Chapter 3

Sigma Level and Process Shift: A SSM Perspectives

Complete understanding of Six Sigma approach is not possible without focusing upon sigma level and process shift. As explained in Chapter 1 sigma level of the process is number of standard deviations covered between process centres to nearest specification limit. And process shift is drifting of process mean over long run that result into higher defective units. This chapter examines both these aspects in detail. Thereafter, we discuss how these concepts are important in SSM.

3.1 Sigma estimation

Since sigma level of the process is based on process standard deviation, it is important to understand different estimation methods of standard deviation. Muralidharan and Raval (2012) proposed different estimators for process standard deviation. Majority of standard deviations estimators are sensitive to normality assumption. Of estimators can be suggested for the estimation of standard deviation (sigma) ranging from traditional square distance estimator of standard deviation to score estimators.

For the observations, $X = (X_1, X_2, X_3 .. X_n)$ the most common estimators of sigma is

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2} \tag{3.1}$$

Where $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

Equation (3.1) can also be written as,

$$\sigma = \sqrt{\frac{1}{n(n-1)}\sum_{i<j}\left(X_i - X_j\right)^2} \tag{3.2}$$

The sample variance s^2 is the minimum variance unbiased estimator for the population variance σ^2. However, sample standard deviation is a biased estimator of underlying population standard deviation. A vague estimator of process standard deviation can be defined in terms of Range as the difference between highest and lowest observation. Unfortunately this estimator is highly influenced by extreme values and hence is less useful.

The most commonly used robust scale estimator is the Inter Quartile Range (IQR) which is defined as,

$$IQR = \frac{1}{2}(Q_3 - Q_1) \tag{3.3}$$

Where Q_1 and Q_3 are respectively the first and third quartiles. An estimator similar to (2) is given by Gini's mean difference which is defined as,

$$G(X) = \frac{2}{n(n-1)} \sum_{i<j} |X_i - X_j| \tag{3.4}$$

Use of absolute value in above estimator instead of squared distance reduces the impact of large distance. But this statistic is also not very robust. Another important estimator of standard deviation is given by Median Absolute Deviation (MAD) as,

$$MAD(X) = median_i |X_i - median_j(X_j)|\} \tag{3.5}$$

Two alternate estimators of MAD is given as S_n and Q_n,

$$S_n = median_i \{median_j |X_i - X_j|\} \tag{3.6}$$

$$Q_n = median_i \{ |X_i - X_j|; i < j \}_{(k)} \tag{3.7}$$

Which is the k^th^ largest of the $|X_i - X_j|$ for i<j, where $k = \binom{[n/2]+1}{2}$.Thus Q_n is the k^th^ order statistics of the $k = \binom{n}{2}$. The Score estimator for scale is defined as,

$$S_c = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \frac{1}{(1 + Z_i^2)} (X_i - M)^2} \tag{3.8}$$

Where $Z_i = \frac{X_i - M}{\sigma_0}$, M is an auxiliary estimate of location and σ_0 is an auxiliary estimate of scale. Muralidharan and Raval (2012) provided efficiency comparison between all above estimators for normal and non normal data given in Table 3.1.

Table 3.1. *Sigma Estimators and their efficiencies*

Formula	3.1	3.2	3.3	3.5	3.6	3.7	3.8
Efficiency (Normal data)	100%	100%	97.41%	98.38%	88.18%	72.68%	102.79%
Efficiency (Non-normal data)	100%	100%	83.85%	87.98%	61.22%	56.49%	97.62%

3.2. Sigma level estimation

Method through which process variation can be measured is known as sigma level of the process. Sigma level of the process measures number of standard deviation process can produce between process mean and nearest specification limit. Estimation of sigma level depends upon type of variable you are dealing with which is discussed below.

3.2.1 Sigma level estimation for discrete distribution

Calculation of sigma level for discrete distribution is based on DPMO approach. This approach involves concept of unit, opportunity and defects. *Unit* is any product, part, assembly, process or service for which quality is desired. *Opportunity* is a value added feature of a unit that should meet specifications proposed by customer. *Defect* is the characteristic of the product that fails to meet customer requirements and *Defectives* are the total number if units containing different types of defects. Thus, *total opportunity* (TOP), *defect per unit* (DPU) and *defect per million opportunities* (DPMO) is defined as,

TOP= Number of units checked*Number of opportunities of failure (3.9)

$$\mathrm{DPU} = \frac{\text{No.of Defects}}{\text{No.of units checked}} \tag{3.10}$$

And

$$\text{DPMO} = \frac{\text{DPU}}{\text{No.of opportunity per unit}} * 10^6 \tag{3.11}$$

Then,

$$\sigma_{LT} = \Phi^{-1}\left(1 - \frac{DPMO}{10^6}\right) \tag{3.12}$$

As an illustration, consider, 250 units are selected from the production process and each are tested for 20 possible opportunities, then from (3.9), TOP = 250*20 = 5000. If suppose, 75 defects are observed, then, the defects per unit is $\text{DPU} = \frac{75}{250} = 0.3$, and the $\text{DPMO} = \frac{0.3}{20} * 10^6 = 15000$ according to (3.10) and (3.11) respectively. That is, process fails to meet specification for 15000 opportunities out 1,000,000 opportunities. Hence, from (3.12), through normal inverse long term sigma level of the process is 2.17σ.

