Synopsis of the thesis entitled

STUDY OF EMBEDDED RANDOM MATRIX ENSEMBLES FOR FINITE INTERACTING PARTICLE SYSTEMS

submitted to

The Maharaja Sayajirao University of Baroda

for the degree of

Doctor of Philosophy

in

Applied Physics

by

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August 2021

Study of Embedded Random Matrix Ensembles for Finite Interacting Particle Systems

With the recent experimental developments on ultra-cold quantum gases and production of Bose-Einstein (BE) condensates in an Earth-orbiting Cold Atom Lab, there is a renewed interest in theoretical investigations of finite interacting many particle quantum systems. These investigations in turn are useful in addressing various open problems of quantum statistical physics such as BE condensation, quantum many-body chaos and thermalization in isolated finite interacting many-particle quantum systems. In order to address these problems, it is extremely important to analyze the spectral and wavefunction properties of these systems. It is now well established that Random Matrix Theory (RMT), due to its universality, successfully describes the spectral as well as wavefunction properties of isolated finite many-particle quantum systems.

In 1928, Wishart's historical paper on multivariate statistics gave birth to the field of RMT. Later in 1950, Wigner introduced RMT in the field of Physics, while addressing compound nucleus resonances. Wigner worked on the spectra of neutron excitation of heavy nuclei obtained using neutron resonances. Dyson gave the tripartite classification of classical random matrices (Gaussian Orthogonal Ensemble (GOE), Gaussian Unitary Ensemble (GUE) and Gaussian Symplectic Ensemble (GSE)) on the basis of the type of symmetry preserved by the system, viz. time reversal, rotational, etc. Over the coming years, rigorous research in the field by Dyson, Mehta, Pandey, Bohigas, Gaudin, Porter, Rosenzweirg, French, Berry and many others resulted in tremendous development of RMT. In 1984, the famous Bohigas-Giannoni-

Schmit (BGS) conjecture established the Universality of RMT. Due to its universality, accompanied with a great deal of mathematical work done over the years, RMT is now not only limited to the fields of Science, but also has emerged as a multidisciplinary research area with numerous applications in fields like economics, finance, stock markets, etc.

However, constituents of isolated quantum systems interact via few-body interactions whereas the classical random matrix ensembles (and in particular the GOE) take into account many-body interactions. This motivated French and co-workers to introduce random matrix models accounting for few-body interactions, called embedded ensembles (EE). With two-body interaction (i.e. $k = 2$), and in the presence of mean-field one-body part they are called EE(1+2). This thesis focuses on the orthogonal variant of EE called the Embedded Gaussian Orthogonal Ensemble (EGOE). It is now well established that these EGOE(1+2) models are paradigmatic models to study the dynamical transition from integrability to chaos in isolated finite interacting many-body quantum systems. These models were initially analyzed for isolated finite interacting spin-less fermion systems (denoted by EGOE(1+2)) and later for spin-less boson systems (denoted by BEGOE(1+2)), as they are generic models for finite isolated interacting many-particle systems. For m fermions in N single particle (sp) states, there exists a constraint called dilute limit (i.e $m \rightarrow \infty$, $N \rightarrow \infty$ and $m/N \rightarrow 0$), which allows only one fermion per sp state. Hence, obeying the Pauli's exclusion principle. For bosons also there exists dilute limit, however one cannot distinguish fermions and bosons in dilute limit. For bosons, it is more interesting to study the dense limit (i.e $m \rightarrow \infty$, $N \rightarrow \infty$ and $m/N \rightarrow \infty$) which allows more than one boson to occupy a particular sp state. Moreover, EGOE(1+2) models with spin degree of freedom for isolated interacting fermion and boson systems have also been developed and analyzed in detail.

In these investigations, the main focus was on one- plus two-body part of the interaction as inter-particle interaction is known to be only one-body and two-body in nature.

