frac{C_O}{T}-\frac{T_1\xi}{T} \qquad (5.3.1.17)$$

5.3.2 Case-2: Non-instantaneous deterioration

As shown in figure 5.3.2, during the interval $[0, T_d]$ the inventory level will decrease only due to the demand, and there will be no deterioration in this time period. The deterioration starts at the time T_d. During the interval $[T_d, T_1]$ the inventory level will continuously decrease due to demand and deterioration with preservation technology.

Figure 5.3.2: Graphical representation of Non-instantaneous deterioration inventory system



The rate of change in the inventory level can be described by the differential equations as given below.

$$\frac{dI_1(t)}{dt} = -D, \qquad 0 \le t \le T_d \tag{5.3.2.1}$$

$$\frac{dI_2(t)}{dt} + \alpha\beta(t - T_d)^{\beta-1}(1 - m(\xi))I_2(t) = -D, \quad T_d \le t \le T_1 \tag{5.3.2.2}$$

$$\frac{dI_3(t)}{dt} = -De^{-\delta(T-t)}, \qquad T_1 \le t \le T \tag{5.3.2.3}$$

Solution of equation 5.3.2.1 with boundary condition $I_1(0) = I_0$ is

$$I_1(t) = -Dt + I_0 \tag{5.3.2.4}$$

Solution of equation 5.3.2.2 with boundary condition $I_2(T_1) = 0$ is

$$I_2(t) = D\left[(T_1 - t) + \frac{\alpha(1 - m(\xi)}{(\beta + 1)}\left[(T_1 - T_d)^{\beta+1} - (t - T_d)^{\beta+1}\right]\right] \times \left(1 - \alpha\big(1 - m(\xi)\big)(t - T_d)^{\beta}\right) \tag{5.3.2.5}$$

Using the condition $I_1(T_d) = I_2(T_d)$ the initial inventory I_0 is

$$I_0 = D\left[T_1 + \frac{\alpha\big(1 - m(\xi)\big)}{\beta + 1}(T_1 - T_d)^{\beta+1}\right] \tag{5.3.2.6}$$

Deterioration cost:

$$DC = C_d\left[I_2(T_d) - \int_{T_d}^{T_1} D\, dt\right]$$

$$= \frac{C_d D\alpha(1 - \mathrm{m}(\xi))}{\beta + 1}(T_1 - T_d)^{\beta+1} \tag{5.3.2.7}$$

Solution of equation 5.3.2.3 with condition $I_3(T_1) = 0$ is

$$I_3(t) = \frac{-D}{\delta}\left[e^{-\delta(T-t)} - e^{-\delta(T-T_1)}\right] \tag{5.3.2.8}$$

Maximum amount of demand backlogged per cycle is obtained by putting $t = T$.

$$I_B = \frac{D}{\delta}\left[1 - e^{-\delta(T-T_1)}\right] \tag{5.3.2.9}$$

$$Q = I_0 + I_B$$

$$= D\left[T_1 + \frac{\alpha(1 - m(\xi))}{\beta + 1}(T_1 - T_d)^{\beta+1} + \frac{1}{\delta}\left[1 - e^{-\delta(T-T_1)}\right]\right] \tag{5.3.2.10}$$

Purchasing cost:

$$PC = C * Q \tag{5.3.2.11}$$

Lost sale cost (opportunity cost due to lost sale including loss of good will):

$$LSC = C_s \int_{T_1}^{\mathrm{T}} \left[\mathrm{D} - \mathrm{De}^{-\delta(\mathrm{T}-\mathrm{t})}\right] \mathrm{dt}$$

$$= C_s D\left[T - T_1 - \frac{1}{\delta} + \frac{\mathrm{e}^{-\delta(\mathrm{T}-\mathrm{T}_1)}}{\delta}\right] \tag{5.3.2.12}$$

Holding cost:

$$HC = C_h\left[\int_0^{T_d} \mathrm{I}_1(\mathrm{t})\ \mathrm{dt} + \int_{T_d}^{T_1} \mathrm{I}_2(\mathrm{t})\ \mathrm{dt}\right]$$

$$= C_h D\left[\frac{\alpha\big(1-m(\xi)\big)}{\beta+1} T_d (T_1 - T_d)^{\beta+1} + \frac{T_1^2}{2} + \frac{\alpha\beta\big(1-m(\xi)\big)}{(\beta+1)(\beta+2)}(T_1 - T_d)^{\beta+2} - \frac{\alpha^2\big(1-m(\xi)\big)^2}{(\beta+1)(2\beta+2)}(T_1 - T_d)^{2(\beta+1)}\right] \tag{5.3.2.13}$$

Total sales revenue:

$$SR = P\left[\int_0^{T_1} D\ dt + \int_{T_1}^{T} D e^{-\delta(T-t)}\ dt\right]$$

$$= PD\left[T_1 + \frac{1}{\delta}\left(1 - e^{-\delta(T-T_1)}\right)\right] \tag{5.3.2.14}$$

Preservation technology investment:

$$PTI = (T_1 - T_d)\xi \tag{5.3.2.15}$$

Total profit function is

$$TP_2(T_1, T, \xi) = \tfrac{1}{T}[SR - PC - DC - LSC\ - HC - OC - PTI]$$

$$TP_2(T_1, T, \xi)\ = \frac{P}{T}\left[T_1 + \frac{1}{\delta}\left(1 - e^{-\delta(T-T_1)}\right)\right]$$

$$-\frac{CD}{T}\left[T_1 + \frac{\alpha\big(1-m(\xi)\big)}{\beta+1}(T_1 - T_d)^{\beta+1} + \frac{1}{\delta}\left[1 - e^{-\delta(T-T_1)}\right]\right]$$

$$-\frac{C_d D}{T}\left[\frac{\alpha\big(1-m(\xi)\big)}{\beta+1}(T_1-T_d)^{\beta+1}\right]$$

$$-\frac{C_s D}{T}\left[T-T_1-\frac{1}{\delta}+\frac{e^{-\delta(T-T_1)}}{\delta}\right]$$

$$-\frac{C_h D}{T}\left[\frac{\alpha\big(1-m(\xi)\big)}{\beta+1}T_d(T_1-T_d)^{\beta+1}+\frac{T_1^2}{2}\right.$$
$$+\frac{\alpha\beta\big(1-m(\xi)\big)}{(\beta+1)(\beta+2)}(T_1-T_d)^{\beta+2}$$
$$\left.-\frac{\alpha^2\big(1-m(\xi)\big)^2}{(\beta+1)(2\beta+2)}(T_1-T_d)^{2(\beta+1)}\right]$$

$$-\frac{C_O}{T}-\frac{T_1\xi}{T} \tag{5.3.2.16}$$

To maximize the total profit TP the necessary and sufficient conditions are given below.

$$\frac{\partial\, TP}{\partial T_1}=0,\frac{\partial\, TP}{\partial T}=0\,\frac{\partial\, TP}{\partial \xi}=0; \tag{5.3.2.17}$$

The Hessian matrix H is a negative definite. Where,

$$H=\begin{bmatrix}\frac{\partial^2\, TP}{\partial T_1^2} & \frac{\partial^2\, TP}{\partial T_1\partial T} & \frac{\partial^2\, TP}{\partial T_1\partial \xi}\\ \frac{\partial^2\, TP}{\partial T\partial T_1} & \frac{\partial^2\, TP}{\partial T^2} & \frac{\partial^2\, TP}{\partial T\partial \xi}\\ \frac{\partial^2\, TP}{\partial \xi\partial T_1} & \frac{\partial^2\, TP}{\partial \xi\partial T} & \frac{\partial^2\, TP}{\partial \xi^2}\end{bmatrix} \tag{5.3.2.18}$$

Putting $T_d=0$ in equation (5.3.2.16) we get the equation (5.3.1.17). So, the case-1 is a particular case of case-2. This means, instantaneous deterioration inventory model is a particular case of non-instantaneous deterioration inventory model.

