Chapter 2

Correlation and its implementation in Speckle Metrology

2.1 Background of Speckle Correlation techniques

Discovery of speckle phenomenon and the ability to harness its properties for quantifying various physical and optical quantities has immensely changed the face of metrology sciences. Speckle being a statistical phenomenon, has to be handled with some statistical functions or algorithms. A very popular and extensively used function is the Correlation (or decorrelation) algorithm [11–16]. Speckle correlation technique has emerged as an effective, efficient, fast, sensitive, simple, rugged and non-contact method for various metrological applications [17–44]. Some of the reported engineering and industrial applications include measurement and inspection of strain (in the fields ranging from aviation to micro-electronics) [17,18,29,38–41], roughness [42–44], deformation [19–21], vibration [22–25], crevice corrosion [26], complex thermo-mechanical properties of thin multilayered structures [27], angular displacement [28], depth of layered structure [30], temperature [31], fingerprint acquisition etc.[32]. In the area of biophotonics, the speckle correlation technique has marked its strong presence as it has been reportedly used for evaluation of blood plasma coagulability [33], measurement of eye tremor [34], estimation of global and regional Left Ventricular function [35], and blood flow in the brain [36]. The speckle correlation technique has also been presented as a tool for evaluating the viscosity of dairy products which can be very helpful in judging the product quality and its consumption [37]. The simplistic, effective, and swift analysis of speckle data using the correlation technique has been one of the prime motivating factors for pursuing this work.

Correlation theory based-technique has been reportedly used for determining phase difference [45,46]. The existing methods for estimating phase difference can broadly be classified into categories: (1) the frequency-domain methods (2) the time-domain methods. The frequency-domain methods work by transforming the time-domain signal to the frequency-domain signal, by using Fourier transforms. While the time-domain methods work on the cross-correlation between two signals.

These methods include the normalized cross-correlation (NCC) method, data extension-based correlation (DEC) method and quadrature delay estimator (QDE) [45]. One of the approaches to apply the correlation method is the fringe pattern correlation where an equi-spaced fringe pattern is employed and two fringe patterns with equal spacing are matched (correlated). The fringe patterns with equi-spaced fringes associated with two different states; before and after introducing a phase change are recorded. The phase difference can be computed by shifting the pattern back to match the initial position. A fixed image patch is selected from the first fringe pattern which acts as a reference image patch and it is shifted in the second fringe pattern back, pixel by pixel. The maximum value of the correlation is obtained by the number of pixels required to be shifted for attaining the best matching position. This is a very highly sensitive technique but it has a disadvantage. A shift of more than one fringe creates an ambiguity in the correlation coefficient values as it will repeat cyclically [47–49]. Using the speckle correlation technique resolves this problem as the speckle pattern is random and does not repeat itself periodically. This makes speckle correlation a preferable technique, especially for motion detection and recognition. The following section describes the basics of correlation function which will be extensively used in examining various optical and physical properties.

2.2 Basics of Correlation

Correlation is a statistical method that determines the degree of similarity between two variables. It is also known as a *bivariate* statistic, where bi- meaning two and variate indicates variables [11]. There can be only three possibilities regarding the relationship between two variables; strong, weak and none. Thus, correlation can be defined as the relationship between two variables such that a change in one variable results in the positive or negative or no change in the other. A greater change in one variable may result in a corresponding greater or smaller change in the other variable.

Correlation can broadly be classified depending on how variable changes with respect to the other variable: (1) Positive or negative (2) Simple and Multiple (3)

Partial and total. If two variables tend to change together in the same direction (direction here refers to the way in which the variables change) then the correlation is called *positive* correlation and the correlation is called *negative* correlation when there is an increase in the value of one variable and a decrease in the value of the other. When we study only two variables or data series the relationship is said to be a *simple* correlation whereas the correlation involved in the study of more than two variables or data series simultaneously, is described as *multiple* correlations. *Partial* correlation is a measure of the strength and direction of a linear relationship between two continuous variables while controlling for the effect of one or more other continuous variables (also known as 'covariates' or 'control' variables) [12].

