CHAPTER - 1

INTRODUCTION

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1.1 INTRODUCTION

A common problem that many businesses are facing in today's uncertain market conditions is that of maintaining optimal inventory levels. Inventory theory is one of the most developed fields of operations research. Inventory modeling involves modeling of several aspects of inventory system such as demand pattern, replenishment pattern, storage mechanism, and costs associated with these aspects. Retail firms, wholesalers, manufacturing companies, blood banks etc. generally maintain a stock of goods on hand in order to meet future demand. How such a facility should decide its "inventory policy", has been the question of interest in almost every work on inventory modeling.

Stock of inventory differs from seller to seller. Malls, departmental stores, retail outlets, etc store a variety of goods ranging from consumer user goods, electronic goods, clothes, toys, etc. It also depends on space availability. Small retail shop stores limited variety of goods. Single unit shop sells only single variety of goods (items).

The seller has to replenish his stock from time to time. Lesser stock will lead to loss of consumers and loss of goodwill. Storing more than adequate stock may incur financial obligations such as loss of interest on tied up money, depreciation, cost of storage, cost of insurance of goods, etc. All these costs are usually put together as inventory holding cost. So seller has to achieve a balance between too less and too large amounts of inventory in the form of an inventory policy. An inventory policy answers the following two questions:

1) How much to order?

2) When to order?

There are several basic considerations involved in determining an inventory policy that must be reflected in a mathematical model of inventory system. The basis for the decision is a model that captures different aspects of inventory system and associated costs as pointed out above. Most commonly considered costs are purchasing cost (inventory cost), ordering cost, holding cost and shortage cost. A detailed discussion of these costs is given in E.Naddor (1966).

The optimal inventory policy is the one that minimizes the operational cost of inventory system per unit time. The most significant factor affecting the inventory policy is the demand pattern. The patterns of demand are generally uncertain and depend on factors such as items (goods), price, stock, time, place, people, fashion, habits, preferences, market etc. Inventory models are usually classified into two categories based on demand, as described in Hiller and Lieberman (1974), as 1) Deterministic models (where the demand for the period is known) and 2) Stochastic models (where the demand follows a Stochastic behavior).

An interesting area of inventory theory is the mathematical modeling of inventory for deteriorating items. One of the implicit assumptions of the traditional inventory models has been the "infinite shelf life" of products that are stored. i.e, a product once in stock remains unchanged and fully usable for satisfying future demand as long as it is in stock. However, many items are known to deteriorate over time, and hence do not have infinite shelf life. If the rate of deterioration or decay is low and negligible with respect to the cycle length, its effect can be safely ignored. However, in many situations this effect plays a significant role and its impact must be considered explicitly. Several inventory models have been proposed, by different authors that consider the effect of deterioration on inventory management.

Deteriorating items can be classified into two categories as described in Ruxian et al (2010). One category is based on reduction in quantity and other is based on decrease in its economic value. The first category refers to the items that decay, get damaged, evaporate, or expire through time, like meat, vegetables, fruit, medicine, flowers, films and so on. The second category refers to the items that lose part or total value over time for one or other reasons. For example, due to the new technology, introduction of alternative products, loss of relevance, etc. Also the fast changing trends in fashion and seasonal goods have a significant impact on the reduction in the value of the inventory over time. We use the term “deteriorating item” for the items of first category, whereas the items in second category will be referred to as **value-deteriorating items.** In either, case the items have limited shelf life, the length of which will be random in most real life situations.

The remaining of this chapter is organized as follows.

In section 1.2, the available literature on the inventory models for deteriorating items is reviewed. The Inventory models with different types of demands patterns, different types of deterioration rates, and different types of cycle length are also summarized. In section 1.3, the research problems addressed in the present work are discussed, and in section 1.4, we provide overview of the thesis.

1.2 LITERATURE REVIEW

Different inventory models for deteriorating items have been developed and studied extensively by many authors in the literature of inventory theory. The details of which can be found in the comprehensive review articles of Raafat (1991), Goyal & Giri (2001) and recently by Ruxian et al. (2010). Whitin (1952) was reportedly the first author who proposed a model for deteriorating items. He assumed that fashion goods on hand at the end of the period must be liquidated as a loss (i.e. value deteriorated). He assumed that the cost of liquidating an unsold item dominates the set up cost and holding cost and hence did not consider these costs in his analysis. Ghare and Schrader (1963) proposed a model with exponentially decaying inventory. They demonstrated that considering inventory decay as a cost factor during inventory analysis results in cost savings and also improvements in inventory reordering policy.

