Chapter 5

Strength Function (or Local Density of States (LDOS)) and its Width

5.1 Introduction

The energy spectra (or eigenvalues) and wave functions of isolated finite interacting many-body quantum systems carry signatures of quantum chaos. Various spectral statistics like nearest neighbor spacing distribution (NNSD) and its higher orders, ratio of spacings and their higher orders, ordered level spacings, etc. are popular measures of chaos, which use the energy spectra of such systems. Analyzing the structure of wave functions of these systems, can give more information about their chaotic dynamics. Measures like number of principle components (NPC), localization length (l_H - defined by information entropy S^{info}), fidelity decay (or survival probability), etc. which use the wavefunctions of such systems can give more information about their chaotic dynamics. In the chapters so far, we have analyzed the spectral properties of isolated finite interacting many particle systems. The spectral properties given by the ordered level spacing distribution and the higher order ratio of spacings distribution are discussed in chapter 3. However if we know the structure of eigenfunctions in addition to the spectral properties of these systems, we can study various other interesting quantities like information entropy (S^{info}), number of principal components (NPC), occupational entropy (S^{occ}), etc. These quantities also quantify chaos in these systems. In this chapter as well as in the upcoming chapter 6, we will analyze the wavefunction properties of these systems. Now when we talk about the wavefunction properties, it becomes very important to analyze the strength functions (also known as local density of states). They play a crucial role in the analysis of wavefunction properties as they give information about how a particular basis state spreads over the eigenstates. The importance of studying strength functions has increased enormously after they have become experimentally measurable [8]. These strength functions are known to exhibit delta function to Breit Wigner (BW) to Gaussian transition in their shape, with increase in the strength of two body inter-particle interaction in fermion and boson systems described by EGOE(1+2) [24, 150–152] and BEGOE(1+2) [63] respectively. Now with the recently known importance of higher body rank k in interacting particle systems, the strength functions for higher k have been studied. It is found that as k increases and reaches $k = m$, the strength functions exhibit a transition from Gaussian to semi-circle for strong enough interaction strength (i.e. in the thermodynamic region) [30, 79].

In this chapter, we analyze the strength function and its width for both fermions and bosons using EGOE(1+k) and BEGOE(1+k) respectively. Firstly in section 5.2, we define the strength functions and present the strength function results as body rank k of the interaction increases in the strong coupling limit (i.e. $\lambda > \lambda_t$). Further, in section 5.3, a complete analytical description of the variance of the strength function in terms of the correlation coefficient ζ, as a function of λ and k is derived. Also, in section 5.4 the (m, N, k) dependence of marker λ_t, defining thermalization region is derived analytically from the analytical expression of ζ. Also, the lower order moments of the strength functions are studied in section 5.5. Finally section 5.6 gives the concluding remarks of this chapter. The work on bosons presented in this chapter is based on [30] and that of fermions is under preparation to be published.

5.2 Strength Function

When m-particles are distributed in N sp states the m-particle Hamiltonian H is generated with basis state $|\kappa\rangle$. We take the basis states $|\kappa\rangle$ to be eigenstates of $h(1)$. The diagonal matrix elements of H are denoted as energy ξ_κ, so that $\xi_\kappa = \langle\kappa|H|\kappa\rangle$. By diagonalization of the full Hamiltonian H, we obtain the eigenstates $|E_i\rangle$ and eigenvalues E_i,

$$H|E_i\rangle = E_i|E_i\rangle \tag{5.1}$$

The eigenstates $|E_i\rangle$ can be expressed as a linear combination of the basis states $|\kappa\rangle$,

$$|E_i\rangle = \sum_i C^i_\kappa\,|\kappa\rangle \tag{5.2}$$

Also, the basis states $|\kappa\rangle$ can be expressed as a linear combination of the eigenstates $|E_i\rangle$ with the same transformation coefficients C_κ^i,

$$|\kappa\rangle = \sum_i C_\kappa^i \, |E_i\rangle \tag{5.3}$$

Here the coefficients C_κ^i are real due to the time reversal invariance. With this we can define the strength function corresponding to a particular basis state $|\kappa\rangle$ as,

$$F_{\xi_\kappa}(E) = \sum_i |C_\kappa^i|^2 \; \delta(E - E_i). \tag{5.4}$$

The basis state $|\kappa\rangle$ starts spreading into the eigenstates $|E_i\rangle$ as soon as the inter-particle interaction is switched on. The strength function shows how this basis state spreads. This gives us an idea about how much of the m-body space is occupied by the eigenstate. Since the strength functions are experimentally measurable quantities they are of great importance [8].

The shape of strength function demonstrates various transitions as we switch on and then gradually increase the two-body inter particle interaction strength λ. The chaos markers λ_δ, λ_c and λ_F show these transitions. Initially when there is no inter particle interaction i.e. $\lambda = 0$, only the one-body interaction is present in the system. In this case the basis states have not yet started spreading into the eigenstates of the system i.e. the basis states coincide with the eigensates and they are fully localized. In this case the shape of the strength function is represented by the delta functions. Now as we switch on the inter particle interaction i.e. increase λ, the states start mixing due to this interaction. As a result the basis states start to spread over the eigenstates of the system i.e. they start to delocalize. As a result the strength functions make a transition from delta function to BW form at $\lambda = \lambda_\delta$. This region from $\lambda = 0$ to $\lambda = \lambda_\delta$ can be called the δ domain. With further increase in λ, the strength function continues to exhibit BW form even after crossing $\lambda = \lambda_c$. As we further increase λ, at point $\lambda = \lambda_F$, the strength functions will take up the Gaussian form. This region from $\lambda = \lambda_\delta$ to $\lambda = \lambda_F$ is called the BW domain. If we increase λ even beyond $\lambda = \lambda_F$, after a point $\lambda = \lambda_t$ the system will thermalize i.e. the different basis dependent quantities like temperature, entropy, specific heat, etc. will behave alike irrespective of their basis. The region $\lambda \sim \lambda_t >> \lambda_F$ is called the thermodynamic region [34]. With further increase in λ, the eigenstates become fully delocalized (or chaotic) and in this case the strength functions are represented by the Gaussian form. Hence the strength functions exhibit a delta to BW to Gaussian transition with increase in inter particle interaction λ and this is a generic feature of EE. Now when we also increase the body rank k from 2 to m, these strength functions further exhibit a Gaussian to semi-circle transition in the thermodynamic domain.

In order to study the strength functions in EGOE($1{+}k$) and BEGOE($1{+}k$), we choose the value of k-body interaction strength such that $\lambda > \lambda_t$, i.e. the system exists in the region of thermalization [10, 27]. Now to construct ensemble averaged strength functions, the eigenvalues E_i and κ-energies ξ_κ of the system are first scaled to have zero centroid and unit variance. For each member, all $|C_\kappa^i|^2$ are summed over the basis states κ with energy ξ in the energy window $\xi \pm \Delta$. Then, the ensemble averaged $F_\xi(E)$ vs. E are constructed as histograms by applying the normalization condition $\int_{s(q)} F_\xi(E)\, dE = 1$. The numerical results represented by histograms are compared with the conditional q-normal density function given by,

$$F_\xi(E) = f_{CqN}(x = E|y = \xi; \zeta, q). \tag{5.5}$$

5.2.1 Results for Fermion Systems

Now in order to study these strength functions, we consider two examples:(i) a 500 member EGOE($1 + k$) ensemble with $m = 6$ fermions in $N = 12$ sp states and (ii) a 20 member EGOE(1+ k) ensemble with $m = 7$ fermions in $N = 14$ sp states. Here $\lambda = 0.5$. The strength function $F_\xi(E)$ results for both these examples are presented in Fig. 5.1 and Fig. 5.2. Results are presented for $\xi = 0, \pm 0.5, \pm 1.0, \pm 1.5, \pm 2.0$ for $k = 2, 3, 4$ and $k = m$. In these figures histograms correspond to the numerical results while the smooth black curves for each k are obtained via Eq.(5.5) using corresponding ensemble averaged ζ and q values. The dotted lines for each k value correspond to their respective centroids. For all k values, the numerical histograms are in very good agreement with the f_{CqN} curves. This agreement is obtained for both $\xi = 0$ and $\xi \neq 0$.

5.2.2 Results for Boson Systems

Now moving further we study strength function in BEGOE($1 + k$). Histograms in Figs. 5.3, 5.4 and 5.5 represent ensemble averaged $F_\xi(E)$ results using a 250 member BEGOE(1+ k) ensemble with a system of $m = 10$ bosons in $N = 5$ sp states for various body rank k values. In this analysis we take $\lambda = 0.5$. The $F_\xi(E)$ results for $\xi = 0$ are given in Fig. 5.3. Going further, we also obtain $F_\xi(E)$ results for $\xi \neq 0$. These results are shown in Fig. 5.4 for $\xi = \pm 1.0$ and in Fig. 5.5 for $\xi = \pm 2.0$. The smooth black curves in these figures for each k are obtained via Eq.(5.5) using corresponding ensemble averaged ζ and q values. With $\lambda >> \lambda_t$, $\zeta^2 << 1/2$, the q value in Eq.(5.5) can fairly be given from chapter 4 [84]. The results in Figs. 5.3, 5.4 and 5.5 clearly show very good agreement between the numerical histograms and continuous black curves for all body rank k.

