Chapter 5

Porous Infinitely Long Rectangular Plates Squeeze Film-Bearing

Contents

5.1 Introduction

The fluid film between two surfaces with a relative normal velocity (known as squeeze velocity) is important in many frictional devices in industry as well as in human body, for example in machine tools, gears, rolling elements, hydraulic systems, engines, clutch plates, skeletal joints (as bio-lubrication), etc. Moore [1] gave excellent review on squeeze films up to 1965. Squeeze film with the attachment of porous layer (region or plate or matrix or surface) are widely used in industry because of its advantageous property of self-lubrication and no need of exterior lubricant supply. But the disadvantage of the porous layer is that, only a part of the fluid is squeezed and the rest flows out through the porous media, which results in the reduction of the load carrying capacity. Wu [2,3] analyzed and discussed the squeeze film performance of a porous bearing for two types of geometries such as annular and rectangular disks (or plates or surfaces) with one porous surface. The results are presented in the form of an infinite series involving Bessel's and trigonometric functions. Prakash and Vij [4] analyzed lower porous plate squeeze film-bearing of different shapes (annular, circular, elliptic, rectangular and cone) using Morgan-Cameron approximation. The effects of the shape of plate and porosity on the bearing performance are calculated. Gupta *et. al*. [5] analyzed squeeze film behaviour between rotating annular plates when the curved upper plate with a uniform porous facing approaching normally to impermeable flat lower plate. The result shows that the load carrying capacity decreased when the speed of the rotation of the upper plate increased up to certain extent.

In recent years, many theoretical and experimental inventions are made on the improvement of the bearing design systems as well as on the lubricating substances in order to improve the efficiency of the bearing performances. One of the major revolutions in the direction of lubricating substances is an invention of ferrofluids (FFs) as lubricant. With the advent of

FFs[6], many researchers have tried to find its application as lubricant in bearing design systems. Tipei [7] analyzed the general momentum equations under the assumption of FFs as Newtonian fluids. The velocity and pressure fields for thin FF films are then obtained. The short-bearing case is studied. It is shown that the effect of magnetic particles increases load carrying capacity under the values of applied magnetic fields as 10^5 - 10^6 *A/m*. Also, it is shown that ferrofluid (FF) lubricant improves the bearing stability and stiffness. Agrawal [8] studied the effects of magnetic fluid (MF) on a porous inclined slider bearing and he showed that the magnetization of the magnetic particles in the lubricant increases load carrying capacity without affecting the friction on the moving slider. Chi *et. al.* [9] studied new type of FF lubricated journal bearing consisting of three pads. One of them is a deformable elastic pad. The theoretical analysis and experimental investigation show that the performance of the bearing is better than that of ordinary bearings. Moreover, the bearing operated without leakage and any feed system. Prajapati [10] analyzed various designed bearings like circular, annular, elliptic, conical, etc. It is shown that with the increase of magnetization parameter « *, the load carrying capacity increases. Thus, concluded that the superiority performance of the bearings with MF as lubricant. It is also concluded that the bearing with MF can support a load even when there is no flow. Shah *et. al.* [11] theoretically studied MF based squeeze film behaviour between rotating porous annular curved plates in the presence of external magnetic field oblique to the lower plate. It is shown that the increase of pressure and load carrying capacity depended only on the magnetization parameter. However, the increase in response time depended on magnetization, fluid inertia and speed of rotation of the plates. Ahmad and Singh [12] discussed MF based porous pivoted slider bearings with slip velocity to study the effect of parameters like load carrying capacity and centre of pressure. It is shown that load carrying capacity increases as the magnetic parameter increases, whereas it decreases as the slip parameter and permeability parameter increases. Also, it is shown that centre of pressure increases with the increase of magnetic parameter, slip parameter and permeability parameter. Andharia and Deheri [13] studied longitudinal roughness effect on MF based squeeze film between conical plates. It is shown that the performance of the bearing gets enhanced due to negative skewed roughness. Also, it is shown that the standard deviation increases the load carrying capacity which is unlike the case of transverse surface roughness. Recently, Shah and Patel [14] discussed effects of various and arbitrary porous structure in the study of squeeze step bearing lubricated with MF using variable magnetic field. It is concluded that the load carrying capacity increases with the increase of length of the first step as well as with the increase of magnetic field strength. The magnetic field strength which gives better results is of the order of 10^4.