To quantify the correlation, the correlation coefficient is computed which acts as an indicator of prediction. A correlation coefficient is a numerical measure that indicates the strength of the relationship between the variables and has a range of -1 to +1. Thus, a strong positive (value close to +1) or negative (value close to -1) correlation implies that the association between the two variables has a high predictive relationship. It is a very common tendency and mistake to think correlation as a cause-effect relationship, where a strong positive or strong negative correlation between two variables can be considered as a sign of dependence of one variable on the other. Correlation does not imply causation and a strong correlation never implies a cause-effect relationship between two variables [11].

2.2.1 Types of Correlation based on the class of variables involved

There are four common types of correlation depending upon the class of variables and their relationship with other variables [11,13].

(1) Pearson's r_p : A measure of the strength of a relationship between two continuous variables.

(2) Spearman's r_s : A measure of the similarity between two ordinal rankings of a single set of data.

(3) Point-biserial r_{pb} :A measure of the strength of a relationship between one

continuous variable and one dichotomous variable.

(4) Phi (Φ) correlation: A measure of the strength of a relationship between two dichotomous variables.

(1) The Pearson product-moment correlation coefficient (r_p)

It is the most commonly used method to compute the correlation coefficient, which measures the degree of relationship between two continuous variables. A continuous variable here is a variable that can be measured along a line scale.

The formula for calculating r_p is [11]

$$r_p = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \tag{2.1}$$

where, n is the sample size, x and y are two individual sample points while $\bar{x}$ and $\bar{y}$ are their mean values respectively.

The above equation is used in this thesis to calculate various optical and physical properties by comparing two speckle patterns associated with two different states of the measured quantity and computing the correlation coefficient.

(2) Spearman's Correlation

Spearman's correlation coefficient (r_s)which is also known as Spearman's rho(ρ),determines the degree of association for two sets of ranked data. Spearman's correlation is also called the rank-order correlation coefficient. It is not as common as the Pearson's coefficient but can be very handy while dealing with variables that are ordered according to ranks or variables that may be subsequently ranked based on a continuous variable.

The Spearman's correlation coefficient can be calculated using [11]

$$r_s = 1 - \frac{6\sum D^2}{N(N^2 - 1)} \tag{2.2}$$

where, N is the number of pairs of scores and D is the difference between the pairs of ranks. Spearman's correlation can be very useful when we have to measure the linear correlation between ranks. Pearson's r_p will produce the same value as

Spearman's r_s on the same set of ranked data. If the variables are expressed in their original form as continuous measures, Pearson's r_p will not equal Spearman's r_s after they have been converted to ranks, but they will have similar values [11].

(3) Point-Biserial Correlation

The point-biserial correlation (r_{pb}) gives an estimate of the degree of relationship between a dichotomous variable and a continuous variable. Before the invention of modern computing devices, one had to use the older devices and had to dichotomize one of the two continuous variables and then run the point-biserial correlation instead of Pearson's because the formula was simpler. However, Pearson's correlation coefficient yields the same value as the point-biserial correlation formula.

Generally, r_{pb} is used when a single test item (dichotomous) is correlated with the overall test score (continuous). The point-biserial correlation (r_{pb}) can show whether an individual item is a good predictor of the overall test score. The formula for it is as follows [11]

$$r_{pb} = \frac{\overline{x_1} - \overline{x_2}}{S} . \sqrt{pq} \tag{2.3}$$

where, $\overline{x_1}$ is the mean score on the continuous variable of just the participants in level 1 of the dichotomous variable, $\overline{x_2}$ is the mean score of the continuous variable of just the participants in level 2 of the dichotomous variable, S is the standard deviation of all the participants on the continuous variable, p is the proportion of persons in Level 1 of the dichotomous variable and $q = 1 - p$.

