Abstract of the Thesis entitled

CERTAIN GENERALIZED MATRIX FUNCTIONS AND THEIR PROPERTIES

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The work carried out in the thesis entitled "*Certain generalized matrix functions and their properties*" contains matrix analogues of several polynomials and a generalized function.
To be precise, certain properties such as the inverse series relation, generating function relation, summation formulas involving the matrix polynomials have been derived and for the generalized function, contiguous function relation, differential formulas, integral forms and fractional integral and derivatives formulas have been obtained.

The thesis comprised of total six chapters. Chapter 1 incorporates some preliminaries of the matrix function such as Gamma matrix function, Beta matrix function, polynomial matrix function which included Laguerre matrix polynomial, Konhauser matrix polynomial, Jacobi matrix polynomial etc. and the matrix analogue of Mittag-Leffler function.

Let $\mathbb{C}^{p\times p}$ be a family of square matrices of order p having in general, complex entries. For a matrix A in $\mathbb{C}^{p\times p}$, let $\sigma(A)$ be the set of all eigenvalues of A. The matrix A is said to be positive stable matrix if $\Re(\lambda) > 0$ for all $\lambda \in \sigma(A)$. A matrix polynomial of degree n in x is a polynomial of the form [8]:

$$F(x) = P_n x^n + P_{n-1}x^{n-1} + P_{n-2}x^{n-2} + ... + P_1 x + P_0, \tag{1}$$

where the matrices $P_0, P_1, P_2, \ldots, P_n$ are in $\mathbb{C}^{p\times p}$ and P_n is a non-zero matrix.
If A is matrix in $\mathbb{C}^{p\times p}$ such that $\Re(\lambda) > 0$ for all eigenvalue λ of A, then $\Gamma(A)$ is well defined as

$$\Gamma(A) = \int_0^\infty e^{-x}t^{A-I}\, dx, \qquad x^{A-I} = exp((A-I)\ln x). \tag{2}$$

The reciprocal gamma function denoted by $\Gamma^{-1}(z) = (\Gamma(z))^{-1} = \frac{1}{\Gamma(z)}$ is an entire function of complex variable z [6, p. 253] and thus for any matrix A in $C^{p\times p}$, the functional calculus [5] shows that $\Gamma^{-1}(A)$ is a well defined matrix.
If I denotes the identity matrix of order p and $A+nI$ is invertible for every integer $n \geq 0$ then ([9])

$$\Gamma^{-1}(A) = A(A+I)\cdots(A+(n-1)I)\Gamma^{-1}(A+nI). \tag{3}$$

From this, the functional equation of the gamma matrix function occurs in the form

$$A\,\Gamma(A) = \Gamma(A+I) \tag{4}$$

which readily follows for $n = 1$. For a matrix A in $\mathbf{C}^{p\times p}$, the Pochhammer matrix symbol is defined by [9]

$$(A)_n = \begin{cases} I, & if \quad n = 0 \\ A(A+I)\cdots(A+(n-1)I), & if \ n \geq 1. \end{cases} \tag{5}$$

For A is an arbitrary matrix in $\mathbb{C}^{p\times p}$ and using (3) we obtain

$$(A)_{m+k} = (A)_m (A+mI)_k. \tag{6}$$

Also, if $I - A - nI$ is invertible for all $n \geq 0$, then

$$(A)_{n-k} = (-1)^k \; n! \; (A)_n \; (I - A - nI)_k^{-1}. \tag{7}$$

The generalized hypergeometric matrix function is defined as follows [16, Eq. (2.2), p. 608].

Definition 1. If $\{A_i; i = 1, 2, \ldots, p\}$, and $\{B_j; j = 1, 2, \ldots, q\}$, are sequences of matrices in $C^{p\times p}$ such that $B_j + nI$ are invertible for all $n \geq 0$, then the generalized hypergeometric matrix function is defined as [16]

$$\begin{aligned} & {}_rF_s\left(A_1, A_2, \ldots, A_r; B_1, B_2, \ldots, B_s; x\right) \\ = & \sum_{n=0}^{\infty} (A_1)_n (A_2)_n \ldots (A_r)_n \; [(B_1)_n]^{-1} [(B_2)_n]^{-1} \ldots [(B_s)_n]^{-1} \; \frac{x^n}{n!}. \end{aligned} \tag{8}$$

Here, the series converges for all x if $r \leq s$. If $r = s+1$, then the series converges for $|x| < 1$. If $r > s+1$, then the series diverges for all $x \neq 0$.

The following polynomials are among those occurring in the literature hitherto on matrix polynomials.

- Konhauser matrix polynomial [18]:

$$Z_m^{(A,\lambda)}(x;k) = \Gamma(kmI + A + I) \sum_{n=0}^{m} \frac{(-1)^n (\lambda x)^{nk}}{(m-n)!n!} \; \Gamma^{-1}(knI + A + I). \tag{9}$$

- Jacobi matrix polynomial [3]:

$$P_n^{(A,B)}(x) = (-1)^n \frac{[(B+I)_n]}{n!} \sum_{k=0}^{\lfloor n/s \rfloor} \frac{(-nI)_k}{k!}(A+B+nI+I)_k \times [(B+I)_k]^{-1} \left(\frac{1+x}{2}\right)^k . \tag{10}$$

- The Gegenbauer matrix polynomials [2] :

$$C_n^A(x) = \sum_{k=0}^{[n/2]} (-1)^k \frac{(-A)_{n-k}}{(n-2k)!\; k!} \;(2x)^{n-2k}. \tag{11}$$

From the generalized Humbert matrix polynomial studied by M.A.Pathan [11, Eq.(2.3), p.210]. We get following generalized Humbert matrix polynomial when $a = y = 1$.

