PREFACE

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The content of the present thesis is based on the research work that I have been carrying out at The Maharaja Sayajirao University of Baroda, Vadodara, on "**RECENT TRENDS IN CONVERGENCE AND SUMMABILITY OF GENERAL ORTHOGONAL SERIES".**

Since the 18th century, the research of Bernoulli D., Bessel F., Euler L., Laplace P., and Legendre A. contains special orthonormal systems and expansion of a functions with respect to orthonormal systems in the subjects like astronomy, mathematics, mechanics, and physics. Research of some of the researchers mentioned below suggest the strong requirement of development of theory of orthogonal series:

- The research of Chebyshev P.L. on the problem of moments and interpolation.
- The research of Hilbert D. on integral equation with symmetric kernel.
- The research of Fourier J. on the Fourier method for solving the boundary value problems of mathematical physics.
- The research by Lebesgue H. for measure theory and the Lebesgue integral.
- The research of Sturm J. and Liouville J. in the area of partial differential equations.

There was a remarkable progress of the theory of general orthogonal series during 20th century. Several researchers have made use of orthonormal systems of functions and orthogonal series in the most areas like computational mathematics, functional analysis, mathematical physics, mathematical statistics, operational calculus, quantum mechanics, etc..

Looking to this scenario: the question of convergence and summability of general orthogonal series have made an important impact on several researchers. The researchers like Alexits G., Fejér L., Hardy G. H., Hilbert D., Hobson E., Kaczmarz S., Lebesgue H., Leindler L., Lorentz L., Meder M., Menchoff D., Riesz F., Riesz M., Steinhaus H., Tandori K., Weyl H., Zygmund A., and many other leading mathematicians have made an important contribution in the areas of convergence, summability and approximation problems of general orthogonal series.

The convergence of orthogonal series was studied by Gapoškin, V. F. (1964), Jerosch, F. and Weyl, H. (1909), Hobson, E.W. (1913), Kolmogoroff, A. and Seliverstoff, G. (1925), Menchoff, D. (1923), Plancherel, M. (1910), Plessner, A. (1926), Rademacher, H. (1922), Salem, R. (1940), Talalyan, A. A.(1956), Tandori, K. (1975) , Walfisz, A. (1940), and Weyl, H. (1909). In recent years significant contribution in this area is observed.

Besides the question of convergence, Menchoff, D. (1925), Menchoff, D. (1926), Kaczmarz, S. (1929) and several others have discussed the Cesàro summability of orthogonal series.

The order of approximation was studied by Agrawal, S. R. and Kantawala, P. S. (1986), Alexits, *G.* and Kralik, D. (1960), Alexits, *G.* and Kralik, D. (1965), Bolgov, V. and Efimov, A. (1971), Kantawala, P. S. (1986), Leindler, L. (1964), Sunouchi, G. (1944), Wlodzimierz, L.(2001) and several others.

Absolute summability of orthogonal series was studied by Alexits, G and. Kralik, *D.* (1965), Bhatnagar, S. C. (1973), Grepachevskaya, L. V. (1964), Leindler, L. (1961), Leindler L.(1963), Meder, J. (1958), Móricz, F.(1969), Patel, C. M. (1967), Patel, D. P. (1989), Patel, R. K. (1975), Prasad, B. (1939), Shah, B. M.(1993), Sapre, A. R. (1970), Srivastava, P.(1967), Tandori, K. (1957), Tsuchikura, T. (1953) and others.

It is important to note that Euler summability, de la Vallée-Poussin summability, Nörlund summability, Riesz summability and $(\overline{N},p_n)$ summability are discussed by several researchers. However, it is observed that certain summability requires more study for research in orthogonal series.

Chapter I is an introduction of complete thesis.

It involves history related to the origin of orthogonal series, some fundamentals and definitions like Banach summability, absolute Banach summability, Cesàro summability, absolute Cesàro summability, Euler Summability, Lambert summability, generalized Lambert summability, Nörlund summability, absolute Nörlund summability, $(\bar{N}, p_n)$ summability, absolute $(\bar{N}, p_n)$ summability, (N, p, q) summability, $|N, p, q|$ summability, $(\bar{N}, p, q)$ summability, $|\bar{N}, p, q|$ summability, $|N, p_n|_k$ summability, $|\bar{N}, p_n|_k$ summability $|N, p, q|_k$ summability, $|\bar{N}, p, q|_k$ summability, where $k \geq 1$, (N, p_n^{α}) summability, $|N, p_n^{\alpha}|$ summability, $(N, p_n^{\alpha}, q_n^{\alpha})$ summability, $|N, p_n^{\alpha}, q_n^{\alpha}|$ summability, $(\bar{N}, p_n^{\alpha}, q_n^{\alpha})$ summability, $|\bar{N}, p_n^{\alpha}, q_n^{\alpha}|$ summability, where, $\alpha > -1$, matrix summability i. e. $|\mathrm{A}|_{\mathrm{k}}; \mathrm{k} \geq 1$ summability, where $A = (A_{nv})$ be a normal matrix, generalized matrix summability, $|(N, p_n, q_n), (N, q_n, p_n)|_k$ summability for $k \geq 1$, and $|(\bar{N}, p_n, q_n), (\bar{N}, q_n, p_n)|_k$ summability for $k \geq 1$. All these summabilities are related to single infinite series.

This chapter also covers definitions of $\left|N^{(2)}, p, q\right|_k$ summability for $k \geq 1$ and $\left|\bar{N}^{(2)}, p, q\right|_k$ summability for $k \geq 1$ of double infinite series. The history of convergence and summability of general orthogonal series is also incorporated in this chapter.

This chapter also cotains the chapterwise description of our investigated results and its connected known results.

In Chapter II, we are focusing on absolute Banach summability of an orthogonal series and absolute $(\bar{N}, \mathrm{p}_{\mathrm{n}}^{\alpha}, \mathrm{q}_{\mathrm{n}}^{\alpha})$ summability i.e. $|\bar{N}, p_n^{\alpha}, q_n^{\alpha}|$ summability of an orthogonal series for $\alpha > -1$.

Chapter III deals with absolute indexed generalised Nörlund summability or $|\bar{N}, p, q|_k$ summability of an orthogonal series for $k \geq 1$. This chapter also includes some our corollaries.

In chapter IV, we have investigated the absolute general matrix summability of an orthogonal series i. e.$\Phi - |\mathrm{A}; \delta|_{\mathrm{k}}$, $\mathrm{k} \geq 1, \delta \geq 0$, summability of an orthogonal series, where $\{\Phi_n\}$ be sequence of positive real numbers and $A = (A_{nv})$ be a normal matrix.

In Chapter V, we have extended the result of Móricz F. et. al.1992. The result is based on approximation of Walsh- Fourier series by $(E, 1)$ mean.

Chapter VI covers absolute indexed generalized Nörlund summability i.e. $\left|\bar{N}^{(2)}, p, q\right|_k$ for $k \geq 1$, of double orthogonal series.

Bellman, R. (1943) discussed Lambert summability of an orthogonal series. In chapter VII, we have discussed generalized Lambert summability of an orthogonal series.

There are some investigations which reflect that under some conditions on coefficients, an orthogonal series may be product summable almost everywhere. Chapter VIII is devoted to generalized product summability of an orthogonal series. In this chapter, we have generalized the theorem of Krasniqi, Xh. Z. 2013 to $|(\bar{N}, p_n, q_n), (\bar{N}, q_n , p_n)|_k$ summability for $k \geq 1$ of an orthogonal series.

The references are provided at the end.