Chapter - 3

The Shrutis

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The Theory of Grams

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The oldest system of notes and sub-notes called Shrutis has been described by Bharat *(5th. Century A.D.)* who seems to be the earliest authority on the subject. Bharat has divided a Saptak into twenty-two Shrutis but has regarded only seven notes as fundamental called Shadaj, Rishabha, Gandhar, Madhyam, Pancham, Dhaivat, and Nishad. The collection of these seven notes is termed as *"Gram"*. In each *"Gram"*, the seven notes are situated on different Shrutis, which means that the seven notes of one Gram may differ from those of another Gram. These Grams have been mentioned by Bharat-Shadaj Gram, Madhyam Gram and Gandhar Gram. Shadaj Gram and Madhyam have been discussed in detail by Bharat and other musicians of later ear, but Gandhar Gram has not been explored at all beyond the remark that it is in use in *"heaven"* and not earth.

In the Shadaj Gram, which is the most fundamental Gram, the situation of the seven notes is as given in the chart below :

1.	Teevra
2.	Kumudvati
3.	Manda
4.	Chhandovati-Shadaj
5.	Dayavati
6.	Ranjani
7.	Raktika-Rishabha
8.	Raudri
9.	Krodhi - Gandhar
10.	Vajrika
11.	Prasarini
12.	Priti
13.	Marjani - Madhyam
14.	Kahiti
15.	Rakta
16.	Sandipani
17.	Alpini - Pancham
18.	Madanti
19.	Rohini
20.	Ramya - Dhaivat
21.	Ugra
22.	Kshobhini - Nishad

Thus the seven notes of Shadaj Gram are situated on 4th., 7th., 9th., 13th., 17th., 20th. and 22nd. Shrutis respectively. Bharat has prescribed that Shadaj, Madhyam and Pancham have four Shrutis each, Rishabha and Dhaivat three Shrutis each and Gandhar and Nishad two Shrutis each. Thus, all the Shrutis between a note and the preceding note *(including the Shruti on which the note is situated)* are said to belong to the note. The note Shadaj, for example, has an interval of 4 Shrutis from its preceding note Nishad and hence it is said that Shadaj has 4 Shrutis.

Thus, there are three kinds of intervals between the consecutive notes :

1) Interval of four Shrutis - Ni *(Mandra Saptak)* - Sa, Ma-Pa, Ga-Ma
2) Interval of three Shrutis - Sa-Re, Pa-Dha
3) Interval of two Shrutis - Re-Ga, Dha-Ni

According to Bhatkhande, all the Shrutis mentioned by Bharat are equal, which means that the ratio of the frequency of second Shruti to the first is equal to that of the third Shruti to the second and so on. Thus, each Shruti bears a constant frequency ratio with its preceding one. If this constant ratio be x, then it must be such that multiplying it 22 times one gets the number two *(starting with Madhya Sa, after raising it by a Shruti 22 times, we must come to Sa of Tar Saptak which has a frequency double that of the Sa of Madhya Saptak).*

Thus XxXxXxXxX .... 22 times = 2
or $X^{22} = 2$
or $X = 2^{1/22}$ or twenty second root of 2.

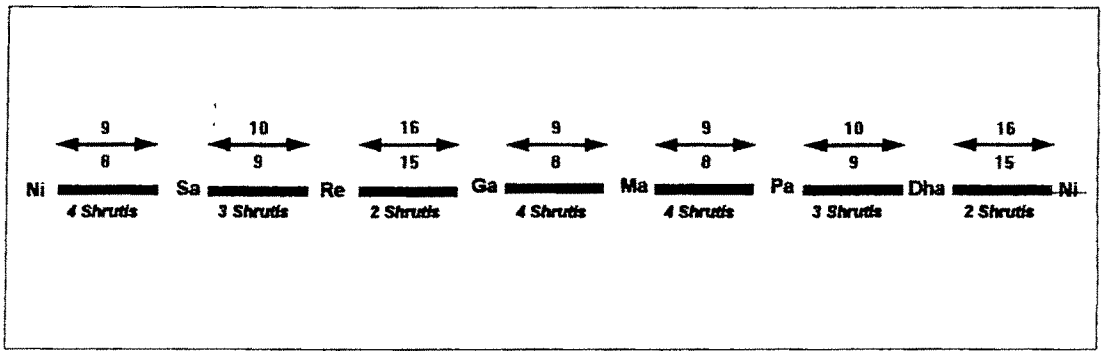
This only gives the equal tempered scale which was discussed in the last chapter. We have already concluded that this cannot be a musical scale. Accordingly equal Shrutis cannot be musical.

However, Pandit Omkarnath Thakur, a well known musician from Gujarat has offered a brilliant interpretation of Bharat's Shrutis which proves that these Shrutis were not equal and he has even arrived at the exact value of the frequency of each Shruti. We give a short account of his treatment of the subject.

It has been presumed that, in Shadaj Gram, the interval of 4 Shrutis between the two notes is equal to a frequency ratio of 9/8 an interval of 3 Shrutis is equal to a frequency ratio of 10/9 and an interval of two Shrutis is equal to a frequency ratio of 16/15.

