Introduction

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The subject matter of the research is mathematical aspect of North Indian Classical music. While it can hardly be claimed that all the aspects of music *(North Indian or otherwise)* can be explained on a mathematical or a scientific basis, it stands to reason that sound and its effect on human ears being a physical phenomenon, the origin of Notes, Shrutis, Grams, Thatas and Ragas must have a mathematical explanation also.

It is well known that sound is produced as a result of physical vibrations, whether they are directly visible or not. The vibrations have three aspects - amplitude, frequency and the pattern. Accordingly, any sound *(musical or otherwise)*, has three characteristics - loudness *(related to the frequency of vibrations)* and quality *(related to the pattern of vibrations)*. For the purpose of the present research work, it is the frequency of vibrations which plays a pivotal role in the origin of Notes, Shrutis, etc. and the determines the inter relationship of different Notes, Shrutis and also the rules for the construction of Gram, Thata and Raga. A mathematical inquiry into the structure of classical music reveals that not only can we explain the exact pitch or frequency of the accepted Notes and Shrutis but can also derive new Notes and Shrutis and new Grams and Ragas based on certain mathematical principles. This has been the purpose of the present research work which has resulted in a number of significant conclusions which go a long way in proving a strong mathematical basis of the system of classical music and may also pave the way for inventing new pattern of music.

The first chapter explains the background of physics and mathematics which is needed for the present study and the research work. The chapter begins with the physical concept of sound and how the sound is produced by vibrations and the theory of wave propagation in which energy is transferred from particle to particle in an elastic media. It has been explained that one kind of sound is distinguished from another by three qualities, namely, loudness, pitch and quality which depend on the characteristics of vibrations. The concepts of frequency, wave-length, amplitude etc. have been explained. It has been explained how the *"Swara"* or a Note depends upon the frequency of vibrations which, in its turn, depends upon the length of the vibrations which, in its turn, depends upon the length of the vibrated chord or vibrated ear column. while the frequency of vibrations distinguishes one *"Note"* or shruti from another, it is the quality of the sound *(depending upon the pattern of vibrations)* which distinguishes a musical sound from a non-musical one and also distinguishes a voice of one singer from another. Contrary to what may appeal to the common sense, a *"pure tone"* which has a regular shape of sine/cosine - curve shape - does not constitute a musical note because it is not pleasant to the human ears. A *"compound note"* which has an irregular shape on the vibration graph consists of a series of pure notes which are suitably superimposed and give rise to harmony. The human ears are familiar with this harmony because most of the sound produced in nature consists of compound notes. This familiarity manifests as *"pleasantness"* In short, musical notes are compound notes rather than *"Pure tones"*. It has been explained that the human ear perceives ratio of frequency rather than their differences. This quality is called logarithmic response. This quality is called logarithmic response.

However, when two notes having almost *(but not exactly)* similar frequencies are sounded simultaneously, the ear perceives the difference of these frequencies which gives rise to the phenomenon of beats. It is this phenomenon of beats which is utilized by musicians to tune their musical instruments. First, the two notes to be tuned to each other are sounded separately, when the ratio of their frequencies is observed and the two notes are adjusted until they have sounded simultaneously and re-adjusted until the beats disappear. The phenomenon of resonance has also been explained in this chapter which arises when two or more notes having exactly the same frequency are sounded together. This chapter, therefore, lays down the foundation of a mathematical analysis into the structure of classical music.

In the second chapter, the origin of musical scale has been studied purely from a mathematical point of view. The special question which has been examined is why only certain frequencies produce pleasant or musical effect in combination with the base note. It must be remembered that in any interval between two notes, there are infinite musical Shrutis and infinite *"non-Shrutis"* which are not musical. The evolution of a musical scale has been explained with the help of a theory of harmonics. Construction of a musical scale has been studied by adopting *"third harmonic method"* and *"fifth harmonic method"*. Since it is not possible mathematically to construct "perfect musical scale" by adopting third harmonic or fifth harmonic method or any other method, at no stage of our construction of a musical scale, do we come back to the same musical note from which we started. On account of this, compromise will have to be made to close the musical scale and this has given rise to a multiplicity of a system of musical scales. The different types of musical scales have been discussed in this chapter including those invented by ancient musicians. The tone tempered scale has also been discussed and it has been established mathematically how it makes unwarranted compromise with the accuracy of notes for the sake of achieving a practical convenience. The origin of Shruti has also been discussed from the mathematical point of view. The exact frequencies of the 22 Shrutis as derived by the Veena experiment described by Bharat have been calculated and it has been established that all the Shrutis are not equal. The musical properties of Shadaj Gram and Madhyam Gram have been compared with respect to musical relationships between different Shrutis. Musical relationships are no longer subjective but dependent on exhibiting certain mathematical ratios between the Shrutis concerned, which is a scientific basis. These ratios are called Shadaj-Pancham Bhav *(equivalent to the ratio of 3/2)*, Shadaj-Madhyam Bhav *(equivalent to the ratio of 4/3)*, Shadaj-Gandhar Bhav *(equivalent to the ratio of 5/4)* and Shadaj-Komal Gandhar Bhav *(equivalent to the ratio of 6/5)* respectively. The Shrutis separated by these ratios are bound to be musical. Apart from Shadaj-Madhyam Gram, a third gram, namely Gandhar Gram has also been discuses. Although no accounts of Gandhar Gram are available in any ancient literature. The notes and Shrutis of this Gram have been derived on mathematical principles which can be said to be the original work of this research study. As has been explained in the chapter, the Shrutis of the Gandhar Gram turn out to be such that a very large number of them exhibit Shadaj-Gandhar Bhav *(either equivalent to ratio of 5/4 or 6/5)*. Since Gandhar Gram is altogether new to the field of Indian Classical music, this opens up a new vista of Ragas arising out of Gandhar Gram. One such Raga has been constructed as a part of this research work on mathematical principles taking care to maximizing the number of notes exhibiting musical relationship with each other.