- Generalized Humbert matrix polynomial :

$$P_n^A(m,x,\eta,c) = \sum_{k=0}^{[n/m]} \eta^k \frac{c^{A-(n-(m-1)k)I}}{(n-mk)!\; k!} \Gamma^{-1}(A+(1-n+mk-k)I) \times \Gamma(A+I)(-mx)^{n-mk}. \tag{12}$$

The fractional order integral of x^A is defined as follows [1].

Definition 2. Let A be a positive stable matrix in $\mathbb{C}^{p\times p}$ and $\mu \in C$ such that $\Re(\mu) > 0$. Then, the Riemann-Liouville fractional integrals of order μ may be defined as follows

$$\mathbf{I}^\mu(x^A) = \frac{1}{\Gamma(\mu)} \int_0^x (x-t)^{\mu-1} t^A dt. \tag{13}$$

In Chapter-2, we introduce a general class of matrix polynomials $\{H_n(C,r,s;x); n = 0,1,2,\ldots\}$ defined by

$$H_n(C,r,s;x) = \sum_{k=0}^{\lfloor n/s \rfloor} \frac{(C+rkI)_{n-sk}}{(n-sk)!} \; M_k \, x^k. \tag{14}$$

This class of polynomials encompasses many matrix polynomial such as the extended Jacobi matrix polynomial:

$$\mathcal{F}^{C}_{n,l,s}[(A);(B):x] = \sum_{k=0}^{\lfloor n/s \rfloor} (-nI)_{sk}(C+nI)_{lk}\,(A_1)_k\cdots(A_p)_k[(B_1)_k]^{-1}\cdots[(B_q)_k]^{-1} \times \frac{x^k}{k!},$$

and hence the extension of the Jacobi matrix polynomial [3]:

$$P^{(A,B)}_{n,s,l}(x) = (-1)^n\,\frac{[(B+I)_n]}{n!}\sum_{k=0}^{\lfloor n/s \rfloor}\frac{(-nI)_{sk}}{k!}(A+B+nI+I)_{lk} \times[(B+I)_k]^{-1}\left(\frac{1+x}{2}\right)^k, \tag{15}$$

and several other polynomials namely, Brafman matrix polynomial, extended Gegenbauer matrix polynomial, extended Legendre matrix polynomial, extended Laguerre matrix polynomial etc.

The inverse matrix series of these polynomials are derived form a general inversion theorem which is stated below.

Theorem 3. *Let A and B be positive stable matrices in $\mathbb{C}^{p\times p}$ such that $A+\jmath B-\ell I\neq O$ for every non negative integers $\jmath,\ell$, and $n\neq sm, m\in \boldsymbol{N}\cup\{0\}, s\in\boldsymbol{N}\backslash\{1\}$, then*

$$\boldsymbol{F}(n)=\sum_{k=0}^{\lfloor n/s \rfloor}(-1)^{n-sk}\,\frac{\Gamma^{-1}(A+skB-nI+I)}{(n-sk)!}\,\boldsymbol{G}(k) \tag{16}$$

if and only if

$$\boldsymbol{G}(n)=\sum_{k=0}^{sn}\frac{(A+kB-kI)\Gamma(A+snB-kI)}{(sn-k)!}\,\boldsymbol{F}(k) \tag{17}$$

and

$$\sum_{k=0}^{n}\frac{(A+kB-kI)\Gamma(A+nB-kI)}{(n-k)!}\,\boldsymbol{F}(k)=O, \tag{18}$$

in which the floor function $\lfloor r\rfloor$ = floor r, represents the greatest integer $\leq r$.

From this theorem we find the inverse series of above polynomials as follows.

- Inverse series relation of $H_n(C, r, s; x)$:

$$M_n\ x^n = \sum_{k=0}^{sn} \frac{(-C-(rk/s)I)}{(sn-k)!}\ (I-C-rnI)_{sn-k-1} H_k(C, r, s; x).$$

- Inverse series relation of $\mathcal{F}^C_{n,l,s}[(A);(B):x]$:

$$(A_1)_n \cdots (A_p)_n\ [(B_1)_n]^{-1} \cdots [(B_q)_n]^{-1} \frac{x^n}{n!}$$

$$= \sum_{k=0}^{sn} \frac{(-snI)_k}{(sn)!\ k!} (C+LkI+kI)[(C+kI)_{ln+1}]^{-1} \mathcal{F}^C_{k,l,s}[(A);(B):x],$$

where $L = l/s, l = r - s$.

- Inverse series relation of $P^{(A,B)}_{n,s,l}(x)$:

$$\frac{[(B+I)_n]^{-1}}{n!} \left(\frac{1+x}{2}\right)^n = \sum_{k=0}^{sn} (-1)^k \frac{(-snI)_k}{(sn)!} (A+B+LkI+kI+I)$$

$$\times [(A+B+kI+I)_{ln+1}]^{-1} [(B+I)_k]^{-1} P^{(A,B)}_{k,s,l}(x)$$

Extended Wilson and Extended Racah matrix polynomials

It is interesting to note that the matrix extension to the Wilson polynomial and the Racah polynomial occur as the special cases of the above theorem. Their matrix forms together with the inverse series are stated below.