5.4 Examples

Example-1: Consider an instantaneous deterioration case (Case-1) with the following parameters in appropriate units: $\alpha = 0.5$, $\beta = 4$, $\delta = 0.4$, $\eta = 0.03$, $D = 500$, $C_O = 400$, $C_h = 1$, $C_s = 10$, $C = 50$, $P = 80$, $C_d = 0$ (i.e. no salvage value and no disposal cost). Using DEoptimR packge in R programming the global optimal solution is $T_1^* = 0.9631, T^* = 1.0636, \xi^* = 139.3494$, $Q^* = 531.4361, TP_1(T_1, T, \xi) = 14212.07$.

Example-2: Consider a non-instantaneous deterioration case (Case-2) with the same parameter values as in example-1 including $T_d = 0.2$. Using DEoptimR packge in R programming the global optimal solution is $T_1^* = 0.9281, T^* = 1.0214$, $\xi^* = 103.1003$, $Q^* = 510.3124$, $TP_2(T_1, T, \xi) = 14266.63$.

5.5 Concavity

Figures 3-10 depicts concavity of the total profit functions $\mathrm{TP_1(T_1, T, \xi)}$ and $\mathrm{TP_2(T_1, T, \xi)}$ with respect to different decision variables. The JDEoptim function of the DEoptimR package uses the differential evolution stochastic algorithm to find a global optimal solution. The solution point for example 1 was $(T_1^*, T^*, \xi^*) = (0.9631, 1.063, 139.3494)$. At this solution point, the gradient is (-0.1315935, 0.1363421, -0.000002), the Hessian matrix is $\begin{bmatrix} -8.35121319 & 7226.13 & 3773.18 \\ 7226.13 & -7226.263 & 0.0000019 \\ 3.77318 & 0.0000019 & -0.0027167 \end{bmatrix}$ and its eigenvalues are (-

0.0145137, -540.7618, -15036.73).Here, the gradient is very close to (0, 0, 0) and since all the eigenvalues are negative, the Hessian matrix is a negative definite. Also, the optimal solution for example 2 was $(T_1^*, T^*, \xi^*) =$ $(0.9281, 1.0214, 103.1003)$, At this solution point, the gradient is (-0.716741, 0.71674, -0.00025), the Hessian matrix is $\begin{bmatrix} -8928.303604 & 7546.17 & 3.902467 \\ 7546.171144 & -7546.8.61 & 0.0002488 \\ 3.902467 & 0.0002488 & -0.0213 \end{bmatrix}$ and its eigenvalues are(-0.0103587, -0.06598759, -15815.30). Here, the gradient is close to (0, 0, 0) and since all the eigenvalues are negative, the Hessian matrix is a negative definite. Hence, the obtained solutions a global optimal solutions.

Figure 5.5.1: Concavity of $TP_1(T_1, T, \xi)$ w.r.t ξ.

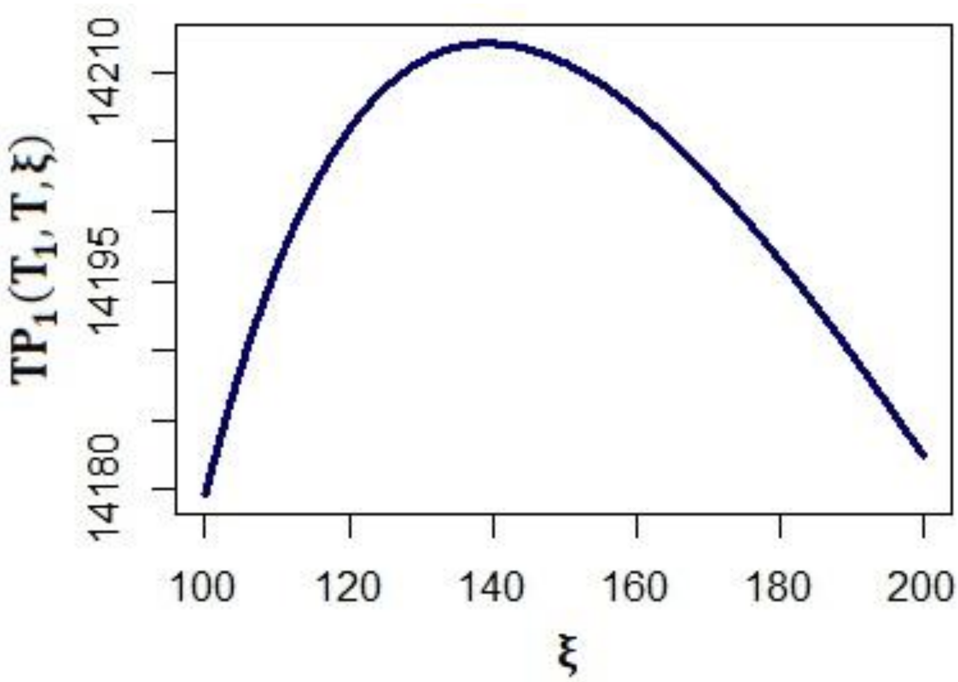


Figure 5.5.2 Concavity of $TP_2(T_1, T, \xi)$ w.r.t ξ.

