Appendices

Appendix A: Measurements of Covariance Analysis

Covariance analysis for the $^{113,115}In(n,xn)$ reaction cross-section data

$^{113}In(n,n')^{113m}In$, $^{115}In(n,2n)^{114m}In$ and $^{115}In(n,n')^{115m}In$ reaction cross-section were measured corresponding to ^{27}Al $(n,\alpha)^{24}Na$ monitor reaction. The experiment was performed using an accelerator facility at BARC-TIFR Pelletron, Mumbai. The detailed calculation for the formation of the covariance matrices is provided in the subsequent sections to give a better understanding of the analysis. These are further divided into two parts, the first part is about covariance and correlation matrix for the detector efficiency of the γ-lines used in the measurements. Further, in the second step, these correlations and errors are taken to measure the resultant uncertainties in the cross-sections using the ratio method.

Part one: Find the covariance matrix for detector efficiency and fitting parameters from the liner fit method [1]

The ^{152}Eu source was used to measure the efficiency of the detector for the present work. Table 1 illustrate data of ^{152}Eu source at different γ-energies.

From the definitions given in equations 3.20-3.25, the partial derivatives can be obtained as,

[1]The present data measured with the measurement of $^{58}Ni(n,x)$ and $^{100}Mo(n,2n)$ of Siddharth Parashari, *Study of nuclear reaction cross sections for reactor applications*, hence the data of part one of the present work is analogous with appendix step: 1 of their work.

Table 1: Data sets of attributes used to obtain the detector efficiency

Energy (keV)	*Cobs*	I_γ	N_0	$T_{1/2}$ (Years)	Efficiency
121.8	439862 ± 3518	0.2853 ± 0.0016	7767.73 ± 88	13.517 ± 0.009	0.10198384
244.7	70828 ± 894	0.0755 ± 0.0004	7767.73 ± 88	13.517 ± 0.009	0.06205470
443.9	18536 ± 296	0.0282 ± 0.0014	7767.73 ± 88	13.517 ± 0.009	0.04305977
964	48429 ± 581	0.1451 ± 0.0007	7767.73 ± 88	13.517 ± 0.009	0.0220777
1112	41140 ± 534	0.1367 ± 0.0008	7767.73 ± 88	13.517 ± 0.009	0.01990730
1408	52162 ± 573	0.2087 ± 0.0009	7767.73 ± 88	13.517 ± 0.009	0.0165328

$$\frac{\partial \epsilon}{\partial Cobs} = \frac{1}{N_0 I_\gamma e^{-\lambda T} \triangle t} \tag{1}$$

The calculated partial uncertainties ε, I_γ, N_0, and $T_{1/2}$, by excluding the negative sign can be calculated and are provided in Table 2. Now we can estimate the total uncertaininty in the detector efficiency taking the quadratic sum of all the individual partial uncertainties from the four attributes, by using the equation 3.24 and data of the Table 2.

Table 2: Partial uncertainties in the detector efficiency

Energy	Partial uncertainty due to attributes				Total uncertainty
(keV)	$r = 1(Cobs)$	$r = 2(I_\gamma)$	$r = 3(N_0)$	$r = 4(T_{1/2})$	$(\sigma_{\varepsilon_{ii}})$
121.8	0.000815663	0.000571939	0.001155367	4.8321E-06	0.001525554
244.7	0.000783262	0.000328767	0.000703013	2.94022E-06	0.001102644
443.9	0.000687618	0.002137719	0.000487821	2.04022E-06	0.002297964
964	0.000264866	0.000106509	0.000250117	1.04607E-06	0.000379549
1112	0.000258398	0.000116502	0.000225528	9.43229E-07	0.000362224
1408	0.000181614	7.12966E-05	0.0001873	7.83345E-07	0.00027046

Now the diagonal matrix (e_{jl}) for partial uncertainty from each attribute can be written as given in Table 3.