3.2.2 Sigma level estimation for continuous distribution

Calculation of sigma level for continuous distribution is based on capability analysis and inverse standard normal distribution. With customer proposed *upper specification limit* (USL) and *lower specification limit* (LSL), capability indices are defined as,

$$C_p = \frac{USL - LSL}{6\sigma} \tag{3.13}$$

$$C_{p_k} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right) \tag{3.14}$$

We can obtain sigma level of the process through corresponding z-score,

$$\text{Upper sigma level}(Z_{USL}) = \frac{USL - \mu}{\sigma} \tag{3.15}$$

$$\text{Lower sigma level}(Z_{LSL}) = \frac{\mu - LSL}{\sigma} \tag{3.16}$$

Consider the following data: 25, 28, 32, 38, 29, 36, 29, 28, 26, 29. Suppose, the USL and LSL of the process are given as 25 and 40 respectively. If the process mean (μ) and standard deviation (σ) are 30 and 5 respectively, then from (3.15), we get the $(Z_{USL}) = \frac{USL - \mu}{\sigma} = \frac{40-30}{5} = 2$ and from (3.16), we get $(Z_{LSL}) = \frac{\mu - LSL}{\sigma} = \frac{30-25}{5} = 1$.

Now suppose a continuous measurement of a process is given with its upper and lower specification values. The long term sigma level of such a process is obtained by computing $\Phi^{-1}[\{P(Z < Z_{LSL}) + P(Z > Z_{USL})\}]$.

For the process discussed above, if the long term sigma level is

$$\Phi^{-1}[\{P(Z < Z_{LSL}) + P(Z > Z_{USL})\}] = \Phi^{-1}[\{P(Z < 1) + P(Z > 2)\}]$$

$$= \Phi^{-1}[0.841 + 0.022] = 1.09$$

Hence, short term sigma level (Z_{ST}) of the process is 1.09+1.5 = 2.59.

3.3 Sigma-shift estimation

To account for certain process variations the proposed concept of 6σ later degraded to 3.4 DPMO with reference to 1.5σ shift in long run. In order to make it comparable over different CTQs, σ level is converted into corresponding Z value.

Estimate of Z_{ST} (short term sigma level) is obtained by adding 1.5 to Z_{LT} (long term sigma level) as given in equation (3.17)

$$Z_{ST} = Z_{LT} + 1.5 \tag{3.17}$$

Hence, 1.5σ shift inflate the number of defects depends on off centring of the process, it is important to understand 1.5σ shift in detail. Harry (2003) consider 1.5σ shift as a one of the mystical pillar of Six Sigma. Effect of 1.5σ shift is indicated in Fig 3.1 and corresponding DPMO are explained in Table 3.2.

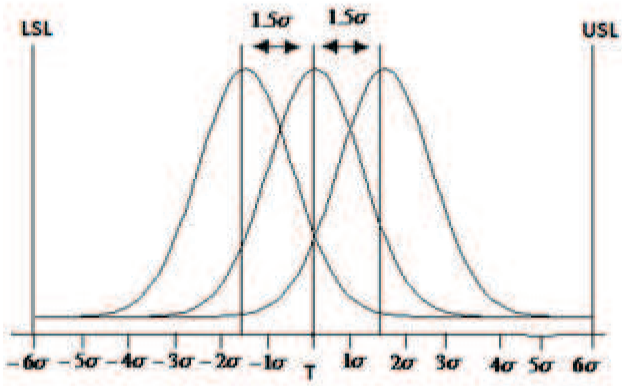

Fig 3.1. *Sigma shift of distribution*

Table 3.2. *DPMO for targeted and shifted distribution*

Specification limit	**Percentage within specification**		**DPMO**	
	Targeted distribution	Shifted distribution	Targeted distribution	Shifted distribution
$\pm 1\sigma$	68.27	30.85	317310	691462
$\pm 2\sigma$	95.45	69.14	45500	308537
$\pm 3\sigma$	99.73	93.32	2699	66807
$\pm 4\sigma$	99.994	99.379	63.34	6210
$\pm 5\sigma$	99.99994	99.9767	0.57	233
$\pm 6\sigma$	99.9999998	99.99966	0.002	3.4

1.5σ shift has its origin in the world of control charts. Understanding of 1.5σ shift requires the understanding of the connection between variation, producibility, process capability and tolerance. Smith identified the connection between how well product did in the field versus how much rework had been required during production. In order to cope up with Japanese quality, Smith proposed to follow 50% design margins for all the key product performance specifications of Motorola, compare to 25% “cushion” of the period (Harry, 2003). Smith believed that 25% cushion” was not sufficient for absorbing a sudden shift in process centring. Figure 3.2 represents Bill’s assertion about 50% design margin. However, at that time Bill’s assertion was statistically undefended. Later on Harry (2003) form the statistical foundation of Six Sigma methodology.

