Chapter 3

Decay Properties of Heavy Baryons

3.1 Introduction

Baryons are strongly interacting three quark fermions. This chapter is dedicated to the study of doubly heavy baryons i.e. baryons having two heavy quarks (c and/or b) as they might prove to be important tool for testing quantum chromodynamics [172, 173]. These states were also predicted long back in the quark model [174]. After the experimental discovery of the first doubly heavy baryon, Ξ_{cc}^{+} by SELEX Collaboration [175] and later confirmed by them [176]. The next doubly heavy baryon Ξ_{cc}^{++} was discovered experimentally by LHCb collaboration in the $\Lambda_c^+ K^- \pi^+ \pi^-$ mass spectrum [177]. LHCb collaboration has also recently reported the life time of Ξ_{cc}^{++} baryon [178]. As an outcome of the LHCb upgrade, one can expect more detailed information on existing doubly heavy baryons and also the discovery of other doubly heavy baryons [177–179].

With the advancement in the detector technology and new results on the properties of doubly heavy baryons, it has created lot of interest for the theoreticians world wide. In the literature, there are two ways in which the spectroscopy and decay properties of the heavy baryons are studied theoretically: quark-diquark picture and other is three quark picture. In quark-diquark picture, the masses and radiative decay properties are studied in Bethe-Salpeter approach [180] and relativistic quark model (RQM) [181, 182]. The spectroscopy of doubly heavy baryons is also studied in the nonrelativistic framework of quark model using the potential of the type Buchmüller and Tye [183], chiral perturbation theory (χPT) [184–188] and also in Ref [189]. In three quark picture, the spectroscopy and decay properties are studied in Bag model [190, 191], effective Lagrangian approach [192], SU(4) chiral constituent quark model (χCQM) [193], relativistic quark model (RQM) [194, 195], chromomagnetic model [196], nonrelativistic quark model (NRQM) [197–200] non-relativistic hypercentral constituent quark model (hCM) [104, 201–204]. The mass spectra are also computed on the model based on first principles such as lattice quantum chromodynamics LQCD [23–28], QCD sum rules [205–208] and NRQCD sum rules [209], light cone sum rules [210]. The mass spectra of heavy baryons are also studied using the Regge phenomenology [211, 212].

In this chapter, we employ the three quark picture of relativistic harmonic model for computing the masses of the doubly heavy baryon (ERHM). The spin dependent part of the one gluon exchange potential employed perturbatively for computing the masses of $1/2^+$ and $3/2^+$ states. The magnetic moments of the doubly heavy baryons are computed using the spin flavor wave function of the baryons. We also compute the radiative transition widths without using additional parameter.

This chapter is organised in the following way: After the brief introduction and survey on doubly heavy baryons in Sec. 3.1, we give the essential components of relativistic harmonic confinement model in Sec. 3.2 and compute the masses of doubly heavy baryons. In Sec. 3.3, we compute the magnetic moments using the spin flavor wave functions. Next, in Sec. 3.4 we compute the radiative decays using the transition magnetic moments. In Sec. 3.5 we present our results of masses, magnetic moments and radiative decay widths. We also compare our results with the available experimental results and other theoretical predictions.

3.2 Methodology

For computation of bound state masses of baryon, we use the relativistic harmonic confinement model in which the quarks are confined through the Lorentz scalar plus vector potential of the form

$$V_{conf}(r) = \frac{1}{2}(1+\gamma_0)A^2r^2 \tag{3.1}$$

Where A is the confinement strength mean field parameter and γ_0 is the Dirac matrix. The Dirac equation is solved using the method of non-relativistic reduction and the eigen energy (ϵ_{conf}) is obtained. The detailed computation technique is given in the Ref. [38, 39], here we provide only the essential components of the model. We perturbatively add contribution due to the Coulomb potential along with state dependent colour dielectric coefficient ω given by

$$V_{coul}(r) = \frac{k\alpha_s(\mu)}{\omega r} \tag{3.2}$$

The mass of a baryon in the different $n^{2S+1}L_J$ state according to different J^{PC} can be written as [38, 39, 134]

$$M_N^J = \sum_{i=1}^{3} \epsilon_N(q_i)_{conf} + \sum_{i<j=1}^{3} \epsilon(q_i, q_j)_{coul} + \sum_{i<j=1}^{3} \epsilon_N^J(q_i, q_j)_{S.D.} \tag{3.3}$$