Table 3: Diagonal matrix ($\times 10^{-3}$) for partial uncertainty from the attribute *Cobs* (r=1).

$$\begin{vmatrix} 0.815663 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.783262 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.687618 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.264866 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.258398 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.181614 \end{vmatrix}$$

The sandwich formula we have used to calculate the covariance matrix for the detector efficiency, written as,

$$(V_\varepsilon)_{jk} = \sum_r e_{jl} S_{jkl} e_{kl} \tag{2}$$

where, V_ε is the covariance matrix for detector efficiency, e_{il} is the diagonal matrix for each r and the S_{jkl} are the micro correlation matrices defined for uncorrelated, partially correlated and correlated cases as,

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & q & \cdots & q \\ 1 & q & \cdots & q \\ \vdots & \vdots & \ddots & \vdots \\ q & q & \cdots & 1 \end{pmatrix} \quad and \quad \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}; 0 < q < 1$$

Now, from the equation 2 the covariance matrix calculated for r=1 (Cobs) as,

$$= \begin{pmatrix} 0.815663 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.783262 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.687618 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.264866 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.258398 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.181614 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} 0.815663 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.783262 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.687618 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.264866 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.258398 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.181614 \end{pmatrix} \times 10^{-6}$$

$$V_{(Cobs)r=1} = \begin{pmatrix} 6.653E-07 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6.135E-07 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.728E-07 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.015E-08 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.676E-08 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.298E-08 \end{pmatrix}$$

Similarly, for r=2, 3 and 4 calculated as,

$$V_{(I_\gamma)r=2} = \begin{pmatrix} 3.271E-07 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.080E-07 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.569E-06 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.134E-08 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.357E-08 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.083E-09 \end{pmatrix}$$

The covariance matrix for r = 3 and 4 will be calculated using the correlated case for S_{jkl} matrix and the result are as follows,

$$V_{(N_0)r=3} \begin{pmatrix} 1.334E-06 & 8.122E-07 & 5.636E-07 & 2.889E-07 & 2.605E-07 & 2.164E-07 \\ 8.122E-07 & 4.942E-07 & 3.429E-07 & 1.758E-07 & 1.585E-07 & 1.316E-07 \\ 5.636E-07 & 3.429E-07 & 2.379E-07 & 1.220E-07 & 1.100E-07 & 9.136E-08 \\ 2.889E-07 & 1.758E-07 & 1.220E-07 & 6.255E-08 & 5.640E-08 & 4.684E-08 \\ 2.605E-07 & 1.585E-07 & 1.100E-07 & 5.640E-08 & 5.0863E-08 & 4.224E-08 \\ 2.164E-07 & 1.316E-07 & 9.136E-08 & 4.684E-08 & 4.2241E-08 & 3.508E-08 \end{pmatrix}$$

$$V_{(T_{1/2})r=4}\begin{pmatrix} 2.334E-11 & 1.420E-11 & 9.858E-12 & 5.054E-12 & 4.557E-12 & 3.785E-12 \\ 1.420E-11 & 8.644E-12 & 5.998E-12 & 3.075E-12 & 2.773E-12 & 2.303E-12 \\ 9.858E-12 & 5.998E-12 & 4.162E-12 & 2.134E-12 & 1.924E-12 & 1.598E-12 \\ 5.054E-12 & 3.075E-12 & 2.134E-12 & 1.094E-12 & 9.866E-13 & 8.194E-13 \\ 4.557E-12 & 2.773E-12 & 1.924E-12 & 9.866E-13 & 8.896E-13 & 7.388E-13 \\ 3.785E-12 & 2.303E-12 & 1.598E-12 & 8.194E-13 & 7.388E-13 & 6.136E-13 \end{pmatrix}$$

The final covariance matrix is calculated by adding the four matrices $V_{r=1,2,3,4}$ and is given in Table 4.