According to Six Sigma philosophy, the variation in Six Sigma model is defined through causal relation as given in equation (3.18)

$$Y = f(X_1, X_2, X_3 \ldots X_n) + \varepsilon \tag{3.18}$$

Here Y is response variable, $X_i's$ are causative variables, ε is error in prediction of Y. In order to examine variation in Y it is important to adopt such a rational sampling plan that fully captures white noise while preserving the effect of assignable causes. Only after removing effect of special causes of variation, it is possible to estimate variation in Y through instantaneous producibility. This can be done by isolating random transient effect and temporal effect.

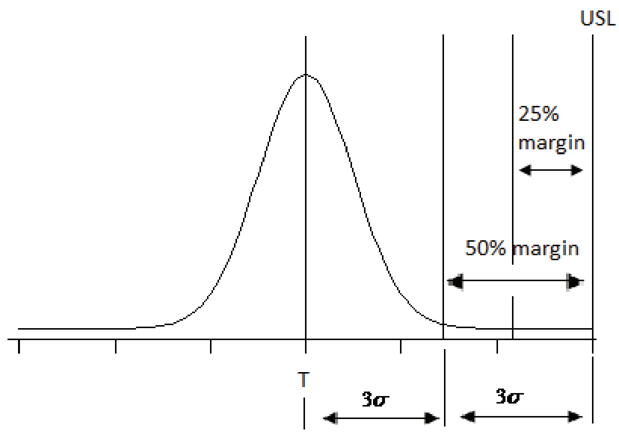

Fig 3.2. *Increased design margin*

Hence, shift of the distribution of X_i away from their normal specifications cause shift in the distribution of response variable Y. This in turn increases the probability of non conformance. Mean off set can be expressed as equivalent inflation (c) of standard deviation. The mean off set equivalent to inflated standard deviation is shown Figure 3.3.

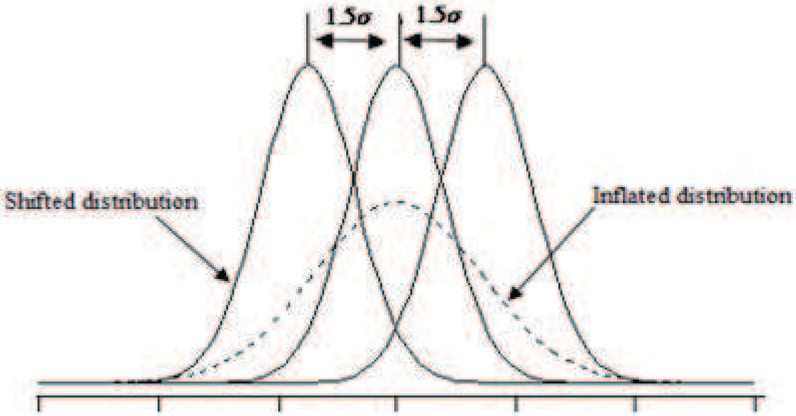

Fig 3.3. *Inflated distribution with equivalent mean shift*

3.3.1 ANOVA approach

Here we discussed a technically sound method of understanding the shift through the Analysis of Variance (ANOVA) approach, which is described as follows: Consider a one-way classification of observations as,

Treatment-1	Treatment-2	…	Treatment-*r*
X_{11}	X_{21}	…	X_{r1}
X_{12}	X_{22}	…	X_{r2}
…	…	…	…
X_{1s}	X_{2s}	…	X_{rs}

A suitable model to represent the above classification is

$$X_{ij} = \mu + \alpha_i + \varepsilon_{ij} \tag{3.19}$$

Where μ is the mean of the population for all treatments and α_i is the effect due to *i*-th treatment. Then the null hypothesis that all treatment means are equal is given by

$$H_0 = \alpha_1 = \alpha_2 = \ldots \alpha_r = \alpha$$

If H_0 is true, we conclude that all treatment means are same. In such a case there is just one treatment population, and chances of shift between populations may be a resultant of chance causes only. If hypothesis is rejected, then we conclude that at least one of the mean is shifted. To test the above hypothesis, we use F-test for equal means, and the test statistic is given as

$$F = \frac{\hat{S}_B^2}{\hat{S}_W^2} \tag{3.20}$$

which has F-distribution with r-1 and r(s-1) degrees o freedom. Where $\hat{S}_B^2$ is the mean square variation between treatments and $\hat{S}_W^2$ the mean square variation within treatments (see Table 3.2. below). The hypothesis of equal means is rejected if $F > F_{\alpha,(r-1,r(s-1))}$. The quantities $\hat{S}_B^2$ and $\hat{S}_W^2$ are calculated as follows:

