9. Inverse relationship, Mirror images of Swaras of the Raaga scales and their degrees of Consonance & Dissonance.

Inverse relations of Swaras and Shrutis:

It has already been explained that the musical scales have evolved due to the swaras contained in the scales following the laws of the musical sounds i.e. the consonances and dissonances and that the musical scale is a group of swaras connected with the prime swara and among themselves with certain ordered degrees of consonance and dissonance. It has also been explained while dealing the musical laws of sounds that the harmonic partials which are nearer to the prime swara are more consonant than the distant ones and that the level/degree of consonance with the prime swara is similar for the ascending series of harmonics and the descending series of harmonics i.e. the level of consonance of prime swar with its harmonics 2:3:4:5 etc. is similar to that of the level of its consonance with the harmonics 1/2:1/3:1/4:1/5 etc.

Now when the above principle is applied on the existing scales of swaras we will find the two the identical groups of swaras of the similar level/degree of consonance and dissonance with prime swara and amongst them, one having the identical mirror image (in audio frame of reference) of the other. Also the mirror image will have all the qualities of the consonance and dissonances of the actual scale. Also, as any scale could be used to represent any one particular system of music, its mirror image would also be able to represent the similar system of music. This fact has been observed as back as 300 BC by Pythagoras, the famous Greek Mathematician, while giving the principle of formation of the musical scales as already discussed. He was the first to make the following Cyclic Musical Scale.

$$F \pm C \pm G \pm D \pm A \pm E \pm B$$

In this the scale the positive ascending order is (5^{th}ascending)

$$F + C + G + D + A + E + B$$

And the negative descending scale is (4^{th} descending)

$$F - C - G - D - A - E - B$$

Let us first check the consonance levels of various Swaras & Shrutis and their audio mirror images with the prime swara as derived from the above principle in the existing most prevalent Scales of Indian Music i.e. the Sadja Grama & the Madhyama Gramas-

Sadja	S 4	R 3	G 2	M 4	P 4	D 3	N 2	S
Grama	240	270	300	320	360	405	450	480

Madhyama	S 4	R 3	G 2	M 4	P 3	D 4	N 2	S
Grama	240	270	300	320	355 45/81	400	450	480

(Here the numbers mentioned along with the name of the Swara are the number of shruties assigned to that Swara as per the prevalent system)

Let us first see the importance of harmonic ratios in the context of musical intervals. We have seen that Music uses a range of more than one octave to create the emotional effects of a large spectrum. It has also been mentioned that the frequencies of all the Swaras of the lower octave are exactly the half of that of middle octave and the frequencies of the Swaras of upper octave are exactly the double of that of the middle octave. Let us now see the musical intervals operating in the musical scales. First of all the most natural and the most commonly used interval is the one which is represented by the ratio 3/2.

By applying the above principle of the positive ascending and the negative descending orders it is obvious that the consonance level of the fifth represented by the ratio of 3/2 is the same as that of 1 divided by 3/2 i.e 2/3. As this ratio i.e 2/3 is less than 1 so the Swara represented by this frequency comes in lower octave. If the same Swara is to be represented in the middle octave then according to the above explained principle its frequency should be doubled, hence the interval 2/3 become equivalent to 2/3 X 2= 4/3 in the middle octave which is the Fourth Swara of the scale. Thus, the degree of consonance of both the Fifth (3/2) and Fourth i.e. 2/3 or 4/3 according to the above mentioned laws of musical sounds are exactly the same.

Middle octave

Lower octave ← ↓ → **Middle octave**

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M_a -3/2 S_a +3/2 P_a

(Negative 5th) **(Positive 5th)**

Thus negative fifth and the positive fourth are basically the Swaras having the same degrees of consonance with the prime swara.