Table 3.1: Model parameters

A	k	m_u	m_d	m_s	m_c	m_b
2166 $MeV^{3/2}$	0.37	240 MeV	243 MeV	450 MeV	1313 MeV	4632 MeV

In above Eq. 3.3, the first term corresponds to total confinement energy of the constituent quarks inside the baryon which is computed in the relativistic harmonic confinement model [139]. In order to obtain the confinement energy, the Dirac equation is reduced to the nonrelativistic case [38]. The confinement energy is given by

$$\epsilon_N(q)_{conf} = \left((2N+3)\Omega_N(q) + m_q^2 - \frac{3m_q}{\sum_i^3 m_{q_i}}\Omega_0(q) \right)^{1/2} \tag{3.4}$$

where m_q is the quark mass, Ω_N is the energy dependent size parameter given by

$$\Omega_N = \sqrt{E_N + m_q} \;\; A \tag{3.5}$$

and the energy eigen value coming from the nonrelativistic reduation of Dirac equation given by

$$E_N^2 = m_q^2 + (2N+3)\Omega_N \quad \text{with} \quad N = 0, 1, 2, 3... \tag{3.6}$$

with the radial solution of Dirac equation

$$R_{n\ell}(r) = \sqrt{\frac{\Omega_N^{3/2}}{2\pi} \frac{n!}{\Gamma(n+\ell+3/2)}} \; (\Omega_N^{1/2} r)^\ell \;\; exp\left(-\frac{\Omega_N r^2}{2}\right) \;\; L_N^{\ell+3/2}(\Omega_N r^2). \tag{3.7}$$

The second term in Eq. (3.3) corresponds to the Coulomb energy which is the expectation value of the Coulomb potential Eq. (3.2). In Eq. (3.2), ω is the state dependent color dielectric constant [38]. α_s is the strong running coupling constant. The Coulomb energy can be computed as

$$\epsilon_N(q_1, q_2)_{coul} = \langle NS|V_{coul}(r)|NS\rangle \tag{3.8}$$

The third term in Eq. (3.3) corresponds to the expectation value of spin dependent part of the confined one gluon exchange potential [213]. The single particle energy given by Eq. (3.6) which is the results of the nonrelativistic reduction of Dirac equation. This methodology also treats the quark and antiquarks on the equal basis. Also the confinement energy of the quarks and antiquarks within the baryons are computed by subtracting the contribution to the centre of mass energy from the single particle energy (last term of Eq. (3.4)). In harmonic confinement model, the residual Coulomb interaction has been introduced for the heavy flavor sector (ERHM) [38,39]. This method is general and applicable to the hadronic state with any number of constituent quarks.

Table 3.2: Masses of doubly heavy baryons (in MeV)

