Synopsis of the thesis entitled

MATHEMATICAL MODELING OF FERROFLUID LUBRICATED BEARING DESIGN PROBLEMS

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Brief Literature Review

A ferrofluid (FF) or magnetic fluid (MF) is a colloidal dispersion of nano magnetic particles in a non-conducting carrier liquid. Neuringer and Rosensweig (NR) [1] invented the concept of ferrofluid and a theory of ferrohydrodynamics is developed. McTague [2] experimentally investigated the influence of a uniform magnetic field **H** on the viscosity of a suspension of cobalt nanoparticles and discovered an increase of viscosity with the increase of magnetic field strength. Hall and Busenberg [3] theoretically studied viscosity of magnetic suspensions and calculated magnetization equation by considering that the torque on spherical particles is exerted by shearing fluid and also due to external magnetic field. But in their development there was no consideration of Brownian rotation of the particles. With this view Shliomis [4] studied FF phenomenon when the angular velocities of rotations of the carrier liquid as well as magnetic particles are different. Due to these differences of angular velocities frictional force arise which cause an increase in the effective viscosity of the FF. Shliomis [5] deals mainly with physical and hydrodynamic properties of magnetic colloid. The mechanisms of relaxation of the magnetization of a suspension are also discussed. Raikher and Shliomis [6] calculated the effective viscosity of a suspension of ferromagnetic particles with anisotropy of the "easy-axis". They established the relationship between the viscosity and the anisotropy field.

Due to the important property of FF to adhere to any desired location on the surface under the influence of an applied magnetic field, it has gained widespread popularity among the researchers working on the lubrication theory of bearings.

The following are some references regarding use of FF lubricant on different bearing design systems using NR model.

Tipei [7] analyzed general momentum equations under the assumption of FFs as Newtonian fluids. The short bearing case is studied. It is shown that the load-carrying capacity increases because of the effect of magnetic particles. The bearing stability and stiffness are also shown to be improved. Agrawal [8] studied the effects of MF on a porous inclined slider bearing. It is shown that the load-carrying capacity increases without affecting the friction on the moving slider due to the effect of magnetization of the magnetic particles in the lubricant. Chi *et. al.* [9] studied new type of FF lubricated journal bearing consists of three pads (one of them is a deformable elastic pad). The theoretical analysis and experimental investigation shows the better performance of the bearing as compared to ordinary bearings (which uses conventional lubricant). Moreover, the bearing is operated without leakage and any feed system. Prajapati [10] studied effect of MF on different porous squeeze film-bearing designs like circular, annular, elliptic, conical, etc. It is concluded that the load-carrying capacity increases with the increase of magnetization parameter, and the bearings can support a load even when there is no flow. Osman *et.al.* [11] studied the static and dynamic characteristics of the hydrodynamic journal bearings lubricated with ferrofluid. They have calculated the bearing characteristics like load carrying capacity, attitude angle, frictional force at the journal surface, friction coefficient and bearing side leakage. Using finite perturbation technique, they also determined the eight oil film stiffness and damping coefficients and used these as input to find the critical speed. Montazeri [12] numerically discussed FF lubricated hydrodynamic journal bearings. It is shown that compared to conventional lubricant, FF improves the hydrodynamic characteristics of journal bearings and provides a higher load capacity with the reduction in friction coefficient. Kuzhir [13] predict the shape of a free boundary of lubricant in the presence of a static load and magnetic field in ferrofluid lubricated journal bearing. The analysis involves simultaneous integration of the Reynolds equation and the free boundary equation using perturbation technique with respect to shaft eccentricity. Magnetic field is shown to flatten ferrofluid free boundaries as well as to reduce cavitation region; both effects diminishing lubricant leakage. Hsu *et. al.* [14] investigated ferrofluid based long journal bearings under the effects of stochastic surface roughness and magnetic field (which is generated by an infinitely long wire). They show that due to placing such infinitely long wire magnetic field at appropriate distance from the center of the bearing can suppress side leakage, thereby extending the life of the bearings. They also show that under a higher power-law index and induced magnetic force, the introduction of transverse roughness can enhance film pressure and load capacity, while reducing the attitude angle and modified friction coefficient. The introduction of longitudinal roughness has the opposite effect. Shah and Kataria [15] theoretically discussed FF based squeeze film between a sphere and a flat plate. It is concluded that loss in the dimensionless load-carrying capacity due to the effect of porosity is almost zero because of using FF as lubricant for smaller values of thickness parameter of the porous layer and radial permeability parameter. Rao *et. al*. [16] analyse porous layered long journal bearing lubricated with ferrofluid using displaced infinitely long wire magnetic field model. Expressions for dimensionless pressure and shear stress are derived using Reynolds boundary conditions. Dimensionless load-carrying capacity and coefficient of friction are evaluated under the influence of permeability of porous media, porous layer thickness, lubricant layer thickness, magnetic field intensity and distance ratio parameter. The results show the increase in load-carrying capacity and reduction in coefficient of friction. Hu and Xu [17] mathematically studied lubrication performance of the journal bearing using cohesion forces & couple stresses of MFs, and the effect of squeeze dynamics. The bearing characteristics like load-carrying capacity, attitude angle, friction coefficient and side leakage studied. The results show that dimensionless load-carrying capacity increases with the increase of squeeze parameter, cohesion force coefficient and couple stress parameter of the MF.