Let

$$T = \sum_{i=1}^{r}\sum_{j=1}^{s} X_{ij}, \quad \overline{X} = \frac{T}{rs}, \quad T_{i.} = \sum_{j=1}^{s} X_{ij}, \quad \overline{X}_{i.} = \frac{T_{i.}}{S}$$

Then,

$$\text{Total variation} = V_T = \sum_{i=1}^{r}\sum_{j=1}^{s}\left(X_{ij} - \overline{X}\right)^2$$

$$= \sum_{i=1}^{r}\sum_{j=1}^{s} X_{ij}^2 - \frac{T^2}{rs} \tag{3.21}$$

Since, $X_{ij} - \overline{X} = \left(X_{ij} - \overline{X}_{i.}\right) + \left(\overline{X}_{i.} - \overline{X}\right)$

Therefore,

$$\sum_{i=1}^{r}\sum_{j=1}^{s}\left(X_{ij} - \overline{X}\right)^2 = \sum_{i=1}^{r}\sum_{j=1}^{s}\left(X_{ij} - X_{i.}\right)^2 + \sum_{i=1}^{r}\sum_{j=1}^{s}\left(X_{i.} - \overline{X}\right)^2$$

$$= \sum_{i=1}^{r}\sum_{j=1}^{s}\left(X_{ij} - \overline{X}_{i.}\right)^2 + s\sum_{i=1}^{r}\left(\overline{X}_{i.} - \overline{X}\right)^2$$

That is,

Total variation = variation within treatments + variation between treatments

$$V_T = V_W + V_B$$

Where

$$V_W = \sum_{i=1}^{r}\sum_{j=1}^{s}\left(X_{ij} - \overline{X}_{i.}\right)^2 \tag{3.22}$$

$$V_B = s\sum_{i=1}^{r}\left(\overline{X}_{i.} - \overline{X}\right)^2 = \frac{1}{s}\sum_{i=1}^{r}T_{i.}^2 - \frac{T^2}{rs} \tag{3.23}$$

The one way analysis of variance for equal number of observations is presented in Table 3.3. In the table, the *mean sum of squares* (MSE) is obtained by dividing the sum of squares by their corresponding degrees of freedom.

Table 3.3. *One way ANOVA for equal number of observations*

Source of variation	**Degrees of freedom**	**Sum of Squares**	**Mean sum of squares**	***F*-ratio**
Between treatment	r -1	$V_B = s\sum_{i=1}^{r}\left(\overline{X}_{i.} - \overline{X}\right)^2$	$\hat{S}_B^2 = \frac{V_B}{r-1}$	$\hat{S}_B^2 / \hat{S}_W^2$
Within treatment	$r(s$-1)	$V_W = \sum_{i=1}^{r}\sum_{j=1}^{s}\left(X_{ij} - \overline{X}_{i.}\right)^2$	$\hat{S}_W^2 = \frac{V_W}{r(s-1)}$	
Total	$rs - 1$	$V_T = \sum_{i=1}^{r}\sum_{j=1}^{s}\left(X_{ij} - \overline{X}\right)^2$		

Note that $V_T/(rs-1)$ is an estimate of the long-term variance $(\sigma_{LT})$and $V_W/r(s-1)$is an estimate of the short-term variance (σ_{ST}).Then the ratio $c = \sigma_{LT}/\sigma_{ST}$ is the ratio of root mean squares (RMS) is a hybrid performance metric that provides us with an insight into how much ‘dynamic centring error’ is occurring in the process over the total period of sampling. When c = 0, then $V_B = 0$, but only after adjusting for differences in degrees-of-freedom, and when c > 0, (i.e. $\sigma_{LT} > \sigma_{ST}$), then $V_B > 0$. Thus c is a measure of capability related to process centring. Given the circumstance that $V_B = 0$, it is quite natural to recognize that all of the subgroup means would necessarily be identical in value, a situation called *state of perfection* of the given process. Now, let us estimate the ratio c as,

$$\hat{c} = \frac{\sigma_{LT}}{\sigma_{ST}} = \frac{\sqrt{V_T/(rs-1)}}{\sqrt{V_w/r(s-1)}}$$