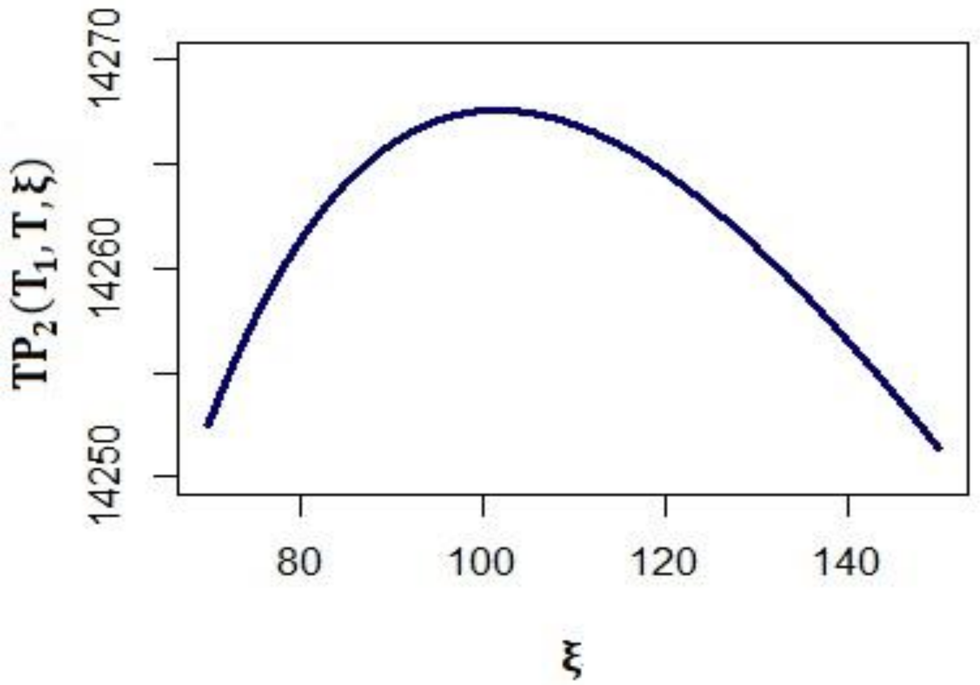


Figure 5.5.3 Concavity of $TP_1(T_1, T, \xi)$ w.r.t T

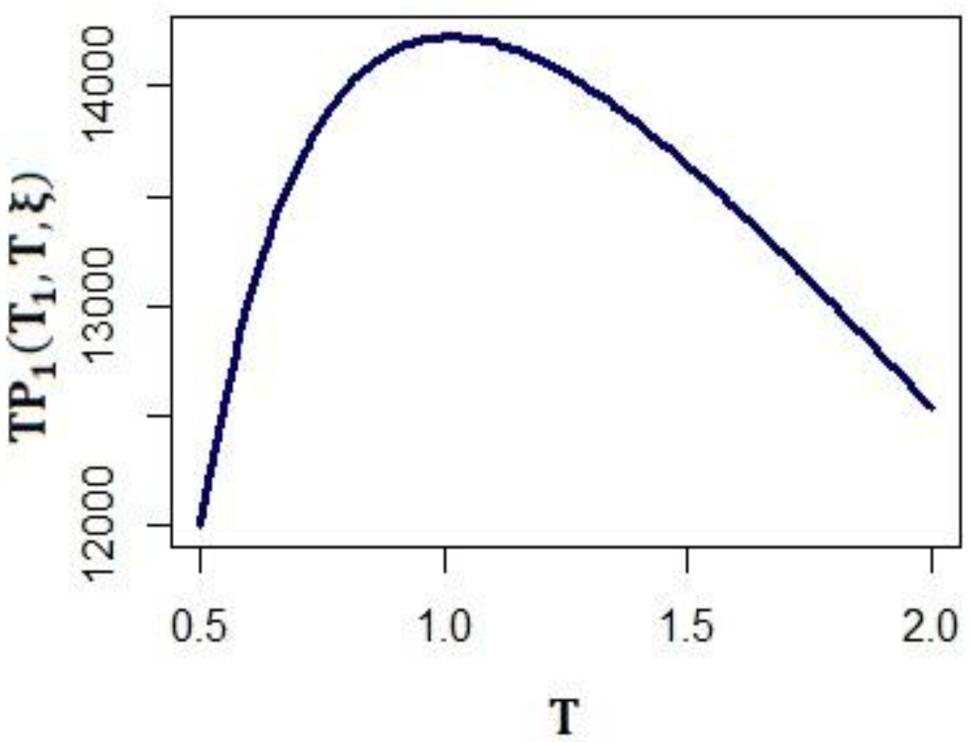


Figure 5.5.4 Concavity of $TP_2(T_1, T, \xi)$ w.r.t T

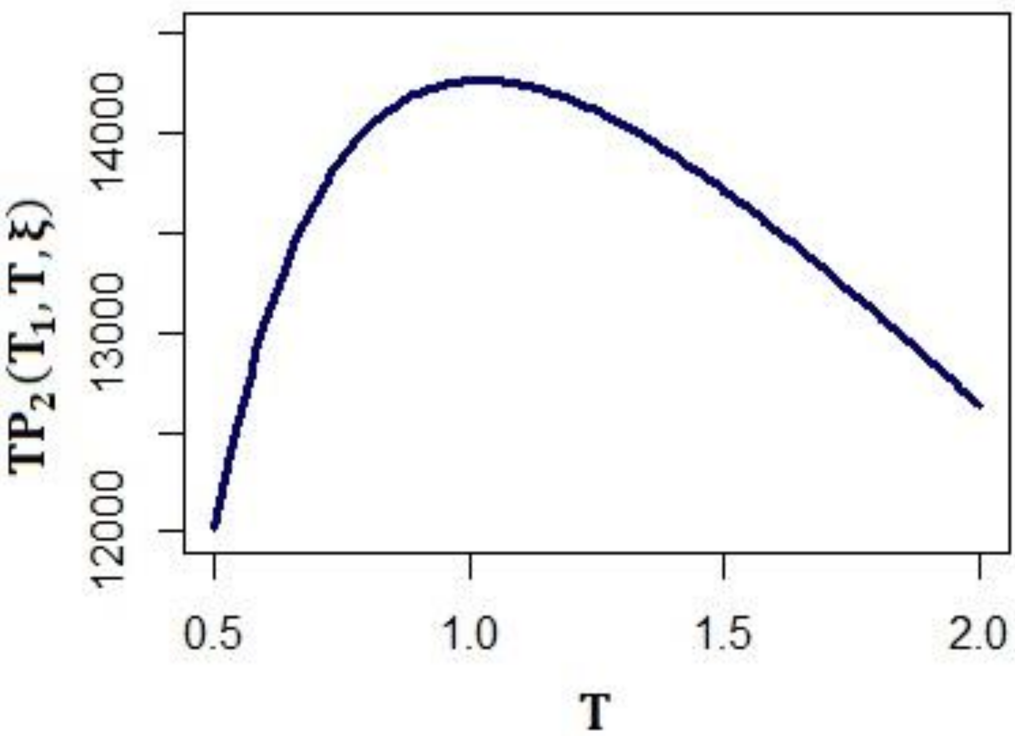


Figure 5.5.5 Concavity of $TP_1(T_1, T, \xi)$ w.r.t T_1