State	Quark Content	Present	RQM [91]	hCM [202–204]	NRQM [197]	RQM [181]	Chromo [196]	LQCD [24]	QCDSR [207]	QCDSR [205, 206]
Ξ_{cc}^{++}	ccu	3621	3620	3511	3676	3606	3633.3 ± 9.3	3610(23)(22)	4260	3720
Ξ_{cc}^{*++}	ccu	3744	3727	3687	4029	3675	3696.1 ± 7.4	3692(28)(21)	3900	3720
Ξ_{cc}^{+}	ccd	3623	3620	3520	3676	–	–	–		–
Ξ_{cc}^{*+}	ccd	3744	3727	3695	4019	–	–	–		–
Ω_{cc}^{+}	ccs	3756	3778	3650	3815	3715	3731.8 ± 9.8	3738(20)(20)	4250	3730
Ω_{cc}^{*+}	ccd	3815	3872	3810	4180	3772	3802.4 ± 8.0	3822(20)(22)	3810	3780
Ξ_{bc}^{+}	bcu	6931	6933	6914	7011	–	6922.3 ± 6.9	6943(33)(28)	6750	6720
Ξ_{bc}^{*+}	bcu	7003	6980	6980	7047	–	6973.2 ± 5.5	6985(36)(28)	8000	7200
Ξ_{bc}^{0}	bcd	6933	6933	6920	7011	–	–	–		–
Ξ_{bc}^{*0}	bcd	7003	6980	6986	7047	–	–	–		–
Ω_{bc}^{0}	bcs	7051	7088	7136	7136	–	7010.7 ± 9.3	6998(27)(20)	7020	6750
Ω_{bc}^{*0}	bcs	7084	7130	7187	7187	–	7065.7 ± 7.5	7059(28)(21)	7540	7350
Ξ_{bb}^{0}	bbu	10205	10202	10312	10340	10138	10168.9 ± 9.2	10143(30)(23)	9780	9960
Ξ_{bb}^{*0}	bbu	10229	10237	10355	10576	10169	10188.8 ± 7.1	10178(30)(24)	10350	10300
Ξ_{bb}^{-}	bbd	10206	10202	10317	10340	–	–	–		–
Ξ_{bb}^{*-}	bbd	10229	10237	10340	10576	–	–	–		–
Ω_{bb}^{-}	bbs	10311	10359	10446	10454	10230	10259.0 ± 15.5	10273(27)(20)	9850	9970
Ω_{bb}^{*-}	bbs	10322	10389	10467	10693	10258	10267.5 ± 12.1	10308(27)(21)	10280	10400
Ω_{ccc}^{*++}	ccc	4465	–	4806	4965	–	–	–		
Ω_{ccb}^{+}	ccb	7720	–	–	8245	–	7990.3 ± 12.2	8007(9)(20)		
Ω_{ccb}^{*+}	ccb	7728	–	–	8265	–	8021.8 ± 9.0	8037(9)(20)		
Ω_{cbb}^{0}	cbb	10965	–	–	11535	–	11165.0 ± 11.8	11195(8)(20)		
Ω_{cbb}^{*0}	cbb	10967	–	–	11554	–	11196.4 ± 8.5	11229(8)(20)		
Ω_{bbb}^{*-}	bbb	14198	–	14496	14834	–	14309.7 ± 11.8	–		

3.3 Magnetic Moments

The magnetic moment can provide the information regarding the structure of the baryons. The magnetic moment of the doubly heavy baryons in terms of constituent quarks as [201]

$$\mu_B = \sum_i \langle \phi_{sf} | \mu_i \vec{\sigma_i} | \phi_{sf} \rangle \tag{3.9}$$

with

$$\mu_i = \frac{e_i}{2m_i^{eff}} \tag{3.10}$$

Where e_i is the charge of the quark and σ_i is the spin of the quark, $|\phi_{sf}\rangle$ is the spin-flavor wave function of the respective baryons and m_i^{eff} is the effective mass of the quarks within the baryons can be computed as

$$m_i^{eff} = m_i \left(1 + \frac{E + \langle V_{spin} \rangle}{\sum_i m_i}\right). \tag{3.11}$$

Using Eqs. 3.9 –3.11 we compute the magnetic moments of doubly heavy baryons and tabulated in Tab. 3.3. We also compare our findings with the other theoretical approaches.