Table 4: Covariance matrix (×100) for the detector efficiency

$$(V_\epsilon)_{jk}=\begin{vmatrix} 0.0030938 & & & & & \\ 0.0008305 & 0.0015126 & & & & \\ 0.0005763 & 0.0003507 & 0.0009057 & & & \\ 0.0002955 & 0.0001798 & 0.0001248 & 0.0001840 & & \\ 0.0002664 & 0.0001621 & 0.0001125 & 0.0000577 & 0.0001593 & \\ 0.0001346 & 0.0001346 & 0.000934 & 0.0000479 & 0.0000432 & 0.0000968 \end{vmatrix}$$

Table 4 is for the covariance matrix measured by taking the square root of the diagonal elements of the uncertainties in the efficiency. The correlation factors among the γ-lines of ^{152}Eu can be calculated from the equation,

$$\mathbf{Corr}(E_{\gamma_j}, E_{\gamma_k}) = \frac{\mathbf{Cov}(E_{\gamma_j}, E_{\gamma_k})}{\sqrt{\mathbf{Var}(E_{\gamma_j})}\sqrt{\mathbf{Var}(E_{\gamma_k})}} \tag{3}$$

where, $E_{\gamma_{j,k}}$ are the j,k^{th} element of the matrix $(V_\epsilon)_{jk}$.

The next step is to calculate the matrix $(V_z)_{ij}$ as equation 3.33, To calculate matrix $(V_z)_{jk}$ with the equation 3.33, first we need to design a diagonal matrix of detector efficiencies as,

$$1/\epsilon_j = \begin{pmatrix} 9.69686 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15.93632 & 0 & 0 & 0 & 0 \\ 0 & 0 & 22.96630 & 0 & 0 & 0 \\ 0 & 0 & 0 & 44.79279 & 0 & 0 \\ 0 & 0 & 0 & 0 & 49.67643 & 0 \\ 0 & 0 & 0 & 0 & 0 & 59.81559 \end{pmatrix}$$

Now the matrix $(V_z)_{jk}$ can be calculated as,

$$(V_z)_{jk} = \begin{pmatrix} 9.69686 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15.93632 & 0 & 0 & 0 & 0 \\ 0 & 0 & 22.96630 & 0 & 0 & 0 \\ 0 & 0 & 0 & 44.79279 & 0 & 0 \\ 0 & 0 & 0 & 0 & 49.67643 & 0 \\ 0 & 0 & 0 & 0 & 0 & 59.81559 \end{pmatrix} \times$$

$$\begin{pmatrix} 3.09377E-06 & 8.30548E-07 & 5.76318E-07 & 2.95492E-07 & 2.66442E-07 & 2.21278E-07 \\ 8.30548E-07 & 1.51262E-06 & 3.50675E-07 & 1.79799E-07 & 1.62124E-07 & 1.34642E-07 \\ 5.76318E-07 & 3.50675E-07 & 9.05744E-07 & 1.24763E-07 & 1.12498E-07 & 9.34284E-08 \\ 2.95492E-07 & 1.79799E-07 & 1.24763E-07 & 1.83963E-07 & 5.76801E-08 & 4.79029E-08 \\ 2.66442E-07 & 1.62124E-07 & 1.12498E-07 & 4.76801E-08 & 1.59302E-07 & 4.31936E-08 \\ 2.21278E-07 & 1.34642E-07 & 9.34284E-08 & 4.79029E-08 & 4.31936E-08 & 9.68049E-08 \end{pmatrix} \times$$

$$\begin{pmatrix} 9.69686 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15.93632 & 0 & 0 & 0 & 0 \\ 0 & 0 & 22.96630 & 0 & 0 & 0 \\ 0 & 0 & 0 & 44.79279 & 0 & 0 \\ 0 & 0 & 0 & 0 & 49.67643 & 0 \\ 0 & 0 & 0 & 0 & 0 & 59.81559 \end{pmatrix}$$

$$= \begin{pmatrix} 0.290905 & 0.128347 & 0.128347 & 0.128347 & 0.128347 & 0.128347 \\ 0.128347 & 0.384155 & 0.128347 & 0.128347 & 0.128347 & 0.128347 \\ 0.128347 & 0.128347 & 0.477736 & 0.128347 & 0.128347 & 0.128347 \\ 0.128347 & 0.128347 & 0.128347 & 0.369103 & 0.128347 & 0.128347 \\ 0.128347 & 0.128347 & 0.128347 & 0.128347 & 0.393117 & 0.128347 \\ 0.128347 & 0.128347 & 0.128347 & 0.128347 & 0.128347 & 0.346359 \end{pmatrix}$$