The following are some references regarding use of FF lubricant on different bearing design systems using Shliomis model.

Shukla and Kumar [18] used Shliomis model to study the slider and squeeze film bearings with uniform transverse magnetic field by neglecting relaxation time of particle rotation. They derived the pressure equation under the assumptions that the FF is saturated so that the saturation magnetization is independent of the applied magnetic field, and the magnetic moment relaxation time is negligible. Shah and Bhat [19] analyzed FF lubricated squeeze film in a long journal bearing and shown that when magnetic field is uniform, the rotational viscosity parameter of Shliomis model causes increase in the load-carrying capacity and response time. The case of non-uniform magnetic field is also studied. Shah and Bhat [20] derived the pressure equation and the case of squeeze film between curved annular plates bearing is studied. It is concluded that the load-carrying capacity and approaching time of squeeze film can be enhanced by increasing the volume concentration of the solid phase in FF and the intensity of external magnetic field. Singh and Gupta [21] studied curved slider bearing with the effect of transverse magnetic field. It is shown that there is an improvement in stiffness and damping capacities of the bearings due to the effects of rotation and volume concentration of magnetic particles. Lin *et. al.* [22] investigated lubrication performance of short journal bearings operating with non-Newtonian ferrofluids model of Shliomis and the micro-continuum theory of Stokes. The results show that bearings give better performance and result in a higher load capacity as compared with the case of conventional non-ferrofluid lubricant. Also, comparing with the Newtonian ferrofluid case, the non-Newtonian effects of couple stresses provide an enhancement in the load capacity, as well as a reduction in the friction parameter. Lin [23] derived Reynolds equation for MF lubricated slider bearings using transverse magnetic field. It is shown that load-carrying capacity, dynamic stiffness and damping characteristics are improved. Shah and Parikh [24] analysed different shapes of slider bearings and compared dimensionless load-carrying capacity for the effect of squeeze velocity. It is concluded that the load-carrying capacity of all bearings remains constant with the increase of Langevin's parameter, whereas it has an increasing tendency with the increase of volume concentration of the particles. Shah and Shah [25] derived Reynolds equation for the study of lubrication of different slider bearings, using Shliomis based ferrofluid flow and continuity equation. They have considered the effects of oblique radially variable magnetic field and squeeze velocity. Using Reynolds equation, general form of pressure equation is derived and expressions for dimensionless load-carrying capacity, frictional force, coefficient of friction and center of pressure are obtained. Using these expressions, results for different slider bearings are computed for different parameters and compared. Shah and Shah [26] derived the modified Reynolds equation using Shliomis ferrofluid flow model and continuity equation for lubrication of circular squeeze film bearings considering the effects of oblique radially variable magnetic field, slip velocity at the film porous interface and rotations of both the discs. Using Reynolds equation, general form of pressure equation is derived and expression for dimensionless load-carrying capacity is obtained. Using this expression, results for different bearing design systems are computed and compared for variation of different parameters.

Basic equations and boundary conditions

Basic equations and boundary conditions used in the study are as follows.

(1) Based on the Shliomis [4, 20] FF flow model, fundamental equations used in the study are as follows.

Equations of Motion

$$-\nabla p + \eta \nabla^2 \mathbf{q} + \mu_0 (\mathbf{M} \bullet \nabla)\mathbf{H} + \frac{1}{2\tau_s} \nabla \times (\mathbf{S} - I\mathbf{\Omega}) = 0 \tag{1}$$

Equation of internal angular momentum

$$\mathbf{S} = I\mathbf{\Omega} + \mu_0 \tau_s (\mathbf{M} \times \mathbf{H}) \tag{2}$$

Equation of magnetization

$$\mathbf{M} = M_0 \frac{\mathbf{H}}{H} + \frac{\tau_B}{I} (\mathbf{S} \times \mathbf{M}) \tag{3}$$

Maxwell's equations

$$\nabla \times \mathbf{H} = 0 \tag{4}$$

$$\nabla \bullet (\mathbf{M} + \mathbf{H}) = 0 \tag{5}$$

Equation of continuity

$$\nabla \bullet \mathbf{q} = 0 \tag{6}$$

(2)

Integral form of the continuity equation in Cartesian coordinate

$$\frac{\partial}{\partial x}\int_0^h u\,dz + w_h - w_0 = 0 \tag{7}$$

Integral form of the continuity equation in cylindrical coordinate

$$\frac{1}{r}\frac{\partial}{\partial r}\int_0^h (ru)\,dz + w_h - w_0 = 0 \tag{8}$$

(3)

Slip boundary conditions

$$u = -\frac{1}{s}\frac{\partial u}{\partial z} + U;\; s = \frac{5}{\sqrt{k\,\eta_r}} \tag{9}$$

where, p is the pressure, η is the viscosity of the suspension, μ_0 is the permeability of free space, $\mathbf{H}$ is the applied magnetic field vector, H is magnetic field strength, $\mathbf{M}$ is the magnetization vector, I is the sum of moments of inertia of the particles per unit volume, $\mathbf{q} = (u, 0, w)$ is the fluid velocity vector, $\mathbf{\Omega} = \frac{1}{2}\nabla\times\mathbf{q}$, τ_B is the Brownian relaxation time, τ_s is the magnetic moment relaxation time and M_0 is the equilibrium magnetization, ρ is the fluid density, s is a slip constant, U is velocity in x- direction, k being permeability of the porous matrix, η_r being porosity of the porous matrix.

