Synopsis of the thesis entitled

SOME CONTRIBUTIONS TO INVENTORY MODELS FOR ITEMS HAVING WEIBULL DISTRIBUTED DETERIORATION

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Introduction

Inventory means the reserved physical stock of items, maintained to accomplish the present and future demand. Here the term "items" is used in a broad sense. These items can be raw materials, semi-finished goods, finished goods, the workforce, or any resources. It is necessary for retailers, manufacturers, companies, states, countries, etc., to maintain the sufficient amount of inventory of required items. But, different costs are associated with the inventory, such as interest on capital investment, ordering cost, handling cost, storage cost, insurance cost, wastage cost, quality inspection cost, healing cost, preservation technology cost. So, an excessive amount of inventory will lead to additional costs, on the other hand, an insufficient amount of inventory will lead to the loss of goodwill and opportunity. To avoid the above two situations and to maintain an adequate amount of inventory, it is necessary to order the correct amount at the right time. So, the problem of inventory arises, which requires a solution to the following questions.

(1) How much to order?

(2) When to order?

Researchers have been developed several economic order quantity (EOQ) models and derived optimal order quantity and optimal length of the order cycle such that the total cost becomes minimum, or the total profit becomes maximum. These models are inherently based on certain assumptions and constraints regarding budget, demand, lead time, inflation, shortages, backlogging, trade credit, price discounts, planning horizon, etc.

Mathematical modelling of deteriorating inventory systems is an interesting subset of inventory modelling. Deterioration is one of the basic characteristics of almost all commodities. Deterioration means damage, decay, loss of utility, spoilage, vaporization, obsolescence, etc. Dairy products, pharmaceutical products, agricultural products, animal husbandry products, and many other products are deteriorating by nature. So, it is necessary to study the inventory systems for deteriorating items. On hand inventory for deteriorating items having demand $\boldsymbol{D}(\boldsymbol{t})$ and deterioration $\boldsymbol{\theta}(\boldsymbol{t})$ can be described by the following differential equation:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t)$$

The above equation laid a foundation for the inventory modelling of deteriorating items, which is a linear differential equation and its solution can be obtained as follows.

$$I(t) = \frac{\int -D(t)\, e^{\int \theta(t)\, dt}\ dt + C}{e^{\int \theta(t)\, dt}}$$

C is the constant and its value can be obtained by using boundary conditions. Demand rate $D(t)$ and deterioration rate $\theta(t)$ may be constant or variable. Our whole research is based on the assumption of variable deterioration, more specifically we assumed the lifetime of the item under consideration follows Weibull distribution. Our objective is to develop some inventory models for items having Weibull distribution deterioration.

Weibull distribution and deterioration:

Two-parameter Weibull distribution for lifetime t with scale parameter $\alpha\ (> 0)$ and shape parameter $\beta\ (\geq 0)$ is given by

$$f(t) = \alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}}$$

Two-parameter Weibull distribution deterioration rate function is given by

$$\theta(t) = \frac{f(t)}{1 - F(t)} = \alpha\beta t^{\beta-1}$$

Researchers often assume that the deterioration process starts as soon as the commodities enter into the inventory system but, it is not true for all commodities. Fruits, vegetables, foodstuffs have a short span of maintaining fresh quality, in which there is almost no spoilage. This phenomenon is known as non-instantaneous-deterioration. If $T_d\ (> 0)$ is the duration in which the item has no deterioration, then the three-parameter Weibull distribution with location parameter T_d is given by

$$f(t) = \alpha\beta(t - T_d)^{\beta-1}e^{-\alpha(t-T_d)^{\beta}}$$

Three-parameter Weibull distribution deterioration rate function is given by

$$\theta(t) = \alpha\beta(t - T_d)^{\beta-1}$$

Demand functions:

During our study we considered different demand functions. In chapter 2 and 3 we considered an exponential demand, in chapter 4 we considered constant demand while in chapter 5, 6 and 7 we considered advertisement and price dependent demand.

Constant demand function:

$$D(t) = D$$

Where, D is a constant.

Exponential demand function:

$$D(t) = ke^{\gamma t}$$

Where, k is a scale parameter γ is a shape parameter.