In this Chapter porous infinitely long rectangular plates squeeze film-bearing studied with the porous matrix attached to the lower surface considering the effects of porosity, permeability, squeeze velocity and oblique variable magnetic field. Expressions for pressure and load carrying capacity are obtained. The results for dimensionless load carrying capacity are computed.

5.2 Mathematical Model and its Solution

Figure 5.1 shows schematic diagram of porous infinitely long rectangular plates squezee film-bearing with dimensions a and b such that $(a/b) >> 1$ (refer [4], [15]). It consists of infinitely long rectangular plates of width $-b/2 \le x \le b/2$ and $-a/2 \le y \le a/2$. The lower plate is attached with a porous matrix of thickness $H*$.

By combining equations (2.18) to (2.22) and under usual assumptions of lubrication theory, neglecting inertia terms and derivatives of fluid velocity across the film predominate, the equation governing the pressure distribution p in the film region using FF as lubricant satisfies the equation

$$\eta \frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right),$$

... (5.1)

where u is a velocity component in x -direction for the film region, η is the fluid viscosity, μ_0 is the free space permeability, $\bar{\mu}$ is the magnetic susceptibility and H is the strength of variable magnetic field.

Integrating equation (5.1) twice with respect to z and using boundary conditions

$$u = 0 \quad \text{when} \quad z = 0 \quad \text{and} \quad u = 0 \quad \text{when} \quad z = h,$$

the expression for velocity profile u in the film region can be obtained, which on substituting in the integral form of continuity equation for the film region

$$\frac{\partial}{\partial x}\int_0^h u \, dz + w_h - w_0 = 0,$$

... (5.2)

yields

$$\frac{\partial}{\partial x}\left[h^3 \frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2 \right)\right] = 12\eta \dot{h} - 12\eta \left. w \right|_{z=0},$$

... (5.3)

where h is the film thickness, $\dot{h} = dh/dt$ is the squeeze velocity of the upper bearing surface, w_0 and w_h are the z -components of the fluid velocity at $z = 0$ and $z = h$ respectively .

Also,

$$\mathbf{q} = u\,\mathbf{i} + v\,\mathbf{j} + w\mathbf{k},$$

… (5.4)

where u, v, w are components of film fluid velocity in *x, y* and *z* -directions respectively.

Also, it is assumed that the porous matrix is homogeneous and isotropic and that the flow satisfies Darcy's law. It can be shown that the pressure P in the porous region satisfies Laplace equation [16]

$$\frac{\partial^2}{\partial x^2}\left(P - \frac{1}{2}\text{«}_0 \overline{\ll} H^2\right) + \frac{\partial^2}{\partial z^2}\left(P - \frac{1}{2}\text{«}_0 \overline{\ll} H^2\right) = 0\,.$$

… (5.5)

Integrating equation (5.5) with respect to *z* over the porous matrix thickness $(-H^*,\, 0)$, yields

$$\left[\frac{\partial}{\partial z}\left(P - \frac{1}{2}\text{«}_0 \overline{\ll} H^2\right)\right]_{z=-H^*}^{z=0} = -\int_{-H^*}^{0}\left[\frac{\partial^2}{\partial x^2}\left(P - \frac{1}{2}\text{«}_0 \overline{\ll} H^2\right)\right] dz\,.$$

… (5.6)

Since

$$\left[\frac{\partial}{\partial z}\left(P-\frac{1}{2}\text{«}_0\bar{\text{«}}H^2\right)\right]_{z=-H*}=0$$

… (5.7)

as $z=-H*$ is a solid surface, that is the porous matrix is press-fitted with a solid housing as shown in Figure 5.1.

Equations (5.6) and (5.7) implies

$$\left[\frac{\partial}{\partial z}\left(P-\frac{1}{2}\text{«}_0\bar{\text{«}}H^2\right)\right]_{z=0}\approx -H*\frac{\partial^2}{\partial x^2}\left(p-\frac{1}{2}\text{«}_0\bar{\text{«}}H^2\right)$$

… (5.8)

using Morgan-Cameron approximation [17].