Now applying this principle on this newly defined negative fifth i.e the one having the Ratio of 4/3 in the middle octave, if we raise the frequency of the Swara by

3/2 i.e by fifth we must get the original Swara Sa. Let us see how it works out to be. Negative Fifth (4/3) when raised by fifth i.e 3/2 becomes 4/3 X 3/2 = 2 which is the first harmonic of the prime swara and exactly the same Swara Sa in the upper octave. As explained above the equivalent Swara of this in middle octave will be 2/2 =1 i.e the same Swara as the prime swara. So we can say that swara 'Ma' is nothing but the negative fifth of the prime. This Swara is incidentally called as Fourth also as it comes at the 4^{th} place in the natural scale. **Thus it is seen that Fourth Swara (Ma) is the mirror image of Fifth Swara (Pa). In other words we can also say that 'Ma' is inversely related to 'Pa' or simply that 'Ma' is the inverse of 'Pa' and vice versa.**

Applying the same principles on other Ratios e.g 5/4 which is the Swara 'Ga', that is called the positive Third , the negative Third , having the same degree of consonance as that of positive Third with the prime swara, becomes 1/5/4 = 4/5 which is less than 1(one) hence belonging to lower octave. The equivalent swara of this in the middle octave will be 4/5 x 2 = 8/5 which is the same swara as Minor Sixth. We can, therefore, say that the negative Major Third is same swara as the positive Minor Sixth. Let us now see when this new Swara is raised by 5/4 again whether we get the original prime Swara or not. By raising the frequency of Negative Third i.e 8/5 by 5/4 we get 8/5 X 5/4 = 2 which is the same as Prime Swara of upper/middle octave as explained above. Hence the intervals of Positive Major Third and Negative Major Third i.e positive Minor Sixth having the same degrees of consonance are the mirror images of one another. In other words they can also be called as inversely related with one another or simply the **inverses** of one another.

Similar principle can also be applied to the other Swaras and it can be seen that the level of consonance of the following Swaras is exactly the same with the prime swara and that they are inversely related to one another.

Set-1		Set-2	
Equivalent Middle Octave Swaras	Lower Octave <------	Middle Oct-ave	Upper Octave ------→
	Negative descending	Prime	Positive Ascending

Ratio	Swara	Swara	Equivelent Ratio	Negative ratio	Swara/ Ratio	Positive ratio	Swara	Swara
4/3	Major 4^{th} (Ma)	Major 4^{th}	2/3	Divided by 3/2	Sa/1	*3/2	Major 5^{th}	Pa
8/5	Minor 6^{th} (Dha K)	Minor 6^{th}	4/5	Divided by 5/4	Sa/1	*5/4	Major 3^{rd}	Ga
16/9	Minor 7^{th} (Ni K)	Minor 7^{th}	8/9	Divided by 9/8	Sa/1	*9/8	Major 2^{nd}	Ra
32/27	Minor 3^{rd} (Ga K)	Minor 3^{rd}	16/27	Divided by 27/16	Sa/1	*27/16	Major 6^{th}	Dha
16/15	Minor 2^{nd} (Ra K)	Minor 2^{nd}	8/15	Divided by 15/8	Sa/1	*15/8	Major 7^{th}	Ni

(DhaK, NiK, GaK &ReK are respectively Komal Dha, Komal Ni, Komal Ga & Komal Re)

Thus the Inverse relationships amongst various Swaras can be summarised below:

Sl-No	Swaras		Inverse Swaras		
	Name	Ratio (x)	Inverse Relation	Ratio (1/x)* 2	Name
1	Prime(Sa) l	1	Prime	1	Sa l
2	Minor 2^{nd} (Re) j	10/9	Negative Minor 2^{nd}	18/10 or 9/5	Major 7^{th} (Ni) fu
3	Major 2^{nd} (Re) j	9/8	Negative Major 2^{nd}	16/9	Minor 7^{th} (Ni) fu
4	Minor 3^{rd} (Ga) x	6/5	Negative Minor 3^{rd}	5/3	Major 6^{th} (Dha) /k
5	Major3^{rd}(Ga) x	5/4	Negative Major 3^{rd}	8/5	Minor6^{th}(Dha)/k
6	Major4^{th}(Ma) e	4/3	Negative Major 4^{th}	3/2	Major 5^{th} (Pa) i
7	Augumented 4^{th} (Ma') e	45/32	Negative Augumented 4^{th}	64/45	Augumented 4^{th} (Ma') e