Covert and Philip (1973) extended Ghare and Schrader's model by assuming a two-parameter Weibull distribution for the deterioration rate. This model is further extended by Philip (1974) by assuming a three-parameter Weibull distribution. This model also allows deterioration to begin at any time. Tadikamalla (1978) considered a similar model by assuming a gamma distribution for the deterioration time. Y. K. Shah (1977) generalized the models proposed in above mentioned papers by introducing a model for deteriorating items where rate of deterioration can be modeled by any well behaved probability distribution.

1.2.1 Classification of the deteriorating inventory models

As proposed by Goyal and Giri (2001), the inventory models can be broadly classified on the basis of shelf life characteristics into the following categories.

- Models With fixed life time.
- Models With random life time.
- Models for inventory which decays in terms of its utility or physical quantity.

Models with fixed life time: In these models, the products are assumed to have deterministic shelf life. i.e if a product remains unused up to its life time, it is considered to be out –dated and must be disposed off. For ex. Human blood used for transfusion, pharmaceutical products etc.

Models with random life time: The products whose exact lifetime, while in stock, is uncertain and cannot be determined in advance are known as random life time products. For example, style goods, mobiles etc.

Models for inventory which decays in terms of its utility or physical quantity: The products whose decay depends on the utility of individual product or physical quantity. Value of the items decrease with decrease in inventory level. For perishable product like fresh vegetables, breads, fruits, dairy products etc., utility decrease continuously with time.

Modeling demand

Demand is another important component, whose modeling is often challenging while developing inventory models. As noted by Goyal and Giri (2001), two types of demand, viz. deterministic demand and stochastic demand are recognized by researchers. Considering the real life situations, deterministic demand are further classified into Uniform demand, time-dependent demand, Stock -dependent demand and price-dependent demand. Stochastic demand may have either a known probability distribution or an arbitrary demand distribution.

1.2.2 Deterministic demand

When demand for item in the inventory can be determined precisely or known, it is referred to as deterministic demand. Deterministic demand can be further classified into Uniform demand, time-dependent demand, Stock -dependent demand and price-dependent demand. Each of these is briefly described below.

a) Uniform demand

When rate of demand is constant during the planning interval, it is referred to as a Uniform demand.

The models proposed by Mishra (1975), Shah (1977) and Heng et al. (1991) consider a finite replenishment rate, constant demand rate and exponential decay in inventory. Raafat et al. (1991) developed an inventory model for deteriorating items with constant demand rate and finite replenishment rate. Sarker, et al. (2000), Chang (2004), Chung and Liao (2006), Huang and Liao (2008), Zhang, et al. (2007a), Zhang, at al. (2007b) all developed inventory models in which both the demand and deteriorating rate are constant. In all these papers, authors assume that the buyer is allowed a delay period to pay for the items purchased. Although the constant demand assumption helps to simplify the problem, it is far from the real life scenarios where demand always changes with fashion & time. Mishra (2012) considered demand rate is constant and deterioration rate is time dependent with weibull's distribution

b) Time varying demand

The assumptions of constant demand rate, in most cases, serve to simplify the model rather than capturing the real scenario. In reality, demand may depend on several factors. In certain type of products, the demand rate follows specific patterns over a period of time. For example, when a new product is launched in market it typically goes through three phases. During the first phase, which may be called the growth phase, the product experiences rising demand. In the second phase the demand may stabilize, and in the third phase demand starts reducing as the product becomes too common, and hence out of fashion. This demand pattern is known as "ramp-type" demand. Hill (1995) was the first to introduce the ramp type demand to the inventory study, followed by Mandal and Pal [1998], Deng et al. (2007), Skouri et al. (2007), Panda et al. (2008), Tend et al. (2011) and Karmakar and Dutta Choudhury (2014).

In other situations, the time dependent demand pattern may be completely different. The age-on-shelf of inventory may make a negative impact on demand because of the loss of confidence of consumers on the quality of such products. As discussed by Goyal and Giri (2001), and Ruxain et al. (2010), in their review, most of the continuous-time inventory models have been developed assuming either linearly varying demand or exponentially varying demand patterns. Resh et al.,(1976), Donaldson (1977), Goyal

(1986), Goswami and Chaudhuri (1991), Chung and Ting (1993), considered linear trend in demand. Dave and Patel (1981), and Dutta and Pal (1992) considered models with time proportional (linearly with time) demand. Hariga and Benkherouf (1994), Wee (1995) considered exponential time varying demand. Giri and Chaudhuri (1997), Papachristos and Skouri (2000), Chu and Chen (2002), Khanra and Chandhuri (2003), Yang (2005), Dye et al. (2006) considered time dependent demand. Mishra (2013) considered demand rate to be a liner function of time. Tripathi (2013), Sing et al (2013) and Mishra et al. (2013) considered time dependent demand. Jagadeeswari and Chenniappan (2014) developed Inventory Model for Deteriorating Items with Time – Quadratic Demand. S.Kumar and Rajput (2015) developed fuzzy inventory model with time dependent demand.