It can be proved mathematically that when $H*\rightarrow 0$, equation (5.8) is valid, that is in the case of thin-walled porous bearings, the approximation will not likely to cause any significant error [4].

Using Darcy's law, the z -component of velocity in the porous region is given by

$$\bar{w}\big|_{z=0}=-\frac{k}{\text{¦}}\left[\frac{\partial}{\partial z}\left(P-\frac{1}{2}\text{«}_0\bar{\text{«}}H^2\right)\right]_{z=0},$$

… (5.9)

where k is the permeability of the porous region.

Assuming the normal components of velocity across the film-porous interface is continuous, therefore

$$w\big|_{z=0} = \bar{w}\big|_{z=0},$$

which yields, using equations (5.3), (5.8) and (5.9)

$$\frac{\partial}{\partial x}\left[h^3 \frac{\partial}{\partial x}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\right] = 12\mu \dot{h} - 12kH^*\left[\frac{\partial^2}{\partial x^2}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right)\right],$$

… (5.10)

implies

$$\frac{\partial^2}{\partial x^2}\left(p - \frac{1}{2}\mu_0 \bar{\mu} H^2\right) = \frac{12\mu \dot{h}}{h^3 + 12kH^*}.$$

… (5.11)

The variable magnetic field **H** can be taken in such a way that it is vanishing at the inlet (or center) and the outlet of the bearings and inclined at an angle ϕ with the x -axis [10, 18]. Thus, for one-dimensional behaviour of the problem, **H** is a function of x only. Moreover, it is also assumed that **H** should be taken in such a way that it attains a maximum at the middle of the bearing producing magnetic pressure. Thus, the components of the applied magnetic field **H** has the form

$$\mathbf{H} = H(x)(\cos\phi,\ 0,\ \sin\phi);\quad \phi = \phi(x,\ z).$$

… (5.12)

The equation (2.20) in the present case becomes

$$\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} = 0.$$

… (5.13)

Combining equations (5.12) and (5.13), the equation for the inclination of the magnetic field can be obtained as

$$\cot - \frac{\partial -}{\partial x} + \frac{1}{H(x)}\frac{\partial H(x)}{\partial x} + \frac{\partial -}{\partial z} = 0,$$

… (5.14)

with a suitable choice of $H(x)$, the solution of the equation (5.14) gives the inclination of —.

Using equation (5.11) and by choosing oblique and variable magnetic field

$$H^2 = K\left(\frac{b}{2} - x\right)\left(\frac{b}{2} + x\right),$$

… (5.15)

where K is chosen to suit the dimensions of both sides.

The Reynolds-type equation for the film pressure can be obtained as

$$\frac{d^2}{dx^2}\left[p - \frac{1}{2}«_0 \bar{«} K\left(\frac{b}{2} - x\right)\left(\frac{b}{2} + x\right)\right] = \frac{12 \brokenvert\, \dot{h}}{h^3 + 12kH*}.$$

… (5.16)

Using boundary conditions $p(\pm b/2)=0$, above equation (5.16) becomes

$$p=\frac{1}{2}\text{«}_0\overline{\text{«}}K\left(\frac{b}{2}-x\right)\left(\frac{b}{2}+x\right)+\frac{6\text{¦}\,\dot{h}b^2}{h^3+12k\,H^*}\left(\frac{x^2}{b^2}-\frac{1}{4}\right),$$

… (5.17)

which can be written in dimensionless form as

$$\overline{p}=\frac{1}{2}\frac{\text{«}^*}{ab}\left(\frac{b}{2}-x\right)\left(\frac{b}{2}+x\right)+\frac{6}{(a/b)(1+12\,\cdot\,)}\left(\frac{1}{4}-\frac{x^2}{b^2}\right),$$

… (5.18)

where

$$\text{«}^*=-\frac{\text{«}_0\overline{\text{«}}Kh^3}{\text{¦}\,\dot{h}},\ \cdot\ =\frac{kH^*}{h^3},\ \overline{p}=-\frac{ph^3}{\text{¦}\,\dot{h}ab}.$$