All these results for fermion and boson systems using EGOE($1+k$) and BEGOE($1+k$) tell us about nature of the strength functions which is as follows. The symmetric nature of strength functions for $\xi = 0$ and the Gaussian to semi-circle transition with increase in k is observed. The smooth form given by Eq.(5.5) interpolates this transition very well. Now as we move away from the center of the spectrum, i.e. $\xi \neq 0$, the $F_\xi(E)$ are asymmetrical about E as demonstrated earlier in [22]. Also, $F_\xi(E)$ are skewed more in the positive direction for $\xi > 0$ and skewed more in the negative direction for $\xi < 0$. From all the above numerical results which are in very good agreement with the f_{CqN} curves for $\xi = 0$ and also for $\xi \neq 0$, we can conclude that the strength functions of many-fermion and many-boson systems with random k-body interactions, follow the conditional q-normal distribution f_{CqN}. The results are also consistent with the analytical forms derived in [84].

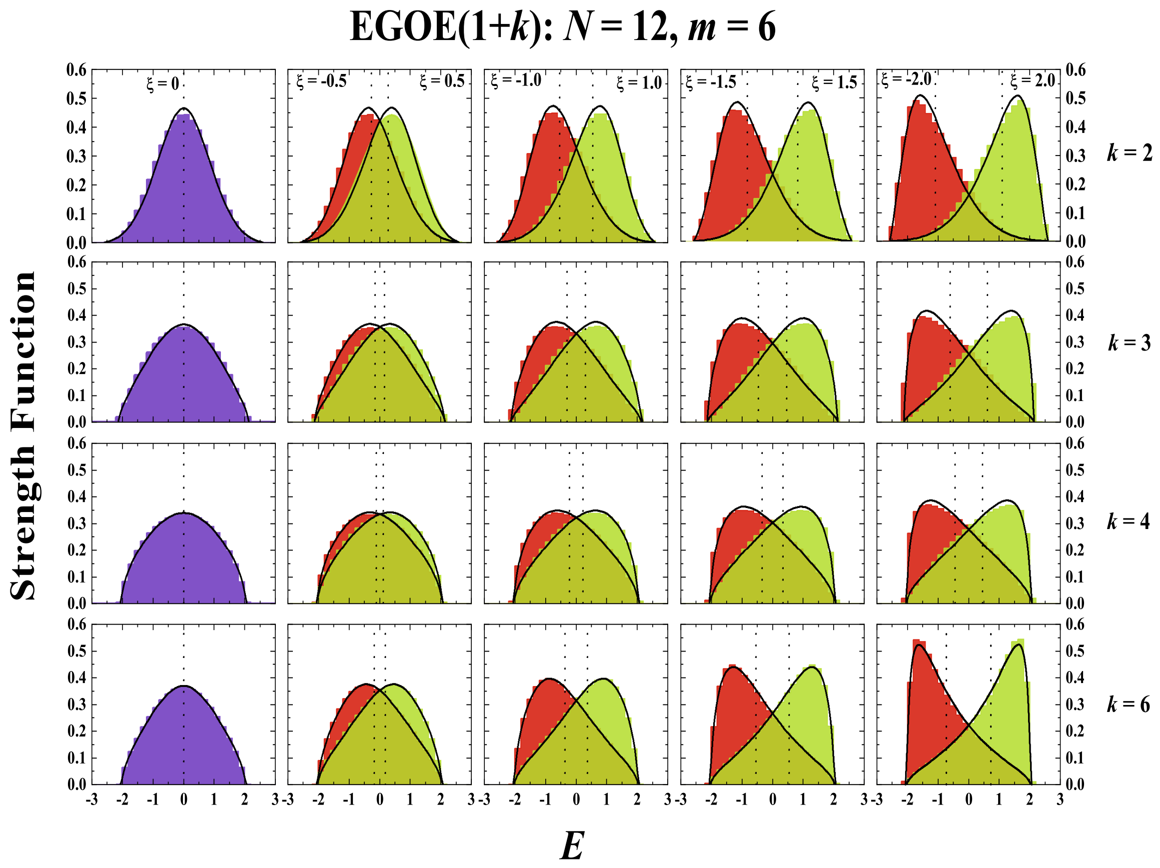

Figure 5.1: Ensemble averaged strength function as a function of normalized energy E for a 500 member EGOE(1+ k) ensemble with $m = 6$ fermions occupying $N = 12$ sp states for body rank k =2, 3, 4 and $k = m = 6$. Here $\lambda = 0.5$. Histograms correspond to strength functions for $\xi = 0, \pm0.5, \pm1.0, \pm1.5$ and ±2.0. In the plots $\int F_\xi(E)dE = 1$. The continuous black curves are due to fitting with f_{CqN} given by Eq. (5.5) with corresponding ensemble averaged q and ζ values. See text for more details.

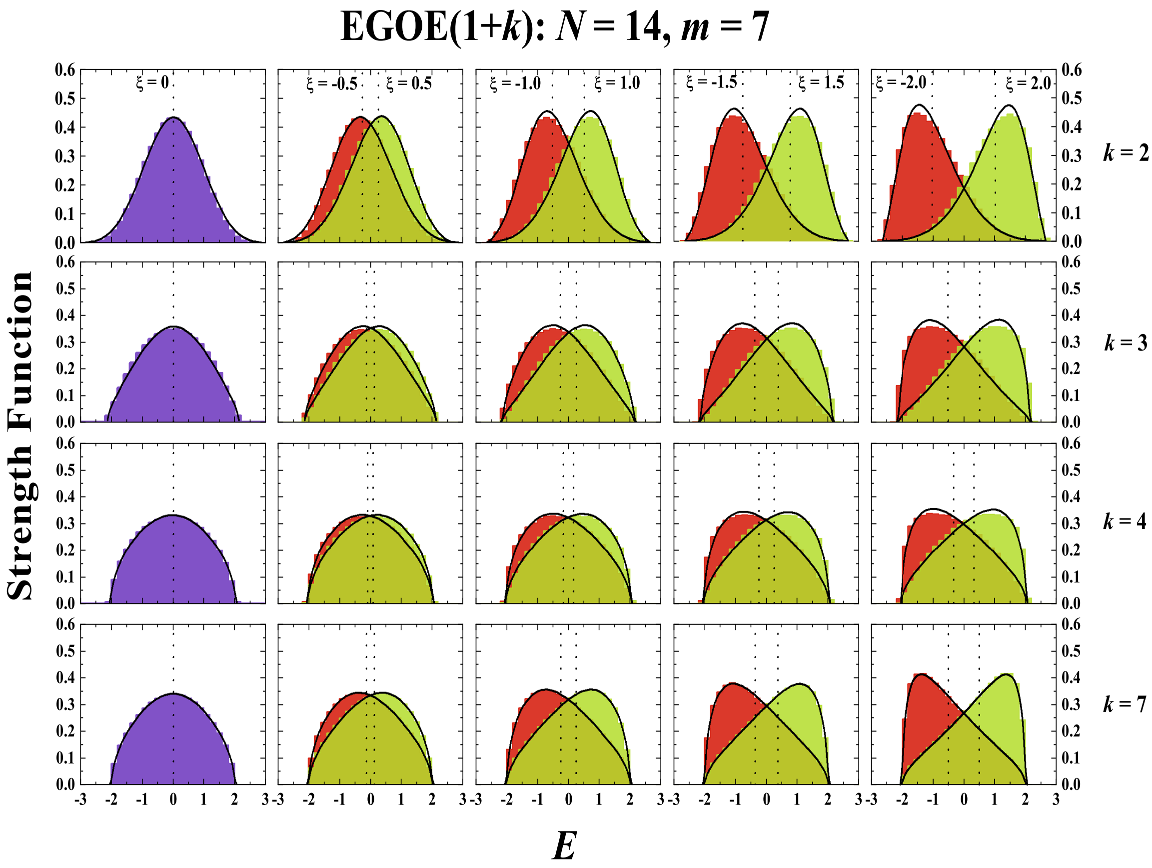

Figure 5.2: Same as Fig. 5.1 but for a 20 member EGOE(1+k) ensemble with $m = 7$ fermions occupying $N = 14$ sp states for body rank k =2, 3, 4 and $k = m = 7$.

