Abstract

Recently notable experimental developments took place in the field of ultra-cold quantum gases and production of Bose-Einstein (BE) condensates in an Earth-orbiting Cold Atom Lab was successful. These experiments allow us to artificially simulate finite interacting many-particle complex quantum systems like atoms, nuclei, interacting spin systems modeling quantum computers, quantum black holes with SYK model, ultra-cold atoms and so on. This in turn has renewed interest in theoretical investigations of these systems and over last few decades it has emerged as an important research area. In spite of the theoretical developments done so far, universal properties which give systematic understanding of behavior of these quantum systems still remains an unsolved problem. The spectral and wavefunction properties of isolated finite interacting particle systems are useful in addressing various open problems of quantum statistical physics like BE condensation, quantum many-body chaos and thermalization. Random matrix theory (RMT) originally introduced by Wishart in Statistics and further introduced by Wigner in Physics to study nuclear spectra, is now established as a good model to describe spectral and wavefunction properties of isolated finite interacting many-particle quantum systems. Dyson gave the tripartite classification of classical random matrix ensembles on the basis of the symmetries preserved by their Hamiltonian viz. Gaussian Orthogonal Ensemble (GOE), Gaussian Unitary Ensemble (GUE) and Gaussian Symplectic Ensemble (GSE). These classical random matrix ensembles (and in particular the GOE) take into account many-body interactions.

However, the constituents of isolated finite interacting many-particle quantum systems interact via few-body interactions in the presence of mean field. This gave rise to a new class of random matrix ensembles called Embedded Ensembles (EE), which incorporate these few-body interactions as well as mean field one body part and hence are more appropriate to describe such systems. For two-body interactions and in the presence of mean-field one body part they are called EE(1+2). The work presented in this thesis focuses on the orthogonal variant of EE called Embedded Gaussian Orthogonal Ensemble (EGOE). It is now well established that these EGOE(1+2) are paradigmatic models to study the dynamical transition from integrability to chaos in isolated finite interacting many-particle quantum systems. For spinless fermion and boson systems they are denoted by EGOE(1+2) and BE-

GOE(1+2) respectively. Moreover, EGOE(1+2) models for spin degree of freedom are also developed and analyzed in detail to study isolated interacting fermion and boson systems with spin degree of freedom.

Recent studies on strongly interacting quantum systems, nuclear physics, quantum black holes and wormholes with SYK model and also quantum transport in disordered networks connected by many-body interactions have shown that higher body interactions i.e. $k > 2$ play a significant role in these systems. This makes it important to extend the study of EE with two-body interactions to higher k-body interactions and they are called EGOE(k) (or BEGOE(k)) for fermions (or bosons) and EGOE(1+k) (or BEGOE(1+k)). The main goal of this thesis is to analyze the spectral and wavefunction properties of finite interacting many-particle fermion and boson systems using EGOEs. This in turn addresses the problems of quantum many-body chaos and thermalization in these systems.

In this thesis the numerical work is carried out by computational simulation by developing FORTRAN codes and the analytical work is done using MATHEMATICA. A brief description of each chapter in this thesis is as follows.

Chapter 1 begins with the recent investigations being carried out on the statistical mechanics of finite interacting particle systems which help in solving various open problems like quantum many-body chaos, BE condensation, thermalization and so on. It is now well established that RMT is a paradigmatic model to study statistical mechanics of finite interacting many particle quantum systems. The field of RMT is introduced along with the three classical random matrix ensembles (GOE, GUE and GSE) classified on the basis of different symmetries, its universality, its applications in various diverse fields, etc. The chapter proceeds by giving a review of EE, which are more appropriate models for realistic systems such as nuclei, atoms, various mesoscopic systems, interacting spin systems and so on. The various developments done in EE over the years are briefly described. Finally the chapter ends with a chapter-wise preview of the entire thesis.

In chapter 2 various EGOEs of one plus two-body interactions which are used in this thesis to model fermion and boson systems with and without spin degree of freedom are introduced. The definition and construction of these ensembles is discussed in detail for the sake of completeness. The chapter begins with spinless fermion and boson systems modeled by EGOE(1+2) and BEGOE(1+2) respectively. Moving further EGOE(1+2)-$\mathbf{s}$ which is used to model fermion systems with spin $\mathbf{s} = 1/2$ degree of freedom is described. Finally, BEGOE(1+2)-F and BEGOE(1+2)-$S1$ used to model boson systems with a fictitious F spin $1/2$ and spin-one degree of freedom respectively are described.