- Wilson matrix polynomial:

$$[(A+B)_n]^{-1}[(A+C)_n]^{-1}[(A+D)_n]^{-1}\ P_{n,l,s}(x^2)$$

$$= \sum_{k=0}^{\lfloor n/s \rfloor} \frac{(-nI)_{sk}}{k!} (A+B+C+D+nI+I)_{lk} (A+ixI)_k (A-ixI)_k$$

$$\times [(A+B)_k]^{-1}[(A+C)_k]^{-1}[(A+D)_k]^{-1};$$

and its inverse series

$$\frac{(A+ixI)_n (A-ixI)_n}{n!} (A+B)_n^{-1} [(A+C)_n]^{-1} [(A+D)_n]^{-1}$$

$$= \sum_{k=0}^{sn} \frac{(-snI)_k}{(sn)!} (A+B+C+D+(r/s)kI+I)$$

$$\times[(A+B+C+D+kI+I)_{ln+1}]^{-1}[(A+B)_k]^{-1}[(A+C)_k]^{-1}$$
$$\times[(A+D)_k]^{-1}P_{k,l,s}(x^2).$$

- Rach matrix polynomial:

$$\begin{aligned}
&R_{n,l,s}(x(xI+D+E+I);A,B,D,E)\\
&\quad= \sum_{k=0}^{\lfloor n/s\rfloor}\frac{(-nI)_{sk}}{k!}(A+B+nI+I)_{lk}(-xI)_k(xI+D+E+I)_k\\
&\qquad\times[(A+I)_k]^{-1}[(B+E+I)_k]^{-1}[(D+I)_k]^{-1};
\end{aligned}$$

and its inverse series

$$\begin{aligned}
&\frac{(-xI)_n(xI+D+E+I)_n}{n!}\,[(A+I)_n]^{-1}(B+E+I)_n^{-1}[(D+I)_n]^{-1}\\
&\quad= \sum_{k=0}^{sn}\frac{(-snI)_k}{(sn)!}(A+B+(rk/s)I+I)\;[(A+B+kI+I)_{ln+1}]^{-1}\\
&\qquad\times R_{k,l,s}(x(xI+D+E+I);A,B,D,E).
\end{aligned}$$

The matrix analogues of Hahn, dual Hahn, Meixner, Krawtchouk, Charlier, Jacobi etc. are occurring from these polynomials together with their inverse series.

Application

The first series of the theorem may be used to obtain the generating function relation. From the general class $H_n(C,r,s,x)$ of polynomials, we get one of the following generating function relations.

$$\sum_{n=0}^{\infty}H_n(C,r,s,x)\frac{t^n}{n!} = \sum_{k=0}^{\infty}(C)_{rk}\;{}_1F_1\left(C+rkI;C+skI;t\right)[(C)_{sk}]^{-1}\,M_k\left((-t)^s x\right)^k.$$

The inverse series of the theorem can be applied to obtained various summation formulas.

For example, we find

$$\sum_{n=0}^{\infty}\frac{x^n}{n!}I=e^xI = \sum_{n=0}^{\infty}\sum_{k=0}^{sn}[M_n]^{-1}\,\frac{(-C-(rk/s)I)}{(sn-k)!n!}\,(I-C-rnI)_{sn-k-1}$$
$$\times H_k(C,r,s;x),$$

and

$$
\begin{aligned}
&{}_2F_3\left(A+ixI, A-ixI; A+B, A+C, A+D; t\right)\\
&\quad=\sum_{n=0}^{\infty}\frac{t^n}{(sn)!}\sum_{k=0}^{sn}(-snI)_k(R+(r/s)kI)[(R+kI)_{ln+1}]^{-1}[(A+B)_k]^{-1}[(A+C)_k]^{-1}\\
&\quad\times[(A+D)_k]^{-1}P_{k,l,s}(x^2). \qquad (19)
\end{aligned}
$$

The above inversion theorem also provides matrix extension to some inverse series relations studied by John Riordan [13]. In order to deduce the inverse pairs, the above theorem is transformed to the following form.

$$
\left.
\begin{aligned}
F(n) &= \sum_{k=0}^{\lfloor n/s\rfloor}(-1)^{n-sk}(A+nB-nI+I)\Gamma^{-1}(A+skB-nI+2I)\\
&\quad\times\Gamma(A+skB-skI+I)\frac{G(k)}{(n-sk)!},\\
G(n) &= \sum_{k=0}^{sn}\Gamma^{-1}(A+snB-snI+I)\Gamma(A+snB-kI+I)\frac{F(k)}{(sn-k)!}.
\end{aligned}
\right\}. \qquad (20)
$$

Table-1: Matrix analogue of Riordan's Pair

$$F(n)=\sum\frac{a_{n,k}}{(n-sk)!}\,G(k);\;\; g(n)=\sum(-1)^{sn-k}\frac{b_{n,k}}{(sn-k)!}\,F(k)$$

Citation	B	$a_{n,k}$	$b_{n,k}$	Matrix analogue of Class (No.)
Theorem -1	B	$\Gamma^{-1}(A+skB-nI+I)$ $\times\Gamma(A+skB-skI+I)$	$\Gamma^{-1}(A+snB-snI+I)$ $\times(A+kB-kI)$ $\times\Gamma(A+snB-kI)$	Gould Class (1)
(20)	B	$(A+nB-nI+I)$ $\times\Gamma^{-1}(A+skB-nI+2I)$ $\times\Gamma(A+skB-skI+I)$	$\Gamma^{-1}(A+snB-snI+I)$ $\times\Gamma(A+snB-kI+I)$	Gould Class (2)
Theorem -1	$C+I$	$\Gamma^{-1}(A+skC+skI-nI+I)$ $\times\Gamma(A+skC+I)$	$\Gamma^{-1}(A+snC+I)$ $\times(A+kC)$ $\times\Gamma(A+snC+snI-kI)$	Legendre -Chebyshev Class (3)
(20)	$C+I$	$(A+nC+I)$ $\times\Gamma^{-1}(A+skC+skI-nI+2I)$ $\times\Gamma(A+skC+I)$	$\Gamma^{-1}(A+snC+I)$ $\times\Gamma(A+snC+snI-kI+I)$	Legendre -Chebyshev Class (7)

The inverse matrix series relations involving finite as well as infinite series are derived in Chapter-3. It is stated below.