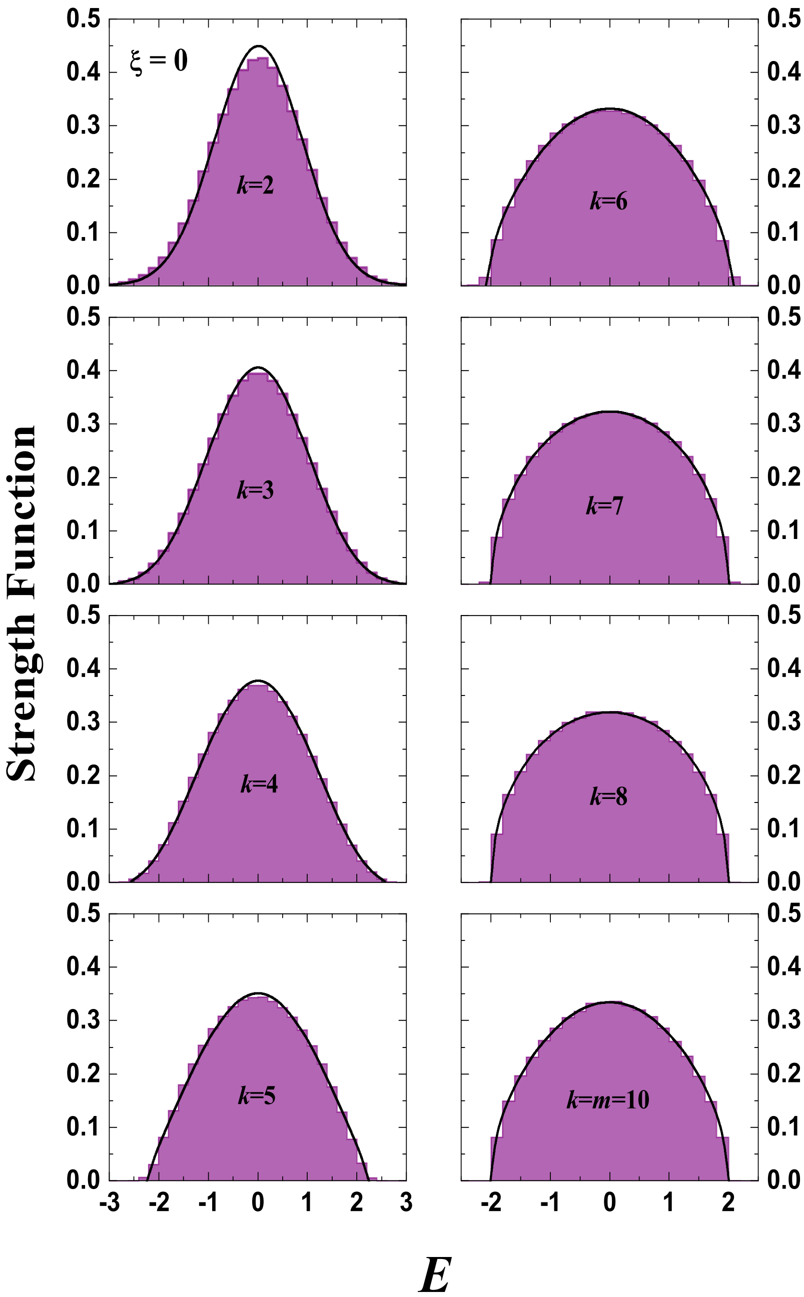

Figure 5.3: Strength function vs. normalized energy E for a system of $m = 10$ bosons in $N = 5$ sp states with $\lambda = 0.5$ for different k values in BEGOE(1+k) ensemble. An ensemble of 250 members is used for each k. Strength function plots are shown for $\xi = 0$. In the plots $\int F_{\xi}(E)dE = 1$. The smooth black curves are due to fitting with f_{CqN} given by Eq. (5.5) using q and ζ values obtained from q vs. λ results (from chapter 4) and Eq. (5.16), respectively. See text for more details.

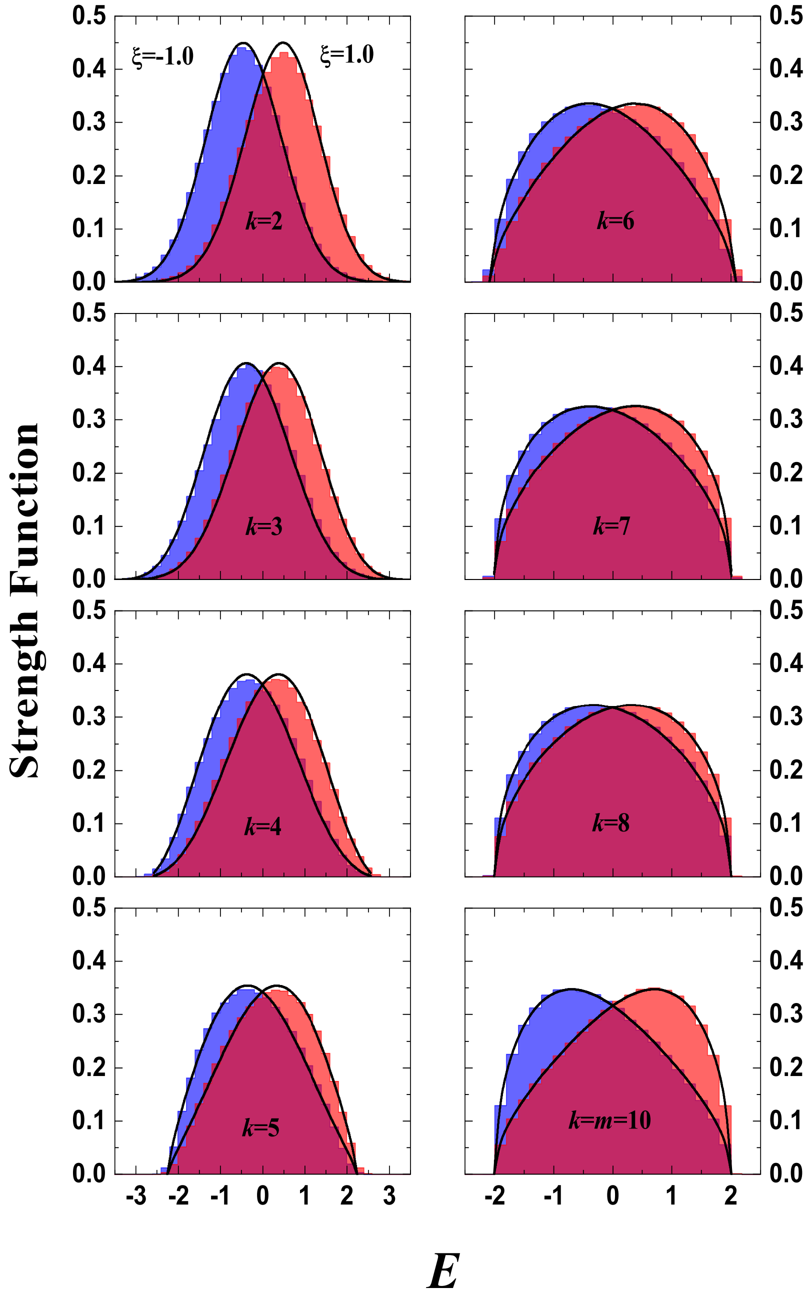

Figure 5.4: Same as Fig. 5.3 but for $\xi = -1.0$ and 1.0.

5.3 Correlation Coefficient ζ

In this section we will proceed by studying the width of the strength functions discussed in the previous section 5.2. The width of the strength functions σ_F is related to the parameter ζ by $\sigma_F^2 = 1 - \zeta^2$. Since ζ and σ_F are related, here we will discuss width of the strength functions in terms of ζ. The correlation between full Hamiltonian H and the diagonal part H_{dia} of the full Hamiltonian can be determined from the correlation