8	Major 5th (Pa) i	3/2	Negative Major 5th	4/3	Major 4th (Ma) e
9	Minor 6th (Dha) /k	8/5	Negative Minor 6th	5/4	Major 3rd (Ga) x
10	Major 6th (Dha) /k	5/3	Negative Major 6th	6/5	Minor 3rd (Ga) x
11	Minor 7th (Ni) fu	16/9	Negative Minor 7th	9/8	Major 2nd (Re) j
12	Major 7th (Ni) fu	15/8	Negative Major 7th	16/15	Minor 2nd (Re) j

In the above relations the following Degrees of Consonances have been defined:

	Sl.No.	
1. Absolute Consonance	1	Sa - Upper Sa
2. Perfect Consonance	6&8	Sa-Major 4th & Major 5th
3. Medial Consonance	5&4	Sa-Major 3rd & Minor 6th
	9&10	Sa-Minor 3rd & Major 6th
4. Imperfect Dissonance	3&11	Sa-Major 2nd & Minor 7th
5. Complete Dissonance	2&12	Sa-Minor 2nd, Major 7th
	7	Augumented 4th

The above mathematical findings are slightly different than what has been indicated by Mr. Joshi in his Book (ref-16-page75). The difference is only in Minor 6th. According to him minor 6th also has complete dissonance. Also Mr Ranade (ref-9), had indicated that both Major 6th & Minor 6th to have imperfect dissonance & absolute dissonance respectively. However, if the consonance level is compared with 'Sa' it is mathematically the same as has been shown above. This practically can be seen from various Raagas when Minor Dha gives a bright appeal with 'Sa' (as in Raaga Bhatiyar) and not as absolute dissonance as mentioned by Mr. Ranade. Same thing is with Sa-Major Dha. In Bhoopali, Major Dha (lowar octave)-Sa gives a 'shringaric' (romantic) appeal rather than pathetic as would be given if the swara 'Dha' is considered as a dissonant Swara.

Now let us see the Inverse Relations in 22 Shrutis:

Shruti No:	Name of the Shruti/Swara			Ratio	Inverse Ratio	Equivalent Shruti in Middle octave	
	Shruti	Ancient Swara	Modern Swara			Equivalent-Ratio	Inverse Shruti No.
0	Kshobhini	Ni.	Sa	1	1	1	22
1	Tivra			135/128	2/(135/128)	256/135	21
2	Kumudvati	Kakli Ni.	Komal Ri	16/15	2/(16/15)	15/8	20
3	Manda			10/9	2/(10/9)	9/5	19
4	Chandovati	Sa	Ri	9/8	2/(9/8)	16/9	18
5	Dayavanti			32/27	2/(32/37)	27/16	17
6	Rajjani			6/5	2/(6/5)	5/3	16
7	Ratika	Ri	Ga	5/4	2/(5/4)	8/5	15
8	Roadri			81/64	2/(81/64)	128/81	14
9	Kroda	Ga	Komal Ma	4/3	2/(4/3)	3/2	13
10	Vajrika			27/20	2/(27/20)	40/27	12
11	Prasarini	Antar Ga	Sankirna Tivratar Ma	45/32	2/(45/32)	64/45 or 45/32	11
12	Preeti			40/27	2/(40/27)	27/40	10
13	Marjini	Ma	Pa	3/2	2/(3/2)	4/3	9

14	Ksiti			128/81	2/(128/81)	81/64	8
15	Rakta			8/5	2/(8/5)	5/4	7
16	Sandipini			5/3	2/(5/3)	6/5	6
17	Alapini	Pa	Dha	27/16	2/(27/16)	32/27	5
18	Madanti			16/9	2/(16/19)	9/8	4
19	Rohini			9/5	2/(9/5)	10/9	3
20	Ramia	Dha	Ni	15/8	2/(15/8)	16/15	2
21	Ugra			256/135	2/(256/135)	135/128	1
22	Kshobhini	Ni	. Sa	1	2/1 . Sa	2	0

(Ref-4-Nizami, page 43)

Thus we see that all the 22 shruties are inversely related with one another in a group such that Shruti no: x is inversely related with Shruti (22-x).