$$= \sqrt{\frac{r(s-1)[V_B + V_W]}{(rs-1)V_W}}$$

$$= \sqrt{\frac{r(s-1)}{(rs-1)}\left[1 + \frac{V_B}{V_W}\right]} \tag{3.24}$$

$$= \sqrt{\frac{r(s-1)}{(rs-1)}\left[1 + \frac{(r-1)}{r(s-1)}\frac{V_B/(r-1)}{V_W/(r(s-1))}\right]}$$

$$= \sqrt{\frac{r(s-1)}{(rs-1)}\left[1 + \frac{(r-1)}{r(s-1)}F\right]}$$

$$= \sqrt{\left[1 + \frac{(F-1)(r-1)}{r(s-1)}\right]} \tag{3.25}$$

From (3.24)

$$\hat{c}^2 = \frac{r(s-1)}{(rs-1)}\left[1 + \frac{V_B}{V_W}\right]$$

$$\Rightarrow 1 + \frac{V_B}{V_W} = \frac{(rs-1)}{r(s-1)}\hat{c}^2$$

$$\Rightarrow V_B = V_W\left[\frac{(rs-1)}{r(s-1)}\hat{c}^2 - 1\right]$$

$$\Rightarrow s\sum_{i=1}^{r}\left(\overline{X}_{i.} - \overline{X}\right)^2 = \frac{V_W}{r(s-1)}\left[(rs-1)\hat{c}^2 - r(s-1)\right]$$

$$= \sigma_W^2\left[(rs-1)\hat{c}^2 - r(s-1)\right]$$

$$\Rightarrow \frac{1}{r}\sum_{i=1}^{r}\left(\overline{X}_{i.} - \overline{X}\right)^2 = \frac{\sigma_W^2}{rs}\left[(rs-1)\hat{c}^2 - r(s-1)\right]$$

$$\Rightarrow \sqrt{\frac{1}{r}\sum_{i=1}^{r}\left(\overline{X}_{i.}-\overline{X}\right)^2} = \sigma_W\sqrt{\frac{1}{rs}\left[(rs-1)\hat{c}^2 - r(s-1)\right]}$$

$$\Rightarrow \frac{1}{\sigma_W}\sqrt{\frac{1}{r}\sum_{i=1}^{r}\left(\overline{X}_{i.}-\overline{X}\right)^2} = \sqrt{\frac{1}{rs}\left[(rs-1)\hat{c}^2 - r(s-1)\right]}$$

$$\Rightarrow Z_{shift} = \sqrt{\frac{1}{rs}\left[(rs-1)\hat{c}^2 - r(s-1)\right]} \qquad (3.26)$$

Note that for c = 1, (3.26) reduces to $Z_{shift} = \sqrt{(r-1)/rs}$. According to Harry (2003) it is highly desirable to set $Z_{shift} = 0$ for c = 1. When c = 1.5, the value of Z_{shift} will be 1.5. Also as $rs \to \infty$, Z_{shift} asymptotically approaches to the quantity c^2-1. Typically, the range of r and s are considered to be $25 \le r \le 100$ and $4 \le s \le 6$. The best commonly employed combination is that of r = 50 and s = 5. Z_{shift} values for different combination of rs is shown in Figure 3.4.

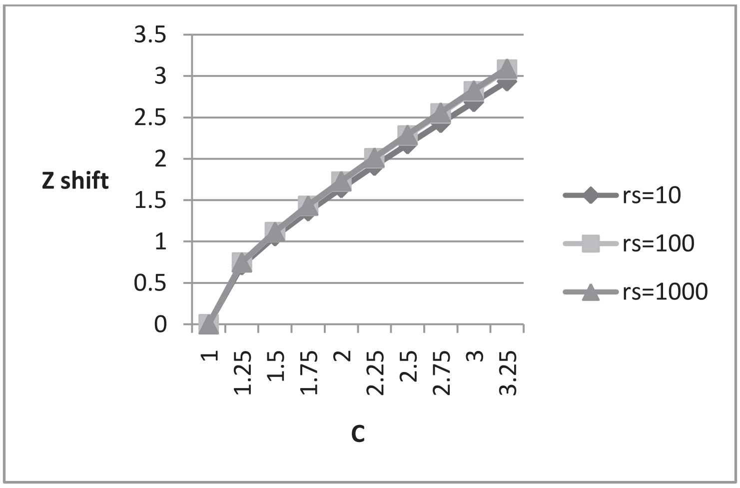

Fig 3.4 Z_{shift} *for c=1 to 3 and different combination of rs*

3.3.2 Equivalence of distributions

Now consider any two set of treatments T_A and T_B having same or different samples. If $H_0 : \mu_A = \mu_B = \mu$ is rejected, then it is obvious that the mean is shifted. Suppose for convenience, the treatment B is inflated with respect to the treatment A, then the total variation of treatment B is explained in terms of A as