Theorem 4. *Let $A - sI,\ s \in \{0\} \cup \boldsymbol{N}$, be a positive stable matrix in $\boldsymbol{C}^{p\times p}$ then*

$$\boldsymbol{U}(n) = \sum_{k=0}^{M} \frac{\eta^k \Gamma^{-1}(A + (1 - nr - brk - k)I)}{k!}\ \boldsymbol{V}(n + bk) \tag{21}$$

if and only if

$$\boldsymbol{V}(n) = \sum_{k=0}^{M} (-\eta)^k\ \Gamma(A - (nr - k)I)\ \frac{A - (nr + brk)I}{k!}\ \boldsymbol{U}(n + bk). \tag{22}$$

As a particular instance, the generalized Humbert matrix polynomial is deduced in the form:

$$\begin{aligned} P_n^A(m, x, \eta, c) \quad &= \quad \sum_{k=0}^{[n/m]} \eta^k \frac{c^{A-(n-(m-1)k)I}}{(n - mk)!\ k!} \Gamma^{-1}(A + (1 - n + mk - k)I) \\ & \qquad \times \Gamma(A + I)(-mx)^{n-mk} \end{aligned} \tag{23}$$

together with its inverse series relation:

$$\begin{aligned} \frac{(-mx)^n}{n!} I \quad &= \quad \sum_{k=0}^{[n/m]} (-\eta)^k \frac{(A - nI + mkI)(A - nI + kI)^{-1} c^{nI-kI-A}}{k!} \\ & \qquad \times \Gamma(A + (1 + k - n)I)\ P_{n-mk}^A(m, x, \eta, c). \end{aligned}$$

From this, the matrix polynomials of Humbert, Pincherle, Kinney, Gegenbauer etc. can be obtain along with their inverse series by specializing the parameters appropriately. Two worth mentioning special cases are the matrix versions of Wilson polynomial and the Racah polynomial. They occur from the theorem with the aid of the substitutions $b = -1, \eta = 1$ and $r = 2$.

- Wilson matrix polynomial and its inverse series:

$$\begin{aligned} & P_n(x^2)(A + B)_n^{-1}(A + C)_n^{-1}(A + D)_n^{-1} \\ & \quad = \sum_{k=0}^{n} \frac{(-nI)_k}{k!} (A + B + C + D + nI - I)_k (A + ixI)_k (A - ixI)_k \\ & \qquad \times (A + B)_k^{-1}(A + C)_k^{-1}(A + D)_k^{-1}. \end{aligned}$$

$$\frac{(A + ixI)_n (A - ixI)_n}{n!} (A + B)_n^{-1}(A + C)_n^{-1}(A + D)_n^{-1}$$

$$= \sum_{k=0}^{n}(-1)^{n-k}\frac{(-nI)_k}{n!}(A+B+C+D+2kI-I)$$

$$\times(A+B+C+D+kI-I)_{n+1}^{-1}(A+B)_k^{-1}(A+C)_k^{-1}(A+D)_k^{-1}P_k(x^2).$$

- Racah matrix polynomial and its inverse series:

$$R_n(x(xI+D+E+I);A,B,D,E) = \sum_{k=0}^{n}\frac{(-nI)_k}{k!}(A+B+nI+I)_k(-x)_k$$

$$\times\,(xI+D+E+I)_k(A+I)_k^{-1}(B+E+I)_k^{-1}(D+I)_k^{-1};$$

$$\frac{(-xI)_n(xI+D+E+I)_n}{k!}\,[(A+I)_n]^{-1}(B+E+I)_n^{-1}[(D+I)_n]^{-1}$$

$$= \sum_{k=0}^{n}(-1)^{n-k}\frac{(-nI)_k}{n!}(A+B+2kI+I)\,(A+B+kI+I)_{n+1}^{-1}$$

$$\times R_k(x(xI+D+E+I);A,B,D,E).$$

From these inverse pairs, it can be readily obtained the inverse pairs of the matrix polynomials of Dual Hahn, Hahn, Jacobi, Krawtchouk, Meixner etc. As an application of the first series in the theorem, we find the generating function relations where as from the second series of the theorem, we derived certain summations formulas.

- Generating function relations:

$$\sum U(n)\;t^n = (1-t)^A\sum_{k=0}^{\infty}(-A)_{2k}(1-t)^{-2kI}\;V(k)\;t^k \tag{24}$$

- Summation Formula:

$$e^{mx}I = \sum_{n=0}^{\infty}(A-nI)\sum_{k=0}^{\infty}\frac{\eta^k}{k!}(A-nI-mkI+kI)^{-1}(-A)_{n+mk-k}^{-1}$$

$$\times c^{nI-kI+mkI-A}P_n^A(m,x,\eta,c) \tag{25}$$

It is interesting to note that most of the Riordan's classes of inverse pairs assume matrix extensions by means of the above theorem. For example, we take $\eta = 1$ and replace $V(n)$ by $\Gamma(A - nrI + I)V(n)$ to get

$$\begin{aligned} \mathbf{U}(n) &= \sum_{k=0}^{M} \frac{\Gamma(A-(nr+brk-1)I)}{k!}\Gamma^{-1}(A+(1-nr-brk-k)I) \\ &\quad \times \mathbf{V}(n+bk) \\ &\Leftrightarrow \\ \mathbf{V}(n) &= \sum_{k=0}^{M}(-1)^k\frac{(A-(nr+brk)I)}{k!}\,\Gamma(A-(nr-k)I) \\ &\quad \times\Gamma^{-1}(A-(nr-1)I)\ \mathbf{U}(n+bk). \end{aligned} \tag{26}$$

From this we illustrate the Gould matrix class, simpler Legendre class-I and Legendre Chebyshev matrix class in the following table.