The next chapter is on general mathematical theory of Grams in which Shrutis have been derived purely from mathematical considerations and also taking into account the requirement of maximizing musical relationship and eliminating those notes which are musically inconsistent with the others. Starting from these numbers, the different Grams have been constructed on theoretical basis and it has been shown that although only 3 Grams have been mentioned in the ancient literature out of which only two have been described *(Shadaj and Madhyam Gram)*, many more Grams can be constructed on a theoretical basis which are musically as rich as the two Grams in the ancient literature. If we discard the requirement of musical relationship *(unthinkable in any system of music)*, the number of Grams which can be constructed out of 22 Shadaj is more than 12 crore, but of course, all these Grams are not musical. When different conditions are imposed on the location of Shadaj in order to make them musical, the number comes down drastically. Still, there are 60 Grams which are as musical as the two or three ancient Grams, at least in terms of consecutive note-ratios. When other requirements of musical relationship are taken care of, we are left with still 22 Grams which are at least as rich in musical relationship as the Shadaj Gram. Thus the research work has established that as against only 3 Grams mentioned in the ancient literature *(out of which only 2 have been described)*, there are 22 Grams which merit attention, on the basis of their musical properties.

In the next chapter, Notes and Shrutis in actual use have been discussed. First, the question has been tackled as to how many notes should form octave. It has been calculated that a musical scale may have either 7 notes or 12 notes and both systems have their advantages and disadvantages. Both systems have been discussed in detail. Coming to Shrutis, the different claims made by musicians have been discussed regarding the exact location of 22 Shrutis as described by Bharat. However, the study has made it clear that the Indian classical music cannot be limited by 12 notes or even 22 Shrutis and, in fact, Shrutis more than 22 are actually employed in Indian classical music, consciously or unconsciously. This point has been established with reference to the discussion of several Ragas.

Next, the origin of Shruti, defined as the shortest possible musical interval distinguishable by human ears has been discussed from the mathematical point of view by considering different for different types of scales *(7-note scale or 12-note scale)*, although the octave would be the same. This is because the range available for rearrangement of the basic Shruti ratios is different with different notes of reference. When we take certain notes as reference points, we come to a set of "first order Shruti" and then the number is 88 as against 22 recognized by the ancient literature. However, we can take many more Shrutis by taking these Shrutis as a reference notes and yet more Shrutis by taking new Shrutis as reference point. In this manner we can go on getting the second order, third order, 4th order................ Shrutis and this process will continue indefinitely. That is why it has been stated that musical Shrutis are infinite. Even then any arbitrary frequency is not musical since the musical Shrutis are infinite, there is always a scope for inventing a new Shruti in the same range to produce and entirely new shade of musical effect... In different melodies and in different moods, new notes and Shrutis are always employed according to the exact musical relationship required between the relevant notes.

No musician calculates this relationship and very few even realize which note or Shruties being played or sung, but owing to the musical sense with which every musician is equipped, the right note or Shruti crops upto to fit in with the requirements of the mood and the rules of the melody.

In the next chapter, Thatas have been discussed and it has been explained what is the fundamental difference between the concept of a Gram and that of a Thata. Many Thatas come from the same Gram because the notes of the same Gram are employed using a different note as the fundamental note. Of course, as long as the cyclic order remains the same, it is recognized as the same Gram. The number of Thatas as derived mathematically from 12-note system and each Thata has been established that each Thata has a number of variations in which notes are slightly different *(differing by a Shruti)* which pertain to different Grams. All these variations are employed in practice according to the requirement of the melody in question. In the process, different versions of Thata come to light which are not obvious. In any literature and which can be revealed only when all possible alternatives are discussed keeping the requirement of musical relationship intact.