However, it is seen that the higher body interactions $k > 2$ play an important role in strongly interacting quantum systems, nuclear physics, quantum black holes and wormholes with SYK model and also in quantum transport in disordered networks connected by many-body interactions. Therefore, it is necessary to extend the analysis of EE to higher k-body interactions in order to understand these problems. They are denoted by EGOE(k) (or BEGOE(k)) for fermion (or boson) systems. In the presence of mean field one-body part they are denoted by EGOE(1+k) (or BEGOE(1+k)) for fermion (or boson) systems.

Organization of the thesis:

The thesis entitled "Study of Embedded Random Matrix Ensembles for Finite Interacting Particle Systems" has been organized in total 8 chapters. A chapter-wise brief description of the work done is given ahead.

Chapter 1 - Introduction

This chapter introduces the field of RMT, the three classical random matrix ensembles (GOE, GUE and GSE) classified on the basis of different symmetries, its universality, its applications in various diverse fields, etc. Further it gives a review of various investigations on embedded random matrix ensembles done so far, motivation for the present work and finally gives an outline of the complete work done in the thesis.

Chapter 2 – Embedded Ensembles for Fermion and Boson Systems

Embedded Gaussian Orthogonal Ensembles (EGOE) are random matrix models with two-body interactions among its constituents (fermions or bosons) that model Hamiltonians *H* of finite interacting many-particle quantum systems. As the particles are in an average field generated by other particles, it is appropriate to add a mean-field term in these models. When the mean-field one-body part is added to the Hamiltonian *H*, these models are denoted by EGOE(1+2) (or BEGOE(1+2)) for spin-less fermions(or bosons). In order to analyze universal properties of systems with spin degree of freedom, it is important to include spin as an additional degree of freedom in these models. For fermions with spin $s = 1/2$ degree of freedom, they are denoted by EGOE(1+2)-*s*. For bosons, with a fictitious (*F*) spin 1/2 degree of freedom we have BEGOE(1+2)-*F* and with spin-one degree of freedom we have BEGOE(1+2)-*S1*. In this chapter, we define and describe the construction of these embedded random matrix ensembles.

Chapter 3 – Spacing Distributions

Complex quantum systems from a wide variety of fields give rise to spectral fluctuations, which reveal whether the given complex quantum system is in regular (or integrable) or chaotic domain. The nearest neighbor spacing distribution (NNSD) giving degree of level repulsion and Dyson-Mehta Δ_3 statistic giving long range spectral rigidity are some of the measures to study these spectral fluctuations and are modeled through RMT. Going beyond these, in the first part of this chapter, the probability distributions of the closest neighbor (CN) and farther neighbor (FN) spacings from a given level have been studied in EGOE(1+2) for both fermion and boson systems with and without spin degree of freedom. These measures involve the cumbersome and non-unique procedure of unfolding to remove the variation in the density of eigenvalues. The method of distribution of ratio of consecutive level spacings $P(r)$ introduced by Oganesyan and Huse does not require the unfolding process as it is independent of the form of the density of the energy levels. In the second part of this chapter, the distribution of non-overlapping spacing ratios of higher orders are studied in EGOE(1+2) for both fermion and boson systems including spin degree of freedom (also without spin) that have their origin in nuclear shell model and the interacting boson model. Using spin ensembles, we demonstrate that the higher order spacing ratio distributions can also reveal quantitative information about the underlying symmetry structure (examples are isospin in lighter nuclei and scissors states in heavy nuclei).