Table-2: Matrix analogue of Riordan's Pair

$$F(n)=\sum\ C_{n,k}\ G(k);\quad G(n)=\sum(-1)^{n-k}\ D_{n,k}\ F(k)$$

$$(\ h_{r,s}=qr-s, B=A+I, A+nI=C\ )$$

Inv.	b	r	A	$C_{n,k}$	$D_{n,k}$	Matrix class
(26)	-1	$1-q$	A	$\frac{\Gamma(B+h_{k,k}I)}{(n-k)!}$ $\times\Gamma^{-1}(B+h_{k,n}I)$	$\frac{(A+(h_{k,k})I)}{(n-k)!}$ $\times\Gamma^{-1}(B+h_{n,n}I)$ $\times\Gamma(A+(h_{n,k})I)$	Golud class
(26)	1	-2	A	$\frac{\Gamma(B+2kI)}{(k-n)!}$ $\times\Gamma^{-1}(B+nI+kI)$	$\frac{(A+2kI)}{(k-n)!}$ $\times\Gamma(C+kI)$ $\times\Gamma(C+kI)$ $\times\Gamma^{-1}(B+2nI)$	Simpler Legendre class
(26)	-1	A	$-c$	$\frac{\Gamma(B+ckI)}{(n-k)!}$ $\times\Gamma^{-1}(B+ckI+kI-nI)$	$\frac{A+ckI}{(n-k)!}$ $\times\Gamma(C+cnI-kI)$ $\times\Gamma^{-1}(B+cnI)$	Legendre Chebyshev class

Chapter-4 deals with extension of a generalized Konhauser polynomial to the matrix form.
We define the polynomial as follows.

Definition 5. For a matrix A in $C^{p\times p}$,

$$Z_{m^*}^{(A,\lambda)}(x^k;r)=\frac{\Gamma(A+rmI+I)}{m!}\sum_{n=0}^{\lfloor m/s\rfloor}(-mI)_{sn}\Gamma^{-1}(A+rnI+I)\frac{(\lambda x^k)^n}{n!}, \tag{27}$$

where $r,\lambda,\mu\in\mathbb{C};\ k\in\mathbb{R}_{>0},\ s\in\mathbb{N},\ m\in\mathbb{N}\cup\{0\},\ \Re(\lambda)>0,\ \Re(\mu)>-1$ for all eigen values $\mu\in\sigma(A)$ and the floor function $\lfloor u\rfloor=floor\ u$, represents the greatest integer $\leq u$.

For this polynomial we obtain the differential equation which is stated below.

Theorem 6. *If $r,s\in\mathbb{N}$ and the operator Θ is defined by $\Theta f(x)=\dfrac{x}{k}\dfrac{d}{dx}f(x)$ then $U=Z_{m^*}^{(A,\lambda)}(x^k;r)$ satisfies the equation*

$$\left[\left\{\Theta\prod_{j=1}^{r}\left(\Theta\ I+\frac{A+jI}{r}-I\right)\right\}-\left(\frac{s^s}{r^r}\right)\ \lambda\ x^k \times\left\{\prod_{i=1}^{s}\left(\Theta\ I+\frac{-m+i-1}{s}I\right)\right\}\right]U=O.$$

The inverse series relation is derived in the form of

Theorem 7. *For a matrix $A\in C^{p\times p},\ r,\lambda\in\mathbb{C},\ s\in\mathbb{N},\ m\in\mathbb{N}\cup\{0\}$,*

$$Z_{m^*}^{(A,\lambda)}(x^k;r)=\frac{\Gamma(A+rmI+I)}{m!}\sum_{j=0}^{\lfloor m/s\rfloor}(-mI)_{sj}\Gamma^{-1}(A+rjI+I)\frac{(\lambda x^k)^j}{j!}$$

if and only if

$$\frac{(\lambda x^k)^m}{m!}I=\frac{\Gamma(A+rmI+I)}{(ms)!}\sum_{j=0}^{ms}(-msI)_j\Gamma^{-1}(A+rjI+I)Z_{j^*}^{(A,\lambda)}(x^k;r),$$

and for $m\neq sl,\ l\in\mathbb{N}$,

$$\sum_{j=0}^{m}(-mI)_j\ \Gamma^{-1}(A+rjI+I)\ Z_{j^*}^{(A,\lambda)}(x^k;r)=O,$$

where O is the zero matrix of order p.

Among the several generating function relation derived for this polynomial the following is one of them.

Theorem 8. *Let a and b be complex constants which are not zero simultaneously, then there holds the generating function relation*

$$\begin{aligned}&\sum_{n=0}^{\infty} Z_{n^*}^{(A,\lambda)}\left(\frac{x^k}{(a+bn)^s};r\right)(a+bn)^n\ \Gamma^{-1}(A+rnI+I)\ t^n\\ =&\ e^{ax}\ (1-bte^{bx})^{-1}\ E_{rI,A+I}(\lambda x^k(-t)^s e^{bsx}),\end{aligned}$$

where

$$E_{rI,A+I}^{B,sI}(z)=\sum_{n=0}^{\infty}(B)_{sn}\Gamma^{-1}(A+rnI+I)\frac{z^n}{n!}$$

is matrix analogue of the Mittag-Leffler function [17].
One of the summation formulas implied by the inverse series is given below.

$$\begin{aligned}\lambda^M x^{kM(M+1)/2}I &= \prod_{m=1}^{M}\frac{m!\ \Gamma(A+rmI+I)}{(ms)!}\sum_{j=0}^{ms}(-msI)_j\Gamma^{-1}(A+rjI+I)\\ &\quad\times Z_{j^*}^{A,\lambda}(x^k;r).\end{aligned}$$

The mixed relation for the polynomial is stated as

Theorem 9. *There holds the mixed relation:*

$$\begin{aligned}&sx\ x^{A-I}\ \Gamma(A+rmI+I)\ \Gamma^{-1}(A+rmI)\ Z_{m^*}^{(A-I,\lambda)}(x^r,r)-Ax^{A-I}Z_{m^*}^{(A,\lambda)}(x^r,r)\\ &\quad -rmx^A\ Z_{m^*}^{(A,\lambda)}(x^r;r)+kx^A(A+mrI+I)_{-r}^{-1}\ Z_{m-1^*}^{(A,\lambda)}(x^r;r)=O.\end{aligned}$$

The Beta matrix transform of this polynomial is stated below.