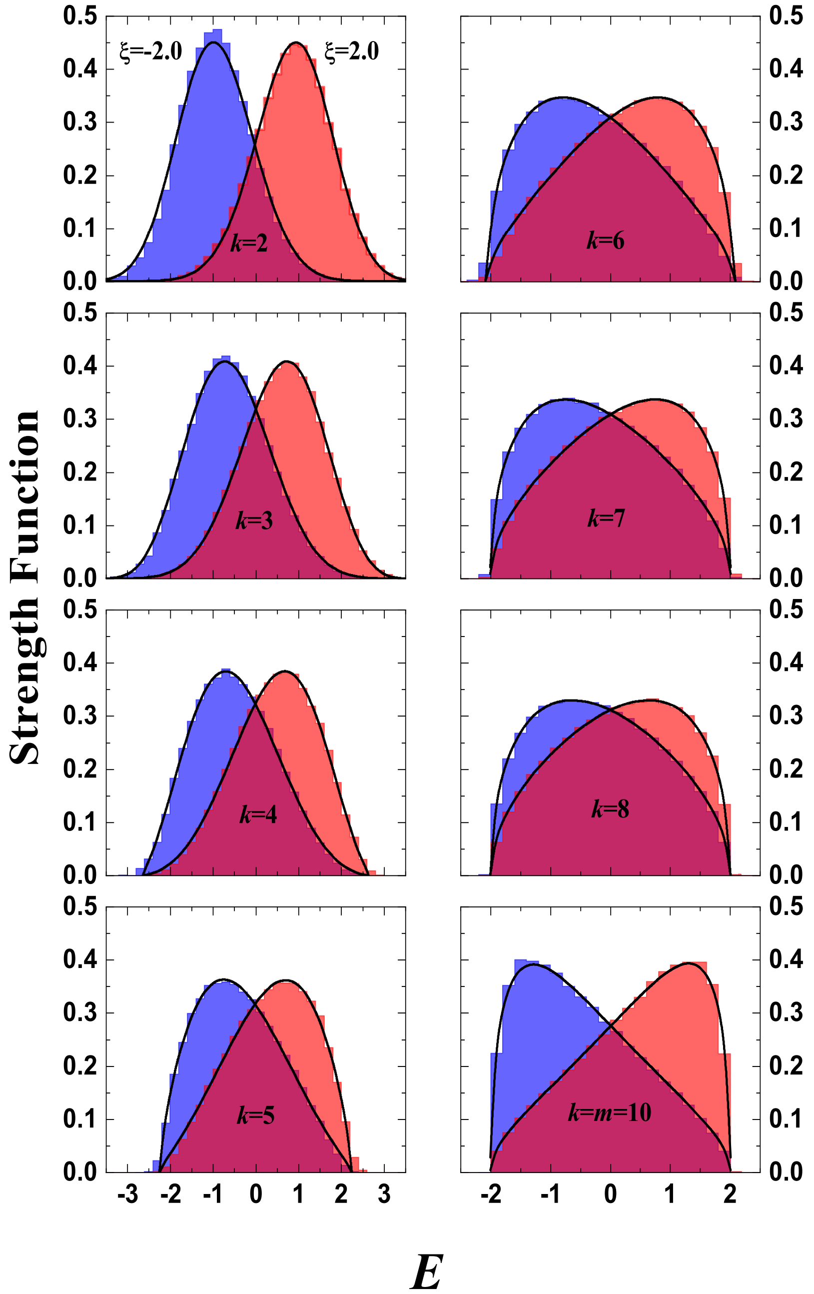

Figure 5.5: Same as Fig. 5.3 but for $\xi = -2.0$ and 2.0.

coefficient denoted by ζ which is defined as,

$$\zeta = \sqrt{1 - \frac{\sigma^2_{H_{\text{off-dia}}}}{\sigma^2_H}} = \sqrt{1 - \sigma_F^2}, \quad \sigma_F = \frac{\sigma_{H_{\text{off-dia}}}}{\sigma_H} \tag{5.6}$$

Here, σ^2_H is the variance of the eigenvalue distribution using full Hamiltonian and $\sigma^2_{H_{\text{off-dia}}}$ is the variance of the eigenvalue distribution by taking all diagonal matrix elements as zero, i.e. by considering only the off-diagonal matrix elements. Now we will derive the analytical expression of ζ for fermionic and bosonic systems modeled by EGOE(1+k) and

BEGOE(1+ k) using the method of trace propagation.

5.3.1 Formula of ζ for Fermion Systems

First let us derive the analytical expression of ζ for fermionic system. For $H = V(k)$ i.e. with all sp energies as degenerate, we have [61],

$$
\begin{aligned}
\sigma^2_{H=V(k)} &= T(N,m,k)\binom{N}{k}^{-1} \sum_{\alpha,\beta} \overline{w^2_{\alpha\beta}}\;, \\
\text{where}, T(N,m,k) &= \Lambda^0(N,m,k)\binom{N}{k}^{-1} \\
\Lambda^0(N,m,k) &= \tbinom{m}{k}\tbinom{N-m+k}{k} \\
T(N,m,k) &= \tbinom{m}{k}\tbinom{N-m+k}{k}\tbinom{N}{k}^{-1}
\end{aligned}
\tag{5.7}
$$

Here, α and β denote k-particle states. We know that in k-particle space, the H matrix is GOE. Therefore, the k-particle matrix elements $w_{\alpha\beta}$ are Gaussian random variables with zero mean and unit variance. The variance of diagonal matrix elements is $\overline{w^2_{\alpha\alpha}} = 2$ while that of off-diagonal matrix elements is $\overline{w^2_{\alpha\beta}} = 1$ for ($\alpha \neq \beta$). With this,

$$
\sigma^2_{H=V(k)} = T(N,m,k)\binom{N}{k}^{-1} \left\{2 \times \text{no-dia} + 2 \times \text{no-offdia}\right\}, \tag{5.8}
$$

For fermions, the number of independent diagonal k-body matrix elements is 'no-dia' $= \binom{N}{k}$ and that of off-diagonal is 'no-offdia' $= \binom{N}{k}\{\binom{N}{k} - 1\}$. Similarly, $\sigma_{H_{\text{off-dia}}}$ is given by removing the contribution of diagonal k-body matrix elements from the above equation. For one body $h(1)$ part of the Hamiltonian defined by the external sp energies ϵ_i we have,

$$
\sigma^2_{h(1)} = \tfrac{m(N-m)}{N(N-1)} \sum \tilde{\epsilon_i}^2. \tag{5.9}
$$

Going further, when we include the one-body part, then for the full Hamiltonian we have,

$$
\begin{aligned}
\sigma^2_H &= \sigma^2_{h(1)} + \lambda^2\, \sigma^2_{V(k)} \\
&= \tfrac{m(N-m)}{N(N-1)} \sum \tilde{\epsilon_i}^2 + \lambda^2\, \sigma^2_{V(k)}.
\end{aligned}
\tag{5.10}
$$

The analytical expression for ζ^2 can be given by,

$$
\zeta^2 = \frac{\tfrac{m(N-m)}{N(N-1)} \sum \tilde{\epsilon_i}^2 + 2\,\lambda^2\, T(N,m,k)}{\tfrac{m(N-m)}{N(N-1)} \sum \tilde{\epsilon_i}^2 + \lambda^2\, T(N,m,k)\, \left\{1 + \tbinom{N}{k}\right\}}\;. \tag{5.11}
$$

In the above equation, the contribution from the diagonal part of $V(k)$ is also included into the numerator term. Now we test the above Eq. (5.11) with numerical ensemble averaged EGOE(1+k) results. We consider two examples for this analysis: (i) a 100 member EGOE(1+k) ensemble with m = 6 fermions in N = 12 sp states and (ii) a 20 member EGOE(1+k) ensemble with m = 7 fermions in N = 14 sp states. The variation of ζ^2 as a function of k body interaction strength λ is studied for body ranks k = 2,3,4 and $k = m$ for both these examples. These ζ^2 vs. λ results are presented in Figs. 5.6 and 5.7. In all the plots in Figs. 5.6 and 5.7, the solid circles represent the numerical ensemble averaged results while the black smooth curves are analytical results obtained using Eq. (5.11). The analytical results are obtained using fixed sp energies $\epsilon_i = i + 1/i$. A very good agreement between the numerical and analytical results for all k values can be observed for both these examples.