Organization of the thesis

Chapter 1 mainly deals with the motivation as well as literature survey of the present work.

Chapter 2 contains physico-mathematical background necessary to understand the subsequent chapters. That means it contains pre-requisite for the problems discussed in the subsequent chapters.

Chapter 3 deals with the derivation of ferrofluid lubrication equation for different shapes of slider bearings considering the effect of oblique variable magnetic field and squeeze velocity. Different slider bearing designs are made up of various stators shapes (inclined, exponential, secant, convex and parallel) and flat parallel sliders. In deriving the modified Reynolds equation for the study of lubrication of different slider bearings, Shliomis FF flow model (equations (1)-(5)) and integral form of the continuity equation (equation (7)) is used. The external oblique radially variable magnetic field vector is chosen as

$$\mathbf{H} = H(x)(\cos\alpha, 0, \sin\alpha) \tag{10}$$

with magnetic field strength

$$H(x) = K_1 \, x \, (A - x) \tag{11}$$

where, A is the length of the lower surface of the slider along the x-axis and α is the inclination of the magnetic field with the x-axis. Also K_1 is a quantity chosen to suit the dimensions of both sides of equation (11). Such a field attains maximum at $x = A/2$ and vanishes at $x = 0$ and $x = A$. In this case we have considered active contact zone in the neighbourhood of $x = A/2$.

With reference to the various stators shapes (inclined pad, exponential pad, secant pad, convex pad and parallel pad), the slider bearing designs referred here as inclined slider bearing, exponential slider bearing, secant slider bearing, convex slider bearing and parallel slider bearing, respectively. Accordingly using subscripts i , e , s , c , p , the following shapes are taken in computation.

$$\bar{h} = \bar{h}_i = a - (a-1)X; \quad 0 \le X \le 1 \tag{12}$$

$$\bar{h} = \bar{h}_e = a\exp(-X \ln a); \quad 0 \le X \le 1 \tag{13}$$

$$\bar{h} = \bar{h}_s = \sec\{\tfrac{\pi}{2}(1-X)\}; \quad 0 < X \le 1 \tag{14}$$

$$\bar{h} = \bar{h}_c = 4\bar{\delta} X^2 - (a - 1 + 4\bar{\delta})X + a; \quad 0 \le X \le 1 \tag{15}$$

$$\bar{h} = \bar{h}_p = 1; \ 0 \le X \le 1 \tag{16}$$

where,

$$a = \frac{h_2}{h_1}, \bar{\delta} = \frac{\delta}{h_1}, X = \frac{x}{A}, \bar{h} = \frac{h}{h_1} \tag{17}$$

and δ is the central thickness of the convex pad, h_1, h_2 are minimum and maximum values of film thickness h respectively.

Using Reynolds equation, general form of dimensionless pressure equation is derived as

$$\bar{p} = \mu^* \ln\left(\frac{\sinh \xi}{\xi}\right) + 6\int_0^X \frac{\bar{h} - \beta X + Q}{G} dX, \tag{18}$$

where,

$$Q=\frac{\int_0^1 \frac{\beta X-\bar{h}}{G}dX}{\int_0^1 \frac{dX}{G}} \quad , \; G=\frac{\bar{h}^3}{(1+\tau)(1+\frac{5}{2}\phi)}$$

and

$$\beta=-\frac{2A\dot{h}_1}{Uh_1},\; \mu^*=\frac{nk_BTh_1^2}{\eta_0 UA}$$

where, ξ is dimensionless field strength (Langevin's parameter), β is squeeze velocity parameter, n is the number of particles per unit volume, k_B is Boltzmann constant, T is temperature, η_0 is the viscosity of carrier liquid and U is slider velocity.

Expression for dimensionless Load-carrying capacity ($\overline{W}$) can be obtained as

$$\overline{W}=\mu^* I^* - 6\int_0^1 \frac{X}{G}(\bar{h}-\beta X+Q)dX \tag{19}$$

where,

$$I^*=\lambda\int_0^1 X(1-2X)\left(\frac{1}{\xi}-\coth\xi\right)dX,$$

Frictional force ($\overline{F}$) can be obtained as

$$\overline{F}=\left(1+\frac{5}{2}\phi\right)\int_0^1\left[\frac{1}{\bar{h}}+\frac{3}{\bar{h}^2}(\bar{h}-\beta X+Q)\right]dX \tag{20}$$

where, ϕ is the volume concentration of the particles.

Coefficient of friction ($\bar{f}$) can be obtained as

$$\bar{f} = \frac{\bar{F}}{\overline{\overline{W}}} \quad (21)$$

and Center of pressure ($\overline{\overline{x}}$) can be obtained as

$$\overline{\overline{x}} = \frac{1}{2\overline{\overline{W}}}\left[\mu^* I^{**} - 6\int_0^1 \frac{X^2}{G}(\bar{h} - \beta X + Q)dX\right], \quad (22)$$

where,

$$I^{**} = \lambda \int_0^1 X^2 (1-2X)\left(\frac{1}{\xi} - \coth\xi\right) dX.$$

Using these expressions, results for different slider bearings are computed for different parameters and compared. In the present analysis, the case of sample magnetic field is considered in such a way that it is maximum at the middle of the bearing. However, the study with other forms of magnetic field because of different requirements can be discussed similarly. From the results and discussion, following conclusions can be made.