the inverse of the matrix $(V_z)_{jk}$ can be written as,

$$(V_z)^{-1}_{jks} = \begin{pmatrix} 5012.8877 & & & & & \\ -723.6418 & 3449.3234 & & & & \\ -529.8212 & -336.6848 & 2615.6325 & & & \\ -768.8848 & -488.6022 & -357.7347 & 3634.4313 & & \\ -699.1473 & -444.2863 & -325.2883 & -472.0636 & 3347.6061 & \\ -849.0981 & -539.5753 & -395.0551 & -573.3102 & -521.3112 & 3953.7803 \end{pmatrix}$$

Now to fit the efficiencies an interpolation model is used as given in equation 3.28 with the simple liner solution, Z=AP. Where the matrices of Z and A are as follows,

$$Z = ln(\varepsilon_j) = \begin{pmatrix} -2.28294 \\ -2.77973 \\ -3.14516 \\ -3.81318 \\ -3.91666 \\ -4.10240 \end{pmatrix};$$

$$A = (lnE_j)^{m-1} = \begin{pmatrix} 1 & -2.105375 & 4.432604 & -9.332292 & 19.647974 \\ 1 & -1.407722 & 1.981682 & -2.789658 & 3.927064 \\ 1 & -0.812156 & 0.659597 & -0.535696 & 0.435069 \\ 1 & -0.036664 & 0.001344 & -0.000049 & 0.000002 \\ 1 & 0.106160 & 0.011270 & 0.001196 & 0.000127 \\ 1 & 0.342170 & 0.117080 & 0.040061 & 0.013708 \end{pmatrix}$$

Now, with the help of the matrix A, Transpose of A and V_z^{-1} the covariance matrix for parameters P can be written as,

$$= V_{\hat{P}} = (A' V_z^{-1} A)^{-1}$$

$$= \begin{pmatrix} 0.268541 & -0.10068 & -0.763573 & -0.673544 & -0.165564 \\ -0.10068 & 1.224617 & 0.301311 & -1.1193217 & -0.499049 \\ -0.763573 & 0.301311 & 8.935453 & 9.790935 & 2.70637 \\ -0.673544 & -1.193217 & 9.790935 & 12.987068 & 3.869357 \\ -0.165564 & -0.499049 & 2.70637 & 3.869357 & 1.183991 \end{pmatrix} \times 10^{-3}$$

Now, the fitting parameters $\hat{P}$ can be calculated as,

$$\hat{P} = V_{\hat{P}}(A' V_z^{-1} Z) = \begin{pmatrix} -3.83706 \\ -0.86929 \\ 0.16981 \\ 0.32085 \\ 0.10003 \end{pmatrix}$$

Now the value of χ_m^2 can be calculated as,

$$\frac{\chi_m^2}{n-m} = \frac{(Z-AP)' V_z^{-1} (Z-AP)}{n-m} = \frac{0.72}{6-5} = 0.72$$

where, n is the number of γ-lines and m is order of the fitting, until, the condition $\chi_m^2/(n-m) \approx 1$ fulfills.

Find the efficiencies, their covariance & correlation matrix for the sample/monitor γ-lines by using the fitting parameters

To find the covariance and correlation matrices for the sample and monitor reaction γ-lines we have pursued the same methodology again.

For that, first we find the efficiencies for the relevant γ-lines using the fitting parameters from the equation 3.28,

$$ln\varepsilon_j = \sum_n p_n (lnE_j)^{m-1} \tag{4}$$

which can be expended for the 0.391, 0.190, and 0.336 MeV γ-lines of $^{113}In(n,n')^{113m}In$, $^{115}In(n,2n)^{114m}In$ and $^{115}In(n,n')^{115m}In$, respectively,

$$ln\varepsilon_1 = p_1(lnE_1)^0 + p_2(lnE_1)^1 + p_3(lnE_1)^2 + p_4(lnE_1)^3 + p_5(lnE_1)^4 \tag{5}$$