… (5.19)

The definition of load carrying capacity

$$W=\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}p\,dx\,dy,$$

… (5.20)

implies

$$W = \frac{1}{12}«_0 \bar{«} Kab^3 - \frac{¦\,\dot{h}ab^3}{h^3 + 12kH*},$$

… (5.21)

which can be written in dimensionless form as

$$\bar{W} = \frac{1}{12}«*\frac{b}{a} + \frac{1}{(a/b)(1+12\,·\,)},$$

… (5.22)

where

$$\bar{W} = -\frac{Wh^3}{¦\,\dot{h}a^2b^2},$$

and $«*$, $·$ are defined in equation (5.19).

5.3 Results and Discussion

It is clear from equation (5.22) that the increase in load carrying capacity is due to the first term of the equation, when FF is used as lubricant. If $«* = 0$, then the results for the porous squeeze film-bearings analyzed by Prakash and Vij [4] are obtained. Further if $· = 0$, the results reduces to those for the squeeze film-bearings for non-porous case [19].

If *h* in equation (5.11) is independent of *x*, then the governing equation in both porous (refer equation (5.11)) as well as non-porous (refer equation (5.11) with $H* = 0$) cases includes free term $h^3 + 12kH*$ and h^3, respectively in denominator. Thus, the effect of porosity can be inserted through the term $12kH^*$ in the non-porous case. If h is dependent on *x*, then various shapes of squeeze film-bearings are obtained (for example, exponential, secant, etc.).

Figures 5.2 and 5.3 shows the variations in $\overline{W}$ for porous infinitely long rectangular plates squeeze film-bearing as a function of magnetization parameter « * for different values of permeability parameter · considering $\overline{W} = -Wh^3 /$ ¦ $\dot{h} a^2 b^2$. Again, it is observed that $\overline{W}$ only moderately increases as « * increases. But with the decrease of · , $\overline{W}$ increases significantly. The better performance of $\overline{W}$ (in the sense of $\overline{W}$ increases) is obtained when $0.0001 \leq$ · ≤ 0.01. Table 5.1 shows interesting results obtained for different values of the plate dimensions a and b. It should be noted here that the dimension of the width of infinitely long rectangular plates bearing along x -axis is $-b/2 \leq x \leq b/2$, while along y -axis is $-a/2 \leq y \leq a/2$. Moreover, the oblique variable magnetic field considered is with respect to x -axis. When $a > b$ considered, that is width of the rectangular plates is smaller along x -axis than y -axis, then the load carrying capacity is significantly less as compared to $a < b$ (that is width of the rectangular plates is smaller along y -axis than x -axis). In this case, instead of neglecting the pressure gradient along the infinitely longer x -dimension [4, 15], which is conventionally assumed by other authors [4, 15] as well as by us in column 2, we have considered it in our study -column 3 (because we do not aware about such study). The load carrying capacity $\overline{W}$ increases up to 300% when $a < b$ (refer Table 5.1). This is because FF in the presence of magnetic field (here magnetic field is taken along x -axis) generates spikes and greater the generation of spikes may lead to better load carrying capacity. Thus, it is suggested to consider the direction of the oblique variable magnetic field along the longer width of the bearing design.

5.4 Conclusions

Based on the ferrohydrodynamic theory given by R.E. Rosensweig, equation of continuity in film as well as porous region and validity of the Darcy's law in the porous region, a Reynolds equation for porous infinitely long rectangular plates squeeze film-bearing, formed by solid upper surface and lower porous plate, is theoretically derived considering the effects of porosity, permeability, squeeze velocity and variable magnetic field. The variable magnetic field considered here is oblique to the lower disk or plate. Moreover, the porous surface is considered because of its advantageous property of self-lubrication and no need of exterior lubricant supply.

The following important conclusions can be made for dimensionless load carrying capacity $\overline{W}$ from results and discussion.

(1) It is suggested to keep permeability parameter · ≤ 0.01.

(2) Because of presence of additional term containing «* in the expression for $\overline{W}$, load carrying capacity increases moderately with respect to magnetization parameter «* when FF is used as lubricant.