Table 3.3: Magnetic moment in μ_N

State	μ [104, 201]	Present	hCM [202, 203]	hCM [104, 201]	NRQM [198]	NRQM [199, 200]	exBag [191]	LCSR [210]
Ξ_{cc}^{++}	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_u$	−0.185	0.031	−0.133	$-0.208^{+0.035}_{-0.086}$	–	−0.110	0.23 ± 0.05
Ξ_{cc}^{*++}	$2\mu_c + \mu_u$	2.724	2.218	2.663	$2.670^{+0.27}_{-0.25}$	2.52	2.35	–
Ξ_{cc}^{+}	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_d$	0.843	0.784	0.833	$0.785^{+0.050}_{-0.030}$	–	0.719	0.43 ± 0.09
Ξ_{cc}^{*+}	$2\mu_c + \mu_d$	−0.256	0.068	−0.163	$-0.311^{+0.052}_{-0.130}$	0.035	−0.178	–
Ω_{cc}^{+}	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_s$	0.710	0.692	0.756	$0.635^{+0.012}_{-0.015}$	–	0.645	0.39 ± 0.09
Ω_{cc}^{*+}	$2\mu_c + \mu_s$	0.208	0.285	0.120	$0.139^{+0.009}_{-0.017}$	0.21	0.0475	–
Ξ_{bc}^{+}	$\frac{2}{3}\mu_b + \frac{2}{3}\mu_c - \frac{1}{3}\mu_u$	−0.532	−0.204	−0.394	$-0.475^{+0.040}_{-0.088}$	−0.369		–
Ξ_{bc}^{*+}	$\mu_b + \mu_c + \mu_u$	2.663	1.562	2.017	$2.270^{+0.27}_{-0.14}$	2.022	1.88	–
Ξ_{bc}^{0}	$\frac{2}{3}\mu_b + \frac{2}{3}\mu_c - \frac{1}{3}\mu_d$	0.626	0.354	0.469	$0.518^{+0.048}_{-0.020}$	0.48		–
Ξ_{bc}^{*0}	$\mu_b + \mu_c + \mu_d$	−0.776	−0.372	−0.558	$-0.712^{+0.059}_{-0.133}$	−0.508	−0.534	–
Ω_{bc}^{0}	$\frac{2}{3}\mu_b + \frac{2}{3}\mu_c - \frac{1}{3}\mu_s$	0.457	0.439	0.389	$0.368^{+0.010}_{-0.011}$	0.407		–
Ω_{bc}^{*0}	$\mu_b + \mu_c + \mu_s$	−0.258	−0.181	−0.310	$-0.261^{+0.015}_{-0.021}$	−0.309	−0.329	–
Ξ_{bb}^{0}	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_u$	−0.893	−0.663	−0.650	$-0.742^{+0.044}_{-0.091}$	−0.63	−0.581	0.51 ± 0.09
Ξ_{bb}^{*0}	$2\mu_b + \mu_u$	2.302	−1.607	1.559	$1.870^{+0.27}_{-0.13}$	1.507	1.40	–
Ξ_{bb}^{-}	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_d$	0.316	0.196	0.188	$0.251^{+0.045}_{-0.021}$	0.215	0.171	0.28 ± 0.04
Ξ_{bb}^{*-}	$2\mu_b + \mu_d$	−1.324	−1.737	−0.941	$-1.110^{+0.06}_{-0.14}$	−1.029	−0.880	–
Ω_{bb}^{-}	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_s$	0.133	0.108	0.107	$0.101^{+0.007}_{-0.007}$	0.138	0.112	0.42 ± 0.05
Ω_{bb}^{*-}	$2\mu_b + \mu_s$	−0.782	−1.239	−0.702	$-0.662^{+0.022}_{-0.024}$	−0.805	−0.697	–
Ω_{ccc}^{*++}	$3\mu_c$	1.261	–	1.189	–	1.16	0.989	–
Ω_{ccb}^{+}	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_b$	0.618	–	0.502	–	0.522	0.455	–
Ω_{ccb}^{*+}	$\mu_b + 2\mu_c$	0.831	–	0.651	–	0.703	0.594	–
Ω_{cbb}^{0}	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_c$	−0.24	–	−0.203	–	−0.2	−0.187	–
Ω_{cbb}^{*0}	$2\mu_b + \mu_c$	0.329	–	0.216	–	0.225	0.204	–
Ω_{bbb}^{*-}	$3\mu_b$	−0.198	–	−0.195	–	−0.198	−0.178	–