$$ln\varepsilon_1 = -3.060104183$$

$$\varepsilon_1 = exp(-3.060104183) = 0.046882811$$

$$\varepsilon_2 = exp(-2.634831024) = 0.071731089$$

$$\varepsilon_3 = exp(-2.962125459) = 0.051708895$$

likewise, the efficiency of the $^{27}Al(n,\alpha)^{24}Na$ monitor reaction for the 1.368 MeV γ-line can be measured as,

$$\varepsilon_4 = exp(-4.082272134) = 0.016875$$

now the covariance/correlation matrix for the efficiencies ϵ_1 and ϵ_2 can be calculated by following the method of liner fitting and reconstructing the matrices Z, A

and using $V_{\hat{P}}$. The matrices Z and A for ϵ_1 and ϵ_2 can now be written as,

$$Z_{\epsilon_j} = \begin{pmatrix} -3.060104183 \\ -2.634831024 \\ -2.962125459 \\ -4.082233815 \end{pmatrix};$$

$$A = \begin{pmatrix} 1 & -0.937264144 & 0.878464076 & -0.823352881 & 0.771699133 \\ 1 & -1.658943332 & 2.75209298 & -4.565566299 & 7.57401577 \\ 1 & -1.089927114 & 1.187941114 & -1.294769231 & 1.411204091 \\ 1 & 0.31378832 & 0.098463109 & 0.030896574 & 0.009694984 \end{pmatrix}$$

$$A' = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -0.937264144 & -1.658943332 & -1.089927114 & 0.31378832 \\ 0.878464076 & 2.75209298 & 1.187941114 & 0.098463109 \\ -0.823352881 & -4.565566299 & -1.294769231 & 0.030896574 \\ 0.771699133 & 7.57401577 & 1.411204091 & 0.009694984 \end{pmatrix}$$

After that, the covariance matrix ($V_{z_{\epsilon_j}}$) for the column matrix Z is calculated as,

$$V_{z_{\epsilon_j}} = A' V_{\hat{P}} A \tag{6}$$

$$V_{z_{\epsilon_j}} = \begin{pmatrix} 0.000422899 & 1.95082E-05 & 0.000368373 & 0.000124765 \\ 1.95082E-05 & 0.000611912 & 0.000135084 & 0.00012426 \\ 0.000368373 & 0.000135084 & 0.000347172 & 0.000131182 \\ 0.000124765 & 0.00012426 & 0.000131182 & 0.000289378 \end{pmatrix}$$

and the covariance matrix for efficiencies ϵ_1, ϵ_2, ϵ_3 and ϵ_4 is now calculated from the equation,

$$V_{\epsilon_{jk}} = \epsilon_j V_{z_{\epsilon_{jk}}} \epsilon_k \tag{7}$$

where, $\epsilon_{j,k}$ is the diagonal matrix,

$$\begin{pmatrix} 0.046882811 & 0 & 0 & 0 \\ 0 & 0.071731089 & 0 & 0 \\ 0 & 0 & 0.051708895 & 0 \\ 0 & 0 & 0 & 0.01686974 \end{pmatrix}$$

and the final covariance ($V_{\epsilon_{jk}}$) and correlation ($Corr[V_{\epsilon_{jk}}]$) matrix 3 for ϵ_1, ϵ_2, ϵ_3 and ϵ_4 is,

$$V_{\epsilon_{jk}} = \begin{pmatrix} 9.2953E-07 & 6.56052E-08 & 8.9303E-07 & 9.86768E-08 \\ 6.56052E-08 & 3.1485E-06 & 5.01E-07 & 1.50366E-07 \\ 8.9303E-07 & 5.01044E-07 & 9.28271E-07 & 1.14432E-07 \\ 9.86768E-08 & 1.50366E-07 & 1.14432E-07 & 8.23537E-08 \end{pmatrix};$$

$$Corr[V_{\epsilon_{ij}}] = \begin{pmatrix} 1 & & & \\ 0.038349022 & 1 & & \\ 0.96138431 & 0.293080207 & 1 & \\ 0.356649587 & 0.000141898 & 0.413873561 & 1 \end{pmatrix}$$

The uncertainties in the efficiencies ϵ_1, ϵ_2, ϵ_3 and ϵ_4 can be calculated by considering the square root of the diagonal elements of the matrix $V_{\epsilon_{jk}}$. Hence, the final efficiencies of the γ-lines along with uncertainties and correlation coefficients are listed in Table 5.

