Chapter 4

Study of Exotic States as Dimesonic Molecules

4.1 Introduction

$Z_c^{\pm}(3900)$ is the charged charmonium-like state observed first time by BESIII [216] and then Belle [217] collaboration in the channel $e^+e^- \to \pi^+\pi^- J/\psi$. This state was also confirmed by CLEO collaboration [218]. BESIII have also determined parity to be $J^P = 1^+$ using the partial wave analysis [219]. Also the charged bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$ observed in Belle Collaboration [220] and later also confirmed by them [221, 222]. These states are also identified with the parity to be $J^P = 1^+$. These states (Z_c and Z_b) don't fit into the conventional quark model and their minimal quark content to be $c\bar{c}d\bar{u}$ or $b\bar{b}d\bar{u}/b\bar{b}u\bar{d}$ which are beyond the conventional $q\bar{q}$ or qqq model. These states have masses nearer to threshold of two heavy flavor mesons and gained lot of attentions for both experimentalists and theoreticians world wide. There are different ways in which these states are studied theoretically based on tetraquark states [223–231], hadro-quarkonium state [232–234] in which the exotic states are considered as coupling to the light and heavy quarkonium state to intermediate open-flavor mesons. These states are also studied on the basis of hadronic composite molecular pictures [235–247]. These states are studied in the different approaches such as chiral quark model [248], relativistic quark model [249], effective field theory [250, 251], holographic QCD [252] and QCD sum rules [253]. The comprehensive reviews on the status of these exotic states are given in the literature [246, 254, 255].

In this chapter, we restrict our study to the exotic states namely Z_c^+, Z_b^+ and

Z_b' considering them as a hadronic composite molecule of $D^+\bar{D}^*$, $B\bar{B}^*$ and $B^*\bar{B}^*$ respectively as their masses are below these threshold. The bound state masses are computed by sloving the Schrödinger equation nemerically for the generalized Woods - Saxon potantial. We also compute the two body strong decays of these states using the phenomenological Lagrangian mechanism. We compare our findings with the available experimental data and other theoretical predictions. We have presented this work in the XXII DAE High energy Physics Symposium held at University of Delhi during December 12-16, 2016 and published in a conference proceeding [256].

4.2 Methodology

There are various approaches available in the literature for studying these exotic states but since their masses are nearer to the $D^*\bar{D}$, $B^*\bar{B}$ and $B^*\bar{B}^*$ threshold, these states are considered as a hadronic composite molecules of these mesons. We consider here the potential of the form modified Woods Saxon potential for the confinement of the exotic state along with the Coulomb replusive term. The potential equation is given by [257, 258],

$$V(r) = \frac{V_0}{1+e^{\frac{r-R_0}{a}}} + \frac{Ce^{\frac{r-R_0}{a}}}{\left(1+e^{\frac{r-R_0}{a}}\right)^2} - \frac{b}{r} \tag{4.1}$$

where, V_0 is the potential strength, b is the strength of Coulomb interaction. R_0 is the radius of the molecule. a is the diffuseness of the surface [257], C is the depth of the potential which range from $0 < C < 150$ MeV [258], where $C = 0$ MeV corresponds to the standard Woods-Saxon Potential. The plot of the potential is also shown in the Fig. 4.1 with the variation in the depth of the potential C.

Table 4.1: Fitted parameters for computing the masses

Potential Strength V_0	15 MeV
Radius of the molecule R_0	1.75 fm
Strength of coulomb interaction b	0.08
Diffuseness of the potential	-0.51 fm
Potential Depth Range	$0 < C < 150$ MeV [258]
Size Parameter Λ:	500 MeV

For computing the bound state masses of the exotic states the Schrödinger equation is sloved nemerically for the potential Eq. (4.1) using the *Mathematica* notebook utilizing the Runge–Kutta method [137] and the binding energy is obtained. The