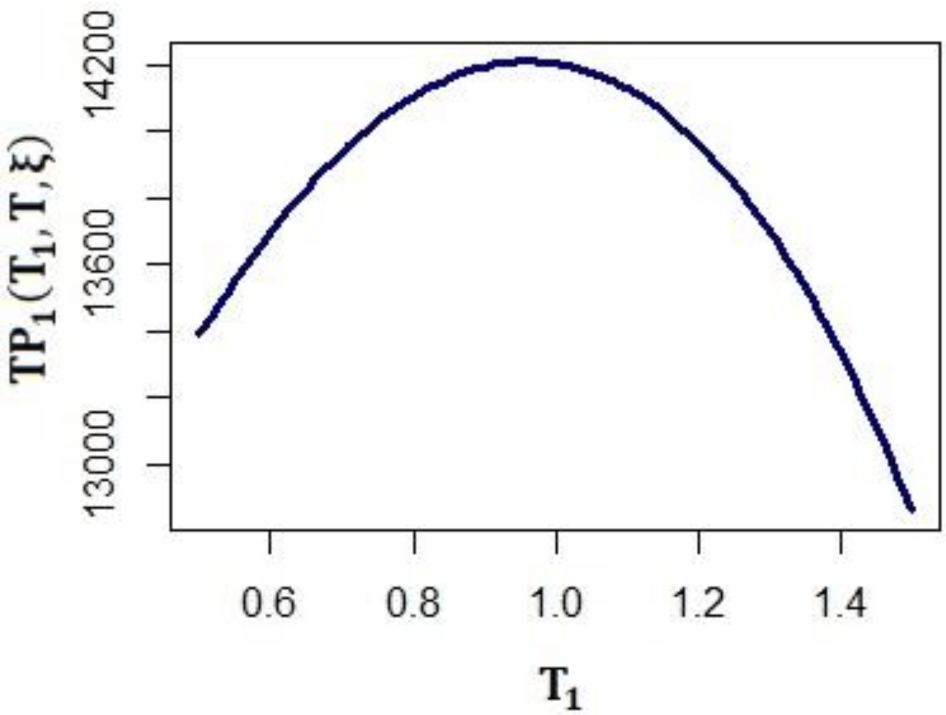


Figure 5.5.6 Concavity of $TP_2(T_1, T, \xi)$ w.r.t T_1

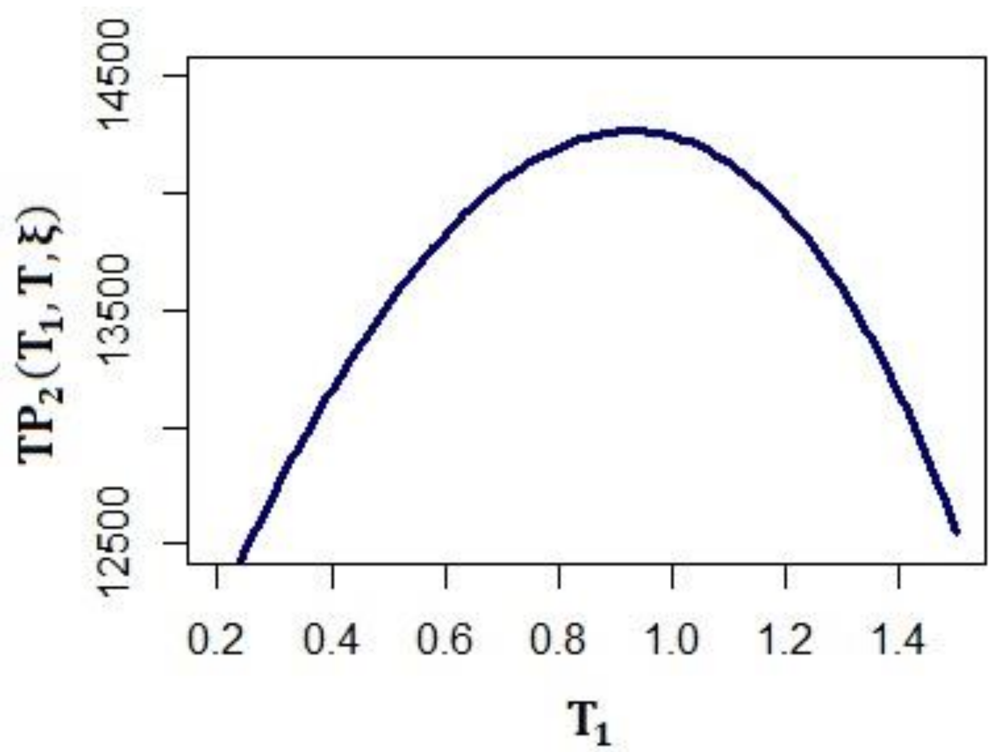


Figure 5.5.7 Concavity of $TP_1(T_1, T, \xi)$ w.r.t T and ξ

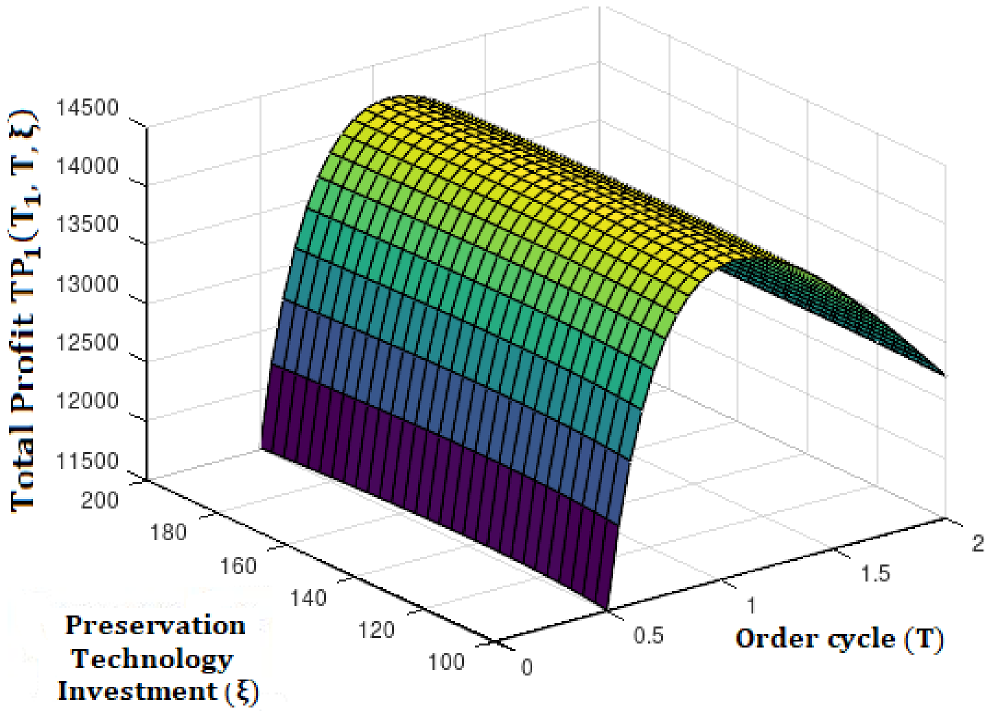
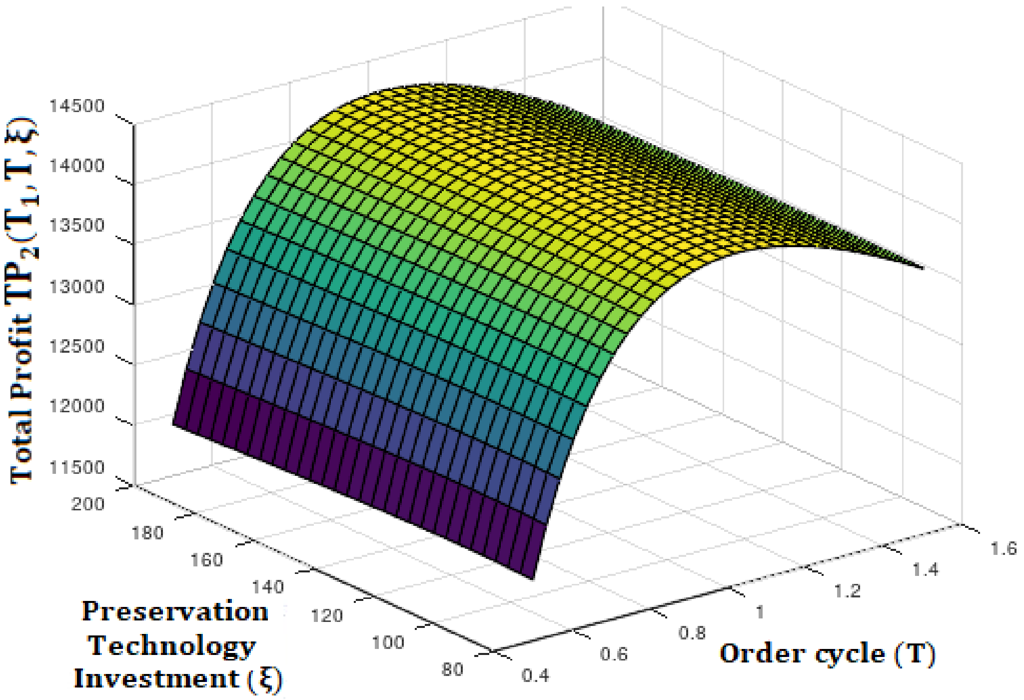