Chapter 4 – Random *k*-body Ensembles and *q*-Hermite Polynomials

It is now known that higher body interactions i.e. $k > 2$ play an important role in finite interacting many-body quantum systems, which makes it necessary to extend the analysis of EE with two-body interactions (discussed in chapter 2) to higher *k*-body interactions. Firstly, this chapter defines and describes the construction of EGOE(*k*) (or BEGOE(*k*)) for fermions (or bosons) and EGOE(1+*k*) (or BEGOE(1+*k*)). Very recently, it is found that *q*-Hermite polynomials can be used to study spectral densities of various finite interacting particle systems. This chapter introduces *q*-Hermite polynomials along with their generating function, the so-called *q*-normal distribution f_{qN} and formulas for parameter q in terms of *m*, *N* and *k* for both EGOE(*k*) and BEGOE(*k*). Also the conditional *q*-normal distribution f_{CqN} and bivariate *q*-normal distribution f_{biv-qN} are discussed. We present the spectral density results for EGOE(1+*k*) and BEGOE(1+*k*). The analytical formula of q considering only the one-body part is also obtained for both fermions and bosons. Furthermore, the variation of parameter q is studied as the interaction strength λ varies in EGOE(1+*k*) (or BEGOE(1+*k*)) for a fixed body rank *k*.

<u>Chapter 5</u> – Strength Function (or Local Density of States (LDOS)) and its Width

The strength functions (also known as local density of states) play a very crucial role in the analysis of wavefunction properties, as they give information about how a particular basis state spreads onto the eigenstates. In this chapter, we analyze the strength function and its width for both fermions and bosons using EGOE(1+*k*) and BEGOE(1+*k*) respectively. A complete analytical description of the variance of the strength function in terms of the correlation coefficient ζ, as a function of λ and *k* is derived. Further, the (m, N, k) dependence of marker λ_t, defining thermalization region is derived. In the strong coupling limit ($\lambda > \lambda_t$), the conditional q-normal density f_{CqN} describes Gaussian to semi-circle transition in strength functions as body rank *k* of the interaction increases.

<u>Chapter 6</u> – Number of Principal Components, Information Entropy and Fidelity Decay

In finite interacting many-particle systems, the chaos measures like number of principal components (NPC), information entropy, fidelity decay etc. can be determined by examining the general features of the strength functions. In this chapter, using EGOE(1+*k*) (or BEGOE(1+*k*)) for fermions (or bosons), a two-parameter (ζ and q) analytical formula for NPC is derived as a function of energy for k-body interaction utilizing the interpolating form f_{CqN} of the strength functions. This formula is tested with numerical EE results. Further the localization length l_H related to the information entropy S_{info} is studied numerically using EGOE(1+*k*) (or BEGOE(1+*k*)) as a function of energy for k-body interaction utilizing the interpolating form f_{CqN} of the strength functions and results are compared with ensemble averaged EE results. Also, for bosons using BEGOE(1+*k*), this interpolating form of the strength function is utilized to describe the fidelity decay after *k*-body interaction quench.

Chapter 7 - Effect of Symmetry on Quantum Transport across Disordered Networks Connected by Many-Body Interactions

The efficient transport of particles or excitations (known as quantum efficiency) within a quantum system is a very important as well as a challenging part of nanotechnology. In this chapter, we study influence of centrosymmetry on transport efficiencies of an initial localized excitation in disordered finite network, modeled by EGOE(*k*). Firstly, we analyze disordered fermionic network of *d* sites, modeled by three different ensembles that include many-body interactions: (i) EGOE(*k*) without centrosymmetry, (ii) EGOE(*k*) with centrosymmetry present in both *k* as well as in *m* particle spaces [denoted as csEGOE(*k*)] and (iii) EGOE(*k*) with centrosymmetry present in *k* particle space (not in the *m*-particle space) [denoted as EGOE(*k*-cs)]. Similarly, we also analyze disordered bosonic network modeled by these three ensembles. We found that presence of centrosymmetry enhances quantum efficiency both for fermionic as well as bosonic networks. The results agree with those obtained in the past.

Chapter 8 - Conclusion and Future Outlook

This chapter is an accomplishment of the whole work done in the thesis. Along with that, we also discuss the future prospects of research in the area of quantum many-body chaos and thermalization in isolated finite interacting particle systems.

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Endorsement of the Supervisor:

This synopsis is approved by me.

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