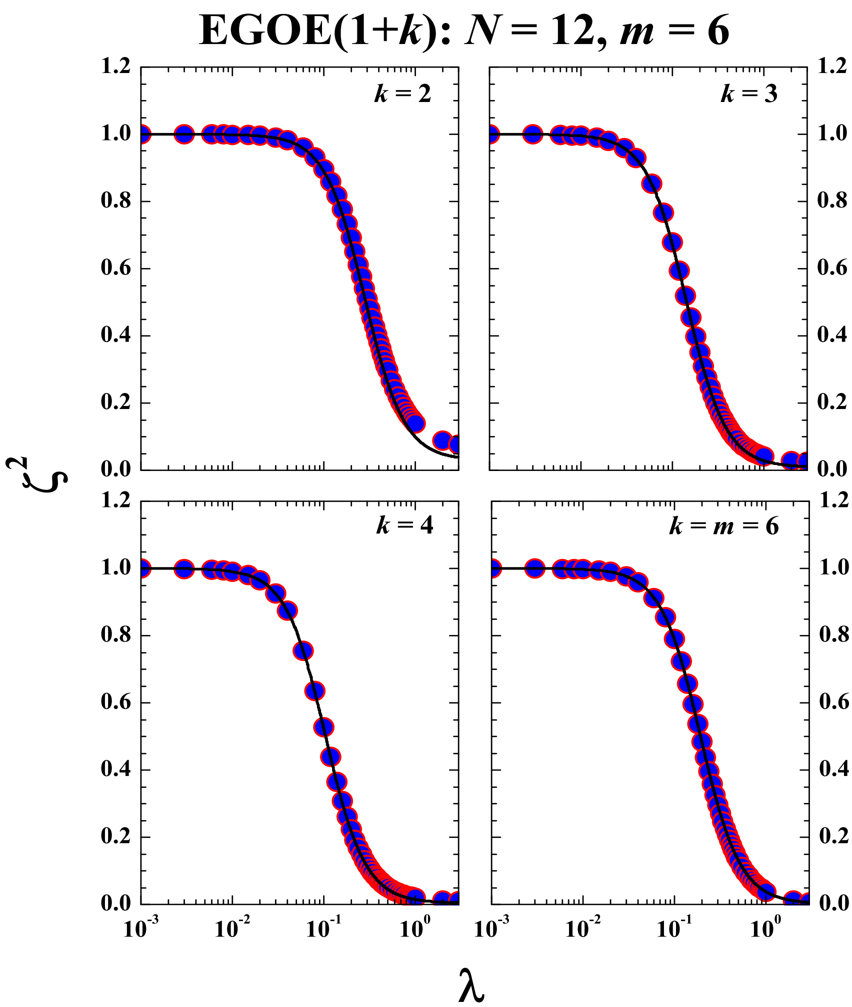

Figure 5.6: Ensemble averaged ζ^2 (solid circles) as a function of interaction strength λ for a 100 member EGOE(1+k) ensemble with $m = 6$ and $N = 12$ are shown for body ranks k = 2,3,4 and $k = m = 6$. The ensemble averaged results are compared with the analytical curves given by Eq.(5.11) (smooth black curves) and a very good agreement between both the curves can be seen.

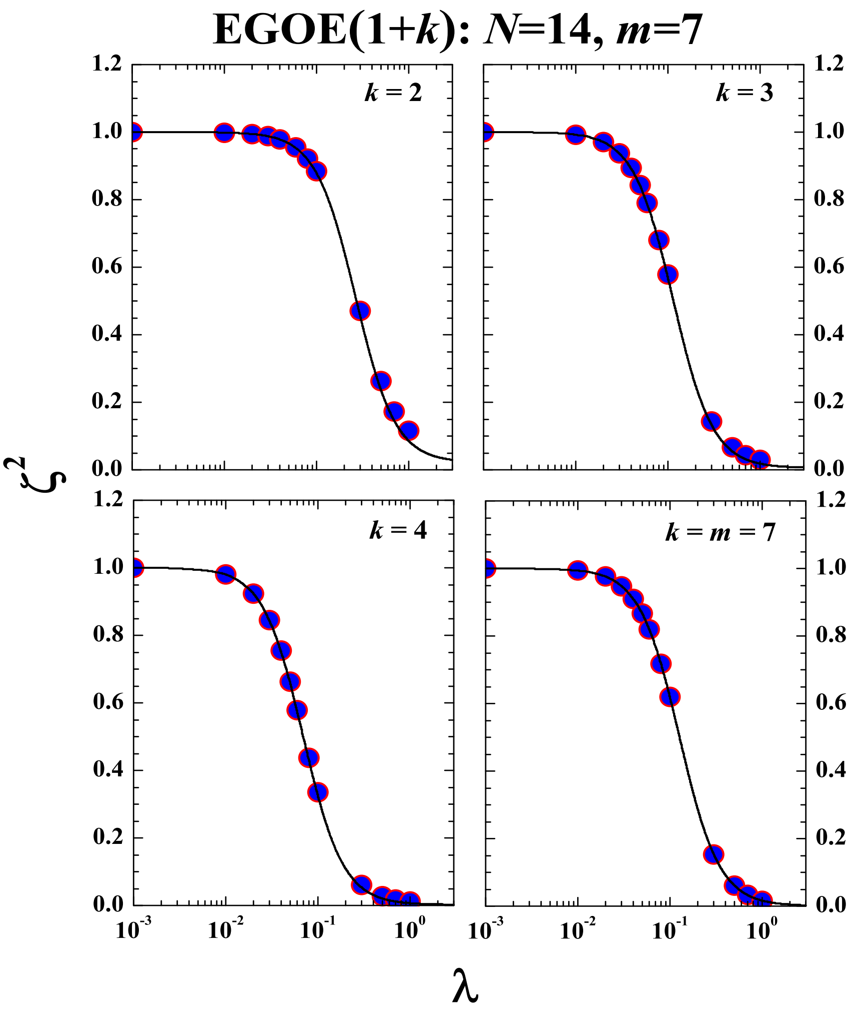

Figure 5.7: Same as Fig. 5.6 but for a 20 member EGOE(1+k) ensemble with $m = 7$ and $N = 14$. Results are shown for body ranks k = 2,3,4 and $k = m = 7$.

5.3.2 Formula of ζ for Boson Systems

In a similar manner, the analytical expression of ζ for bosonic system can be obtained. From [61] we have

$$\begin{aligned}
\sigma^2_{H=V(k)} &= T(N,m,k)\binom{N+k-1}{k}^{-1} \sum_{\alpha,\beta}\overline{w^2_{\alpha\beta}}\,, \\
\text{where, } T(N,m,k) &= \Lambda^0(N,m,k)/\binom{N+k-1}{k}\,, \\
\Lambda^0(N,m,k) &= \tbinom{m}{k}\tbinom{N+m-1}{k} \\
T(N,m,k) &= \tbinom{m}{k}\tbinom{N+m-1}{k}/\tbinom{N+k-1}{k}.
\end{aligned} \tag{5.12}$$

Since the variance of diagonal matrix elements is twice the variance of off-diagonal matrix elements, we have

$$\sigma^2_{H=V(k)} = T(N,m,k) \binom{N+k-1}{k}^{-1} \{2 \times \text{no-dia} + 2 \times \text{no-offdia}\}\,, \tag{5.13}$$

Here the number of independent diagonal and off-diagonal k-body matrix elements are 'no-dia' $= \binom{N+k-1}{k}$ and 'no-offdia' $= \frac{1}{2}\binom{N+k-1}{k}\{\binom{N+k-1}{k} - 1\}$ respectively. Similarly, $\sigma_{H_{\text{off-dia}}}$ is given by removing the contribution of diagonal k-body matrix elements from the above equation. Then using Eq.(5.6) for $H = V(k)$,

$$\zeta^2 = \frac{4}{\binom{N+k-1}{k} + 1}\,. \tag{5.14}$$

Here, it can be immediately seen that ζ^2 is independent of m for BEGOE(k). In the dense limit, with $k << m$, $\zeta^2 \propto 1/N^k$, hence as $N \to \infty$, $\sigma_F \to 1$ which implies $\zeta \to 0$ [62]. Going further, with inclusion of one-body part defined by the external sp energies (ϵ_i), and with $H = h(1) + \lambda V(k)$, we have

$$\begin{aligned} \sigma^2_H &= \sigma^2_{h(1)} + \lambda^2\, \sigma^2_{V(k)}, \\ &= \tfrac{m(N+m)}{N(N+1)} \sum \tilde{\epsilon}_i^{\,2} + \lambda^2\, \sigma^2_{V(k)}. \end{aligned} \tag{5.15}$$

The analytical expression for ζ^2 can be given by,

$$\zeta^2 = \frac{\frac{m(N+m)}{N(N+1)} \sum \tilde{\epsilon}_i^{\,2} + 2\,\lambda^2\, T(N,m,k)}{\frac{m(N+m)}{N(N+1)} \sum \tilde{\epsilon}_i^{\,2} + \lambda^2\, T(N,m,k)\, \left\{1 + \binom{N+k-1}{k}\right\}}\,. \tag{5.16}$$

The analytical expression for ζ^2 given by Eq.(5.16) is tested with the numerical ensemble averaged results obtained using an example of 100 member BEGOE(1+k) ensemble with $m = 10$ and $N = 5$. The results of ζ^2 as a function of k-body interaction strength λ for different body rank k are presented in Fig. 5.8. The red solid circles represent ensemble averaged numerical results while the continuous black smooth curve in each plot is obtained using Eq.(5.16) with fixed sp energies employed in the present study. It can be seen from the results that there is a very good agreement between the ensemble averaged results and the smooth forms obtained by Eq.(5.16) for all k values. Small difference with large λ, for $k < 5$, is due to neglect of induced sp energies. The contribution of induced sp energies reduces as λ and k increases.

The results of ζ^2 for both fermion and boson systems presented in Figs. 5.6, 5.7 and 5.8 show that the width of the strength function strongly depends on λ. For $\lambda \to 0$, $\zeta^2 \to 1$ for all k and the strength functions are known to be represented by δ functions.