Figure 5.5.8 Concavity of $TP_2(T_1, T, \xi)$ w.r.t T and ξ



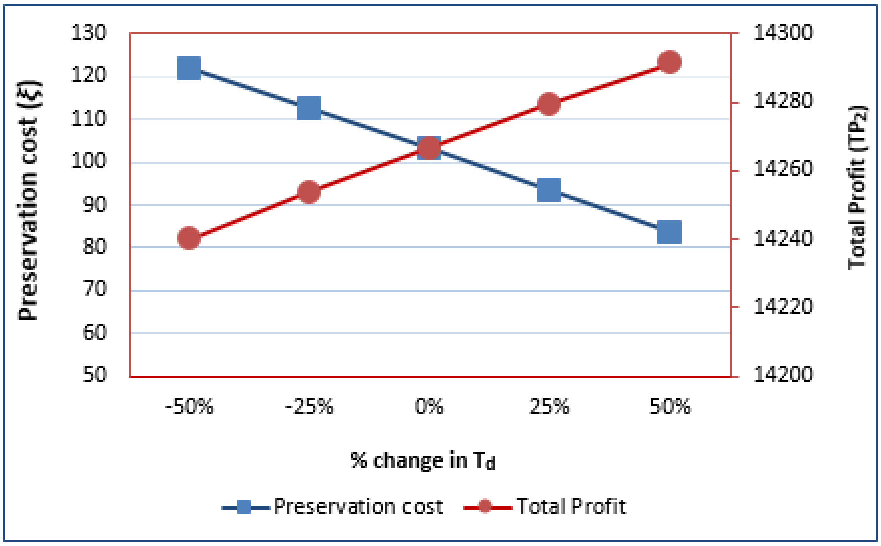
5.6 Sensitivity Analysis

We can observe the effectiveness of each parameter on T_1, T, ξ, Q and $TP_2(T_1, T, \xi)$ in below table and figures.

Table 5.6.1 Sensitivity table of T_d.

Parameter		% change	T_1^*	T^*	ξ^*	Q^*	Total Profit (TP_2)
T_d	0.1	-50%	0.9424	1.0393	121.8385	519.2526	14240.04
	0.15	-25%	0.9343	1.0294	112.6126	514.2953	14253.54
	0.2	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	0.25	25%	0.9237	1.0155	93.3262	507.3321	14279.23
	0.3	50%	0.9219	1.0121	83.5606	505.6393	14291.24

Figure 5.6.1 Effect of T_d on ξ and $TP_2(T_1, T, \xi)$.

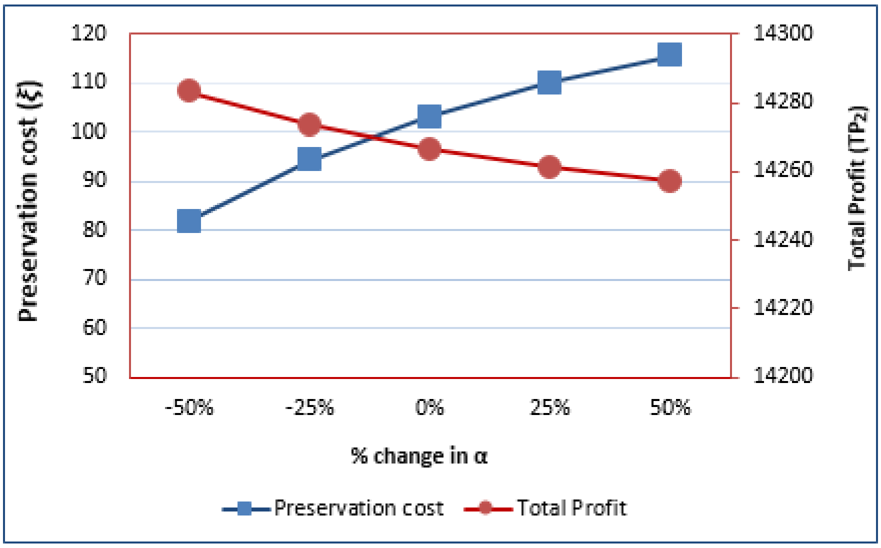


Observations from the table 5.6.1 and figure 5.6.1: When the starting point of deterioration (T_d) increases, T_1^*, T^*, ξ^*, Q^* decrease; however, the total profit increase. This implies that when the items maintain its original form for longer time then less preservation cost is required, which leads an increment in total profit.

Table 5.6.2 Sensitivity table of α.

Parameter		% change	T_1^*	T^*	ξ^*	Q^*	Total Profit
α	0.25	-50%	0.9376	1.0289	81.7410	514.0969	14283.15
	0.375	-25%	0.9321	1.0246	94.2134	511.9307	14273.47
	0.5	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	0.625	25%	0.9249	1.0189	109.9347	509.0594	14261.33
	0.75	50%	0.9222	1.0168	115.5410	507.9144	14257.01

Figure 5.6.2 Effect of α on ξ and $TP_2(T_1, T, \xi)$.