Theorem 10. *If $A,P,Q\in C^{p\times p}$, P,Q are positive stable matrices, for $q=0,1,2,\dots,$ the matrices $P+qI$, Q are commutative, $P+qI,Q+qI$, $P+Q+qI$ are invertible and $k,r,s,m\in\mathbb{N}$, then*

$$\begin{aligned}\mathfrak{B}\left\{Z_{m^*}^{(A,\lambda)}(tx^k;r):P,Q\right\} &= \frac{(A+I)_{rm}}{m!}\ \Gamma(Q)\Gamma^{-1}(P)\Gamma^{-1}(P+Q)\\ &\quad\times\ {}_{s+k}F_{r+k}\left[\begin{matrix}\prec s;-mI\succ, & \prec k;P\succ; & \dfrac{s^s}{r^r}t\\ \prec r;A+I\succ, & \prec k;P+Q\succ; & \end{matrix}\right].\end{aligned}$$

- Generalized Mittag -Leffer function

  Chapter-5 incorporates the matrix version of a generalized Mittag -Leffer function [10]. It is defined as follows.

  Definition 11. For A, B, C to be positive stable matrices in $\mathbb{C}^{p\times p}$, $\alpha, \lambda, z \in \mathbb{C}$ with $\Re(\alpha) > 0$, $\delta, \mu > 0, r \in \{-1, 0\} \cup \mathbb{N}$ and $s \in \{0\} \cup \mathbb{N}$,

$$E^{A,B,C}_{\alpha I,\delta I,\mu I}(\lambda z; s, r) = \sum_{n=0}^{\infty} \Gamma^{-1}(\alpha nI + B) \, [(A)_{\delta n}]^s \, [(C)_{\mu n}]^{-r} \, \frac{(\lambda z)^n}{n!}. \quad (28)$$

This function yields Bessel-Maitland matrix function, Dotsenko matrix function, Saxena and Nishimoto's matrix function and The Elliptic matrix function.

This function satisfies the differential equation which is given as

Theorem 12. *Let $\alpha, \mu, \delta \in \mathbb{N}$ then $\boldsymbol{Y} = E^{B,C,A}_{\alpha I,\mu I,\delta I}(\lambda z; s, r)$ satisfies the equation*

$$\Upsilon^{(\alpha,B;1)}_{j} \; \theta \; \boldsymbol{Y} \, \Upsilon^{(\mu,C;r)}_{k} - (u\lambda z) \, \boldsymbol{Y} \, \Delta^{(\delta,A;s)}_{m} = O, \quad (29)$$

wherein the matrices $AC = CA$.

Here

$$\frac{\delta^{s\delta}}{\alpha^{\alpha} \; \mu^{r\mu}} = u, \;\; \frac{d}{dz} = D, \;\; zD = \theta, \;\; \prod_{m=0}^{\delta-1} \left[\left(\theta I + \frac{A + mI}{\delta} \right) \right]^{s} = \Delta^{(\delta,A;s)}_{m}, \quad (30)$$

and

$$\prod_{j=0}^{\alpha-1} \left(\theta I + \frac{T + jI}{\alpha} - I \right)^{m} = \Upsilon^{(\alpha,T;m)}_{j}. \quad (31)$$

The Eigen function property involving the operators

$$\Theta^{(\delta,A;-s)}_{m} = \prod_{m=0}^{\delta-1} \left[\left(-\theta I + \frac{A + mI}{\delta} - I \right) \right]^{-s},$$

is derived as

Theorem 13. *Let $\alpha, \mu, \delta \in \mathbb{N}$ then $\boldsymbol{W} = E^{B,C,A}_{\alpha I,\mu I,\delta I}(\zeta\lambda z; s, r)$ possesses the eigen function property given by*

$$(\lambda u)^{-1} \; D \; \Upsilon^{(\alpha,B;1)}_{j} \; \boldsymbol{W} \, \Upsilon^{(\mu,C;r)}_{k} \; \Theta^{(\delta,A;-s)}_{m} = \zeta \; E^{B,C,A}_{\alpha I,\mu I,\delta I}(\zeta\lambda z; s, r), \quad (32)$$

where $AC = CA$.

- Extended Konhauser matrix polynomial

  The Mittag-Leffler Matrix function (28) is capable of providing the Konhauser matrix polynomial as define below.

  Definition 14. For the matrices A and B in $C^{p\times p}$,

$$\begin{aligned} Z_{m^*}^{(A,B,\mu,r)}(\lambda x^k; s,p) &= \Gamma(A+rmI+I)\sum_{n=0}^{\lfloor m/\delta\rfloor}(-m)_{\delta n}^{s}\ [(B)_{\mu n}^{p}]^{-1} \\ &\quad \times\Gamma^{-1}(A+rnI+I)\ \frac{(\lambda x^k)^n}{n!\ (m!)^s}, \end{aligned}$$

  where $r \in \mathbb{C};\ \delta, k \in \mathbb{N},\ \ s,p \geq 0,\ \ \Re(\mu) > -1$, for all eigen values $\mu \in \sigma(A)$, λ is a complex number with $\Re(\lambda) > 0$ and the floor function $\lfloor u\rfloor = floor\ u$, represents the greatest integer $\leq u$.