3.4 Radiative decays

The radiative decay width can be expressed in terms of transition magnetic moment (in nuclear magneton μ_N) as [214]

$$\Gamma_{B^* \to B\gamma} = \frac{\omega^3}{4\pi} \frac{2}{2J+1} \frac{e^2}{m_p^2} \mu^2_{B^* \to B\gamma} \tag{3.12}$$

where, m_p is the mass of proton, μ is the transition magnetic moment that can be written in terms of magnetic moment of constituent quark of final and initial state of baryons as $\mu_{B^* \to B\gamma} = \langle B | \hat{\mu}_{B^*z} | B^* \rangle$. Our results for the transition magnetic moment and radiative decay widths are tabulated in Tab. 3.4 and 3.5 in comparison with other theoretical predictions such as Bag models, chiral perturbation theory and different quark model results.

Table 3.4: Transition magnetic moment in μ_N

Transition	$\mu_{B' \to B\gamma}$ [184]	Present	χPT [184]	Bag [190]	exBag [191]	χCQM [193]	NRQM [199]
$\Xi_{cc}^{*++} \to \Xi_{cc}^{++}$	$\frac{2\sqrt{2}}{3}(\mu_u - \mu_c)$	1.563	-2.35	-0.787	-1.27	1.33	1.35
$\Xi_{cc}^{*+} \to \Xi_{cc}^{+}$	$\frac{2\sqrt{2}}{3}(\mu_d - \mu_c)$	-1.295	1.55	0.945	1.07	-1.41	1.06
$\Omega_{cc}^{*+} \to \Omega_{cc}^{+}$	$\frac{2\sqrt{2}}{3}(\mu_s - \mu_c)$	-0.897	1.54	0.789	0.869	-0.89	0.88
$\Xi_{bc}^{*+} \to \Xi_{bc}^{+}$	$\frac{\sqrt{2}}{3}(\mu_c + \mu_b - 2\mu_u)$	-2.010	-2.56	0.695	1.12	–	–
$\Xi_{bc}^{*0} \to \Xi_{bc}^{0}$	$\frac{\sqrt{2}}{3}(\mu_c + \mu_b - 2\mu_d)$	1.249	1.34	-0.747	-0.919	–	–
$\Omega_{bc}^{*0} \to \Omega_{bc}^{0}$	$\frac{\sqrt{2}}{3}(\mu_c + \mu_b - 2\mu_s)$	0.769	1.33	-0.624	-0.748	–	–
$\Xi_{bb}^{*0} \to \Xi_{bb}^{0}$	$\frac{2\sqrt{2}}{3}(\mu_b - \mu_u)$	-4.631	-2.77	-1.039	-1.45	–	–
$\Xi_{bb}^{*-} \to \Xi_{bb}^{-}$	$\frac{2\sqrt{2}}{3}(\mu_b - \mu_d)$	2.199	1.13	0.428	0.643	–	–
$\Omega_{bb}^{*-} \to \Omega_{bb}^{-}$	$\frac{2\sqrt{2}}{3}(\mu_b - \mu_s)$	1.174	1.12	-0.624	0.478	–	–

Table 3.5: Radiative decay width (in keV)