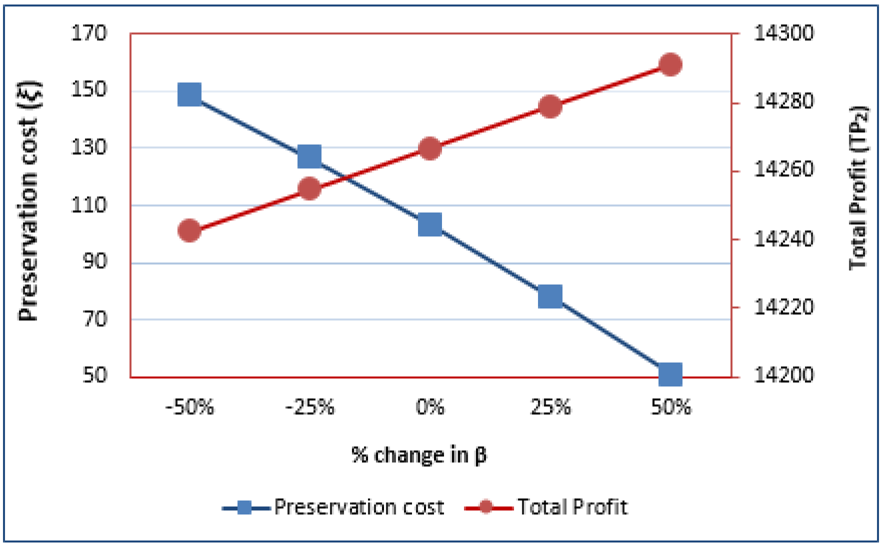


Observations from the table 5.6.2 and figure 5.6.2: When α increases, T_1^*, T^*, Q^*, TP_2^* decrease; however, the preservation cost ξ^* increase. An increment in α will increase the deterioration rate. Hence, when α increase, the spending on preservation will increase to reduce the deterioration rate. Also, the higher deterioration rate will decrease the order quantity and reduce the order cycle.

Table 5.6.3 Sensitivity table of β.

Parameter		% change	T_1^*	T^*	ξ^*	Q^*	Total Profit
β	2	-50%	1.0182	1.1147	148.3718	556.9842	14242.30
	3	-25%	0.9712	1.0662	126.4788	532.6902	14254.65
	4	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	5	25%	0.8892	0.9810	77.8408	490.1169	14278.72
	6	50%	0.8542	0.9445	50.5779	471.8297	14291.11

Figure 5.6.3 Effect of β on ξ and $TP_2(T_1, T, \xi)$.

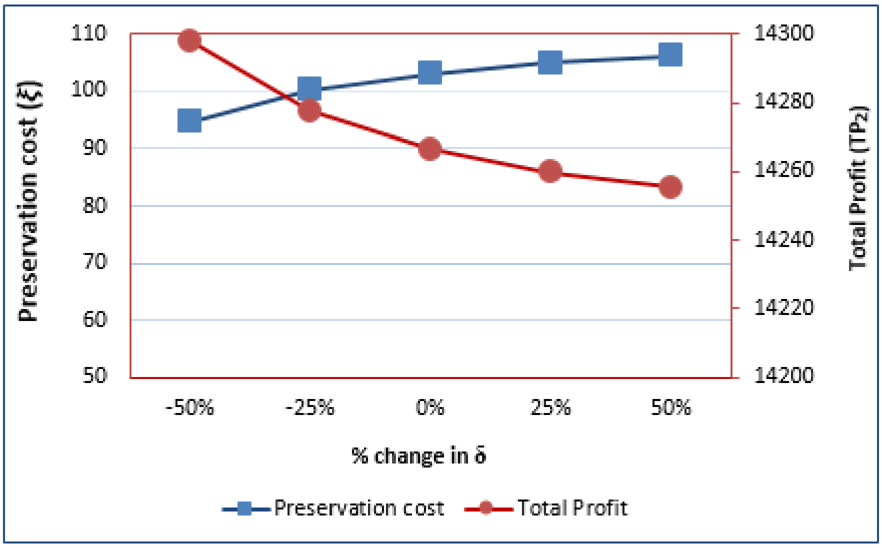


Observations from the table 5.6.3 and figure 5.6.3: When β increases, T_1^*, T^*, ξ^*, Q^* decrease; however, the total profit TP_2^* increase. When $\beta > 1$, an increment in β will increase the deterioration rate rapidly. Hence, instead of spending on preservation, reduce the order cycle and order quantity which will reduce the preservation cost and finally increase the total profit.

Table 5.6.4 Sensitivity table of δ.

Parameter		**% change**	T_1^*	T^*	ξ^*	Q^*	**Total Profit**
δ	0.2	-50%	0.8827	1.0614	94.5366	529.5348	14297.91
	0.3	-25%	0.9120	1.0347	100.1415	516.6931	14277.50
	0.4	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	0.5	25%	0.9380	1.0134	104.8621	506.4741	14259.88
	0.6	50%	0.9448	1.0081	106.1081	503.9204	14255.28

Figure 5.6.4 Effect of δ on ξ and $TP_2(T_1, T, \xi)$.

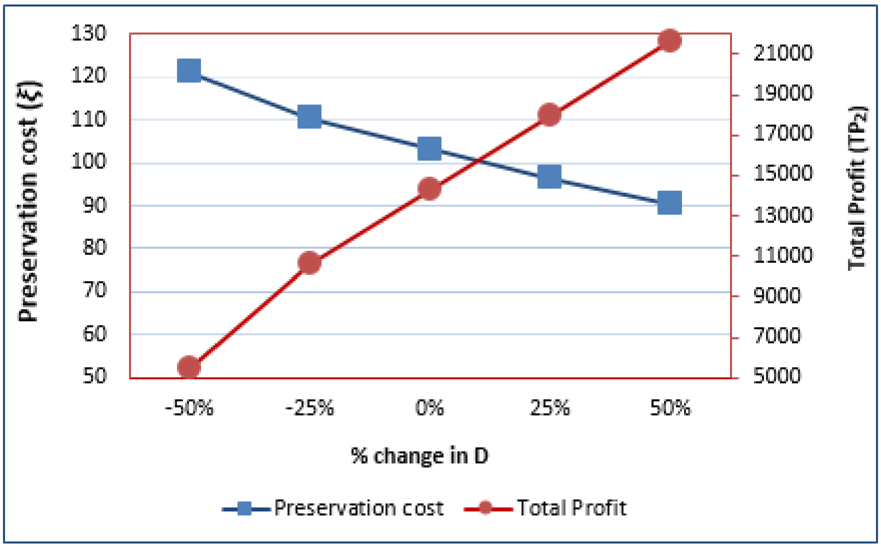


Observations from the table 5.6.4 and figure 5.6.4: When δ increases, T_1^*, T^*, Q^*, TP_2^* decrease but the preservation cost ξ^* increase. Since, $De^{-\delta(T-t)}$ is the partial backlogging rate, when δ increases the backlogging amount decrease. This implies that if customers are impatient, retailers are suggested to reduce the shortage period $(T^* - T_1^*)$. Here, the preservation technology cost increase, because the on-hand inventory period T_1^* increases.

Table 5.6.5 Sensitivity table of D.

Parameter		% change	T_1^*	T^*	ξ^*	Q^*	Total Profit
D	250	-50%	1.2496	1.4236	121.2349	284.2133	5462.11
	375	-25%	1.0267	1.1396	110.4006	426.3998	10588.24
	500	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	625	25%	0.8549	0.9359	96.4431	584.5626	17953.07
	750	50%	0.7979	0.8701	90.4512	652.2096	21645.24

Figure 5.6.5 Effect of D on ξ and $TP_2(T_1, T, \xi)$.