(3) Better load carrying capacity can be obtained when magnetic field should be considered along the longer width of the bearing.

It should be noted here that the variable magnetic field is used because it can be seen from equation (5.11) that uniform magnetic field cannot produce magnetic pressure, and hence does not enhance bearing performances.

5.5 Figures

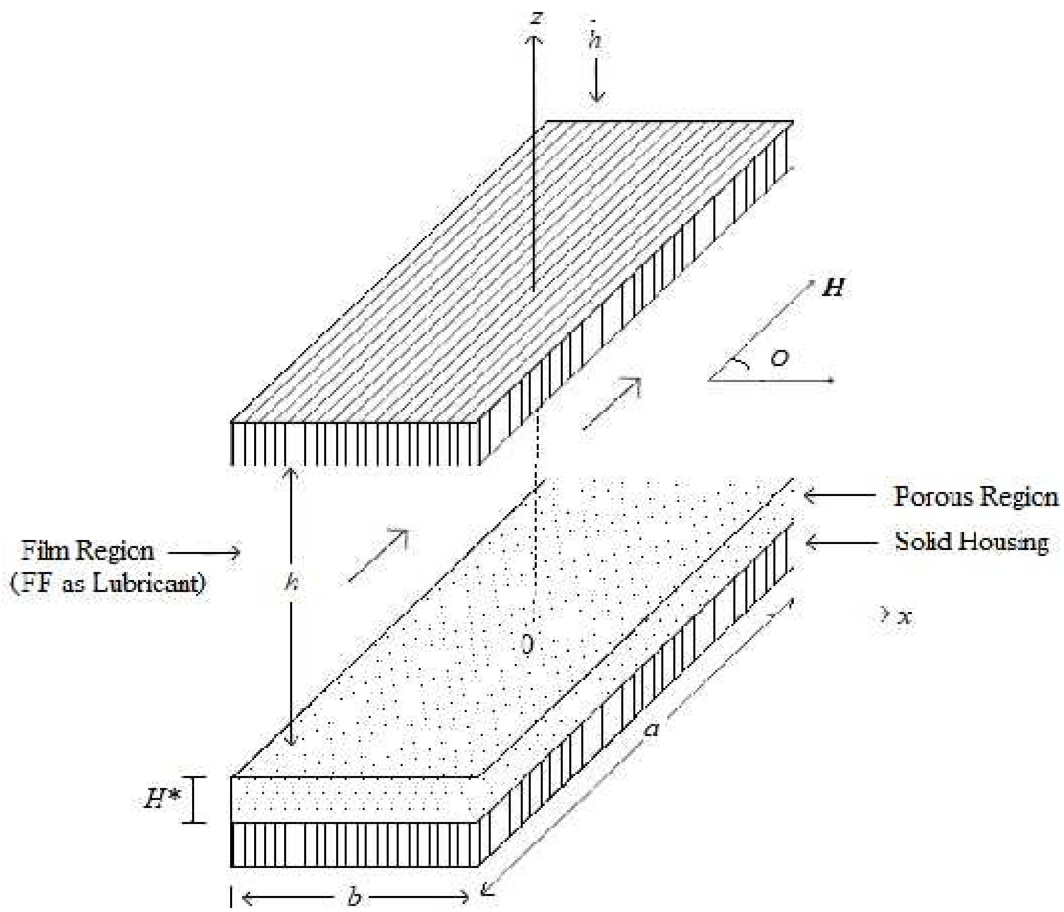

Figure 5.1 Schematic diagram of porous infinitely long rectangular plates squeeze film geometry.