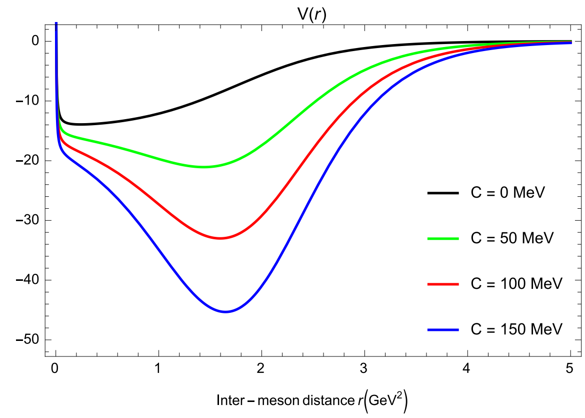

Figure 4.1: Wood-Saxon potential with variation in potential depth

masses of the dimesonic states are obtained using constituent mesons and binding energy

$$M_{12} = M_1 + M_2 - BE. \tag{4.2}$$

The model parameters are fitted to obtain the masses of the respective exotic states.

Table 4.2: Masses of $Z_c^+(D^+\bar{D}^*)$, $Z_b^+(B\bar{B}^*)$ and $Z_b'(B^*\bar{B}^*)$ molecular states (in MeV) with the variation in potential depth C (in MeV)

C	$D^+\bar{D}^*$		$B\bar{B}^*$		$B^*\bar{B}^*$	
	Binding Energy	Mass	Binding Energy	Mass	Binding Energy	Mass
0	11.82	3864.74	5.58	10598.9	5.54	10644.9
50	11.96	3864.61	7.05	10597.4	7.01	10643.4
100	12.07	3864.5	8.04	10596.4	8.02	10642.4
150	12.15	3864.42	8.72	10595.7	8.70	10641.7
PDG [1]		3883.9±4.5		10607.2±2.0		10652.2 ± 1.5

4.3 Strong decay width

The strong two body decay widths are computed using the phenomenological Lagrangian mechanism given in Ref. [235, 236]. The the Lagrangian corresponding to the coupling of Z_c and Z_b states to its constituent can be written as [235, 236],

$$\begin{aligned}
\mathcal{L}_{Z_c}(x) &= \frac{g_{Z_c}}{\sqrt{2}} M_{Z_c} Z_c^{\mu}(x) \int d^4y \Phi_{Z_c}(y^2) \left\{ D\left(x+\frac{y}{2}\right) \bar{D}^*_{\mu}\left(x-\frac{y}{2}\right) + D^*_{\mu}\left(x+\frac{y}{2}\right) \bar{D}\left(x-\frac{y}{2}\right) \right\} \\
\mathcal{L}_{Z_b'}(x) &= \frac{g_{Z_b'}}{\sqrt{2}} i\epsilon_{\mu\nu\alpha\beta} \int d^4y \Phi_{Z_b'}(y^2) B^{*\alpha}\left(x+\frac{y}{2}\right) \bar{B}^{*\beta}\left(x-\frac{y}{2}\right) \\
\mathcal{L}_{Z_b}(x) &= \frac{g_{Z_b}}{\sqrt{2}} M_{Z_b} Z_b^{\mu}(x) \int d^4y \Phi_{Z_b}(y^2) \left\{ B\left(x+\frac{y}{2}\right) \bar{B}^*_{\mu}\left(x-\frac{y}{2}\right) + B^*_{\mu}\left(x+\frac{y}{2}\right) \bar{B}\left(x-\frac{y}{2}\right) \right\}
\end{aligned} \tag{4.3}$$

where y is the relative Jacobi coordinate, g_{Z_c}, g_{Z_b} and $g_{Z_b'}$ are the dimensional coupling constants of Z_c, Z_b and Z_b' to the molecular $D^+\bar{D}^*$, $B\bar{B}^*$ and $B^*\bar{B}^*$ components, respectively. $\Phi_{Z_c}(y^2)$, $\Phi_{Z_b'}(y^2)$ and $\Phi_{Z_b'}(y^2)$ are the correlation functions, which describes the distributions of the constituent mesons in the bound states.

