Chapter 8

Conclusions and Future Outlook

8.1 Conclusions

The work presented in this thesis has addressed the two very recently popular open problems of quantum statistical physics i.e. quantum many-body chaos and thermalization in isolated finite interacting many-particle systems. The main motivation for this study comes from the recent experimental developments on ultra-cold quantum gases and production of Bose-Einstein (BE) condensates, which allow us to artificially simulate these complex quantum systems. In order to address these problems the spectral and wavefunction properties of these systems are analyzed. Embedded random matrix ensembles being paradigmatic models to study finite interacting many-body quantum systems are used for this study. Throughout this thesis we have modeled these systems using orthogonal variant of these embedded ensembles called the Embedded Gaussian Orthogonal Ensemble (EGOE). The conclusions of this entire study are as follows.

In chapter 2, various EGOEs of one plus two-body interactions [EGOE(1+2)] used to model interacting fermion and boson systems with and without spin degree of freedom in the present study are introduced. Spinless fermion (or boson) systems are modeled using EGOE(1+2) (or BEGOE(1+2)). Moving ahead with spin degree of freedom, fermion systems with spin $\mathrm{s} = 1/2$ are modeled using EGOE(1+2)-s. For bosons with a fictitious F spin $1/2$ degree of freedom we have BEGOE(1+2)-F and with spin-one degree of freedom we have BEGOE(1+2)-$S1$. The basic construction of the Hamiltonian of all these ensembles is given in this chapter.

Now having defined these ensembles, we proceed with the study of their spectral properties using eigenvalues obtained from their respective Hamiltonian. The spectral fluctuations reveal whether the complex system under study is in regular (or integrable) or chaotic domain. Spacing distributions are measures to study these spectral fluctuations.

In chapter 3, we use the eigenvalues obtained from various EGOE(1+2) discussed in chapter 2 to study these spectral fluctuations using two spacing distributions. In the first part of this chapter, the distributions of closest neighbor spacings $P_{CN}(s)$ and farther neighbor spacings $P_{FN}(s)$ for interacting fermion and boson systems with and without spin degree of freedom are studied. Our numerical results for various examples of fermion and boson systems and shell model, are consistent with the recently derived analytical expressions using a 3×3 random matrix model and other related quantities [107]. This concludes that these analytical expressions are universal and the local level fluctuations generated by EE follow the results of classical Gaussian ensembles when the two-body interaction is strong enough. Going further, for EGOE(1+2) and BEGOE(1+2) using $\langle s_{CN} \rangle$ and $\langle s_{FN} \rangle$ we have also obtained the transition marker λ_C. Using this marker it is possible to study a transition from Poisson to GOE in level fluctuations with increase in the strength of the two-body interaction λ [24,62,65,115,135]. The analysis of these distributions involve the cumbersome and non-unique procedure of unfolding to remove the variation in the density of eigenvalues. However if we use the method of ratio of spacings [109] we can escape from this unfolding procedure. In the second part of this chapter going beyond ratio of spacings, we have analyzed the distributions of higher order level spacing ratios $P^k(r)$ for interacting fermion and boson systems with and without spin degree of freedom. Our numerical results for higher order spacing ratios are in excellent agreement with the recently derived Wigner surmise like scaling relation. From this we can conclude that, this scaling relation is universal to understand higher order spacing ratios in complex many-body fermionic as well as bosonic quantum systems preserving rotational and time-reversal symmetry with and without spin degree of freedom. We have also shown that the higher order spacing ratio distributions can reveal quantitative information about underlying symmetry structure. This further concludes that this analysis of higher order spacing ratios is not only useful in studying spectral fluctuations but also reveals quantitative information about symmetry structure of complex quantum systems.