Using subscripts *i*, *e*, *s*, *c*, *p* for the concerned quantities when the slider bearings are inclined, exponential, secant, convex, and parallel respectively.

(1) $\overline{W}$ is maximum for parallel slider bearing, whereas it is least for secant slider bearing. For all other bearings, $\overline{W}$ almost remains the same. Thus,

$$\overline{W}_p > \overline{W}_i \approx \overline{W}_e \approx \overline{W}_c > \overline{W}_s .$$

The behaviour of $\overline{W}$ increases with the increasing values of squeeze velocity parameter β for all bearing designs.

(2) $\overline{F}$ is maximum for inclined, exponential and convex slider bearings, whereas it is least for parallel slider bearing. Thus,

$$\overline{F}_p < \overline{F}_s < \overline{F}_i \approx \overline{F}_e \approx \overline{F}_c \,.$$

The behaviour of $\overline{F}$ remains constant for parallel slider bearing, whereas it increases for all other bearings with the increasing values of β.

(3) $\overline{f}$ is maximum for secant slider bearing, whereas it is least for parallel slider bearing. For all other bearings, $\overline{f}$ almost remains the same. Thus,

$$\overline{f}_p < \overline{f}_i \approx \overline{f}_e \approx \overline{f}_c < \overline{f}_s.$$

The behaviour of $\overline{f}$ decreases with the increasing values of β for all bearing designs.

(4) The position of $\overline{\overline{x}}$ shift towards the outlet for all bearing designs except parallel. The shifting is maximum in the case of secant slider bearing. For parallel slider bearing, $\overline{\overline{x}}$ is in the middle of the bearing. For inclined, exponential and convex slider bearings, the position of $\overline{\overline{x}}$ remains the same, and it is inbetween secant and parallel slider bearings. The behaviour of $\overline{\overline{x}}$ remains constant with the increasing values of β for all bearing designs.

Chapter 4 deals with the study of lubrication of circular squeeze film-bearings using Shliomis FF flow model with the effects of oblique radially variable magnetic field, porosity, slip velocity at the film-porous interface and rotations of both the discs. The squeeze film-bearings are made up of circular porous upper disc of different shapes (exponential, secant, mirror image of secant and parallel) and circular impermeable flat lower disc. While deriving the modified Reynolds equation, the validity of the Darcy's law is assumed in the porous region (matrix or layer). The use of continuity equation is also made in the film as well as porous region.

The magnetic field taken in this case is

$$\mathbf{H} = H(r)\,(\cos\theta\,,0\,,\sin\theta),\;\; \theta = \theta\,(r\,,z) \tag{23}$$

where, θ is the inclination to the radial direction and is assumed to be small.

In order to consider active contact area in the neighbourhood of $r = 2a_1/3$, the magnetic field strength of radially VMF should be chosen as

$$H = K_2 r^2 (a_1 - r)\,, \tag{24}$$

where, K_2 being the quantity chosen to suit the dimensions of both sides of equation (24). Such a field attains maximum at $r = 2a_1/3$ and vanishes at $r = 0$ and $r = a_1$. For other active contact areas, suitable form of magnetic field strength should be chosen.

Slip boundary condition is used as

$$u=0 \text{ when } z=0,\;\; u = -\frac{1}{s}\frac{\partial u}{\partial z};\; s = \frac{5}{\sqrt{k\,\eta_r}} \text{ when } z=h \tag{25}$$

where, k is permeability of the porous matrix, η_r is porosity of the porous matrix in r-direction and s is the slip constant.

Using Reynolds equation, general form of dimensionless pressure equation is derived as

$$\bar{p} = \mu^* \ln\frac{\sinh\xi}{\xi} + \int_1^R \frac{F}{G}\,dR \tag{26}$$

where,

$$G = 12\psi + \frac{\bar{h}^3(4+\bar{s}\bar{h}) + \delta\bar{s}\tau\bar{h}^2}{(1+\bar{s}\bar{h})(1+\tau)},$$

$$F = (-6+12\psi S)R + \frac{\delta\bar{s}\tau\bar{h}^2 SR(3+2\Omega_f+\Omega_f^2)}{6\,(1+\bar{s}\bar{h})(1+\tau)} + \frac{SR\bar{h}^3\{(18+3\bar{s}\bar{h})+(14+4\bar{s}\bar{h})\Omega_f+(8+3\bar{s}\bar{h})\Omega_f^2\}}{10\,(1+\bar{s}\bar{h})(1+\tau)},$$

where,

$$\psi = \frac{kH^*}{h_0^3}, \bar{h} = \frac{h}{h_0},\ \bar{s} = sh_0,\ R = \frac{r}{a_1},\ V_{sq} = -\frac{\dot{h}_0}{\Omega_u h_0}, S = \frac{\rho\Omega_u h_0^2}{\eta V_{sq}},$$

$$\Omega_f = \frac{\Omega_l}{\Omega_u}, \qquad \mu^* = -\frac{nk_B T h_0^3}{\eta a_1^2 \dot{h}_0}, \qquad \tau = \frac{3}{2}\phi\frac{\xi - \tanh\xi}{\xi + \tanh\xi}$$

where, H^* is a thickness of the porous matrix, k is a permeability of the porous matrix, h is the film thickness, h_0 is the central film thickness, s is a slip constant, a_1 is the radius of the circular disc, $\dot{h}_0$ is squeeze velocity dh_0/dt, Ω_u is a rotational velocity of the upper disc, Ω_l rotational velocity of the lower disc, ρ is fluid density, η is viscosity of the suspension, n is the number of particles per unit volume, k_B is Boltzmann constant, T is temperature, ϕ is the volume concentration of the particles, ξ dimensionless field strength(Langevin's parameter).