c) Stock dependent demand

Practically, an increase in stock level for an item induces more consumers to buy it. Conversely, low stocks of certain goods might raise the perception that they are not fresh or not of good quality. Gupta and Vrat (1986) are the first who proposed an inventory model in which demand rate is stock dependent. Dutta and Pal (1990), Giri et al. (1996), and Giri and Chaudhuri (1998), Padmanabhan and Vrat (1990), Pal et al. (1993), and Sarkar et al. (1997) have studied inventory models for a single deteriorating item with stock dependent demand rates. Padmanabhan and Vrat (1995) presented inventory models for deteriorating items with stock-dependent demand rate. Balkhi and Benkherof (2004) also consider stock dependent demand. Pal et al. (2006), and Chang et al. (2010) proposed a model in which demand rate depends on the stock level on-display.

d) Price dependent demand

In reality, Observations in the market show that a price reduction results in an increase in demand. The retailers in turn offer a price discount to their customers to increase demand. Moreover, in order to reduce the losses due to deterioration, a discount pricing policy is frequently, implemented by the retailers. This policy is normally used in supermarkets, malls etc. When the expiry date of the items are near, they are also sold at a discount price. Wee and Yu (1997) considered the effects of the temporary discount sale when the items deteriorate exponentially with time. Peng and Peng (2004), Teng et al. (2005), and Qin et al. (2006) consider price dependent demand.

1.2.3 Stochastic demand

In the market with growing uncertainty in the modern business environment, the assumption of deterministic demand is far from reality. Models with stochastic demand are closer to real systems and have received more and more attention by both, researchers and practitioners. Assumption of Poisson demand is the most common assumption made by researchers when modeling a stochastic demand. Scarf (1958), Karlin and Scarf (1958) and Galliher et al. (1959) have considered Poisson demand.

Federgruen & Schechner (1983) have studied continuous review models with a fixed delivery lag, where the stochastic demand is characterized by the renewal reward process.

Krishnamoorthy and Varghese (1995) proposed a continuous review (s, S) inventory system with Poisson demand where commodities are assumed to be damaged due to decay and disaster. Liu and chaung (1997) proposed a single item inventory models with Poisson demand and exponentially distributed lifetime. Kalpakam and Shanthi (2000) analyzed inventory systems with Poisson demands for deteriorating items. Johan & Inneke (2005) proposed a model with Poisson demand, where the average inventory holding costs are related to the inventory on shelf and the average cost of lost sales.

Sobel & Zhang (2001) and Presman & Sehi (2006) proposed models with demand that is a mixture of deterministic and stochastic component.

Katy,Julia & Udayabhanu (2012) also proposed models where demand is a either a compound Poisson process or a mixture of deterministic component and a random component which follows a compound Poisson process.,

For several types of products, like electronic goods, home appliances, mobiles, cars, etc., customers demand items only in discrete units, and one unit in most cases. The demand for such items must be modeled accordingly. Poisson demand is generalized by Finch (1961) and Sivazlian (1974) to unit demands arriving at epochs following a renewal process.

1.3 THE RESEARCH PROBLEM

From above literature review, it is clear that most of the inventory models assume deterioration rate to be distributed as exponential or Weibull or gamma. Inventory models for deteriorating items that are proposed in the literature assume that the items are good and usable in the beginning, and after deterioration these items are completely useless, hence are removed from the inventory. However, for many items this may not be the case, and the items may either continuously lose their value or may be still good enough to be able to get sold at a reduced price. For example, in case of items such as mobiles, fashion goods, electronic goods, etc.

The products whose exact lifetime is uncertain while in stock are considered as the products with random life time. For example, fashion/style goods, mobiles, electronic items, fruits, vegetables etc. In the present research work we focus on developing models for such items. The shelf life of this type of the product is not fixed. For example, it is very hard to predict when the fashion of certain clothes may change thus the shelf life of such products is random.

Items that lose their value through time because of technology or the introduction of new product or low stock can also be considered as deteriorating items. For example, Price of the style goods must be reduced after some time. It is necessary develop and investigate suitable models for such items.

Normally sellers reduce the price in the following situations

- When the demand rate reduces due to low inventory levels
- To release money through selling the unsold units if any, after a specified duration T_0 ,
- When new product with latest technology or a new model of the product is introduced, the existing stock is sold at reduced price

Inventory models encompassing these practices need to be developed and investigated.