With increase in λ i.e. $\lambda \geq \lambda_C$, the strength functions are known to be described by the Briet-Wigner (Lorentz) form. With further increase in λ i.e. $\lambda >> \lambda_F$, ζ^2 goes on decreasing smoothly leading to a fully chaotic domain giving the Gaussian or semi-circle or intermediate to Gaussian and semi-circle character of the strength functions depending upon the values of λ and k. One can also observe the BW to Gaussian to semi-circle transition in strength functions by changing both λ and k. Therefore, it is possible to have a shape intermediate to BW and semi-circle for some values of λ and k [153].

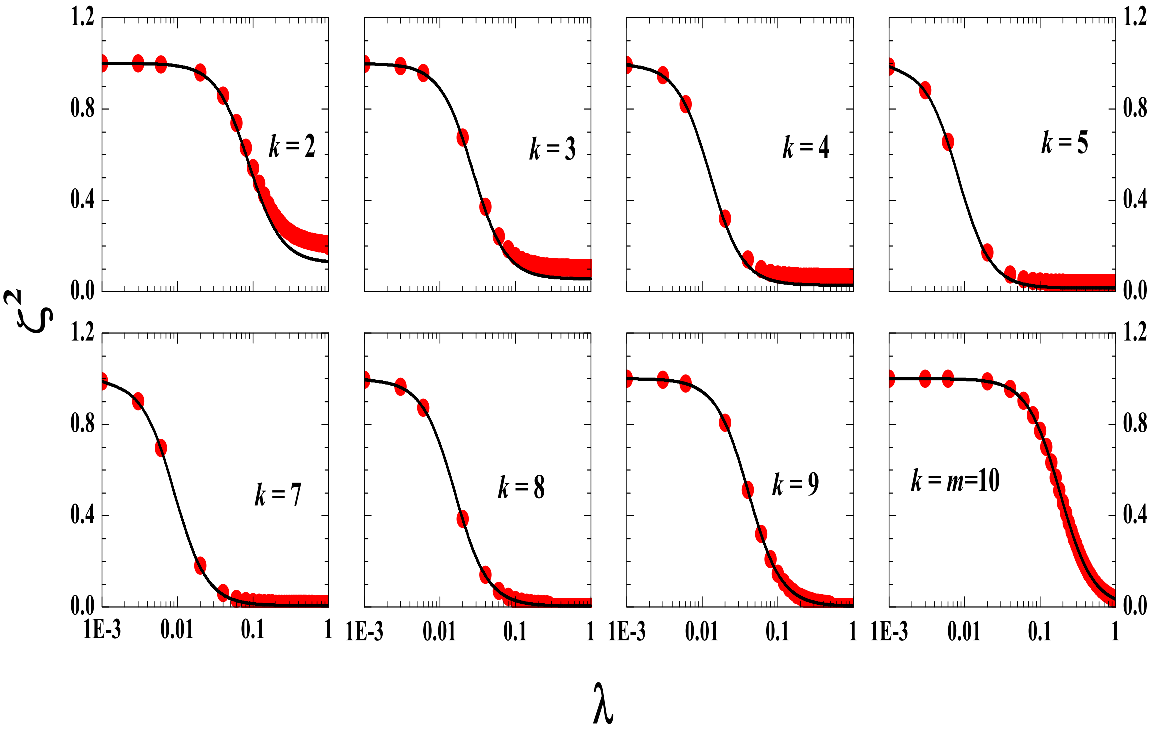

Figure 5.8: Ensemble averaged ζ^2 vs. λ calculated for BEGOE(1+k) ensemble with $N = 5, m = 10$ example for different k values. Solid circles represent numerical ensemble averaged results and the smooth black curves are obtained using Eq.(5.16). Refer text for more details.

5.4 Chaos marker λ_t

The thermodynamic region can be identified from chaos marker λ_t. This is the region where different quantities defining the eigenstate properties like entropy, strength functions, temperature, etc. give the same values irrespective of the defining basis. This means that once the interaction strength between the particles λ goes beyond λ_t, the quantum system thermalizes and then we can define quantities like temperature, entropy, etc. of the system. We can obtain λ_t from the correlation coefficient ζ discussed in previous section 5.3. In [27, 152] for two-body interaction, it is shown that λ_t can be obtained from ζ^2 using the condition $\zeta^2 = 0.5$ i.e. the spreading produced by one-body part and two-body part are equal. Using this condition, we can also obtain the analytical expression for marker λ_t for k-body interactions in presence of mean field by considering the spreading produced by one-body part and k-body part equal, for both fermion and boson system.

5.4.1 Formula of λ_t for Fermion Systems

First let us obtain the analytical expression of λ_t for fermion system. We put $\zeta^2 = 0.5$ in Eq. (5.11) and solve it for λ. In this way we can obtain the marker λ_t in terms of m, N and k which is given by,

$$\lambda_t = \sqrt{\frac{m(N-m)\ \sum \tilde{\epsilon_i}^2}{N(N-1)\Lambda^0(N,m,k)(1-3\ \binom{N}{k}^{-1})}}\ . \tag{5.17}$$

The results of the variation of marker λ_t as a function of m for fermion systems modeled by EGOE(1+k) are presented Fig. 5.9. This analysis is done for k = 2,3 and 4 by taking m/N values 0.1 and 0.5.

5.4.2 Formula of λ_t for Boson Systems

Now moving further let us derive the analytical expression for boson system with BEGOE(1+k). As in the case of fermions, we put $\zeta^2 = 0.5$ in Eq. (5.16) and solve it for λ to obtain (m, N, k) dependence of λ_t. In this way we obtain the analytical expression of λ_t given by,

$$\lambda_t = \sqrt{\frac{m(N+m)\ \sum \tilde{\epsilon_i}^2}{N(N+1)\Lambda^0(N,m,k)(1-3\ \binom{N+k-1}{k}^{-1})}}\ . \tag{5.18}$$

Fig. 5.10 shows the variation of marker λ_t in dense boson systems as a function of N for the fixed sp energies used in the present study. The results are shown for body rank values $k = 2, 3$ and 4, and with $m/N = 2$ and 5.

These results for both fermion and boson systems show that as the body rank of interaction k among the particles increases, the value of λ_t decreases i.e. the thermalization sets in faster in the system.

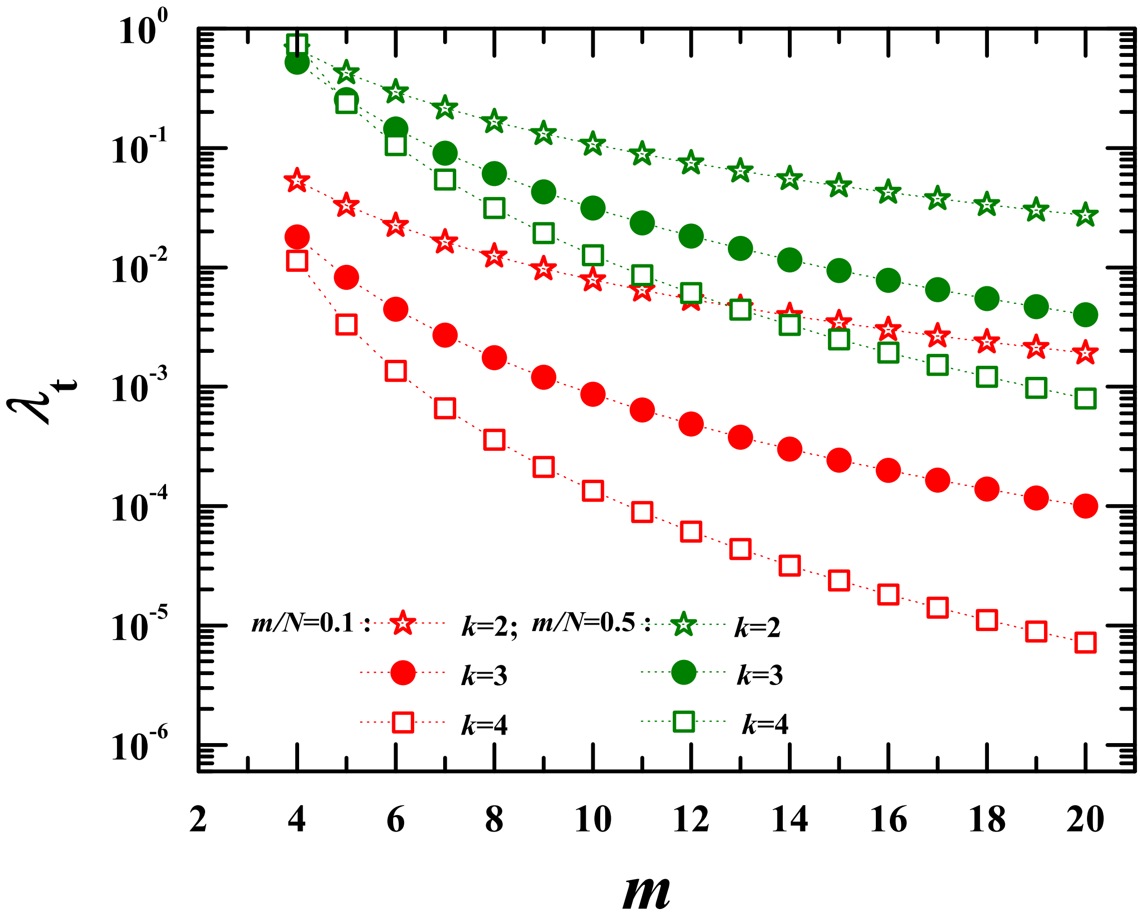

Figure 5.9: Marker λ_t vs. m for fermion system. Results are obtained using Eq.(5.17) for k =2,3 and 4 and for various values of m/N .