Expression for dimensionless load-carrying capacity can be obtained as

$$\overline{W} = \int_0^1 R\bar{p}dR = \mu^* I^* - \frac{1}{2}\int_0^1 \frac{R^2 F}{G}dR \qquad (27)$$

where,

$$I^* = \frac{\lambda}{2}\int_0^1 R^3(2-3R)\left(\frac{1}{\xi} - \coth\xi\right)dR$$

and

$$\lambda = \frac{\mu_0 m K_2 a_1^3}{k_B T}$$

where, μ_0 is the permeability of free space, m is the magnetic moment of a particle. Using this expression, results for different bearing design systems (due to different shapes of the upper disc) are computed and compared for variation of different parameters.

Using subscripts *e*, *s*, *is*, *p* for the concerned quantities when the squeeze film-bearing designs are of exponential, secant, mirror image of secant and parallel shapes, respectively, equations for the film thickness for computation are taken as

$$\bar{h} = \bar{h}_e = e^{-\bar{\beta} R^2}; \bar{\beta} = \beta_1 a_1^2 \quad 0 \le R \le 1 \tag{28}$$

$$\bar{h} = \bar{h}_s = \sec(\bar{\gamma} R^2); \; \bar{\gamma} = \gamma_1 a_1^2 \quad 0 \le R \le 1, \tag{29}$$

$$\bar{h} = \bar{h}_{is} = 2 - \sec(\bar{\alpha} R^2); \; \bar{\alpha} = \alpha_1 a_1^2 \quad 0 \le R \le 1, \tag{30}$$

$$\bar{h} = \bar{h}_p = 1; \; 0 \le R \le 1. \tag{31}$$

where, α_1 is the curvature of the mirror image of the secant upper disc, β_1 is the curvature of the exponential upper disc, γ_1 is the curvature of the secant upper disc.

The results for the dimensionless load-carrying capacity $\bar{W}$ are computed using Simpson's one-third rule with step size 0.1.

The following conclusions can be made from the results and discussion.

1. $\overline{W}$ is maximum when $\Omega_f = -1$; that is, either Ω_u is rotated in counterclockwise direction and Ω_l in clockwise direction, or Ω_u is rotated in clockwise direction and Ω_l in counterclockwise direction with the same speed. But getting $\Omega_f = -1$ for faster rotation results moderate reduction in $\overline{W}$.

2. Using subscripts *e*, *s*, *is*, *p* for the concerned quantities when the bearing design are of exponential, secant, mirror image of secant and parallel respectively, maximum load carrying capacity $\overline{W}$ is obtained in the case of exponential squeeze film-bearing while minimum in the case of secant shape with $\overline{W}_e > \overline{W}_{is} > \overline{W}_p > \overline{W}_s$.

3. Concave (with respect to lower flat disc) shape of the upper disc with more curvature at the center has more impact on the increase of $\overline{W}$ as compared to convex shape. Moreover, in convex shape with less curvature at the center in upward direction has more impact on the increase of $\overline{W}$.

4. $\overline{W}$ increases even if rotation of the lower disc is zero and irrespective of the decrease of rotation of the upper disc.

5. $\overline{W}$ increases when dimensionless porous thickness parameter (ψ) approaches to 0.

6. $\overline{W}$ almost remains constant when squeeze velocity parameter increases.

Chapter 5 theoretically derived generalized ferrofluid lubrication equation from basic theory for variable as well as uniform (transverse) strong magnetic field, and studied application of variable and strong magnetic field to axially undefined porous journal bearing (AUPJB) using equation of continuity in film as well as porous region and assuming validity of the Darcy's law in the porous region. The variable magnetic field (VMF) is used to retain all magnetic terms of the Shliomis model. Moreover, it has advantage of generating maximum magnetic field at the required active contact zone of the bearing design system. Following equations are derived theoretically.

Equation of motion:

$$\rho \frac{D\mathbf{q}}{Dt} = -\nabla p + \eta \nabla^2 \mathbf{q} + (\mathbf{M} \cdot \nabla)\mathbf{H} + \frac{1}{2}\nabla \times (\mathbf{M} \times \mathbf{H}) \tag{32}$$

where, p is fluid pressure, η is the viscosity of the suspension, **q** is the fluid velocity vector, **M** is the magnetization of the suspension, **H** is the magnetic field vector, ρ is the fluid density.

Moreover, two well-known concept of magnetic relaxation time are discussed, namely Brownian relaxation time described by

$$\tau_B = \frac{3V\eta}{k_B T}, \tag{33}$$

and Neel relaxation time described by

$$\tau_N = f_0^{-1} \exp\left(\frac{KV}{k_B T}\right), \tag{34}$$

where, k_B Boltzmann's constant, T is the absolute temperature, V particle volume, f_0 is the Larmour frequency of the magnetization vector in the anisotropy field of the particle and K is the anisotropy constant of the magnetic particle.