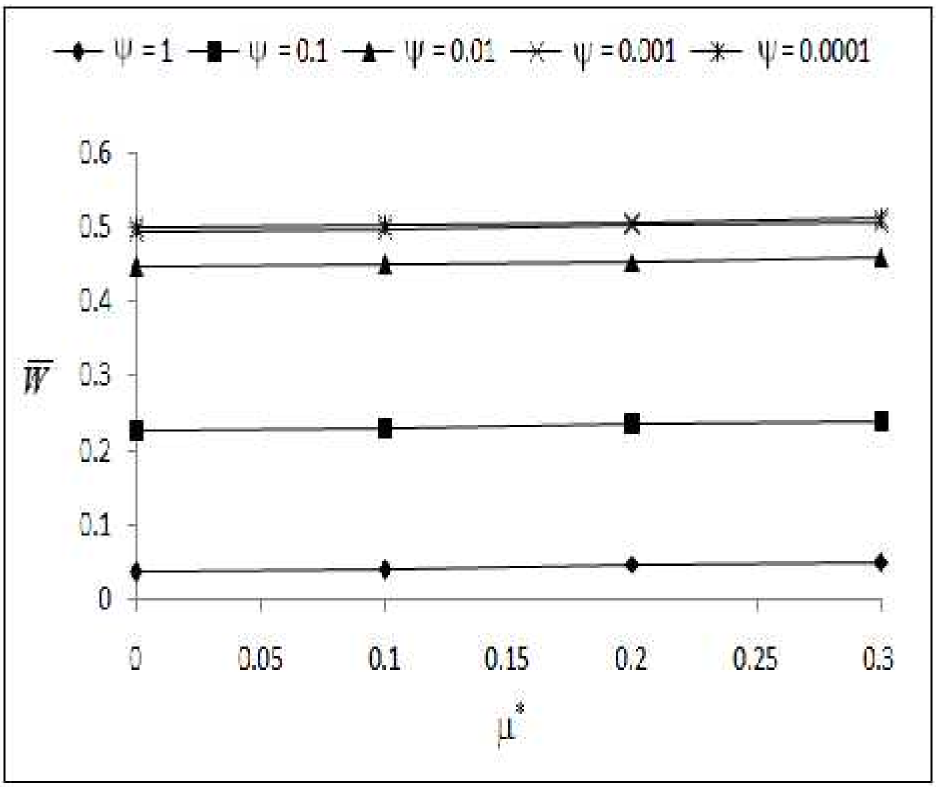

Figure 5.2 Variations in dimensionless load carrying capacity $\bar{W}$ for different values of magnetization parameter « * and permeability parameter · considering $a/b=2$.

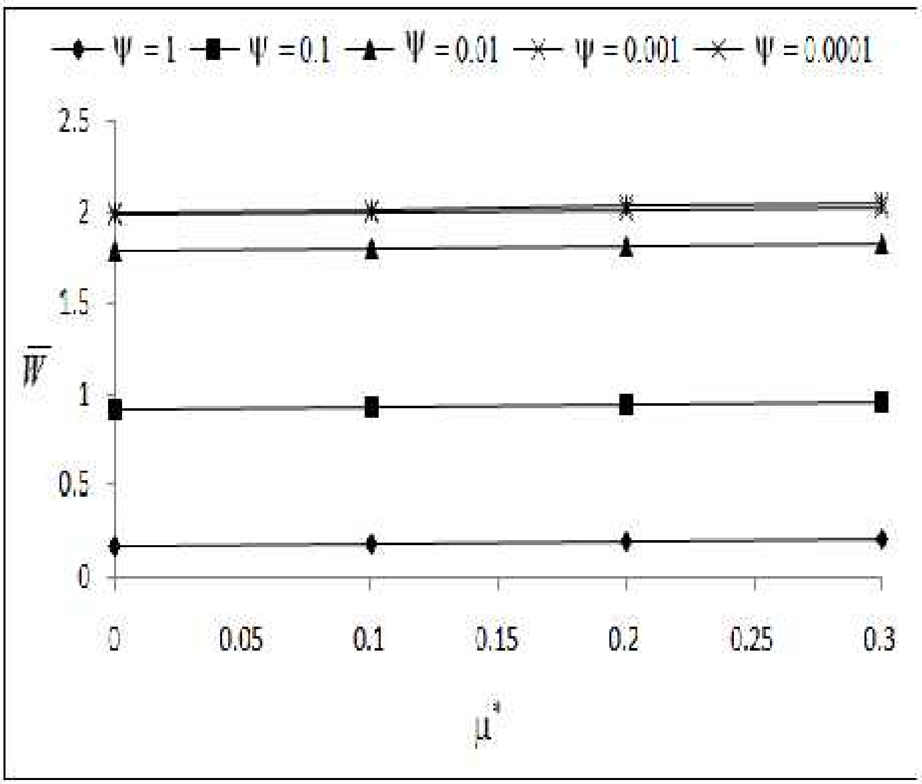

Figure 5.3 Variations in load carrying capacity $\bar{W}$ for different values of magnetization parameter « * and permeability parameter · considering $b/a = 2.$