The strong two body decay widths are given by [235, 236]

$$\begin{aligned}
\Gamma_{Z_c^+\to\Psi(ns)\pi^+} &\simeq \frac{g^2_{Z_c\Psi(ns)\pi}}{96\pi M^3_{Z_c}}\lambda^{3/2}(M^2_{Z_c}, M^2_{\psi(ns)}, M^2_\pi)\left(1+\frac{M^2_{\psi(ns)}}{2M^2_{Z_c}}\right)\\
\Gamma_{Z_b^+\to\Upsilon(ns)\pi} &\simeq \frac{g^2_{Z_b\Upsilon(ns)\pi}}{16\pi M_{Z_b}}\lambda^{1/2}(M^2_{Z_b}, M^2_{\Upsilon(ns)}, M^2_\pi)\\
\Gamma_{Z_b'^+\to\Upsilon(ns)\pi} &\simeq \frac{g^2_{Z_b\Upsilon(ns)\pi}}{16\pi M_{Z_b'}}\lambda^{1/2}(M^2_{Z_b'}, M^2_{\Upsilon(ns)}, M^2_\pi)
\end{aligned} \tag{4.4}$$

where $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2zx$ is the Källen function, $g_{Z_c\Psi(ns)\pi}$, $g_{Z_b\Upsilon(ns)\pi}$ and $g_{Z_b'\Upsilon(ns)\pi}$ are the decay coupling constants, expressed as [235, 236]

$$\begin{aligned}
g_{Z_c\Psi(ns)\pi} &= 8g_{Z_c}\frac{g_F g_H}{F_\pi M_J}J_{Z_c}M_{Z_c}\\
g_{Z_b\Upsilon(ns)\pi} &= g_{Z_b}g_{BB^*\Upsilon(ns)\pi}J_{Z_b}\\
g_{Z_b'\Upsilon(ns)\pi} &= g_{Z_b'}g_{B^*B^*\Upsilon(ns)\pi}M_{Z_b'}J_{Z_b'}
\end{aligned} \tag{4.5}$$

with $g's$ and $J's$ are the coupling constants and loop integrals respectively given by

$$\begin{aligned}
\frac{1}{g^2_{Z_c}} &= \frac{M^2_{Z_c}}{32\pi^2\Lambda^2}\int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_1^3}(\alpha_{12}+2\alpha_1\alpha_2)\left(1+\frac{\Lambda^2}{2M^2_{D^*}\Delta_1}\right)\\
&\quad\times\exp\left(-\frac{M^2_{D^*}\alpha_1+M^2_D\alpha_2}{\Lambda^2}+\frac{M^2_{Z_c}}{2\Lambda^2}\frac{\alpha_{12}+2\alpha_1\alpha_2}{\Delta_1}\right)\\
\frac{1}{g^2_{Z_b}} &= \frac{M^2_{Z_b}}{32\pi^2\Lambda^2}\int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_1^3}(\alpha_{12}+2\alpha_1\alpha_2)\left(1+\frac{\Lambda^2}{2M^2_{B^*}\Delta_1}\right)\\
&\quad\times\exp\left(-\frac{M^2_{B^*}\alpha_1+M^2_B\alpha_2}{\Lambda^2}+\frac{M^2_{Z_b}}{2\Lambda^2}\frac{\alpha_{12}+2\alpha_1\alpha_2}{\Delta_1}\right)\\
\frac{1}{g^2_{Z_b'}} &= \frac{M^2_{Z_b}}{32\pi^2\Lambda^2}\int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_1^2}\left(\frac{\Lambda^2}{M^2_{Z_b'}}+\frac{\alpha_{12}+2\alpha_1\alpha_2}{2\Delta_1}\right)\left(1+\frac{\Lambda^2}{M^2_{B^*}\Delta_1}\right)\\
&\quad\times\exp\left(-\frac{M^2_{B^*}\alpha_{12}}{\Lambda^2}+\frac{M^2_{Z_b'}}{2\Lambda^2}\frac{\alpha_{12}+2\alpha_1\alpha_2}{\Delta_1}\right)
\end{aligned} \tag{4.6}$$