Magnetization dynamic equation:

$$\frac{D\mathbf{M}}{Dt} = \frac{1}{I}(\mathbf{S}\times\mathbf{M}) - \frac{1}{\tau_B}\left(\mathbf{M} - M_0\frac{\mathbf{H}}{H}\right). \tag{35}$$

where, $\mathbf{S}$ is the internal angular momentum density of particles, τ_B is the Brownian relaxation time, M_0 is the equilibrium magnetization given by Langevin formula, H is the magnitude of the magnetic field.

Also discussed the expression for rotational and effective viscosity of the suspension in the case of applied magnetic field and it is found to be

For rotational viscosity,

if the magnetic field is oriented parallel to the flow and perpendicular to the flow,

$$\eta_{rv} = \frac{3}{2}\phi\eta\frac{\xi - \tanh\xi}{\xi + \tanh\xi} \quad \text{and} \quad \eta_{rv} = \frac{3}{4}\phi\eta\frac{\xi - \tanh\xi}{\xi + \tanh\xi} \tag{36}$$

respectively.

For effective viscosity

$$\eta_e(\xi) = \eta_0\left(1 + \frac{4\xi + \tanh\xi}{\xi + \tanh\xi}\phi\right) \tag{37}$$

where, η_0 is the viscosity of the carrier liquid, ϕ is the volume concentration of all suspended material, ξ is the magnetic field parameter.

In Mathematical Modelling of the ferrofluid based axially undefined porous journal bearing, external oblique radially variable magnetic field taken similar to

$$\mathbf{H} = H(x)(\cos\alpha, 0, \sin\alpha), \tag{38}$$

where, α is the inclination of the magnetic field with the *x*-axis.

The magnetic field strength as

$$H^2 = K_3 x\left(2\pi R - x\right) \tag{39}$$

where, R is the journal radius, K_3 is the quantity chosen to suit the dimensions of both sides of equation (39).

According to the Darcy's law, the velocity components in the porous region in *x* and *z*- directions, respectively taken as

$$\bar{u} = -\frac{\varphi_x}{\eta}\left\{\frac{\partial}{\partial x}\left[P - nk_B T \ln\frac{\sinh\xi_1}{\xi_1}\right] - \frac{1}{4}\frac{\partial}{\partial z}\left(\mu_0 M_1 \bar{\tau} H\frac{\partial u}{\partial z}\right)\right\} \tag{40}$$

$$\bar{w} = -\frac{\varphi_z}{\eta}\left\{\frac{\partial}{\partial z}\left(P - nk_B T \ln\frac{\sinh\xi_1}{\xi_1}\right) + \frac{1}{4}\frac{\partial}{\partial x}\left(\mu_0 M_1 \bar{\tau} H\frac{\partial u}{\partial z}\right)\right\} \tag{41}$$

where,

$$M_1 = nm\left(\coth\xi_1 - \frac{1}{\xi_1}\right), \quad \xi_1 = \frac{\mu_0 m H}{k_B T}, \bar{\tau} = \frac{\tau_B}{1 + \frac{\mu_0 \tau_B \tau_s}{I} M_1 H}$$

Here, φ_x and φ_z are permeability components of the porous facing in the *x* and *z*-directions, respectively. *u*, *w* are the velocity components in the film region in *x* and *z*- directions, respectively. $\bar{u}, \bar{w}$ are the velocity components in the porous region in *x* and *z*- directions, respectively. *P* is the fluid pressure in the porous region, *n* is the number of magnetic particles, μ_0 is the permeability of free space, *m* is the magnetic moment of the particle.

Slip boundary condition taken as

$$u = 0 \quad \text{when} \quad z = 0, \qquad \text{and}$$
$$u = -\frac{1}{s}\left(\frac{\partial u}{\partial z}\right)_{z=h} \text{ when } z = h\,;\, s = \frac{5}{\sqrt{\varphi_x \eta_x}} \tag{42}$$

where, s is the slip constant and η_x is the porosity in x-direction.

The modified Reynolds equation for AUPJB is derived considering continuity equation in film as well as porous region and by assuming the validity of the Darcy's law. The pressure equation derived as

$$\bar{p} = \mu^* \ln \frac{\sinh \xi_1}{\xi_1} + 12\int_0^{\theta} \frac{\sin\theta}{G} d\theta \tag{43}$$

where,

$$G = \frac{12(\psi_x + \psi_z \tau)}{1+\tau} + \frac{\bar{h}^3(\bar{s}\bar{h}+4) + \gamma\tau\bar{s}\bar{h}^2}{(1+\tau)\left(1+\bar{s}\bar{h}\right)}, \ \mu^* = \frac{nc^2 k_B T}{\eta R^2 \dot{\varepsilon}}$$

$$\bar{s} = sc, \ \bar{h} = \frac{h}{c}, \ \ \psi_x = \frac{H^*\varphi_x}{c^3}, \ \psi_z = \frac{H^*\varphi_z}{c^3}, \ \gamma = \frac{6\varphi_z}{c^2}, \ \dot{\varepsilon} = \frac{d\varepsilon}{dt}$$

Here, H^* is the thickness of the porous region, c is radial clearance, s is slip constant, ε is eccentricity ratio, τ is rotational viscosity parameter, h is the film thickness.

The expression for dimensionless load carrying capacity can be obtained as

$$\overline{W} = \mu^* I^* + 12\int_0^{2\pi} \frac{\sin^2\theta}{G} d\theta, \tag{44}$$

where,

$$I^* = \lambda \int_0^{2\pi} (\pi - \theta) \frac{\sin\theta\left(\coth\xi_1 - \dfrac{1}{\xi_1}\right)}{\sqrt{\theta(2\pi-\theta)}} d\theta.$$

and studied for different parameters.