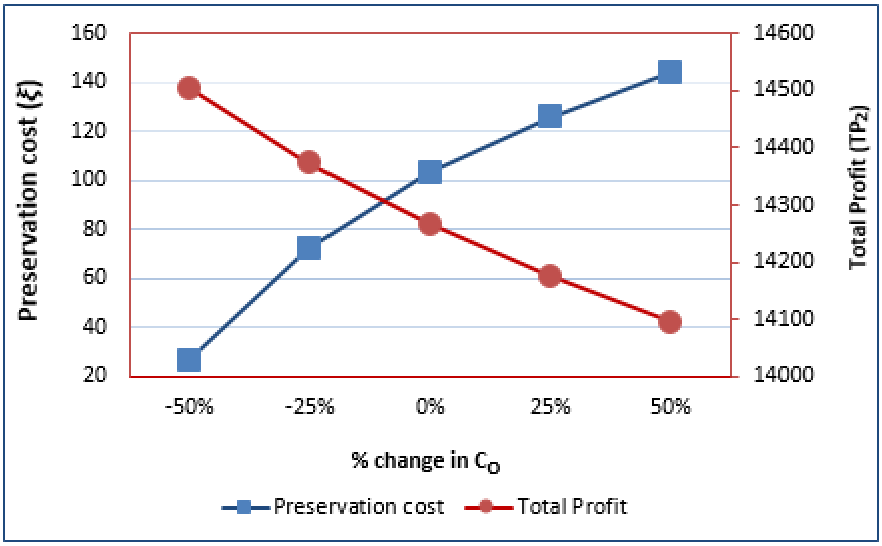


Observations from the table 5.6.5 and figure 5.6.5: When the demand D increases, the order quantity Q^* and the total profit TP_2^* drastically increases while the order cycle $\boldsymbol{T^*}$ and the preservation cost $\boldsymbol{\xi^*}$ decrease. Hence, when the demand is high the retailers are suggested to reduce the order cycle and reduce the shortage period $(T^* - T_1^*)$. High demand will reduce the on-hand inventory period T_1^* and hence the preservation cost $\boldsymbol{\xi^*}$.

Table 5.6.6 Sensitivity table of C_O.

Parameter		**% change**	$\boldsymbol{T_1^*}$	$\boldsymbol{T^*}$	$\boldsymbol{\xi^*}$	$\boldsymbol{Q^*}$	**Total Profit**
C_O	200	-50%	0.6083	0.6710	26.5774	335.3858	14504.41
	300	-25%	0.7773	0.8569	72.2869	428.2126	14373.21
	400	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	500	25%	1.0644	1.1696	125.8886	584.2533	14175.31
	600	50%	1.1897	1.3056	143.9698	652.1126	14094.50

Figure 5.6.6 Effect of C_O on ξ and $TP_2(T_1, T, \xi)$

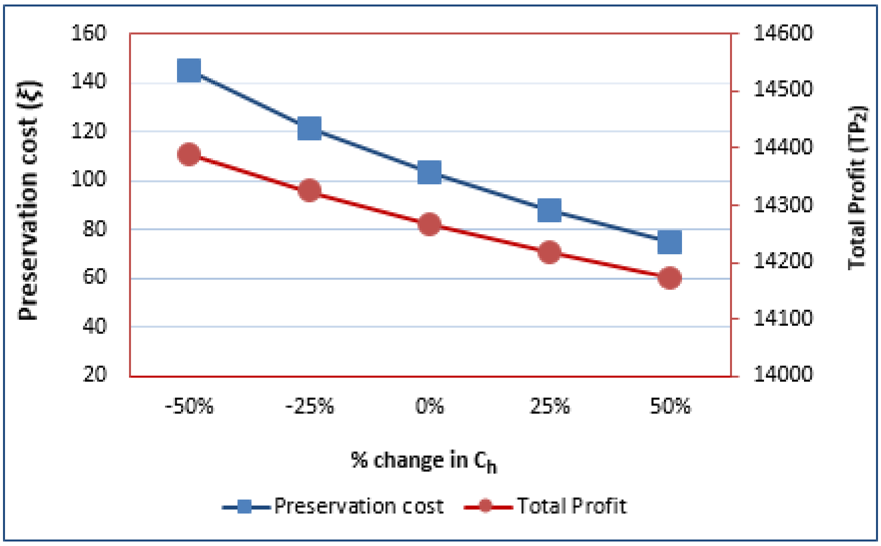


Observations from the table 5.6.6 and figure 5.6.6: When the ordering cost (C_O) increases, T_1^*, T^*, ξ^*, Q^* increase significantly; however, the total profit decrease. This implies that the high ordering cost increases the on-hand inventory period, order cycle and order quantity. In this case, there will be more loss due to deterioration. Hence, retailers are suggested to spend more money on preservation technology to reduce the deterioration rate.

Table 5.6.7 Sensitivity table of C_h.

Parameter		% change	T_1^*	T^*	ξ^*	Q^*	Total Profit
C_h	0.5	-50%	1.2018	1.2794	144.8757	639.7719	14388.06
	0.75	-25%	1.0376	1.1237	121.4024	561.6572	14322.98
	1	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	1.25	25%	0.8476	0.9475	87.8977	473.1620	14216.45
	1.5	50%	0.7849	0.8908	74.7965	444.6593	14170.97

Figure 5.6.7 Effect of C_h on ξ and $TP_2(T_1, T, \xi)$.

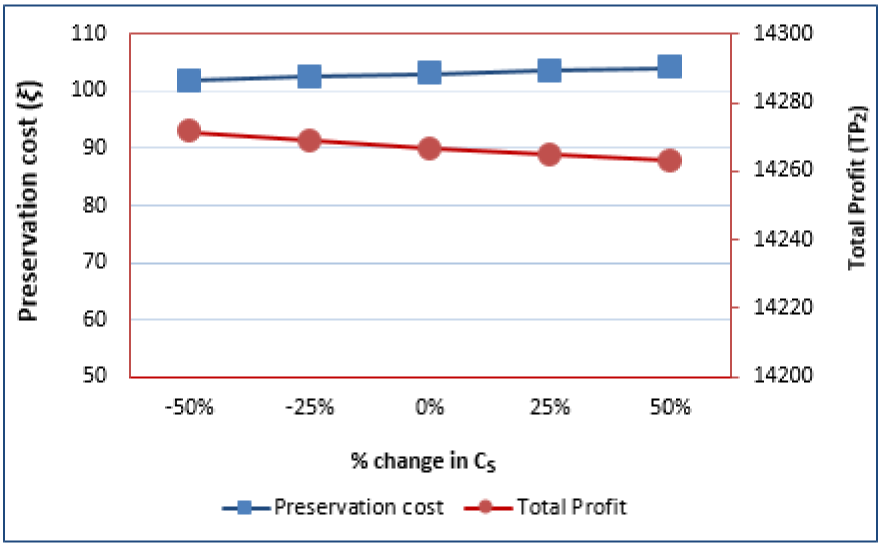


Observations from the table 5.6.7 and figure 5.6.7: When the holding cost (C_h) increases, T_1^*, T^*, ξ^*, Q^* and TP_2^* decrease. This implies that when the holding cost is high, the retailers should reduce the on-hand inventory period, and the order quantity to avoid high holding charges. Also, when the holding cost increase, the retailer is suggested to decrease the preservation cost to avoid additional cost.