Demand function depend on frequency of advertisement and price:

$$D(A, P) = A^m a P^{-b}$$

Where, A (> 0) is the frequency of advertisement, P is the selling price, $a(> 0)$ is the scaling factor, $b(\geq 1)$ is the index of price elasticity and m is the shape parameter, where $0 \leq m < 1$. Since $\frac{\partial D(A,P)}{\partial A} > 0$ and $\frac{\partial D(A,P)}{\partial P} < 0$, the demand function is an increasing function of the advertisement frequency (A) and decreasing function of price (P), which reflect a real situation.

Preservation Technology (PT) investment:

Most of the studies on deteriorating inventory systems assumed that the deterioration rate of items is not controllable. But nowadays different preservation technologies are available to control the deterioration of products. Cooling, freezing, and drying are some methods to control the deterioration of vegetables, fruits, meat, dairy products, medicines, etc. Vacuuming, Irradiation, High pressure, Biopreservation, etc. are modern preservation technologies. Total profit can be maximized by opting preservation technology for items having significant deterioration. Also, in today's competitive environment, preservation technology investment has become necessary for producers, suppliers, and retailers to reduce the deterioration rate and economic losses due to deterioration.

We considered the proportion of reduced deterioration rate as given in Hsu et al. (2010)

$$m(\xi) = 1 - e^{-k\times\xi}$$

Where, $k(\geq 0)$ is the simulation coefficient representing the percentage increase in $m(\xi)$ per dollar increase in ξ.

When $\xi = 0$, the reduced deterioration rate $m(\xi) = 0$, and $\lim_{\xi\to\infty} m(\xi) = 1$. We can set constraint on PT investment $0 \leq \xi \leq \xi'$, where, ξ' is the maximum PT investment allowed.

In chapter 5 and 6 we considered preservation technology investment.

Permissible delay in payment (Trade credit):

One of the best practices in businesses is the supplier allows some credit period (permissible delay period) to the retailer for payment. If the retailer can't settle the account before the given credit period, then the supplier will charge interest at some rate on the remaining amount. This policy is beneficial for both the supplier and retailer. By allowing the credit period, the supplier can increase sales and potential customers by attracting and motivating new customers. The retailer can take advantage of it because it is not always true that the retailer has adequate capital. Also, the retailer can earn interest on sales revenue till settlement.

In chapter 3 and 6 we considered preservation technology investment.

Shortages and partial backlogging:

In chapter 1 and 2, shortages are not allowed, while in chapter 3-6, shortages are allowed and partially backlogged.

The fraction of unsatisfied demand backlogged is given by

$$De^{-\delta(T-t)}$$

where backlogging parameter δ is a positive constant and $(T - t)$ is the waiting time.

Literature review

Deteriorating inventory systems are studied extensively in the past few years. In traditional economic order quantity (EOQ) model the deteriorating nature of the items was not considered. The research work in the field of deteriorating inventry modeling begun with Whitin (1957), who proposed an inventory model for fashion items deteriorating at the end of prescribed storage period. Ghare and Schrader (1963) first proposed an exponential deteriorating inventory model with constant demand. Rafat (1991), Nahmias (1982), Goyal and Giri (2001), Li et al. (2010), Bakker et al. (2012), and Janssen et al. (2016) from time to time provided the detailed review of the literature on deteriorating inventory systems.