(4) Phi (Φ) Correlation

The phi correlation gives an estimate of the degree of relationship between two dichotomous variables. The value of the phi (Φ) correlation coefficient has the range similar to the Pearson's r; that is, it can vary from -1.00 to +1.00. The phi correlation can be explained using an example where the occurrence of two diseases, Disease 1 and Disease 2, with total of 183 people under study. A person affected with Disease 1 may get affected by Disease 2. So, the classification is based on a person having Disease 1, Disease 2, both or neither. The frequencies of these diagnostic combinations are presented in the tabular form as below

Table 2. 1: Frequencies of diagnostic combinations (Phi correlation)

		Event 2	
		No	Yes
Event 1	No	110[a]	19[b]
	Yes	22[c]	32[d]

The individual cells in this matrix of numbers have been labeled *a* to *d* to identify the cells in the table. The formula for Φ is [11]

$$\Phi = \frac{ad - bc}{\sqrt{(a+b)(c+d)(a+c)(b+d)}} \tag{2.4}$$

using which Φ for the aforementioned example can be calculated to be 0.45.

2.2.2 Types of correlation based on how the variables are compared (Autocorrelation and Cross-correlation)

Before moving on to the topics of cross-correlation and autocorrelation which play a significant role in the present work, one similar statistical and signal processing tool, convolution, needs to be addressed.

Convolution works on the principle of observing how a given system responds to an impulse, for any possible input signal. It is a mathematical way of combining two signals to form a third signal. In linear systems, convolution is used to describe the relationship between three signals: the input signal, the impulse response, and the output signal. In the convolution approach, impulse decomposition is used where a system is characterized by knowing how it responds to an impulse. This helps in calculating the system’s output for any given input [14]. Mathematically a convolution is integral that expresses the amount of overlap of one function when it is shifted over another function.

The convolution of two functions $\phi(t)$ and the impulse function $\delta(t)$ is represented by an integral [15] as

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-t_0)dt = \phi(t_0) \qquad (2.5)$$

The response of a linear time-invariant (LTI) system to an impulse response of the system is obtained by the convolution of the input signal and the impulse response of the system [15].

Convolution is very similar to correlation, with a very slight difference in how both these mathematical operations are perceived. Unlike correlation, in convolution one of the two functions is reversed and shifted before computing the product of the two functions. Convolution is generally between a signal and a filter, which can be thought of as a system with a single input and stored coefficients whereas correlation is usually between two signals, where the system has two inputs and no stored coefficients [16]. Convolution can work the same way as correlation and produce similar results when the filter is symmetric as the symmetric filter nullifies the effect of folding or reversing in convolution.

Cross-correlation provides a measure of similarity between two signals, while the autocorrelation is the measure of how similar a signal is to itself. Cross-correlation is a comparison between functions $x(\tau)$ and $h(\tau)$, and is defined as an integral [15] as

$$R_{xh}(\tau) = x(\tau) ** h(\tau) = \int_{-\infty}^{\infty} x(t)h(t+\tau)dt \qquad (2.6)$$

where " $**$ " is the symbol for correlation and $h(t+\tau)$ is the time-shifted version of the function $h(\tau)$. Autocorrelation function can be considered as a special case of cross-correlation function, where $h(\tau)$ is taken to be $x(t)$. The autocorrelation provides a comparison between a function $x(\tau)$ and its time-shifted version $x(t+\tau)$ and is defined as [04]

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt \qquad (2.7)$$

Autocorrelation is associated with power spectral density by Wiener-Khintchine theorem which suggests that the autocorrelation function and the power spectrum are Fourier Transform pair i.e. the autocorrelation of a signal can be obtained by the Fourier transform of the power spectral density of the signal. There is a corollary to this theorem which shows that the autocorrelation is independent of spectral phase which means that two signals with the same power spectral density but different spectral phases will have the same autocorrelation function and hence numerous signals can have the same autocorrelation. Thus, methods of signal analysis that are based on autocorrelation cannot differentiate between such signals, no matter how different they may look in the time domain. So a high order statistic is required to differentiate between such signals [16].

2.3 Motivation behind adopting the Speckle Correlation technique

The technique of speckle correlation works on imaging of the speckle pattern which is formed by an object, generally a rough surface or a volume of interest. Generally, a speckle pattern is observed in arrangements involving a coherent light source, where coherent light is reflected or transmitted through a rough surface [2,50]. The individual speckle can be considered as a reference point from which the change in the phase of the light that is reflected from or transmitted through a rough surface can be tracked [51]. Thus, a change in the physical or optical properties of the interacting surface or volume can be converted into a change in the speckle pattern. This change in the speckle pattern can be quantified by computing the correlation coefficient which involves comparing two speckle patterns associated with two different states of the measurand. It is already established that the speckle correlation technique is one of the many optical techniques which can work remotely and provide accurate measurements with simple arrangements making it a very preferable noncontact, inexpensive, sensitive technique [52–55].