5.6 Table

Dimensionless load carrying capacity	$a/b=2\ (a>b)$	$b/a=2\ (a<b)$	% increase in $\bar{W}$ for $(a<b)$
$\bar{W}$	0.51190	2.04760	300

Table 5.1 Comparison of dimensionless load carrying capacity $\bar{W}$ for $\cdot = 0.0001$ and $«^* = 0.3$.

5.7 References

[1] Moore D F, A review of squeeze films, *Wear*, 8, 245-263 (1965).

[2] Wu H, Squeeze-film behavior for porous annular disks, *Transactions of ASME* , series F, 92(4), 593-596 (1970).

[3] Wu H, An analysis of the squeeze film between porous rectangular plates, *Tranactions of ASME*, F94, 64-68 (1972).

[4] Prakash J and Vij S K, Load capacity and time-height relations for squeeze films between porous plates, *Wear,* 24, 309-322 (1973).

[5] Gupta J L, Vora K H and Bhat M V, The effect of rotational inertia on the squeeze film load between porous annular curved plates, *Wear*, 235-240 (1982).

[6] Rosensweig R E, Ferrohydrodynamics, *Cambridge University Press*, New York, (1985).

[7] Tipei N, Theory of lubrication with ferrofluids: Application to short bearings, *Transactions of ASME*, 104, 510-515 (1982).

[8] Agrawal V K, Magnetic fluid based porous inclined slider bearing, *Wear*, 107, 133-139 (1986).

[9] Chi C Q, Wang Z S and Zhao P Z, Research on a new type of ferrofluid-lubricated journal bearing, *Journal of Magnetism and Magnetic Materials*, 85, 257-260 (1990).

[10] Prajapati B L, Magnetic-fluid-based porous squeeze films, *Journal of Magnetism and Magnetic Materials*, 149, 97-100 (1995).

[11] Shah R C, Tripathi S R and Bhat M V, Magnetic fluid based squeeze film between porous annular curved plates with the effect of rotational inertia, *Pramana-journal of Physics*, 58(3), 545-550 (2002).

[12] Ahmad N and Singh J P, Magnetic fluid lubrication of porous-pivoted slider bearing with slip velocity, *Journal of Engineering Tribology*, 221, 609-613 (2007).

[13] Andharia P I and Deheri G M, Longitudinal roughness effect on magnetic fluid-based squeeze film between conical plates, *Industrial Lubrication and Tribology*, 62(5), 285-291 (2010).

[14] Shah R C and Patel N I, Impact of various and arbitrary porous structure in the study of squeeze step bearing lubricated with magnetic fluid considering variable magnetic field, *Proc I MechE Part J: Journal of Engineering Tribology*, 229(5), 646-659 (2015).

[15] Khonsari M M and Booser E R, Applied Tribology : Bearing design and lubrication, *John Wiley & Sons*, (2001).

[16] Morgan V T and Cameron A, The mechanism of lubrication in porous metal bearings, *Proc Conf on Lubrication and Wear* : *Institute of Mechanical Engineers*, 151-157 (1957).

[17] Shah R C and Bhat M V, Ferrofluid lubrication equation for porous bearings considering anisotropic permeability and slip velocity, *Indian Journal of Engineering and Materials Sciences*, 10, 277-281 (2003).

[18] Shah R C and Bhat M V, Ferrofluid lubrication in porous inclined slider bearing with velocity slip, *International Journal of Mechanical Sciences*, 44, 2495-2502 (2002).

[19] Archibald F R, Load capacity and time relations for squeeze films, *Transactions of the ASME*, 78, 231-245(1956).