If this logic is applied to the Scales and Raagas we get inverse relationship in them also in a similar way.

According to the law of musical consonance we have the following relationships:

1	Absolute consonance		‘Sa’ and ‘Upper Sa’			
2	Perfect Consonance	i	Shruti	13&9	3/2	$4/3^{M}$
3	Medial consonance	ii		7 &15	$5/4^{G}$	$8/5^{DK}$
		iii		6&16	$6/5^{GK1}$	$5/3^{DK2}$
4	Imperfect dissonance	iv		4&18	$9/8^{R}$	$16/9^{NK1}$
		v		3&19	$10/9^{RK}$	$9/5^{NK2}$
5	Dissonances of degrees	vi	I	2&20	$16/15_{RKS}$	$15/8_{NkS}$
		vii		5&17	$32/27_{GK2}$	$27/16_{DKS}$
	I	viii	II	10&12	27/20	40/27
	II	ix		11&11	$45/32_{M1}$	$64/45_{M2}$

	III	x	III	8&14	$81/64_{M1}$	$128/81_{M2}$
		xi		1&21	$135/128_{RK}$ G	$256/135_{NK}$ 4

(Here NK, DK, NK1, NK2 are komal swaras of Ni & Dha & M1 & M2 are Tivra swaras of Ma)

Formation of Inverse Scales:

In Indian music the most commonly used Swara Gramas are as given below: (Numbers in between the swaras indicate the number of shruties between the two swaras)

Sa Grama	S_a 4	R 3	G 2	M 4	P 4	D 3	N 2	S
Frequency	240	270	300	320	360	405	450	480
Ratios	1	9/8	5/4	4/3	3/2	27/16	15/8	2

Positive ascending

Ma Grama	S_a 4	R 3	G 2	M 4	P 3	D 4	N 2	S
Frequency	240	270	300	320	360	400	450	480
Ratio	1	9/8	5/4	4/3	3/2	5/3	15/8	2

As per the above explained principle the negative descending ratios will also give the same degree of consonance/dissonance of various swaras with the prime. With the swaras having the negative descending ratios in the similar order we get the following two grams which are the mirror audio image of the above mentioned Gramas

Inverse Gramas

Sadja Grama	S 2	R 3	G 4	M 4	P 2	D 3	N 4	S
Ratio	1/2	8/15	16/27	2/3	3/4	4/5	8/9	1
Frequency	120	128	142.22	160	180	192	213.33	240
Middle	240	256	288	320	360	384	426.66	480

Octave frequenc

(Double of that of lower octave).

In the above two scales if we take the first Sadja Grama which is equivalent to the modern 'Bilawal Thaat' , the inverse of that becomes equivalent to 'Bhairvi Thaat'. As we know that the Scale is made up of swaras which are in consonance or dissonance with the prime swara in a predetermined manner. For example, in the Sadja Grama we have perfect consonances (2 Nos) as Sa – Pa (1-3/2) & Sa –Ma (1-4/3) , Medial consonances (2 nos) as Sa-Dha(1-27/16) & Sa-Ga (1-5/4) , Imperfect dissosonance (1 no) as Sa-Re and Complete dissonance (1 no) Sa-Ni (1-15/8) . In the Inverse Sadja Grama also the pattern of number of various consonances/dissonances are the same i.e. perfect consonances (2 nos) as (Sa-Pa & Sa-Ma, Medial consonances (2 no) as Sa-Minor Sixth & Sa – Minor 3rd , Imperfect dissonance- (2 no) as Sa - Minor Seventh, Sa –Major Second and the Complete Dissonance-1 no (Sa-Minor 2nd). Therefore the nature of this scale will also be the same as that of its miror image.