Table 5: Measured efficiencies with correlation matrix for the sample and the monitor reaction

E_γ (MeV)	Efficiency	Correlation Matrix			
0.190	0.046882 ± 0.000964	1			
0.336	0.071731 ± 0.001774	0.0383	1		
0.391	0.051708 ± 0.000963	0.9613	0.2930	1	
1.368	0.01686 ± 0.000286	0.3566	0.0001	0.4138	1

The listed uncertainties and correlation coefficient in Table 5 will be considered in the second part to calculate the uncertainty in the measured cross-section. It can be noticed from the Table 5 that the uncertainty in the efficiencies are $\leq 2\%$.

Part two: Covariance analysis for the cross-section data

In this part we have calculated the covariance matrix and correlations among the different measured cross-sections, the detailed procedure is given in 3.7.2. In the present work, the sample reaction cross-section are measured relative to a monitor reaction. The equation for activation analysis takes the form,

$$< \sigma_s >=< \sigma_m > \frac{Cobs_s N_{0_m} \varepsilon_m I_{\gamma_m} f_{\lambda_m}}{Cobs_m N_{0_s} \varepsilon_s I_{\gamma_s} f_{\lambda_s}} \tag{8}$$

with the time factor f defined as,

$$f = (1 - e^{-\lambda t_i})(e^{-\lambda t_c})(1 - e^{-\lambda T_L})/\lambda \tag{9}$$

To calculate the partial uncertainties contributed from the different attributes the Equation 8 is differentiated with respect to them. The uncertainties arise from each attribute of the equation 8 is listed in Tables 6 & 7.

The correlation assigned at the bottom of the Tables 6 & 7 gives the essential knowledge to introduce the micro correlation matrix in the sandwich formula 2. Now covariance matrices can be calculated for each attribute (take an example

Table 6: Fractional uncertainty associated with all the possible parameter, which help us in obtain the uncertainty for ^{113} In(n, n′), ^{115} In(n, 2n) and ^{115}In(n, n′).

Reaction	E_p (MeV)	$Cobs_s$	ε_s	$I_{\gamma s}$	$f_{\lambda s}$	M_s	a_s	A_s
^{113}In(n, n′)	13	10.248	2.056	0.261	0.018	2.054	1.165	1.06E-08
^{113}In(n, n′)	16	4.711	2.056	0.261	0.018	2.211	1.165	1.06E-08
^{113}In(n, n′)	19	0.982	2.056	0.261	0.018	1.985	1.165	1.06E-08
^{113}In(n, n′)	22	1.807	2.056	0.261	0.020	2.310	1.165	1.06E-08
^{115}In(n, 2n)	16	57.216	2.473	0.964	0.001	2.054	0.052	2.63E-06
^{115}In(n, 2n)	19	15.512	2.473	0.964	0.0001	2.211	0.052	2.63E-06
^{115}In(n, 2n)	22	18.586	2.473	0.964	0.001	1.985	0.052	2.63E-06
^{115}In(n, n′)	13	0.015	1.863	0.217	0.073	2.054	0.052	4.35E-06
^{115}In(n, n′)	16	0.007	1.863	0.217	0.075	2.211	0.052	4.35E-06
^{115}In(n, n′)	19	0.003	1.863	0.217	0.077	1.985	0.052	4.35E-06
^{115}In(n, n′)	22	0.011	1.863	0.217	0.080	2.310	0.052	4.35E-06
Corr†		0	0.5	0.5	0.5	0.5	0.5	0.5

† Here 0, 1/2 and 1 are for un-correlated, partial-correlated and fully-corelated.

Table 7: Fractional uncertainty associated with all the possible parameter in flux measurement of ^{27}Al(n, α)^{24}Na.