In classical EOQ models it is assumed that the retailer must pay off as soon as the items are received, that would not be always true in today's competitive business environment. In practice the supplier may provide the retailer some credit period to settle the account in order to attract and motivate new customers and increase sales. Obviously, supplier charges interest at some rate on the remaining amount if retailer is not able to settle before given credit period. Haley and Higgins (1973) investigated the relationship between inventory policy and trade credit policy in the context of the basic lot-size model for the first time. Goyal (1985) introduced an EOQ model under permissible delay in payments. He ignored the difference of selling and purchase price. Later Dave (1985) corrected Goyal's model by assuming that the selling price should be higher than its purchase price. Shah et al. (1988) studied the same model allowing shortages. Shah (1993) extended an EOQ model in which delays in payment are permissible and items in inventory deteriorate at a constant rate. Aggarwal and Jaggi (1995) extended Goyal's model for deteriorating items. Jamal et al. (1997) developed an inventory model for deteriorating items allowing shortages under permissible delay in payments. Davis and Gaither (1985) developed EOQ models for firms offering a one-time opportunity to delay in payments by their suppliers for an order of a commodity. Khouja and Mehrez (1996) discussed the problem that the supplier offers a permissible delay in payments only when the order quantity is larger than a predetermined quantity. Shinn and Hwang (2003) developed model for optimal pricing and ordering policies under order-size dependent delay in payments. Lokhandwala et al. (2005) derived optimal ordering policies under conditions of extended payment privileges. Goyal et al. (2007) introduced optimal ordering policies when the supplier provides a progressive interest scheme. Manna et al. (2008) derived optimal pricing and lot-size policies with weibull deterioration under trade credit. Ouyand et al. (2009)

considered partially permissible delay in payments inked to order quantity. The review article by Molamohamadi et al. (2014) provided a comprehensive survey of published literature for the inventory models with permissible delay in payments or trade credit. Singh and Panda (2015) considered generalized weibull deterioration rate with price dependent demand under inflation and trade credit. Pervin at al. (2016) developed a deteriorating inventory model for declining demand market under trade credit. Sundara Rajan and Uthayakumar (2017) derived optimal pricing and ordering policies for deteriorating items under trade credit and inflation over a finite planning horizon. Kumar and Kumar (2016) provided a genetic algorithm for the solution of weibull deteriorating inventory model under inflation and permissible delay in payments. Shaikh (2017) developed an inventory model for with three parameter wibull deterioration under mixed type financial trade credit. He considered that the demand is dependent on selling price and frequency of advertisement.

Most of the earlier developed models assumed constant demand, but several factors affect the demand. Selling price and advertisement are the major factors for the demand. Cohen (1977), Mukhopadhyay et al. (2004), Dye (2007), Maihami and Kamalabadi (2012), considered price-dependent demand and derived joint pricing and replenishment policies for deteriorating inventory systems. Together with price, advertisement also plays a very crucial role in sales. A regular frequency of advertisement through different mediums such as banners, newspaper, magazine, internet, radio, and television significantly increase the demand of the product. Till now, very few researchers studied the effect of advertisement policies on inventory. Kotler (1972) first incorporated marketing policies into inventory and derived an optimal marketing policy but not EOQ. Subramanyam and Kumaraswamy (1981) further extended the problem of inventory by incorporating demand as a function of price and frequency of advertisement. Also Urban (1992), Bhunia and Maiti (1997), Goyal and Gunasekaran (1995), Pal et al. (2005) studied the impact of pricing and advertisement policies on inventory policies.

Sensitivity analysis of some studies (Yang et al. (2006), Tsao and Sheen (2008), Geetha and Uthayakumar (2010)) shows that the lower deterioration rate is beneficial from an economic point of view. Hsu et al. (2010) for the first time, developed a deteriorating inventory model with constant demand and constant deterioration when the retailer invests on the preservation technology to reduce the deterioration rate of the product. Dye and Hsieh (2012) extended the model of Hsu et al. (2010) with time-varying deterioration and partial backlogging by assuming the preservation technology cost as a function of the replenishment cycle. Lee and

Dye (2012) developed a deteriorating inventory model with stock dependent demand and partial backlogging by allowing preservation technology cost. Chen and Dye (2013) and Dye and Hsieh (2013) obtained optimal solutions for deteriorating inventory models with fluctuating demand and preservation technology cost allowing trade credit. Mishra (2014) formulated an inventory model with preservation technology considering linear holding cost and linear demand. Dem and Singh (2013) formulated an integrated production inventory model with preservation technology. They considered single-manufacturer-single buyer supply chain problem with fluctuating demand and the concept of preservation technology on the buyer's side.