Transition	Present	χPT [184]	Bag [190]	exBag [191]	χQM [215]	RQM [181]	RQM [195]
$\Xi_{cc}^{*++} \to \Xi_{cc}^{++}$	18.545	22	1.43	2.79	16.7	7.21	23.46
$\Xi_{cc}^{*+} \to \Xi_{cc}^{+}$	12.145	9.57	2.08	2.17	14.6	3.90	28.79
$\Omega_{cc}^{*+} \to \Omega_{cc}^{+}$	0.678	9.45	0.949	1.60	6.93	0.82	2.11
$\Xi_{bc}^{*+} \to \Xi_{bc}^{+}$	6.042	26.2	0.533	1.31	–	–	0.49
$\Xi_{bc}^{*0} \to \Xi_{bc}^{0}$	2.22	7.19	0.612	0.876	–	–	0.24
$\Omega_{bc}^{*0} \to \Omega_{bc}^{0}$	0.087	7.08	0.239	0.637	–	–	0.12
$\Xi_{bb}^{*0} \to \Xi_{bb}^{0}$	1.233	31.1	0.126	0.137	1.19	0.98	0.31
$\Xi_{bb}^{*-} \to \Xi_{bb}^{-}$	0.265	5.17	0.022	0.0268	0.24	0.21	0.0587
$\Omega_{bb}^{*-} \to \Omega_{bb}^{-}$	0.008	5.08	0.011	0.0148	0.08	0.04	0.0226

3.5 Results and Discussion

After determining all the required model parameters, we present our numerical results. In Tab. 3.2, we present our results for the masses of doubly heavy baryons. It is observed that our result for Ξ_{cc}^{++} matches perfectly with the LHCb results [177].

In Tab. 3.2 we also compare our results with the other theoretical approaches such as hypercentral model [202–204], nonrelativistic quark model [197] chromomagnetic model [196] that are based on three quark picture of baryons. We also compare with the quark-diquark models such as relativistic quark model [91,181]. In NRQM [197], the authors have used the NRQM Hamilton and the wave function chosen to be on the basis of Harmonic Oscillator wave function. In hypercentral model [202–204], the authors have computed the mass spectra by solving the Schrödinger equation for the hypercentral Cornell potential. Our results are in very good agrement with the RQM [91] for Ξ_{QQ} baryons. For Ω_{QQ} baryons, our results are nearly $40-50$ MeV lower. This is may be because of the different methodology as the authors of Ref. [91] has used the relativistic treatment for the light quarks where as we have treated the all systems as nonrelativistically. Our results are also in good accordance with the chromagnetic model [196], in which the authors have used the effect of color interaction in chromomagnetic model. But for triply heavy baryons, our results are systematically lower than others. We also compare our results with the LQCD data [24] and we found that our results for doubly heavy baryons match well but for triply heavy baryons, our results underestimate LQCD data.

In Tab. 3.3 we present our results of magnetic moment of doubly heavy baryons using the spin flavor wave function of the respective baryons. Note that we have not introduced any additional parameter to compute the magnetic moments of spin 1/2 and 3/2 baryons. Our results are in good agreement with the hypercentral model [201] and also nonrelativistic quark models [198] and [199, 200]. For triply heavy baryons also our predictions are matching well with the NRQM [199, 200].

Next, we compute the radiative decays of doubly heavy baryons and tabulate in Tab. 3.5. We consider here the transition from spin $3/2^+ \rightarrow 1/2^+$ only. The required transition magnetic moments are presented in Tab. 3.4. Still the radiative decays of doubly heavy baryons are not reported in any experimental facility but theoretical results are available in the literature. We compare our findings with the theoretical approaches such as RQM [194] and [181]. We also compare with the results from χQM [215], χPT [184] and Bag model [190] predictions. There are wide range of results predicted in theory. Our results for the radiative decay widths of Ξ_{cc} baryon is very close to those obtained using χPT [184] and χQM [215]. For Ω_{cc} baryons, our results are near to the Bag model [190] and RQM [181]. For Ξ_{bc} and Ω_{bc} baryons, our results are higher than Bag model [190] and RQM [194] where as it is lower than the χPT [184]. For Ξ_{bb} baryons, our results are very nearer to the χQM [215] and RQM [181]. For Ω_{bb}, our result underestimate with χQM [215] by one order. But as discussed earlier, there are a wide range of radiative decay widths available in the literature and also no experimental as well as LQCD results are available, our results might be interesting for the point of view of future experiments.