Monitor	E_p (MeV)	$Cobs_m$	ε_m	$I_{\gamma m}$	$f_{\lambda m}$	M_m	σ_W	A_m
^{27}Al(n, α)	13	9.047	1.701	0.001	0.082	1.878	1.431	4.44E-07
^{27}Al(n, α)	16	4.186	1.701	0.001	0.081	1.823	0.599	4.44E-07
^{27}Al(n, α)	19	1.837	1.701	0.001	0.081	1.968	0.812	4.44E-07
^{27}Al(n, α)	22	1.506	1.701	0.001	0.082	1.884	0.350	4.44E-07
Corr†		0.5	1	1	1	0.5	0.5	1

† Here 0, 0.5 and 1 are for un-correlated, partial-correlated and fully-correlated.

$Cobs_s$) as,

$$V_{Cobs_s} = \begin{pmatrix} 0.1024 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0471 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0098 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0180 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0572 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1551 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1858 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.95E-05 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.18E-05 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 \end{pmatrix} \times$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} 0.1024 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0471 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0098 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0180 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0572 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1551 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1858 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.95E-05 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.18E-05 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 \end{pmatrix}$$

$$= \begin{pmatrix} 0.0105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0022 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9.66E-05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0003 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0032 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0240 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0345 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.34E-08 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6.32E-09 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.01E-09 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.43E-08 \end{pmatrix}$$

Similarly for ε_s, the micro correlation matrix will take the form for the partial correlation case, as,

$$V_{\varepsilon_s} =$$

$$\begin{pmatrix} 0.0205 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0205 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0205 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0205 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0247 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0247 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0247 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0186 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0186 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0186 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0186 \end{pmatrix}$$

$$\times\begin{pmatrix} 1&1&1&1&0&0&0&0&0&0&0\\ 1&1&1&1&0&0&0&0&0&0&0\\ 1&1&1&1&0&0&0&0&0&0&0\\ 1&1&1&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&1&0&0&0&0\\ 0&0&0&0&1&1&1&0&0&0&0\\ 0&0&0&0&1&1&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&1&1&1\\ 0&0&0&0&0&0&0&1&1&1&1\\ 0&0&0&0&0&0&0&1&1&1&1\\ 0&0&0&0&0&0&0&1&1&1&1 \end{pmatrix}$$

$$\times\begin{pmatrix} 0.0205&0&0&0&0&0&0&0&0&0&0\\ 0&0.0205&0&0&0&0&0&0&0&0&0\\ 0&0&0.0205&0&0&0&0&0&0&0&0\\ 0&0&0&0.0205&0&0&0&0&0&0&0\\ 0&0&0&0&0.0247&0&0&0&0&0&0\\ 0&0&0&0&0&0.0247&0&0&0&0&0\\ 0&0&0&0&0&0&0.0247&0&0&0&0\\ 0&0&0&0&0&0&0&0.0186&0&0&0\\ 0&0&0&0&0&0&0&0&0.0186&0&0\\ 0&0&0&0&0&0&0&0&0&0.0186&0\\ 0&0&0&0&0&0&0&0&0&0&0.0186 \end{pmatrix}$$

$$=\begin{pmatrix} 0.00042&0.00042&0.00042&0.00042&0&0&0&0&0&0&0\\ 0.00042&0.00042&0.00042&0.00042&0&0&0&0&0&0&0\\ 0.00042&0.00042&0.00042&0.00042&0&0&0&0&0&0&0\\ 0.00042&0.00042&0.00042&0.00042&0&0&0&0&0&0&0\\ 0&0&0&0&0.00061&0.00061&0.00061&0&0&0&0\\ 0&0&0&0&0.00061&0.00061&0.00061&0&0&0&0\\ 0&0&0&0&0.00061&0.00061&0.00061&0&0&0&0\\ 0&0&0&0&0&0&0&0.00034&0.00034&0.00034&0.00034\\ 0&0&0&0&0&0&0&0.00034&0.00034&0.00034&0.00034\\ 0&0&0&0&0&0&0&0.00034&0.00034&0.00034&0.00034\\ 0&0&0&0&0&0&0&0.00034&0.00034&0.00034&0.00034 \end{pmatrix}$$