5.5 Lower Order Moments

In this section, we analyze the lower order moments of the strength functions discussed in section 5.2. We have computed the first four moments viz. centroid, variance, skewness (γ_1) and excess (γ_2) of the strength functions for both fermion and boson systems. These numerically computed results are also compared with the analytical forms of the first four moments of f_{CqN} [84] discussed in chapter 4.

5.5.1 Results for Fermion Systems

Let us first analyze the first four moments for fermion systems. Figs. 5.11 and 5.12 represent results for centroid, γ_1 and γ_2 for fermion systems. For this calculation, we consider the example of m =6 fermions in N =12 sp states. We study these three moments as a function of λ for various values of $\xi = 0$, ± 1.0 and ± 2.0 for body rank of interaction

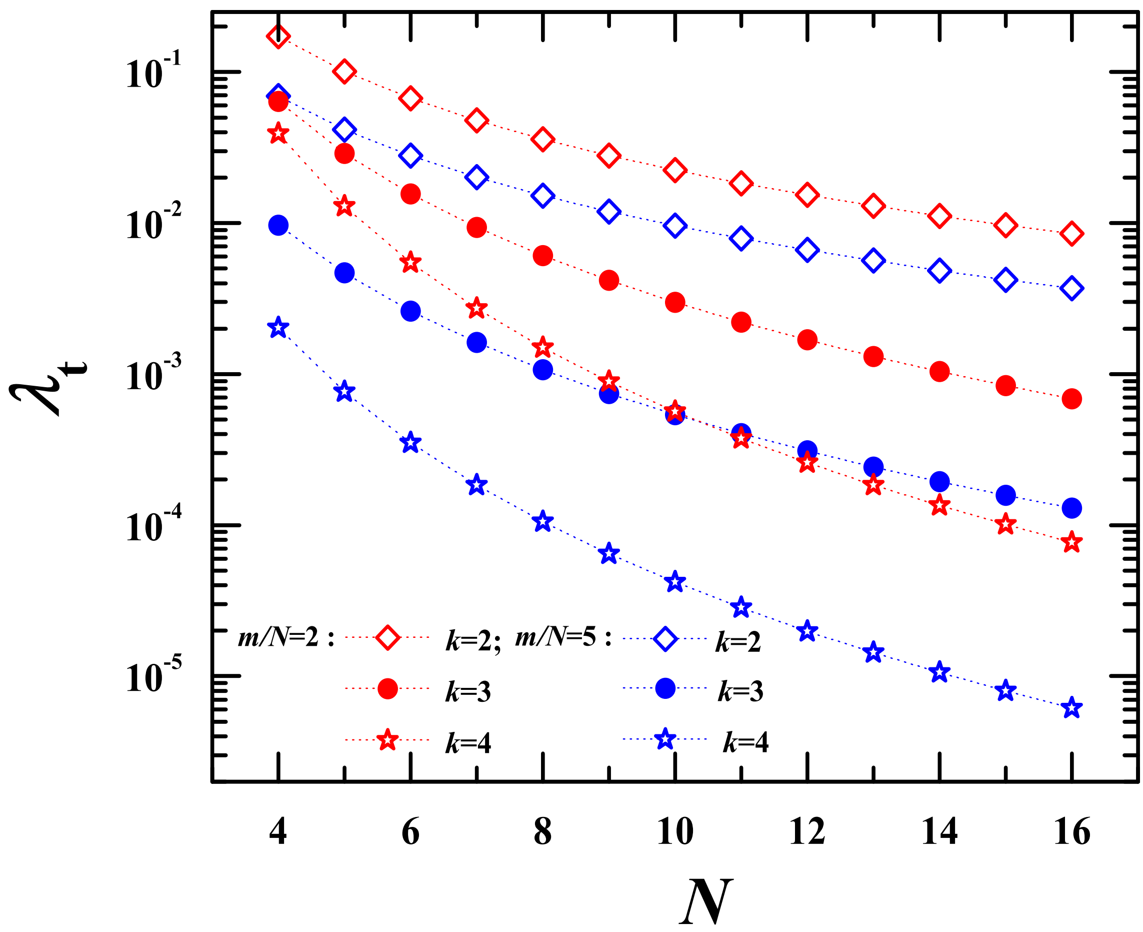

Figure 5.10: Variation of marker λ_t as a function of N for dense boson systems with BEGOE(1+k). Results are shown for body rank values $k = 2, 3$ and 4, for various values of m/N determined from Eq.(5.18).

$k = 2$ and 3. The solid symbols represent the numerical results while the smooth forms are analytical results given by the corresponding equations of moments of f_{CqN} discussed in chapter 4. It can be observed from the plots that no variation is observed between the numerical results and smooth forms for the first moment i.e. centroid, by changing the interaction strength λ. Though not shown here, similarly no variation is observed in the second moment i.e. variance as well. Now the interesting case is of the third and fourth moments i.e. γ_1 and γ_2. These are the moments which can also give us the marker λ_t. Initially when the interaction strength λ is very low (i.e. $\lambda < \lambda_t$), we can observe variation in the numerical and analytical results. But as we approach the λ_t, we no longer observe this variation in both these results. And for $\lambda \geq \lambda_t$ this variation can no longer be observed. In this way we can also obtain the marker λ_t from the moments of strength functions. In Figs. 5.11 and 5.12, the marker λ_t is shown by dotted lines for each k value. These values of λ_t are obtained from the analytical results presented in section 5.4 and the numerical results of λ_t obtained from the the lower order moments are in good agreement with these analytical values.

5.5.2 Results for Boson Systems

Now let us move forward by analyzing the lower order moments for boson systems. Fig. 5.13 represents results of centroid, variance, γ_1 and γ_2 of the strength function results of bosons for the body rank k going from 2 to $k = m = 10$. The results are shown for various values of ξ. As discussed earlier in section 5.2, the variance of the strength functions is independent of ξ and simply related to correlation coefficient. From the numerical results of lower order moments of strength functions, one can clearly see that in the thermodynamic domain, lower order moments follow moments of the conditional q-normal distribution f_{CqN}. The results are also consistent with the analytical forms derived in [84] and discussed in chapter 4.

5.6 Conclusion

The strength functions that play a very crucial role in studying the wavefunction structure in finite interacting particle systems have been analyzed in this chapter. The strength function along with its width has been analyzed for both fermions and bosons using EGOE(1+k) and BEGOE(1+k) respectively. It is shown that in the strong coupling limit (i.e. $\lambda > \lambda_t$), the conditional q-normal density f_{CqN} describes Gaussian to semi-circle transition in strength functions as body rank k of the interaction increases. Further, a complete analytical description of the variance of the strength function in terms of the correlation coefficient ζ, as a function of λ and k is derived. Using this analytical expression of ζ, the (m, N, k) dependence of marker λ_t, defining thermalization region is derived analytically. Finally the lower order moments of the strength functions are studied for BEGOE(1+k) as a function of body rank k. For EGOE(1+k), the lower order moments of the strength functions are studied as a function of interaction strength λ and using these results it is shown that the marker λ_t can also be determined from the lower order moments of the strength functions.