Researchers often assume that the deterioration process starts as soon as the commodities enter into the inventory system but, it is not true for all commodities. Fruits, vegetables, foodstuffs have a short span of maintaining fresh quality, in which there is almost no spoilage. This phenomenon is known as non-instantaneous-deterioration. Dye (2013) studied the effect of preservation technology investment on a non-instantaneous deteriorating inventory model, also established several structural properties on finding the optimal replenishment and preservation technology strategies. Mishra (2014) developed a deteriorating inventory model for non-instantaneous deteriorating items with linear demand and holding cost considering preservation technology cost when deterioration period starts. Tsao (2016) developed a non-instantaneous deteriorating inventory model for joint location, inventory, and preservation decisions under permissible delay in payments. Bardhan et al. (2017) developed a non-instantaneous deteriorating inventory model with stock dependent demand and preservation technology. Pal et al. (2017) derived optimal inventory policies for non-instantaneous deteriorating products with preservation technology and constant demand, assuming random deterioration start time.

Summary of research work

Our work is divided into six chapters, **Chapter 1-6**. C**hapter 1** includes an introduction and a detailed literature review. Our contribution to the inventory models for items having Weibull distribution deterioration is presented in **Chapter 2-6**.

In **chapter 2**, we derived an inventory model for Weibull deteriorating items with exponential demand and time-varying holding cost. Sensitivity analysis shows how the total profit is affected by different parameters. The solution of the given example is obtained by using **optimize** function in **R programming**. This is a simplest model without shortages, preservation technology investment, trade credit, and price and advertisement dependent demand. Paper based on this chapter is **published** in the journal **Global and Stochastic Analysis (**ISNN: **2248-9444)** in 2016.
https://www.mukpublications.com/resources/6_rapolu.pdf

In **chapter 3,** an inventory model is developed for Weibull deteriorating items when the demand is exponential. We assumed the supplier provides some credit period to settle the account, and beyond the credit period, he will charge a certain rate of interest on the outstanding amount. Linear holding cost is considered, and the deterioration cost is taken as the difference between purchasing price and salvage value. A numerical example is provided, and sensitivity analysis made to check the effect of various parameters on the inventory system. We found that as permissible delay period increase, the corresponding cost decrease. That is, as suppliers allow more days to settle the account, the retailer can earn more profit through sales revenue. Also, the supplier can attract and motivate new customers to increase his sales. Paper based on this chapter is **published** in the journal **IAPQR Transactions** (ISNN: **0970-0102**) in 2017.

In **chapter 4**, we extended the developed model of chapter 3 allowing shortages and partial backlogging with price and advertisement frequency dependent demand. We considered the price, frequency of advertisement, length of order cycle and shortage period as decision variables and found optimal values simultaneously to maximize the profit. So, this model provides joint pricing, advertisement and inventory ordering policies to the retailer to maximize the total profit. An algorithm is provided to get the optimal solution. The solution is obtained by using **DEoptimR** package in **R programming**. This package uses the differential evolution stochastic algorithm and gives the approximate global optimum solution.

In **chapter 5**, we formulated an inventory model for items, having Weibull distribution deterioration by allowing preservation technology investment. We assumed the demand is constant, and it is possible to reduce the deterioration rate of the item by investing in preservation technology. Further, shortages are allowed and partially backlogged. Both instantaneous and non-instantaneous cases are considered. This model provides optimal preservation technology investment and optimal ordering policies to the retailer to maximize the total profit. Paper based on this chapter is **published** in the journal **International Journal of Statistics and Reliability Engineering.** ISSN: 2350-0174 (Print), 2456-2378(Online). http://ijsreg.com/index.php/ijsre/article/view/499

In **chapter 6**, we developed an inventory model for joint pricing, advertisement, preservation technology investment and inventory policies for non-instantaneous deteriorating items under trade credit. This is an extended model of the developed model of chapter 4 allowing preservation technology investment for non-instantaneous deteriorating items. This model reveals that instantaneous deteriorating items need more PT investment and the profit for the non-instantaneous deteriorating items is more than the profit for instantaneous deteriorating items. When the advertisement cost is less, the retailer can earn more profit through increasing advertisement frequency. Paper based on this chapter is **published** in the journal **Opsearch** (ISNN: **0030-3887**) in 2019. https://doi.org/10.1007/s12597-019-00427-7

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