$\mu + 3\hat{\sigma}_B = \mu + 3\hat{\sigma}_A + Z_{shift}\hat{\sigma}_A$

$$Z_{shift} = \frac{3(\hat{\sigma}_B - \hat{\sigma}_A)}{\hat{\sigma}_A} \tag{3.27}$$

$$Z_{shift} = 3(\hat{c} - 1)$$

where,

$$\hat{c} = \frac{\hat{\sigma}_B}{\hat{\sigma}_A} \tag{3.28}$$

and is called the dynamic correction used to adjust or compensate the influence of random sampling error over a protracted period of time or many cycles of operation. Again, if $\sigma_A^2 = \sigma^2$ is known and fixed, then $c^2 = \sigma_B^2 / \sigma^2$ follows a Chi-square distribution with n-1 degrees of freedom. That is

$$\left(\frac{\hat{\sigma}_B}{\sigma}\right)^2 \sim \frac{n-1}{\chi^2}$$

That is

$$\hat{\sigma}_B \cong \sigma\sqrt{\frac{n-1}{\chi_\alpha^2}}$$

Therefore,

$$\hat{\sigma}_B \cong \hat{\sigma}_A\sqrt{\frac{n-1}{\chi_\alpha^2}}$$

$$\hat{\sigma}_B \cong \hat{c}\hat{\sigma}_A$$

where,

$$\hat{c} = \sqrt{\frac{n-1}{\chi_\alpha^2}} \tag{3.29}$$

For various levels of α ,one can visualize the shift happening in one variable with respect to the other. For example: Suppose n=30, $\hat{\sigma}_A = 280$, and $\hat{\sigma}_B = 376$.Then $\hat{c} = \hat{\sigma}_B / \hat{\sigma}_A = 1.343$ using (3.28) and $\hat{c} = \sqrt{29/16} = 1.346$ for α =0.025 using (3.29). The Z_{shift} according to (3.26) where r=2 and s=n=30 is 0.99.

3.3.3 Tolerance analysis based approach

Evolution of Tolerance analysis is found in the concept of interchangeability, which focuses upon replacing damaged assembly piece hence making assembly parts interchangeable. Buckingham (1921) examines interchagability over four different phases:

- Design a product: Manufacturing design determines the success and failure of a project. Construction of mechanism that works properly is the major objective of this phase. Perfect manufacturing design is aimed to initiate large scale production and this designing phase last throughout production to examine issues that might be difficult to foreseen in advance.
- Manufacturing model: Testing a model developed in design phase is the focus of this phase. Creating a physical model of accepted design to persuade large scale production.
- Clearance: It is an important consideration in developing manufacturing design. Clearance is the allowable space between operating parts. Minimum clearance should be as small as assembling parts and their proper operation. Maximum clearance should be as great as the functioning permits. Design allowing higher clearance between parts results in to higher degree of interchangeability and greater economic benefits.
- Manufacturing tolerance: Manufacturing tolerance is about trying to hold the product as closely as possible to a fix size. Though practically tolerance value often exceeds its decided design value, it is important to note it down since sometimes it match with its design value.

The issue of 1.5σ shift is discussed extensively in Tolerance analysis literature which is a part of mechanical engineering. Bender (1968), Gilson (1951), Evans (1975) discussed 1.5σ shift in the early tolerance literature. Two important methods discussed in the literature are the Worst case and root sum square (RSS) techniques. The worst case method is based on the consideration that component dimension are taking extreme values.

The concept of tolerance analysis is again based on the model presented in (3.18), where i^{th} product characteristic X_i has upper specification limit (U_i) and lower specification limits (L_i) indicating worst scenario, tolerance for i^{th} product characteristic will be,

$$T_i = U_i - L_i \tag{3.30}$$

According to worst case model, the assembly tolerance, T_a is given by

$$T_a = \sum_{i=1}^{n} T_i \tag{3.31}$$

Worst case tolerance analysis method is considered as the most widely used method from design engineering perspective (Kuo and Tsai, 2011). This method can be used if all product component characteristics are within allowable variability and few rejections are not permissible (Chase and Greenwood, 1988). Since there are very less chance of all product components characteristics to fall at extreme tolerance limits, this method is considered as most pessimistic method.

Root sum square (RSS) method is based on the assumption of statistical distribution of components and generally but not restricted to normal distribution. This method further assumes that component characteristics are within statistical control and process mean is at the centre of specified tolerance. RSS model is provided in (3.32). This method is based on tolerance range corresponds to ±3σ range of component distribution.

$$T_a = \sqrt{T_1^2 + T_2^2 + T_3^2 + \ldots T_n^2} = \left(\sum_{i=1}^{n} T_i^2\right)^{1/2} \tag{3.32}$$

Issues with root sum square model is that, if all component characteristics may not be in control state which leads to more predicted out of control assemblies, then all component characteristics may not be approximated through normal distribution and process centre may not lie exactly at the centre of specified tolerance.