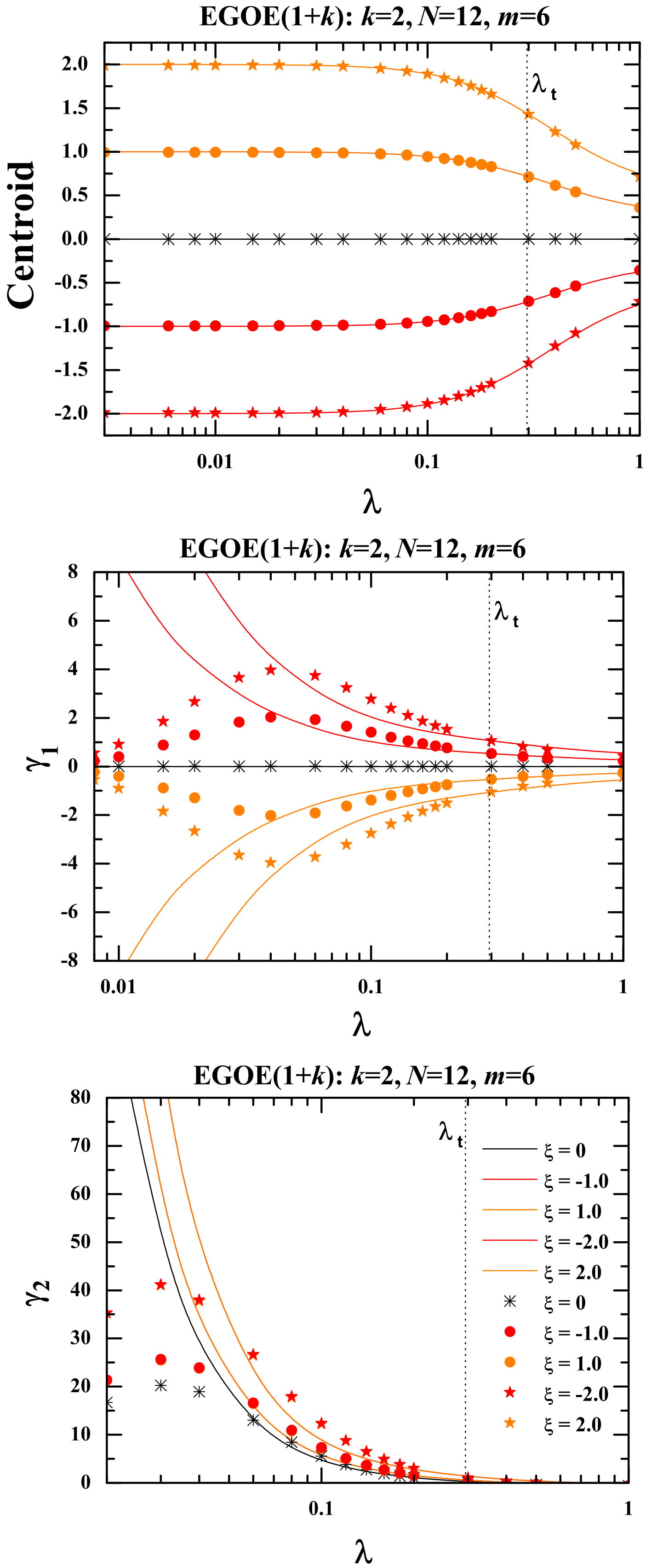

Figure 5.11: Variation of the moments of strength functions i.e. centroid, γ_1 and γ_2 as a function of λ. The results are shown for various values of ξ for a system of m =6 fermions in N =12 sp states for $k = 2$. These results give us the marker λ_t. For more details refer the text.

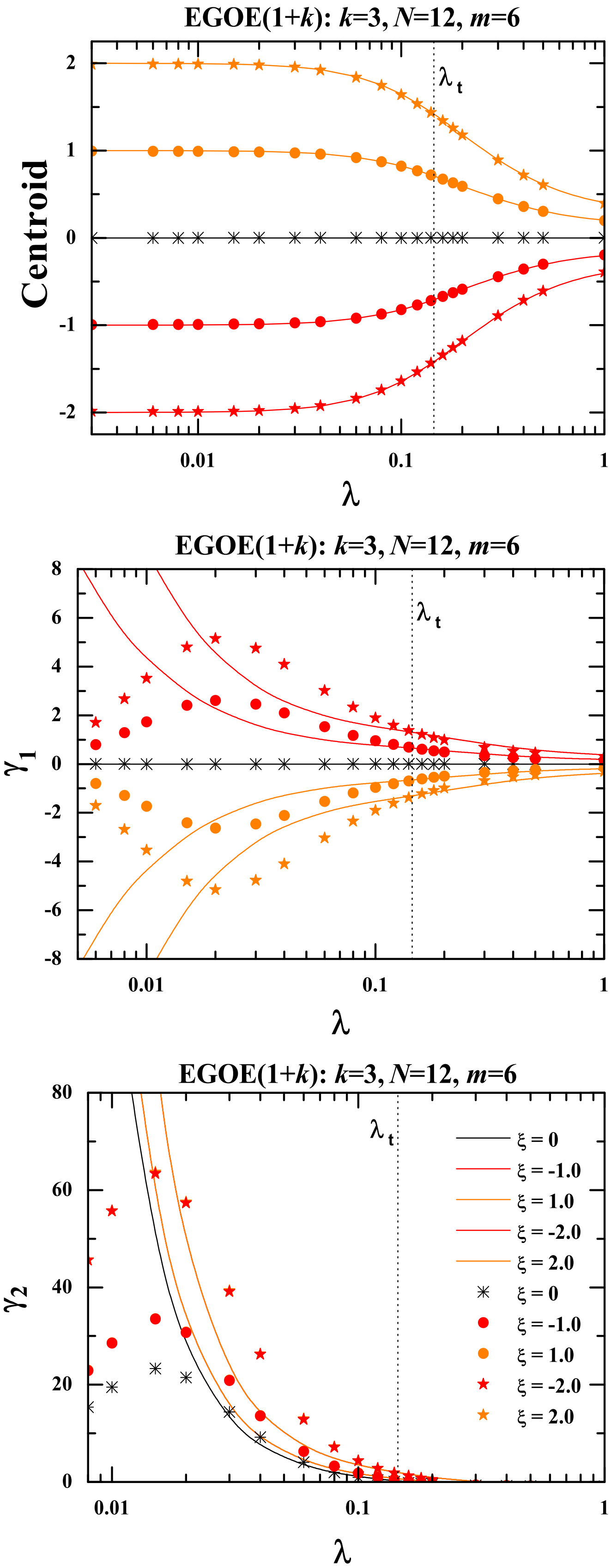

Figure 5.12: Same as Fig. 5.11 but for $k = 3$.

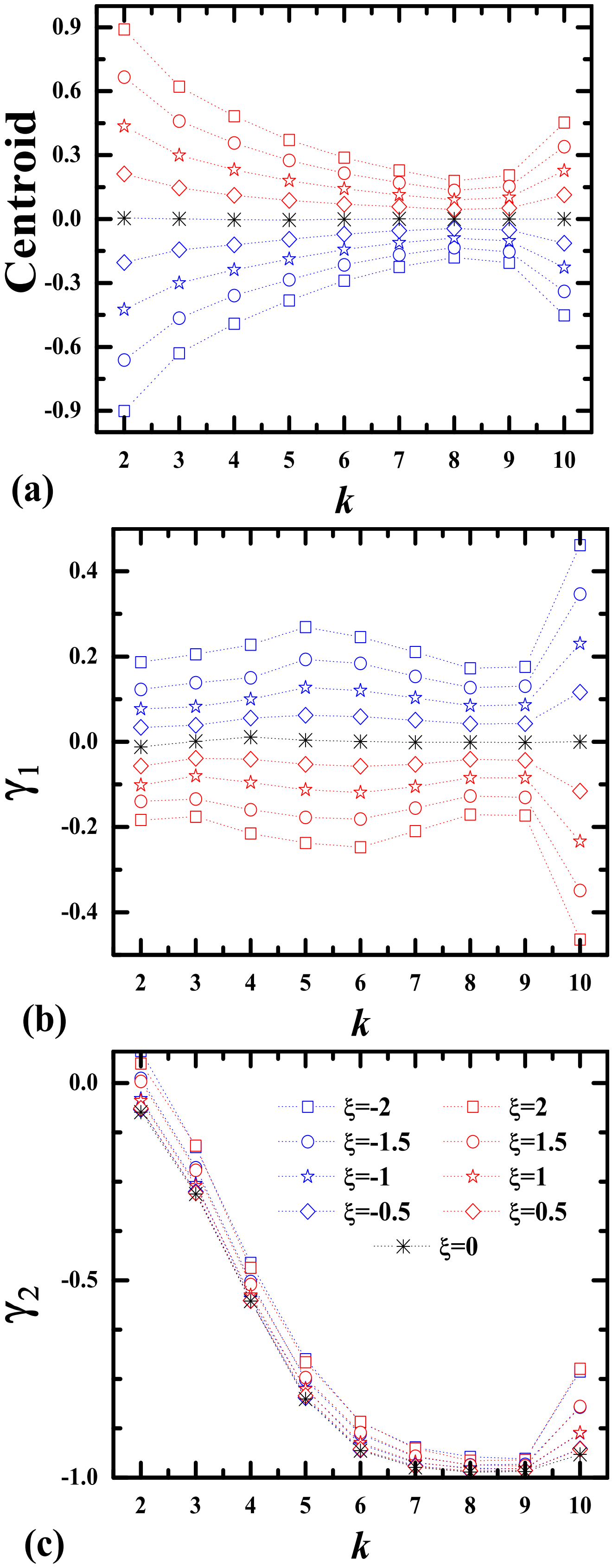

Figure 5.13: Ensemble averaged (a) Centroid, (b) γ_1 and (c) γ_2 as a function of body rank k for the strength function results for bosons. Results are shown for various values of ξ.