Similarly, for $I_{\gamma m}$, the micro correlation matrix will take the form for the full correlation case, as,

$$V_{I_{\gamma m}} = 1.50E-05\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&1 \end{pmatrix}\times\begin{pmatrix} 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&1&1&1&1 \end{pmatrix}$$

$$\times 1.50E-05 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= 2.25E-10 \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Similarly, the covariance matrices for other attributes can be calculated and add up to get the final covariance $[V_{cs_{jk}}]$ and the corresponding correlation matrix, as given in Tables 8 & 9.

Table 8: Covariance matrix for the measured $^{113,115}In(n, xn)$ cross-sections

Reactions	E_p (MeV)	Covariance Matrix ($V_{cs_{jk}}$) (× 100)										
^{113}In(n, n′)	13	2.0525										
	16	0.4636	0.5685									
	19	0.2511	0.1618	0.2137								
	22	0.2211	0.1479	0.1125	0.1936							
^{115}In(n, 2n)	16	0.1295	0.0290	0.0290	0.0719	1.3434						
	19	0.0290	0.1172	0.0290	0.0290	0.4690	2.7668					
	22	0.0290	0.0290	0.1162	0.0290	0.2564	0.1671	3.6725				
^{115}In(n, n′)	13	0.1275	0.0290	0.0290	0.0290	0.1291	0.0290	0.0290	0.9808			
	16	0.0290	0.1153	0.0290	0.0290	0.0290	0.1168	0.0290	0.4426	0.3253		
	19	0.0290	0.0290	0.1143	0.0290	0.0290	0.0290	0.1158	0.2300	0.1407	0.1828	
	22	0.0290	0.0290	0.0290	0.1196	0.0290	0.0290	0.0290	0.2000	0.126	0.0914	0.1770

The uncertainty at a given incident neutron energy can be calculated by taking the square root of the diagonal element of the $V_{cs_{jk}}$ matrix and are given in Table 10.

Table 9: Correlation coefficients for the measured $^{113,115}In(n, xn)$ cross-sections

Reactions	E_p (MeV)	Correlation matrix									
^{113}In(n, n′)	13	1									
	16	0.4292	1								
	19	0.3791	0.4641	1							
	22	0.3508	0.4243	0.5533	1						
^{115}In(n, 2n)	16	0.0779	0.0331	0.0541	0.1410	1					
	19	0.0121	0.0934	0.0377	0.0396	0.2432	1				
	22	0.0105	0.0200	0.1312	0.0343	0.1154	0.0524	1			
^{115}In(n, n′)	13	0.0899	0.0388	0.0633	0.0665	0.1124	0.0176	0.0152	1		
	16	0.0354	0.2680	0.1099	0.1155	0.0439	0.1231	0.0265	0.7835	1	
	19	0.0473	0.0899	0.5783	0.1541	0.0585	0.0408	0.1414	0.5433	0.5771	1
	22	0.0481	0.0914	0.1490	0.6463	0.0595	0.04147	0.0360	0.4801	0.5286	0.5085	1

Table 10: Uncertainty (%) present in the measured $^{113,115}In(n, xn)$ cross-sections at respective neutron energy

Reaction	E_n (MeV)	Diagonal element ($V_{cs_{jj}}$)	Uncertainty (%)
^{113}In(n, n′)	10.95 ± 0.45	0.020525147	14.32
	13.97 ± 0.68	0.005685758	7.54
	16.99 ± 0.53	0.002137525	4.62
	20.00 ± 0.58	0.001936119	0.19
^{115}In(n, 2n)	13.97 ± 0.68	0.013434667	11.59
	16.99 ± 0.53	0.027668212	16.63
	20.00 ± 0.58	0.036725775	19.16
^{115}In(n, n′)	10.95 ± 0.45	0.009808575	9.90
	13.97 ± 0.68	0.003253039	5.70
	16.99 ± 0.53	0.001828065	4.27
	20.00 ± 0.58	0.001770782	4.20

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