It possesses the inverse series relation which is stated as

Theorem 15. *For* $A, B \in C^{p\times p}$ *and* $\delta = 2, 3, \ldots,$

$$\begin{aligned} Z_{m^*}^{(A,B,\mu,r)}(\lambda x^k; 1,p) &= \Gamma(A+rmI+I)\sum_{n=0}^{\lfloor m/\delta\rfloor}(-mI)_{\delta n}\ [(B)_{\mu n}^{p}]^{-1}\Gamma^{-1}(A+rnI+I) \\ &\quad \times\frac{(\lambda x^k)^n}{n!\ m!}, \end{aligned}$$

if and only if

$$\begin{aligned} \frac{(\lambda x^k)^m}{m!}I &= (B)_{\mu m}^{p}\ \Gamma(A+rmI+I)\sum_{j=0}^{m\delta}\frac{(-m\delta I)_j}{(m\delta)!}\Gamma^{-1}(A+rjI+I) \\ &\quad \times Z_{j^*}^{(A,B,\mu,r)}(\lambda x^k; 1,p), \end{aligned}$$

and for $m \neq \delta l,\ \ l \in \mathbb{N}$,

$$\sum_{j=0}^{m}(-mI)_j\ \Gamma^{-1}(A+rjI+I)\ Z_{j^*}^{(A,B,\mu,r)}(\lambda x^k; 1,p) = O.$$

Next using the L-exponential function

$$e_k(x) = \sum_{n=0}^{\infty}\frac{x^n}{(n!)^{k+1}}$$

of order k, due to Ricci and Tavkhelidze [12], the following generating function is obtained.

Theorem 16. *If $A+\ell I, B+\jmath I$ are the matrices in $C^{p\times p}$ which are invertible for all $\ell, \jmath = 0, 1, 2, \ldots$ and if $\mu, r, \delta \in \mathbb{Z}_{>0}$, then there holds the generating function relation:*

$$\sum_{m=0}^{\infty}(A+I)_{rm}^{-1}\, Z_{m^*}^{(A,B,\mu,r)}(\lambda x^k; s, p)\; t^{ms} = e_{s-1}(t^s)$$
$$\times\; {}_0F_{r+\mu p}\left(-; \left(\frac{B}{\mu}\right)^p, \ldots, \left(\frac{B+(\mu-1)I}{\mu}\right)^p, \frac{A+I}{r}, \ldots, \frac{A+rI}{r}; \frac{(-t)^{\delta s}\;\lambda x^k}{r^r\;\mu^{\mu p}}\right).$$

Among the summation formulas derived with the aid of the inverse series relation, the following is stated below.

$$x^{-kA/2}\,\Gamma(A+I)\; J_A(2x^{k/2}) = \sum_{m=0}^{\infty}(B)_{\mu m}^{p}\sum_{j=0}^{m\delta}\frac{(-m\delta I)_j}{(m\delta)!}(A+I)_{rj}^{-1}\; Z_{j^*}^{(A,B,\mu,r)}(-x^k; 1, p),$$

where the function $J_A(z)$ is the Bessel matrix function of first kind defined by [15, Eq. (1.10), p. 268]

$$J_A(z) = \left(\frac{z}{2}\right)^A\; \Gamma^{-1}(A+I)\; {}_0F_1\left(-; A+I; -\frac{z^2}{4}\right). \tag{33}$$

The last chapter, chapter-6 introduces the matrix analogue of the function[4]

$$\begin{aligned}
{}_pR_q(\alpha,\beta;z) &= {}_pR_q\left(\begin{matrix}\gamma_1,\ldots,\gamma_p & \\ & \mid \alpha,\beta;z \\ \delta_1,\ldots,\delta_q & \end{matrix}\right)\\
&= \sum_{n\geq 0}\frac{(\gamma_1)_n\ldots(\gamma_p)_n}{\Gamma(\alpha n+\beta)(\delta_1)_n\ldots(\delta_q)_n}\,\frac{z^n}{n!},
\end{aligned}$$

where $\alpha, \beta \in \mathbb{C}$, $\Re(\alpha), \Re(\beta), \Re(\gamma_i), \Re(\delta_j) > 0\ \forall i$ and $\forall j$. The infinite series on the right hand side converges absolutely if
(i) $p \leq q+1\ \forall z \in \mathbb{C}$,
(ii) $p = q+2$ for those $z \in \mathbb{C}$ such that $|z| < 1$ and
(iii) $p = q+2$ and $|z| = 1$ for $\Re(\sum \delta_j - \sum \gamma_i) > 0$.

The proposed matrix form is defined as follows. We use the notation (P) to denote the array of $p \times p$ matrices $P_1, P_2, \ldots, P_k$ for some $k \in \mathbb{N}$.

Definition 17. For $1 \leq i \leq p$, $1 \leq j \leq q$, let A, B, C_i and D_j, be the positive stable matrices in $\mathbb{C}^{p\times p}$ such that $D_j + kI$ are invertible for all integers $k \geq 0$. Then the matrix function denoted by ${}_pR_q(A, B : (C), (D); z)$ is defined as

$$\begin{aligned} {}_pR_q(A, B : (C), (D); z) &= {}_pR_q \left(\begin{matrix} C_1, \ldots, C_p \\ \\ D_1, \ldots, D_q \end{matrix} \mid A, B; z \right) \\ &= \sum_{n\geq 0} \Gamma^{-1}(nA + B)(C_1)_n \ldots (C_p)_n \\ &\quad \times (D_1)_n^{-1} \ldots (D_q)_n^{-1} \frac{z^n}{n!}, \end{aligned}$$

whenever the series converges absolutely.

The conditions of absolute convergence of the series on the right hand side in the definition are determined in the following theorem.