Spacing distributions are popular measures to study spectral fluctuations arising from various complex quantum systems and are modeled through RMT. In chapter 3, two such spacing distributions are studied using the spectra (i.e. eigenvalues) obtained from the Hamiltonian of various EGOEs defined in chapter 2. In the first part of this chapter, the probability distributions of the closest and farther neighbour spacings from a given level are studied for both interacting fermion and boson systems with and without spin degree of freedom. The construction of this spacing distribution involves a cumbersome and non-trivial procedure called unfolding. However if the method of ratio of spacings is used then, this unfolding procedure is not required. Going beyond the method of ratio of spacings, in the second part of this chapter, the probability distribution of higher orders of the ratio of spacings is studied for both interacting fermion and boson systems with and without spin degree of freedom. Further it is shown that the higher order spacing ratio distributions can also reveal quantitative information about the underlying symmetry structure (examples are isospin in lighter nuclei and scissors states in heavy nuclei). The spin EGOEs are used to demonstrate this.

In chapter 4, the study of EE with two-body interactions is extended to higher k-body interactions called the EGOE(k) (or BEGOE(k)) for fermions (or bosons) and EGOE(1+k) (or BEGOE(1+k)). Recently it is found that q-Hermite polynomials are successful in describing the spectral densities in finite interacting particle systems. This chapter begins with the introduction of q- Hermite polynomials, its mathematical formulation and finally the q-normal distribution, conditional q-normal distribution and bivariate q-normal distribution denoted by f_{qN}, f_{CqN} and f_{biv-qN} respectively are discussed. Firstly the analytical formula of parameter q considering only the mean field one body part, $q_{h(1)}$ is derived for both fermions and bosons. Further, the variation of parameter q is studied as the interaction strength λ varies in EGOE(1+k) (or BEGOE(1+k)) for a fixed body rank k. Lastly, the spectral density of EGOE(1+k) and BEGOE(1+k) is studied by using all this knowledge of q-Hermite polynomials.

Now going beyond the spectral properties, the upcoming two chapters 5 and 6 use the eigenfunctions of these systems firstly to analyze their wavefunction properties and then study various important quantities using these properties. In chapter 5, firstly the strength functions (also known as local density of states) are analyzed which form the basis of wavefunction properties. For both EGOE(1+k) and BEGOE(1+k) the strength functions are analyzed along with its width and it is shown that the strength functions are well described by f_{CqN}. A complete analytical description of the variance of the strength function in terms of the correlation coefficient ζ, as a function of λ and k is derived. Also, analytical expression of the marker λ_t which defines thermalization region is derived in terms of m, N and k using the analytical expression of ζ. Lastly the analysis of the lower order moments of the strength functions is presented.

In chapter 6, the analysis of strength functions presented in chapter 5 is used to study two quantities viz. number of principal components (NPC) and localization length l_H (related to information entropy) that quantify chaos in finite interacting quantum systems. Firstly the analytical formula for NPC as a function of energy is derived for k body interaction in terms of two parameters ζ and q. This formula is derived using interpolating form f_{CqN} of the strength functions and is tested with numerical EE results for both EGOE(1+k) and BEGOE(1+k). Next l_H is studied numerically using both EGOE(1+k) and BEGOE(1+k) as a function of energy for k body interaction utilizing interpolating form f_{CqN} of the strength functions and results are tested with numerical EE results. In the end of this chapter fidelity decay after k-body interaction quench is studied for bosons using BEGOE(1+k) and it is shown that the interpolating form of strength functions describes this fidelity decay.

In chapter 7, EE with k-body interactions are used to study quantum efficiency which is a very important as well as challenging part of nanotechnology. EGOE(k) and BEGOE(k) are used to model disordered fermionic and bosonic networks respectively to study the transport efficiencies in these networks and it is found out that centrosymmetry present in m-particle space enhances transport efficiency in a small network and results are in good agreement with the past results. The results also verify that in order to enhance quantum efficiency across these networks centrosymmetry is essentially needed in both k as well as m particle spaces.

Finally chapter 8 presents the conclusions drawn from each chapter of this thesis and also discusses about the future directions of the entire work presented in this thesis.