The Results show that dimensionless load-carrying capacity increases with the increase of eccentricity ratio ε and for the anisotropic case $\psi_x < \psi_z$ where ψ_x is the dimensionless permeability parameter in x-direction and ψ_z is the dimensionless permeability parameter in z-direction. The above behaviour of dimensionless load-carrying capacity is more significant for thin layer of porous matrix.

Chapter 6 deals with the static and dynamic performance of ferrofluid lubricated long journal bearing with constant magnetic field. Based on ferrofluid flow and continuity equation Reynolds equation is derived for the study of static and dynamic performance of journal bearing under the constant magnetic field. Using Reynolds equation, we have obtained the pressure equation for steady-state (static) as

$$\bar{p} = \frac{12\pi(1+\frac{5}{2}\phi)(1+\tau)\varepsilon\sin\theta(2+\varepsilon\cos\theta)}{(2+\varepsilon^2)(1+\varepsilon\cos\theta)^2} \tag{45}$$

and the pressure equation for dynamic condition as

$$\hat{p} = 12\pi(1+\frac{5}{2}\phi)(1+\tau)\left\{\frac{\varepsilon\sin\theta(2+\varepsilon\cos\theta)}{(2+\varepsilon^2)(1+\varepsilon\cos\theta)^2} + \frac{1}{\varepsilon}\left[\frac{1}{(1+\varepsilon\cos\theta)^2} - \frac{1}{(1+\varepsilon)^2}\right]\left(\frac{\frac{\dot{\varepsilon}}{\omega}}{1-\frac{2\dot{\varphi}}{\omega}}\right)\right\} \tag{46}$$

where, θ is the angular coordinate related to a fixed direction, $\dot{\varphi}$ whirling motion of the journal, ϕ is the volume concentration of particles, τ is the rotational viscosity parameter, ε is the eccentricity ratio, $\dot{\varepsilon}$ is squeeze velocity parameter $\left(d\varepsilon / dt\right)$, ω is angular velocity of the journal.

With the help of these pressure distribution expression, we obtained fluid film reaction forces f_R and f_T in radial and tangential direction, respectively, for both static and dynamic cases separately.

From the fluid reaction forces, for the static case we have obtained dimensionless load carrying capacity as

$$\overline{W} = \left[\frac{12\pi^2 \left(1 + \frac{5}{2}\phi\right)(1+\tau)\varepsilon}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}} \right] \tag{47}$$

dimensionless frictional forces at the journal as

$$\bar{F} = \left(1 + \frac{5}{2}\phi\right) \frac{4\pi^2(1+2\varepsilon^2)}{(2+\varepsilon^2)\sqrt{1-\varepsilon^2}} \tag{48}$$

and dimensionless frictional coefficient as

$$\overline{f} = \frac{(1+2\varepsilon^2)}{3(1+\tau)\varepsilon} \tag{49}$$

For the dynamic loading condition, we have discussed the increase in the fluid force components f_R and f_T in radial and tangential directions, respectively over their equilibrium value and using the Taylor's series expansion we obtained the dimensionless stiffness matrix as

$$\bar{k} = \begin{bmatrix} \frac{\partial f_R}{\partial \varepsilon} & \frac{\partial f_R}{\varepsilon \partial \phi} - \frac{f_T}{\varepsilon} \\ \frac{\partial f_T}{\partial \varepsilon} & \frac{\partial f_T}{\varepsilon \partial \phi} + \frac{f_R}{\varepsilon} \end{bmatrix} \tag{50}$$

and dimensionless damping matrix as

$$\bar{c} = \begin{bmatrix} \frac{\partial f_R}{\partial \dot{\varepsilon}/\omega} & -\frac{2 f_R}{\varepsilon} \\ \frac{\partial f_T}{\partial \dot{\varepsilon}/\omega} & -\frac{2 f_T}{\varepsilon} \end{bmatrix} \tag{51}$$

We also obtained analytical expression for the stiffness and damping coefficients as

Non-dimensional stiffness and damping coefficients	Equivalent derivative in terms of Non-dimensional pressure force components	Analytical expression of stiffness and damping coefficients
$\bar{k}_{RR}$	$\frac{\partial f_R}{\partial \varepsilon}$	0
$\bar{k}_{RT}$	$\frac{\partial f_R}{\varepsilon \partial \phi} - \frac{f_T}{\varepsilon}$	$-\frac{12\pi^2 \left(1+\frac{5}{2}\phi\right)(1+\tau)}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}$
$\bar{k}_{TR}$	$\frac{\partial f_T}{\partial \varepsilon}$	$12\pi^2 \left(1+\frac{5}{2}\phi\right)(1+\tau)\left[\frac{2-\varepsilon^2+2\varepsilon^4}{(2+\varepsilon^2)^2(1-\varepsilon^2)^{3/2}}\right]$
$\bar{k}_{TT}$	$\frac{\partial f_T}{\varepsilon \partial \phi} + \frac{f_R}{\varepsilon}$	0
$\bar{c}_{RR}$	$\frac{\partial f_R}{\partial (\dot{\varepsilon}/\omega)}$	$-\frac{12\pi^2 \left(1+\frac{5}{2}\phi\right)(1+\tau)}{(1-\varepsilon^2)^{3/2}}$
$\bar{c}_{RT}$	$-\frac{2 f_R}{\varepsilon}$	0
$\bar{c}_{TR}$	$\frac{\partial f_T}{\partial (\dot{\varepsilon}/\omega)}$	0
$\bar{c}_{TT}$	$-\frac{2 f_T}{\varepsilon}$	$-\frac{24\pi^2 \left(1+\frac{5}{2}\phi\right)(1+\tau)}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}}$