Mansoor (1963) propose the model combining both worst case and root sum square part. He proposed probabilistic model based on requirement of predicting probable limits of assembly dimension or assignment of component tolerance based on assembly requirement. This need can be cater through selecting appropriate theoretical distribution for component tolerance. Depending upon the shape of distribution the author provide estimate of probable sum of component tolerances (T_{prob}) as,

$$T_{\text{Prob}} = c\left(\sum_{i=1}^{n} T_i\right)^{1/2} \tag{3.33}$$

Here value of constant c depends upon the shape of the distribution. Mansoor (1963) proposed theory based on “relative precision index” (RPI), which examines the matching between natural process tolerance (±3.09 times the distribution standard deviation) and specified tolerance interval.

According to this model tolerance specifications of different dimensions are denoted by $T_1, T_2, \ldots T_n$ and natural process tolerance of dimensions are indicated by t_1, t_2,...t_n.

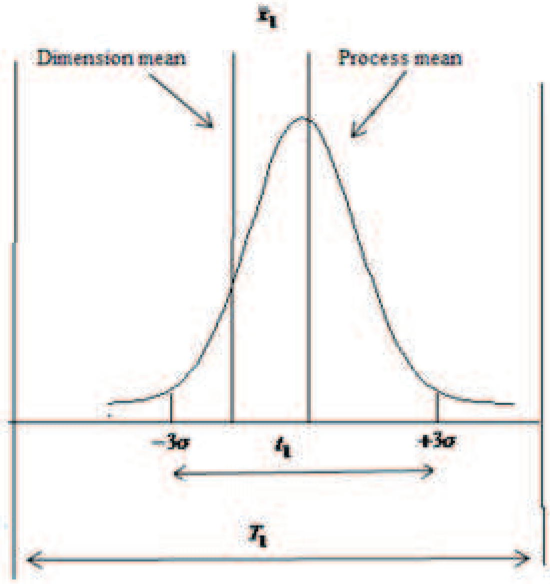

Fig 3.5. *biasness in dimensional position of natural process tolerance*

Biasness in dimensional position of natural process tolerance for first component is denoted by $\bar{x}_1$ similarly biasness in dimensional position of natural process tolerance of rest of the components given by $\bar{x}_2, \bar{x}_3, \ldots \bar{x}_n$. Figure 3.5 shows biasness for fist assembly component, where biasness is the difference between midpoint of natural process tolerance and specified dimension tolerance given as below:

$$\bar{x}_i = \frac{T_i}{2} - \frac{t_i}{2},\ i = 1,2,..n$$

$$2\bar{x}_i = T_i - t_i,\ i = 1,2,..n \tag{3.34}$$

Therefore, the probable sum of assembly dimension comprise biasness in dimensional position of each components and their natural process tolerance,

$$\begin{aligned} T_{prob} &= 2\bar{x}_1 + 2\bar{x}_2 + \ldots 2\bar{x}_n + 6\sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2} \\ &= 2(\bar{x}_1 + \bar{x}_2 + \ldots \bar{x}_n) + 6\sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2} \end{aligned} \tag{3.35}$$

If natural process tolerance is,

$$t_i = 3\sigma - (-3\sigma) = 6\sigma \tag{3.36}$$

Then, upon using (3.34) and (3.36) in (3.35) we get,

$$T_{prob} = (T_1 - t_1) + (T_2 - t_2) + ..(T_n - t_n) + \sqrt{t_1^2 + t_2^2 + ..t_n^2}$$

$$= T_1 + T_2 + \ldots + T_n - (t_1 + t_2 + \ldots + t_n) + \sqrt{t_1^2 + t_2^2 + ..t_n^2}$$

$$= T_{arith} - \left(\sum_{i=1}^{n} t_i\right) + \sqrt{\sum_{i=1}^{n} t_i^2} \tag{3.37}$$

Hence

$$T_{arith} = T_{prob} + \left(\sum_{i=1}^{n} t_i\right) - \sqrt{\sum_{i=1}^{n} t_i^2} \tag{3.38}$$

where

$$T_{arith} = T_1 + T_2 + \ldots + T_n \tag{3.39}$$

Assuming constant ratio for allowable tolerance and natural tolerance we get,

$$\frac{T_1}{t_1} = \frac{T_2}{t_2} = \ldots . \frac{T_n}{t_n} = K \text{ (constant)} \tag{3.40}$$

Therefore,

$$T_2 = \frac{T_1 t_2}{t_1},\ T_3 = \frac{T_1 t_3}{t_1}, \ldots, T_n = \frac{T_1 t_n}{t_1} = K \tag{3.41}$$