Theorem 18. *Let $A, B, C_1, \ldots, C_p, D_1, \ldots, D_q$ be positive stable matrices in $\mathbb{C}^{p\times p}$, then*

1. *if $p \leq q + 1$, the series converges absolutely for all finite $|z|$.*
2. *if $p = q + 2$, the series converges for $|z| < 1$ and diverges for $|z| > 1$.*
3. *if $p > q + 2$, the series diverges for all $z \neq 0$.*

This function gives rise to the matrix analogues of the generalized matrix M-series:

$$\begin{aligned} &{}_{p-1}M_q^{(A,B)}(C_1, \ldots, C_{p-1}, D_1, \ldots, D_q; z) \\ &= \sum_{n\geq 0} \Gamma^{-1}(nA + B)(C_1)_n \ldots (C_{p-1})_n (D_1)_n^{-1} \ldots (D_q)_n^{-1} z^n, \end{aligned} \tag{34}$$

the Mittag-Lefler function:

$$ {}_1R_0\left(\begin{array}{c} I \\ \qquad \mid A,B;z \\ - \end{array}\right) = \sum_{n\geq 0}\Gamma^{-1}(nA+B)\,z^n = E_{A,B}(z), $$

the generalized Mittag-Leffler function in four parameters (cf. [14]):

$$ {}_2R_1\left(\begin{array}{c} C,I \\ \qquad \mid A,B;z \\ D \end{array}\right) = \sum_{n\geq 0}\Gamma^{-1}(nA+B)\,(C)_n\,(D)_n^{-1}z^n = E_{A,B}^{C,D}(z), $$

the generalized Bessel-Maitland function (cf. [7]):

$$ {}_0R_0\left(\begin{array}{c} - \\ \qquad \mid A,B+I;-z \\ - \end{array}\right) = \sum_{n\geq 0}\frac{\Gamma^{-1}(nA+B+I)\,(-z)^n}{n!} = J_A^B(z) $$

etc. Besides this, the matrix polynomials those of Jacobi, Gegenbauer, Konhauser etc. are also contained in this function. For the function (34), the contiguous function relations have been derived; one of these is

$$ R(C_i+) = {}_pR_q\left(\begin{array}{c} C_1,\dots,C_{i-1},C_i+I,C_{i+1},\dots,C_p \\ \qquad \mid A,B;z \\ D_1,\dots,D_q \end{array}\right). $$

The differential equation of the function $y =_p R_q(A,B:(C),(D);z)$ is obtained in the form:

Theorem 19. *Let $\alpha \in \mathbb{N}, A = \alpha I$, then $\boldsymbol{Y} =_p R_q(\alpha I, B : (C), (D); z)$ satisfies the equation:*

$$\Upsilon_j^{(\alpha,B)}\ \theta\ \boldsymbol{Y}\, \Upsilon_k^D - \boldsymbol{Y}\, \Delta_i^C = O. \tag{35}$$

wherein the matrices $C_i D_k = D_k C_i$ for all $i = 1, 2, \ldots, p$ and $k = 1, 2, \ldots, q$.

The operators in the equation are specified below.

$$\begin{aligned}
&\frac{d}{dz} = D,\ zD = \theta,\ \prod_{i=1}^{p} (\theta I + C_i) = \Delta_i^C, \\
&\prod_{k=1}^{q} (\theta I + D_k - I) = \Upsilon_k^D, \prod_{j=0}^{\alpha-1} \left(\theta I + \frac{B + jI}{\alpha} - I\right) = \Upsilon_j^{(\alpha,B)}.
\end{aligned} \tag{36}$$

This is followed by the matrix differntial formulas; one of which is illustrated here.

$$\begin{aligned}
zA\frac{d}{dz}\ &{}_pR_q(A, B : (C), (D); z) \\
&= {}_pR_q(A, B - I : (C), (D); z) - (B - I)\ {}_pR_q(A, B : (C), (D); z),
\end{aligned}$$

where $D_j + kI, j = 1, 2, \ldots, q$, are invertible, $A, B - I$ are positive stable with $AB = BA$.
The integral representation occurs in the form:

$$\begin{aligned}
{}_pR_q(A, B : (C), (D); z) = \int_0^1 t^{C_p - I}\ (1 - t)^{D_q - C_p - I}\ {}_{p-1}R_{q-1}(A, B : (C), (D); tz)\ dt \\
\times \Gamma(D_q)\Gamma^{-1}(C_p)\Gamma^{-1}(D_q - C_p).
\end{aligned}$$

Lastly, the fractional integral and fractional derivative formulas are applied to the function ${}_pR_q(A, B : (C), (D); z)$ which resulted again in ${}_pR_q[z]$. They are stated below.

$$\begin{aligned}
&\mathbf{I}^\mu[{}_pR_q(A, B : (C), (D); z)z^{D_j - I}] \\
&= {}_pR_q\left(\begin{matrix} C_1, \ldots, C_p \\ \\ D_1, \ldots, D_{j-1}, D_j + \mu I, D_{j+1}, \ldots, D_q \end{matrix} \mid A, B; z\right) z^{D_j + (\mu - 1)I}
\end{aligned}$$

$$\times\Gamma(D_j)\Gamma^{-1}(D_j+\mu I).$$

and

$$\begin{aligned}
&\mathbf{D}^{\mu}[{}_pR_q(A,B:(C),(D);z)z^{D_j-I}]\\
=\;& {}_pR_q\left(\begin{array}{c} C_1,\dots,C_p \\ \\ D_1,\dots,D_{j-1},D_j-\mu I,D_{j+1},\dots,D_q \end{array} \mid A,B;z\right) z^{D_j-(\mu-1)I}\\
&\times\Gamma(D_j)\Gamma^{-1}(D_j-\mu I).
\end{aligned} \tag{37}$$

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Paper presented in International/National Conference

- Presented the paper entitled "Generalized Konhauser matrix polynomial and its properties" at the International Conference on Special Functions & Applications (ICSFA 2017) held at College of Engineering & Technology, Bikaner (Rajasthan), India during 2-4 November, 2017.

- Presented the paper entitled " A general matrix inverse series relation and associated polynomials" at the International Conference on Mathematical Modelling, Applied Analysis and Computation (ICMMAAC-2018) held at Department of Mathematics and Statistics , JECRC University, Jaipur, India, from 06-08 July, 2018.

- Presented the paper entitled "A matrix analogue of a general inversion pair and associated matrix polynomials" at International Conference on Mathematical Analysis And Application (MAA-2020) held at the department of Mathematics , National Institute of Technology (NIT), Jamshedpur, India, from 02-04 November, 2020.

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