In the next chapter, the Ragas or melodies have been defined and derived. The definition of a Raga is the same as adopted in the ancient literature but the properties of the Ragas have been discussed form the point of view of the mathematical requirements of musical relationship and it has been established that different Shrutis of the same note are employed in different Ragas and sometimes in the same Raga also according to the requirement of musical relationship between certain notes. A new Raga has been constructed from the Gandhar Gram as developed in the earlier chapter by defining all its characteristics and showing mathematically that it would be rich in musical relationship. In a similar fashion, many other hitherto unexplored Ragas can be constructed from the Thatas which are in existence as well as from new Thatas which have not yet been explored. The Raga is a frame-work of notes arising out of Thata bound by a set of rules in the nature of certain conditions or restrictions leaving a lot of scope for composition of tunes. However, apart from the general set of rules, a Raga has its own special characteristics which are difficult to define but easy to demonstrate. In order to construct a Raga from Thatas, an important step is to select Aaroh and Avaroh in such a way that makes possible a number of musical relationships which are exploited.

In the concluding chapter, the basic conclusions of the research work have been highlighted including the mathematical and physical basis of the notes employed in Indian classical music and their combination into Grams, Thatas and Ragas, the conditions necessary for certain combinations of musical notes to have a pleasant and musical effect on the ears and the method that evaluates musical richness of a particular scale on mathematical basis. A musical note has to be a compound note capable of generating harmonics rather than a pure tone once the fundamental note has thus been established, those notes appear musically related to this note which have a predominant frequency equal to its 3rd., 4th, 5th. and 6th. harmonics. These harmonics give rise to Shadaj-Pancham Bhav *(frequency ratio of 3/2)* and Shadaj-Shuddha Gandhar Bhav *(frequency ratio 5/4)*.

From Shadaj-Pancham Bhav, another relationship known as Shadaj-Madhyam Bhav *(frequency ratio 4/3)* can be derived by raising a lower note by an octave or by lowering the higher note by an octave. Similarly, by raising the lower note by a frequency ratio of 3/2, or by lowering a higher note by the same ratio, Shadaj-Shuddha Gandhar Bhav is converted into Shadaj-Komal Gandhar Bhav *(6/5)*. Thus four fundamental musical relationships were derived having frequency ratio 3/2, 4/3, 5/4 and 6/5.

Ideally, in a musical scale containing seven note, notes separated by three notes *(Sa-Pa, Re-Dha etc.)* are expected to exhibit Sa-Pa type relationship, notes separated by two notes *(Sa-Ga, Re-Ma)* are expected to exhibit one of the Shadaj-Gandhar type relationships. However, the research study has established that theoretically it is impossible for any musical scale to exhibit these relationships in all possible pairs. At least one pair would fail to exhibit the relationship it is expected to. The ideal method of construction of a musical scale could be to follow the successive third harmonic or fifth harmonic method. However, it is mathematically impossible for either of these sequences to close at any stage, although the successive third harmonic method gives 12 notes before coming to a note 13th. fairly close to the starting note. Since a prefect musical scale is impossible in this sense, there has been a multiplicity of scales which have their own advantages and disadvantages.

The Grams *(seven-note scales)* defined by Bharat were constructed on very sound scientific basis. Although a full description of Gandhar Gram is not available, the study has made a conjecture of its notes and Shrutis and how rich it is in terms of Shadaj-Gandhar relationships.

The musical richness of a scale in terms of the umber of airs which exhibit the relationship they are expected to, has been defined. When we proceed in this manner, we get 22 Grams as against 3 mentioned in ancient literature. In fact, if we admit certain unusual frequency ratios between successive notes, we get 17 more Grams.

It has been established in the research study that although Shrutis are infinite, it is possible to derive 88 order Shrutis on theoretical consideration as against 22 Grams as against 3 mentioned in ancient literature. In fact, if we admit certain unusual frequency ratios between successive notes, we get 17 more Grams.

It has been established in the research study that although Shrutis are infinite, it is possible to derive 88 order Shrutis on theoretical consideration as against 22 traditional Shrutis. Many of these 88 Shrutis are in use consciously or unconsciously.

Thatas are seven-note scales in which the notes are in a particular order *(in a Gram, only cyclic order of the note is important)*. The number of mathematically possible Thatas and the number of Thatas which are musically rich on the basis of yardstick developed upon have been discussed. Also musically richness and the prevalent Thatas along with their possible variation has also been discussed.

Sometimes, by varying a note slightly *(by a Shruti),* we move into a different Gram although, Thata appears to be unchanged. As compared to Gandhar, a Thata is a more crude system of defining a musical scale. The development of Murchhana, Taans and Jatis *(Prototype of Ragas)* from ancient Grams have been discussed. It has been explained how various Shrutis are employed in different Ragas or out of the same Raga in order to highlight a particular musical relationships.