and

$$\begin{aligned}
J_{Z_c} &= \frac{1}{8\pi^2}\int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_2^2}\left(1+\frac{\Lambda^2}{2M^2_{D^*}\Delta_2}\right)\exp\left(-\frac{M^2_{D^*}\alpha_1+M^2_D\alpha_2}{\Lambda^2}+\frac{M^2_{Z_c}}{4\Lambda^2}\frac{\alpha_{12}+4\alpha_1\alpha_2}{\Delta_2}\right)\\
J_{Z_b} &= \frac{1}{8\pi^2}\int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_2^2}\left(1+\frac{\Lambda^2}{2M^2_{B^*}\Delta_2}\right)\exp\left(-\frac{M^2_{B^*}\alpha_1+M^2_B\alpha_2}{\Lambda^2}+\frac{M^2_{Z_b}}{4\Lambda^2}\frac{\alpha_{12}+4\alpha_1\alpha_2}{\Delta_2}\right)\\
J_{Z_b'} &= \frac{1}{8\pi^2}\int_0^\infty \frac{d\alpha_1 d\alpha_2}{\Delta_2^2}\left(1+\frac{\Lambda^2}{2M^2_{B^*}\Delta_2}\right)\exp\left(-\frac{M^2_{B^*}\alpha 12}{\Lambda^2}+\frac{M^2_{Z_b'}\alpha_{12}}{4\Lambda^2}\frac{\alpha_{12}+4\alpha_1\alpha_2}{\Delta_2}\right)
\end{aligned}$$

with $\Delta_1 = 2 + \alpha_{12}$, $\Delta_2 = 1 + \alpha_{12}$, $\alpha_{12} = \alpha_1 + \alpha_2$ and Λ is the size parameter which characterizes the effective size of the hadrons. For computation we take $\Lambda = 0.5$ GeV [235, 236].

The strong two body decay widths are computed using Eq. (4.4) and the results are tabulated in Tab. 4.3.

Table 4.3: Hadronic decay widths of Z_c^+, Z_b^+ and Z_b' molecular states (in MeV)

Decay Mode	Decay width								
	C = 0	C = 50	C = 100	C = 50	Exp [259]	[235, 236]	[260]	[261]	[262]
$Z_c \to \psi(1s) + \pi$	11.72	11.76	11.78	11.81	–	10.43 − 23.89	12.00	3.67	
$Z_c \to \psi(2s) + \pi$	2.12	2.11	2.11	2.11	–	1.28 − 2.94	0.9749	8.24	
$Z_b \to \Upsilon(1s) + \pi$	22.84	22.93	23.00	23.06	22.9±7.3	13.3 − 30.8	19.34	–	5.9 ± 0.4
$Z_b \to \Upsilon(2s) + \pi$	26.93	26.99	27.04	27.09	21.1±4.0	15.4 − 35.7	23.54	–	–
$Z_b^{'} \to \Upsilon(1s) + \pi$	23.43	23.51	23.58	23.64	12±10±3	14.0 − 31.7	19.49	–	$9.5^{+0.7}_{-0.6}$
$Z_b^{'} \to \Upsilon(2s) + \pi$	28.77	28.84	28.90	28.95	16.4±3.6	16.9 − 39.3	25.07	–	–

4.4 Results and Discussion

In this chapter we compute the masses of exotic states considering the dimeson molecules considering interaction of type modified Woods - Saxon potential. We have also analysed the nature of potential with the depth of the potential. From the potential plot Fig. 4.1, it is clear that as the depth of the potential increases, the binding energy increases. Solving Schrödinger equation numerically, we obtain the binding energy of the exotic states and the bound state masses are obtained. The bound state masses of the exotic states are in good agreement with PDG data [1]. We have also computed the two body strong decay widths of these states in interaction Lagrangian mechanism from Ref. [235, 236] and compare with the experiments. Our predictions of decay widths are in good agreement with the experimental data [1]. We also compare our findings with the other theoretical approaches such as covariant quark model [262], light front model [261] and potential model [260]. It is observed that our results are also matching well with the theoretical approaches.