These EGOE(1+2) models used in the chapters so far, to study interacting fermion and boson systems with and without spin degree of freedom, focused on one- plus two-body part of the interaction as inter-particle interaction is known to be only one-body and two-body in nature. However, recently the importance of higher body interactions i.e. $k > 2$ is discovered in strongly interacting quantum systems [71,72], nuclear physics [73], quantum black holes [35,74] and wormholes [75] with SYK model and also in quantum transport in disordered networks connected by many-body interactions [76–78]. Therefore, it is necessary to extend the analysis of EE with two-body interactions to higher k-body interactions and study their statistical properties in order to address these problems. They are denoted by EGOE(k) (or BEGOE(k)) for fermion (or boson) systems. In the presence of mean-field they are denoted by EGOE(1+k) (or BEGOE(1+k)) for fermion (or boson) systems. The generating function of q-Hermite polynomials exhibits Gaussian to semi-circle transition. The spectral densities of SYK model and these EGOEs also show this same transition. Hence recently q-Hermite polynomials are used to study spectral density in these models [80–82] and also in EGOE(k) [79].

In chapter 4, we firstly define and describe the construction of Hamiltonian of EGOE(1+k) and BEGOE(1+k). We then introduce q-Hermite polynomials, q-normal distribution f_{qN}, conditional q-normal distribution f_{CqN} and bivariate q-normal distribution f_{biv-qN}. Further we have derived the analytical formula of parameter q considering only the one-body part of the Hamiltonian for both fermions and bosons. Also the variation of parameter q as a function of the interaction strength λ in EGOE(1+k) and BEGOE(1+k) is studied for a fixed body rank k. Lastly, we show that the Gaussian to semi-circle transition in spectral density of EGOE(1+k) and BEGOE(1+k) is well described by f_{qN} for any value of interaction strength λ.

In the chapters so far, the spectral properties of isolated finite interacting many particle systems are analyzed using the eigenvalues of the Hamiltonian. Going beyond the eigenvalues, the eigenfunctions give us knowledge of the structure of wavefunctions of these systems. This in turn is useful to study various other interesting quantities like information entropy (S^{info}), number of principal components (NPC), occupational entropy (S^{occ}), etc. These quantities also quantify chaos in these systems. The strength functions (also known as local density of states) are very fundamental in the analysis of wavefunction structure in finite interacting particle systems. Strength functions carry the information of spreading of a particular basis state over the eigenstates. In chapter 5, we have analyzed the strength function along with its width for both fermions and bosons using EGOE(1+k) and BEGOE(1+k) respectively using the eigenfunctions obtained from their corresponding Hamiltonian. It is shown that for strong enough inter-particle interaction (i.e. $\lambda > \lambda_t$), the shape of strength functions show a transition from Gaussian to semi-circle form as the body rank k of the interaction increases. This transition is very well described by f_{CqN}. Moving ahead, a complete analytical description of the variance of the strength function in terms of the correlation coefficient ζ, as a function of λ and k is derived for both EGOE(1+k) and BEGOE(1+k). Analytical expression of marker λ_t, defining thermalization region is derived in terms of the system parameters m, N and k using the analytical expression of ζ for both EGOE(1+k) and BEGOE(1+k). Lastly the lower order moments of the strength functions are studied for BEGOE(1+k) as a function of body rank k. Using EGOE(1+k), we have shown that the marker λ_t can also be determined from the lower order moments of the strength functions. For this analysis we have studied the lower order moments of the strength functions as a function of interaction strength λ.

Now moving ahead, we apply the analysis of strength functions presented in chapter 5, to study two chaos qauntifiers viz. NPC and S^{info} in chapter 6. We use the interpolating conditional q-normal form f_{CqN} of the strength functions and f_{qN} form to analyze NPC. We have obtained an analytical formula in terms of two parameters ζ and q as a function of energy for k-body interaction. We tested this formula with the numerical EE results for both EGOE(1+k) and BEGOE(1+k) when the interaction strength is strong enough. We found that there is a good match between numerical EE results and the analytical formula in strong interaction domain. Now moving to the next chaos quantifier, we have studied the localization length l_H related to S^{info}. l_H is studied numerically using the interpolating conditional q-normal form f_{CqN} of the strength functions. The study has been carried out for both EGOE(1+k) and BEGOE(1+k) as a function of energy for k-body interaction and these results are in good agreement with the smooth forms. Till now we have studied these quantities as a function of energy. We now end this chapter with study of time evolution in these systems. In the study of time evolution, fidelity decay is an important quantity to investigate. We study the fidelity decay in boson systems after the application of k-body interaction quench. This is studied using BEGOE(1+k) using the interpolating form of strength functions. From all the results presented in chapters 4, 5 and 6 and from the previous studies [79, 83, 84], we conclude that the q-Hermite polynomials play a very significant role in analyzing many-body quantum systems interacting via k-body interaction. Due to the recently known contribution of 3-body and 4-body parts in nuclear interactions and the possibility of higher body interactions becoming prominent in strongly interacting quantum systems in future [35,71,72], the generic results presented in these chapters are important for a complete description of many-body quantum systems interacting via k-body interaction.