In this work, the correlation coefficient is measured using the Pearson product-moment correlation coefficient (r_p) which is given by Eq. 2.1. The Eq. 2.1 is extensively used during this work for measuring optical rotation, refractive index, temperature, and magnetic field. Rearranging Eq. 2.1 for the intensities of the speckle pattern to give

$$\frac{\sum_{k=1}^{N}\sum_{l=1}^{N}\left[I_R(k,l)-\bar{I}_R\right]\left[I_O(k,l)-\bar{I}_O\right]}{\sqrt{\left\{\sum_{k=1}^{N}\sum_{l=1}^{N}\left[I_R(k,l)-\bar{I}_R\right]^2\right\}\left\{\sum_{k=1}^{N}\sum_{l=1}^{N}\left[I_O(k,l)-\bar{I}_O\right]^2\right\}}} \tag{2.8}$$

where $I_R(k,l)$ and $I_O(k,l)$ are the intensities of the recorded speckle patterns before and after the change in the level/state of the measurand respectively with $\bar{I}_R$ and $\bar{I}_O$ being their mean values. The change in correlation coefficient is computed from the obtained correlation value using

$$\Delta C = 1 - C \tag{2.9}$$

Fig. 2.1 shows the flow chart describing the general process involved in the measurement of optical rotation, refractive index, temperature, and magnetic field. The measurement process requires two sets of data, one set of speckle patterns corresponding to the reference level of the measured quantity and the other set of speckle patterns corresponding to that level which needs to be measured.

Any external vibration and physical or electronic noise may lead to a change in speckle pattern which acts as background noise for the system which needs to be taken into account. This background noise can affect the resolution of the system and quantification of which can help in determining the stability of the system. The stability of the measurement system is computed by recording speckle patterns corresponding to the reference level over a period of time and comparing these speckle patterns in sequence to obtain the change in correlation coefficient values. The standard deviation in this change in correlation coefficient values over time is defined as the stability of the system.

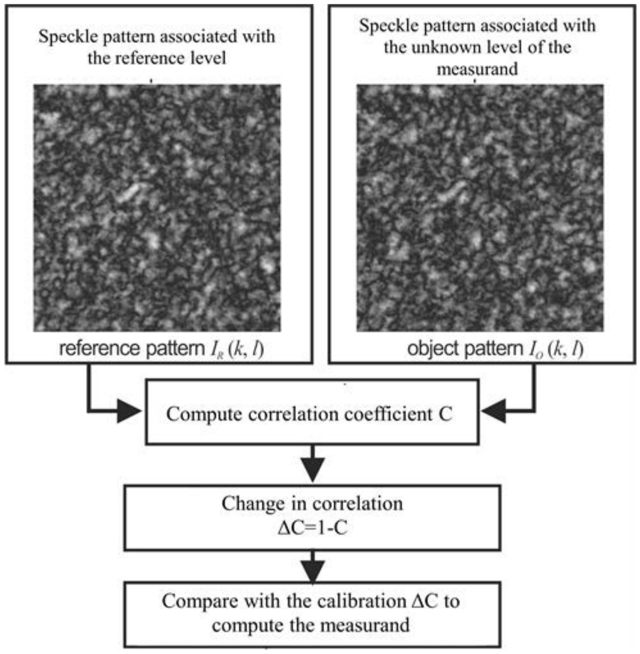

Fig 2. 1: Flow chart exhibiting the process of comparing the speckle pattern to obtain correlation coefficient values

Chapter 3 and Chapter 4 are dedicated to applications of speckle correlation techniques where the former deals with applications wherein speckles that are generated after the incident laser beam is allowed to pass through a volume for optical rotation and refractive index measurement while the latter is dedicated to applications that use speckles that are generated upon reflection of the laser beam from a rough surface which is attached to a cantilever for measurement of temperature and magnetic field.