Hence (3.39) reduces to

$$T_{arith} = T_1 + \frac{T_1 t_2}{t_1} + \frac{T_1 t_3}{t_1} + \ldots + \frac{T_1 t_n}{t1}$$

$$= \frac{T_1}{t_1}\left(\sum_{i=1}^{n} t_i\right) \tag{3.42}$$

Substituting From (3.38) in to (3.42) we get,

$$\frac{T_1}{t_1}\left(\sum_{i=1}^{n} t_i\right) = T_{prob} + \left(\sum_{i=1}^{n} t_i\right) - \sqrt{\sum_{i=1}^{n} t_i^2} \tag{3.43}$$

On substituting (3.41) in (3.43) we get,

$$K\left(\sum_{i=1}^{n} t_i\right) = T_{prob} + \left(\sum_{i=1}^{n} t_i\right) - \sqrt{\sum_{i=1}^{n} t_i^2}$$

$$\Rightarrow K = \frac{T_{prob} + \left(\sum_{i=1}^{n} t_i\right) - \sqrt{\sum_{i=1}^{n} t_i^2}}{\left(\sum_{i=1}^{n} t_i\right)} \tag{3.44}$$

According to Mansoor (1963), the constant K can take different values:

- For $K<1$, process will not meet with tolerance specification
- $1<K<1.33$, process will meet with tolerance specification under strict statistical control
- $1.33<K<2$, process will easily meet tolerance specification but still statistical control is required
- $K>2$, process will easily meet tolerance specification

As it is depicted in Figure 3.3., inflated distribution is equivalent to linear mean shift, it is possible to compare their tail areas based on capability ratios as mentioned in Harry M J (1992), comparing midterm capability ratio with long term ratio we get,

$$C_{Pk} = C_p^* \tag{3.45}$$

In order to examine process capability, the index used is C_P, defined as,

$$C_P = \frac{USL - LSL}{6\sigma}$$

However major shortcoming of this index is, it does not account for shifts in the process centring. Another index developed to indicate process centring is C_{pk} defined as,

$$C_{Pk} = (1 - K)C_p \tag{3.46}$$

where

$$K = \frac{Amount\ of\ target\ off\ centering}{Amount\ of\ process\ mean\ to\ nearest\ specification} \tag{3.47}$$

The above estimate can be used to understand single component off targeting. To estimate K for multiple component assembly (3.44) can be used. As proposed by Evans (1975), long term process standard deviation is inflated by factor c,

$$\sigma_{LT} = c\sigma_{ST} \tag{3.48}$$

Upon substituting from (3.46) and (3.48) in to (3.45), for bilateral case it can be written as,

$$\frac{USL - LSL}{3\sigma_{ST}}(1 - K) = \frac{USL - LSL}{3\sigma_{ST}c}$$

which yields

$$c = \frac{1}{1 - K} \tag{3.49}$$

Upon substituting value of K from (3.47) for single component off targeting and from (3.45) for multiple component assembly in to (3.49), we can get estimate of inflation factor c.

As an illustration, consider a single component manufacturing assembly with Mean=1 and SD=0.002, with natural process variation = 6σ =6*0.02=0.012. Therefore,

$$\text{Natural tolerance interval} = [\text{Mean}-0.012, \text{Mean}+ 0.012]$$

$$= [1-0.012, 1+ 0.012]$$

$$= [0.988, 1.012]$$

Let us take specified tolerance interval as [0.980, 1.014] with Mean=0.097, with amount of off targeting as 0.003 (=1- 0.097). Hence, the amount of process mean to nearest specification is *min*[|0.980-1| , |1.014-1|], which is equal to 0.014. Upon substituting these values in (3.47) we get, K=0.003/0.014 = 0.214. Hence c =1.272.

3.4 How to market sigma level of the process

Improving business processes and bringing them to the notice of others, is the important criteria to establish firm-customer trust building. To stand out of general practitioners many organizations are promoting their different quality initiative like ISO, ASQ etc, to generate feeling of assured quality standards among customers.

Though Six Sigma efforts speaks in itself based on improved quality standards of business processes and significant ROI, no criteria has been proposed to promote organization's Six Sigma efforts so far. Since, performance CTQs is important with reference to firms position in the market, sigma level of these CTQs can be a great criterion to promote company's quality improvement efforts. Promoting Six Sigma efforts based on sigma level of major CTQs, can help organization to establish fact based firm-customer relationship. Companies can device plans to promote sigma level for product branding, service quality and competitive campaign of their product and processes. The task of convincing all types of customers may not be very easy, but with a concerted plan this may be possible. Some of the possible ways through which it can be achieved are:

- Establishing fact based organizational communication system instead of perception based communication system. This can very well done by leveraging use of sigma level at each level of communication - from receiving customer order to delivery.
- Label each process with their corresponding sigma level
- Promote quality healthiness of process through assigning sigma level appropriately
- Promote process improvement through improvement in KPI of the process.