If we apply the same logic with the 'Ma' Grama we will find the following consonances/dissonances:

	Ma Grama	Sa Grama
• Perfect Consonance -	2	2
• Medial Consonance -	1	2
• Imperfect Dissonance -	2	1
• Complete Dissonance -	1	1

Inversal of the Musical Scales of Western Music;

In the Western Music the two scales have been formed by the application of the laws of simple ratios and the resultant notes formed out of these. They are called the Major chord and the Minor chord. If we take the tonic frequency as 240, the frequencies of other notes of the two chords would be -

Major chord:	240	270	300	320	360	400	450	480
Minor chord:	240	270	288	320	360	384	432	480

It is seen from the above that the Major chord is identical with our modern Ma Grama, while the Minor Chord corresponds with our Darbari scale of Indian Music.

These two perfect chords Major & Minor form the key stones of musical system of Western music. But music would be very poor if it were limited to only the few notes that compose them. In order to increase the resources and to acquire greater vigour and strength in the expression of their ideas, at the same time with a view to diminish excessive number of notes and to render practical execution more easy, attempts were made towards the beginning of the sixteenth century, to divide the five full tones D E F A B of the Major chord in to 10 half tones thus forming the Chromatic Scale of 12 tones. Hence the modern Chromatic Scale of equal temperaments has arisen as represented by seven white and five black keys on the key board of ordinary Piano, Organ, Harmonium etc. According to this the musical scale consists of twelve semi tones, each equal to a twelfth part of an octave i.e. all the twelve tones are equidistance from each other. The seven fundamental notes of the temperate scale can be represented as

240, 269 2/5, 302 2/5, 320 2/5, 359 2/5, 403 2/5, 453 3/5, 480

Let us now see how the principles of inverse consonance apply on Western scales. The Major chord is similar to our 'Ma Grama' which has already been discussed earlier; let us see how the inverse of Major Chord looks like

Major Chord

a) Frequency	240	270	300	320	360	400	450	480
b) Ratios	1	9/8	5/4	4/3	3/2	5/3	15/8	2

Inverse Major Chord

a) Ratios	1	16/15	6/5	4/3	3/2	8/5	16/9	2
b) Frequency	240	256	288	320	360	384	426.66	480

Minor chord

a) Frequency	240	270	288	320	360	384	432	480
b) Ratio	1	9/8	6/5	4/3	3/2	8/5	27/15	480

Inverse Minor chord

a) Ratio	1	10/9	5/4	4/3	3/2	5/3	16/9	2
b) Frequency	240	266.66	300	320	360	400	426.66	480

However if one chooses equally tempered scale the Major chord would look like the following;

Major Chord

Ratios (a)	1	$2^{2/12}$	$2^{4/12}$	$2^{5/12}$	$2^{7/12}$	$2^{9/12}$	$2^{11/12}$	2
Inverse- Ratios	1	$2^{1/12}$	$2^{3/12}$	$2^{5/12}$	$2^{7/12}$	$2^{8/12}$	$2^{10/12}$	2
Minor Chord	1	$2^{2/12}$	$2^{3/12}$	$2^{5/12}$	$2^{7/12}$	$2^{8/12}$	$2^{10/12}$	2
Inverse Ratios	1	$2^{2/12}$	$2^{4/12}$	$2^{5/12}$	$2^{7/12}$	$2^{9/12}$	$2^{10/12}$	2

We see that inverse of Minor chord is the same as the Major chord with a difference in one Swara. Similarly the inverse of Major chord is the same as Minor chord with a difference in only one Swara.

In the Chinese system of ascending 5^{th}, the principal on which the musical scale has been formed, the inversals can also be worked out in the similar way and we can see that descending 5^{th} is as melodic in that system as the ascending 5^{th} as already suggested by Pythagorus in very early days.The theory of inversals can then be applied to that system also with the same analogy as explained above. The same will hold good for all the other prevalent systems of music world over, like Arabian, African systems etc.