Finally in the last chapter 7, we have applied the embedded ensembles with k-body interactions to study the role of centrosymmetry in quantum transport across disordered networks connected by many-body interactions. We modeled the fermionic and bosonic networks using EGOE(k) and BEGOE(k) respectively. We studied the transport efficiencies in these networks and found out that the presence of centrosymmetry in m-particle space is responsible for the enhancement of transport efficiency in a small network and results are in good agreement with [76]. Further we have verified that the centrosymmetry is essentially needed in both k as well as m particle spaces to enhance quantum efficiency.

8.2 Future Outlook

Following problems are for future work on the lines mentioned in this thesis:

- In the past, the criterion for the chaos marker λ_C for EGOE(1+2) models [31, 62], based on the perturbation theory was derived by Jacquod and Shepelyansky [196]. The validity of the perturbation theory gives λ_C. Hence, in future it is important to analyze $P_{CN}(s)$ distribution and related measures in the context of onset of chaos in embedded ensembles.

- In future, the higher order spacing ratios can be utilized to get new information about symmetries such as isospin symmetry and F-spin symmetry in the following nuclei: (i) in ^{26}Al and ^{30}P energy levels up to $\sim$ 8 MeV excitation with isospin $T = 0$ and $T = 1$ [197] and they can be analyzed using EGOE(1+2)-s with spin being isospin; (ii) scissors levels in 2.5 to 4 MeV excitation in heavy deformed nuclei with the low-lying levels having $F = N/2$ and scissors levels $F = N/2-1$ [198] and they can be analyzed using BEGOE(1+2)-F; (iii) excited 2^+ and 4^+ levels in nuclei across the periodic table with various symmetries of IBM and its extensions [199]; (iv) analyzing energy levels from large shell model and also IBM-2 and IBM-3 models. The higher order spacing ratios may give new information about symmetries in these nuclei. In the past, all these data have been analyzed using NNSD but the analysis of their higher order spacings still remains to be carried out in future [31].

- It is also known that the strength functions and the entanglement essentially capture the same information about eigenvector structure [155, 200] and therefore in future it is important to study entanglement properties using embedded ensembles with k-body forces.

- The fidelity decay W_0 is expected to surely demonstrate a power-law behavior i.e. $W_0(t) \propto t^{-\gamma}$ with $\gamma \geq 2$ implying thermalization [201], over very long times, no matter how fast the decay may initially be. It is shown in [201], that the power-law behavior appears due to the fact that the energy spectrum is bounded from both the ends. This condition is essentially satisfied by f_{CqN}. Therefore, in future it is important to analyze the long-time behavior of fidelity decay using EE firstly to establish its universality and secondly to test whether it can be explained with the use of f_{CqN}.

- Using these results, in future it is possible to design networks with good efficiency even in presence of certain many-body random perturbations. For example, the efficiency in nano-structure such as quantum wires, it is interesting to check the case with filling factors close to one, where many body interactions lead to very good efficiencies. Another example is of efficient single electron transport in a linear array of tunnelcoupled quantum dots, which can further be used as an ideal quantum channel in quantum computers [185] and efficient transmission of qubits between the different quantum registers in a quantum bus based on semiconductor self-assembled quantum dots [186]. Also, controlling strong interactions between ultra-cold atoms trapped in optical lattices can serve in efficient quantum computation [187]. Finally, the results presented in this chapter can be useful to understand the good efficiency properties experimentally observed in exciton transport in certain biomolecules such as the Fenna- Matthews-Olson complex [176]. Also in future it will be interesting to study the transfer of quantum states from one location to another which is the base of Quantum Information Science, using EE with spin degree of freedom.