For the steady-state (Static) the effect of variation of dimensionless eccentricity over the load carrying capacity, frictional force and coefficient of friction are studied. It is observed that the load carrying capacity is increasing without significantly increase in the frictional force with the use of FF lubricant in the presence of transverse magnetic field. Also for the dynamic condition, variation of dimensionless eccentricity over dynamic coefficients are studied. It is observed that the bearing performance improves with the use of FF lubricant than conventional lubricant, which leads to added advantages to various types mechanical devices.

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The following are some of the list of references which we have referred during the course of investigation.

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List of Symbols

a	$\frac{h_2}{h_1}$
c	radial clearance(m)
A	bearing length (m)
$\overline{f}$	dimensionless coefficient of friction
$\overline{F}$	dimensionless frictional force
h	film thickness (m)
h_1, h_2	minimum and maximum values of h (m)
$\dot{h}_1, \dot{h}_0$	squeeze velocity $\frac{dh_1}{dt}, \frac{dh_0}{dt}$ respectively(ms^{-1})
H	magnetic field strength (Am^{-1})
H	magnetic field vector
I	sum of moments of inertia of the particles per unit volume (Ns^2m^{-2})
m	magnetic moment of a particle ($A\,m^2$)
M	magnetization vector
M_0, M_1	saturation magnetization ($A\,m^{-1}$)
n	number of particles per unit volume (m^{-3})
f_0	Larmour frequency of the magnetization vector in the anisotropy field of the particle
$\overline{p}$	dimensionless film pressure
$\hat{p}$	dimensionless film pressure in dynamic condition

$\mathbf{q}$	fluid velocity vector
t	time (s)
T	temperature (^{0}K)
U	slider velocity (ms^{-1})
$\overline{W}$	dimensionless load-carrying capacity
x,y,z	coordinates
$\overline{\overline{x}}$	dimensionless center of pressure
X	$\frac{x}{A}$
a_1	radius of the circular discs (m)
FF	ferrofluid
h_0	central film thickness (m)
H^*	thickness of the porous matrix (m)
k_B	Boltzmann constant (J (^{o}K)$^{-1}$)
k	permeability of the porous matrix (m^2)
K	anisotropy constant
$\mathbf{S}$	sum of angular momentums of particles per unit volume (K m^2 s^{-1})
V	volume of the particle (m^3)
MF	magnetic fluid
P	fluid pressure in the porous matrix (N m^{-2})
r	radial co-ordinate (m)

s slip constant (m^{-1})

V_{sq} dimensionless squeeze velocity parameter

VMF variable magnetic field

f_R fluid film reaction forces in radial direction

f_T fluid film reaction forces in tangential direction

Greek symbols

α inclination of the magnetic field with the *x*-axis

β squeeze velocity parameter

δ central thickness of the convex pad (m)

η viscosity of the suspension (N s m^{-2})

η_0 viscosity of the liquid carrier (N s m^{-2})

λ magnetic field strength parameter

μ_0 permeability of free space

ξ, ξ_1 dimensionless field strength (Langevin's parameter)

τ, η_{rv} rotational viscosity parameter(N s m^{-2})

η_e effective viscosity parameter(N s m^{-2})

η_x porosity in the *x*-direction.

φ_x permeability of porous facing in *x*-direction (NA^{-2})

φ_z permeability of porous facing in *z*-direction (NA^{-2})

ψ_x dimensonless permeability parameter in the *x*-direction

ψ_z dimensonless permeability parameter in the *z*-direction

τ_N Neel relaxation time (s)

τ_B Brownian relaxation time (s)

τ_s magnetic moment relaxation time (s)

$\mathbf{\Omega}$ $\frac{1}{2}\nabla\times\mathbf{q}$

ϕ volume concentration of the particles

θ inclination of the magnetic field vector to the radial direction and also in the case of journal bearing angular coordinate related to a fixed direction

α_1 curvature of the mirror image of secant upper disc (m^{-2})

β_1 curvature of the exponential upper disc (m^{-2})

γ_1 curvature of the secant upper disc (m^{-2})

$\bar{\alpha}$ $\alpha_1 a_1^2$, dimensionless curvature parameter defined in equation (30)

$\bar{\beta}$ $\beta_1 a_1^2$, dimensionless curvature parameter defined in equation (28)

$\bar{\gamma}$ $\gamma_1 a_1^2$, dimensionless curvature parameter defined in equation (29)

η_r porosity of the porous matrix

ρ fluid density ($N\ s^2 m^{-4}$)

ψ dimensionless porous thickness parameter

Ω_f dimensionless rotational parameter

Ω_u rotational velocity of the upper disc (rad. s^{-1})

Ω_l rotational velocity of the lower disc (rad. s^{-1})

φ attitude angle

$\dot{\varphi}$ whirling motion of the journal

ε eccentricity ratio

ω angular velocity of the journal