Holding cost is also a one of the important component of the inventory models. In a classical EOQ models, Whitin (1952), Ghare and Schrader (1963), Covert and Philp (1973), Y.K.Shah (1977) , and Ravichandran (1993) assume that holding cost is based on average inventory level during the cycle. Giri and Chaudhari (1998) considered non-linear holding cost, whereas Bhathavala & Rathod (2012) considered storage time dependent holding cost. Karmakar and Dutta Choudhury (2014) considered holding cost as a linearly increasing function of time. In present work we assume that holding cost incurs only for the period during which the inventory items are in the stock.

1.4 OVERVIEW OF THE THESIS

There are large varieties of inventory items, where customers typically demand for only one unit. Electronic gadgets, domestic appliances, vehicles and fashion goods are examples of this type. In the present research work, we develop inventory models for this type of the items with a special focus on items having random shelf life. Also we have restricted our research work for only the value deteriorating items as discussed in section 1.1.

In Chapter 2, we present and discuss a model that is developed for a continuous review inventory system with zero lead time. The work presented here differs from the previous work in the literature in the sense that holding cost incurs only for the period during which the inventory items are in the stock. The model assumes deterministic uniform demand. We obtain a closed form expression of optimal inventory level as

$$n^* = \left\lceil \frac{-1 + \sqrt{1 + \frac{8DC_o}{C_h}}}{2} \right\rceil \qquad \dots (1.4.1)$$

We compare above formula with classical EOQ formula

$$Q^* = \sqrt{\frac{2DC_0}{C_h}}$$

We find that n^* = round(Q^*) for every value of $Q^* \geq 1$.

Remark: The two solutions differ only for the case of $Q^* < 1$. However, in that case we note that rounding of Q^* would lead to an infeasible solution, and in practice we need to use the value 1 for the solution, which again would be same as n^*.

In chapter 3, we present a model in which the demand is generated according to a Poisson process with every customer having a unit demand. Due to our model assumptions, the actual holding cost is random and depends on the time points at which actual demands occur. Also the replenishment is assumed to be instantaneous and hence the new order is placed only when (and as soon as) inventory level reaches zero. This also results into the random cycle length. For this model also we obtain a closed form expression of optimal inventory level as given below.

$$n^* = \left\lceil \frac{1 + \sqrt{1 + \frac{8(C_o + C)}{\theta C_h}}}{2} \right\rceil \qquad \dots\dots (1.4.2)$$

We compare this result with that of previous model (1.4.1) and also with classical EOQ model. We also indicate a scenario in which the models proposed in this chapter would provide better results.

In Chapter 4, we present a model for the items such as fashion goods, where the demand is generally influenced by inventory levels. As inventory level decreases, demand of the item also decreases. Normally sellers reduce the price in order to maintain the demand rate when inventory levels go down. We present an inventory model in which the items are initially sold at price p_1 per unit, and later at a reduced price p_2 per unit when stock level drop below a specified level. All other model assumptions are same as those of the model presented in Chapter 3.

The optimal inventory level for this model is the feasible solution to the following cubic equation.

$$n^3C_1 + n\left(C_1(k^2+k-1) - C_2k(k+1) - 2\frac{C_0+c}{\theta}\right) + C_2k(k+1) - C_1k(k+1) - 2\frac{C_0+c}{\theta} = 0$$

Using the approach of solving a cubic equation as given by Francois Viete (2006), we find that

$$n^* = \left\lceil 2 * \sqrt{\frac{-p}{3}} * cos\left(\frac{1}{3}\arccos\left(\frac{3q}{2p}\sqrt{\frac{-3}{p}}\right)\right)\right\rceil \qquad \ldots.(1.4.3)$$

is the only feasible solution out of the three solutions of the equation.

In items such as fashion goods, it is often seen that, shopkeepers also adopt a strategy of selling the unsold goods at a reduced price after specified time duration. This is generally done in order to save carrying cost and to release the blocked money. This also helps to maintain smooth flow of money in the inventory system.

In Chapter 5, we model this phenomenon and obtain optimum inventory levels. In the proposed model we assume that the price reduction does not affect the demand rate, All other assumption are same as those in the models proposed in earlier chapters. This model can also be regarded as a model for the second category for deteriorating items (Ruxian et al (2010)) as discussed earlier section 1.1.

The closed form expression for optimal order quantity is not obtainable for this model.- We, therefore, present an algorithm for obtaining the optimal ordering quantity and a C++ implementation of the same.

In Chapter 6, we present an inventory model in which the items are initially sold at a regular price p_1 per unit. At a random time point T_0, the price is reduced in response to the occurrence of some event, such as new technology or the introduction of alternative products. We assume T_0 to be an exponential variable. Remaining assumptions are same as those in previous model.

For this model also it is not possible to obtain a closed form expression for the optimal order quantity. Hence, we again present an algorithm and its implementation in C++ as earlier.