Table 5.6.8 Sensitivity table of C_s

Parameter		**% change**	T_1^*	T^*	ξ^*	Q^*	**Total Profit**
C_s	5	-50%	0.9212	1.0275	101.7912	513.0992	14271.41
	7.5	-25%	0.9246	1.0241	102.4765	511.5343	14268.87
	10	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	12.5	25%	0.9308	1.0188	103.5543	509.1224	14264.64
	15	50%	0.9335	1.0168	104.0351	508.1666	14262.86

Figure 5.6.8 Effect of C_s on ξ and $TP_2(T_1, T, \xi)$

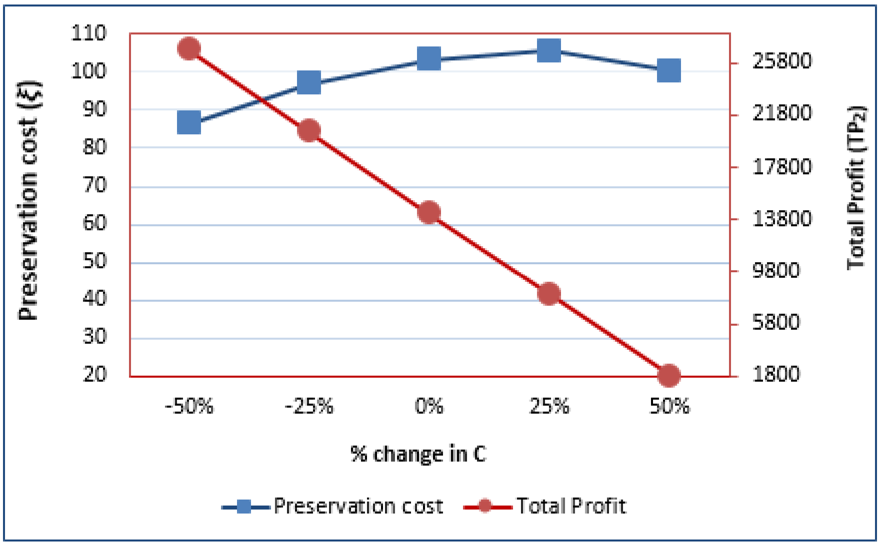


Observations from table 5.6.8 and figure 5.6.8: When C_s increases, T^*, Q^*, TP_2^* decrease while T_1^* and the preservation cost ξ^* increase. In table 5.6.8, we can observe that when the lost sale cost C_s increases, the backlogging period $(T^* - T_1^*)$ decreases. Hence, when the C_s increase, the retailers are suggested to decrease the backlogging period. Also, it can be observed that C_s has less impact on the preservation cost and the total profit.

Table 5.6.9 Sensitivity table of C.

Parameter		**% change**	T_1^*	T^*	ξ^*	Q^*	**Total Profit**
C	25	-50%	0.9560	1.0129	86.3595	507.0424	26769.54
	37.5	-25%	0.9438	1.0148	96.8049	507.5007	20515.16
	50	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	62.5	25%	0.9028	1.0379	105.5326	517.5128	8026.75
	75	50%	0.8462	1.0874	100.2528	538.3280	1810.41

Figure 5.6.9 Effect of C on ξ and $TP_2(T_1, T, \xi)$.

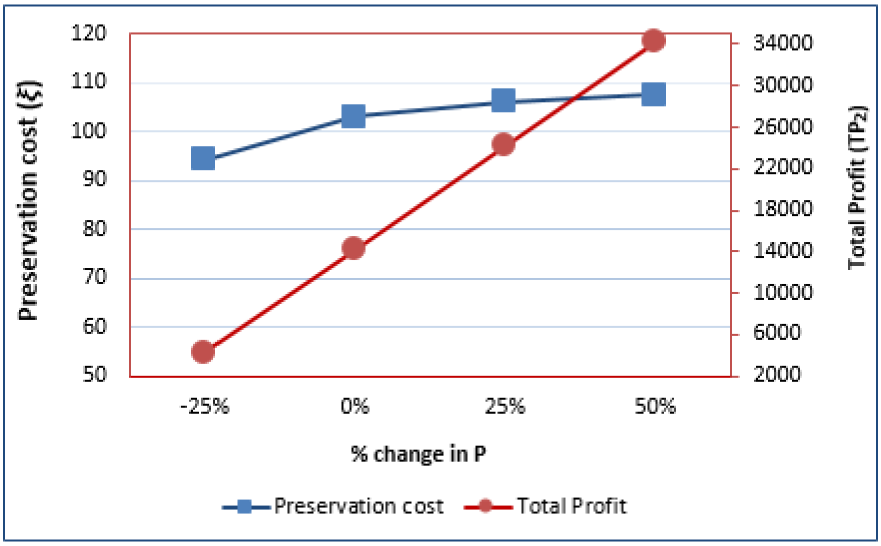


Observations from the table 5.6.9 and figure 5.6.9: When purchase cost C increases, T_1^*, T^*, TP_2^* decrease but Q^* increase while the preservation cost ξ^* increases initially but then decrease. This implies that for costly items retailers need to increase the preservation cost but when the cost (C) goes nearer to the selling price (P) the profit margin becomes less so, retailers need to decrease the preservation cost. Also, we can observe that the total cost is very sensitive with respect to cost.

Table 5.6.10 Sensitivity table of P.

Parameter		**% change**	T_1^*	T^*	ξ^*	Q^*	**Total Profit**
	40	-50%	-	-	-	-	-
	60	-25%	0.8817	1.0635	94.3505	528.9648	4298.62
P	80	0%	0.9281	1.0214	103.1003	510.3124	14266.63
	100	25%	0.9448	1.0076	106.0999	503.9054	24255.18
	120	50%	0.9536	1.0009	107.6526	500.7125	34249.29

Figure 5.6.10 Effect of P on ξ and $TP_2(T_1, T, \xi)$.



Observations from table 5.6.10 and figure 5.6.10: When the selling price P increases, T^*, Q^*, TP_2^* decreases while T_1^*, ξ^* and TP_2^* increase. That means when the selling price increases the profit margin will increase and hence the total profit TP_2^* increases. Also, when the profit margin is high, retailers should increase the on-hand inventory period (T_1^*) and reduce the shortage period $(T^* - T_1^*)$ to avoid any opportunity loss.

5.7 Conclusion

In this chapter, we optimized the preservation technology investment and inventory ordering policies simultaneously. Also investigated the effect of preservation technology investment on optimal ordering policies. Concavity of the total profit function with respect to ξ indicates that the total profit can be increased through investing in preservation technology. Both the instantaneous deterioration case model and non-instantaneous case model were developed separately. It is observed that instantaneous deterioration inventory model is a particular case of non-instantaneous deterioration inventory model. Also, it is observed that the total profit increase when the items maintain original quality for a long time (see table 5.6.1).