Thus the arrangement of the seven notes takes the following form.

Diagram - 11



It may be remarked that following musical ratios exist between various pairs of notes :

Sa - Pa = 10/9 X 16/15 X 9/8 X 9/8 = 3/2

Re - Dha = 16/15 X 9/8 X 9/8 X 10/9 = 3/2

Ga - Ni = 9/8 X 9/8 X 10/9 X 16/15 = 3/2

Ma- Sa = 9/8 X 10/9 X 16/15 X 9/8 = 3/2

Pa - $\overline{\text{Re}}$ = 10/9 X 16/15 X 9/8 X 10/8 = 3/2

Which are third harmonic relationships. Bharat also has mentioned that Shadaj-Pancham Bhav *(third harmonic relationship)* is essential between successive pairs of notes in Shadaj Gram in the same Saptak.

The diatonic scale of western music also is composed of seven notes in such a way that the frequency ratios are referred to as major tone, minor tone and half tone. The Shuddha notes of the modern scale of Indian classical music also are composed of the above ratios only. Thus the following presumption appears to be quite reasonable.

4 - Shrutis interval = a ratio of 9/8
3 - Shrutis interval = a ratio of 10/9
2 - Shrutis interval = a ratio of 16/15

Regarding the frequency of the remaining intervening Shrutis, Bharat has not defined it as a mathematical ratio or the length of a vibrating chord, but he has described the following experiment in four states which throws some light about their magnitude.

A Veena with seven different chords is tuned in such a way that the successive chords are tuned to the seven notes *(Sa, Re, Ga, Ma, Pa, Dha and Ni)* of Shadaj Gram respectively. This Veena is kept tuned in this manner throughout the experiment and is called the fixed Veena. Another Veena is also tuned exactly as the first Veena initially but its tuning is varied during the course of the experiment. This latter Veena is called the variable Veena.

In the first stage of the experiment, the Pancham of the variable Veena *(which was initially turned to the Pancham of the fixed Veena)* is lowered in Pitch by *"a Shruti"* Bharat has not specified what is a Shruti, but value can be inferred by a little consideration.

In the original notes of Shadaj Gram,

the ratio Re-Pa = Re - Ga X Ga-M X Ma - Pa
= 16/15 X 9/8 X 9/8 = 54/40 = 27/20

which is approximately but not equal to 4/3.

The ratio 4/3 called Shadaj-Madhyam Bhav and is in fact the third harmonic relationship in reverse. For example, there is a third harmonic relationship between Pa and Sa *(of the original notes of the Shadaj Gram)*. If Pa is reduced by an octave, then its ratio with Sa become 2/ 3/2 = 4/3. Similarly, if Sa is raised by an octave, then also the ratio of its frequency with Pa becomes 4/3. Moreover, the relationship of Ma with the Sa of Tar Saptak is 3/2 which is third harmonic relationship while the relationship of Ma with the Sa of Mandra Saptak is 4/3. Hence, it follows that when two notes are related by third harmonic relationship *(frequency ratio 3/2)*, raising the lower note by an octave *(doubling the frequency)* or lowering the higher note by an octave *(halving the frequency)* transforms this relationship to 4/3 which is called Shadaj-Madhyam Bhav. Thus the frequency ratio of 4/3 or Shadaj-Pancham Bhav, is as fundamental and musical as the third harmonic relationship or Shadaj-Pancham Bhav, because it is obtained by changing the octave of one of the notes related by a frequency ratio 3/2.

It follows that if we lower the frequency of the initial Pancham note by a ratio 81/80, its relationship with Re becomes 27/20 X 80/81 = 4/3 which is Shadaj-Madhyam Bhav.

Therefore, in the first stage of the experiment, we have to tune Pancham by lowering its pitch until a Shadaj-Madhyam relationship is established between Re and the new Pa. This process is very simple for a musician because the Shadaj-Madhyam relationship is easily recognized by the ears. Mathematically, we have shown that Pancham will have to be lowered by a frequency ratio 81/80 in this process.

The *"Shruti"* by which the Pancham is lowered at this stage must be equal to a ratio of 81/80. This, then is the ratio between 17th. Shruti and 16th. Shruti. It may be remarked that this new arrangement of notes, in which Pancham is lowered by a Shruti *(81/80)* and other notes are the same as in the original Shadaj Gram, is called Madhyam Gram by Bharat, the only difference between Shadaj Gram and Madhya Gram is that Pancham is on 17th. Shruti in Shadaj Gram, but on 16th. Shruti in Madhyam Gram.

Now, the other notes of the variable Veena also are to be depressed by a Shruti each *(Shadaj should be depressed from 4th. shruti to 3rd. Shruti, Rishabh from 7th. to 6th. Gandhar from 9th. to 8th. Madhyam from 13th. to 12th., Dhaivat from 20th. to 19 the and Nishad from 22nd. to 21st.)* so that the notes of the variable Veena again get tuned according to the Shadaj Gram system. To be more precise, the frequency ratio between any two notes must be the same as between the corresponding notes of the fixed Veena. This is possible only if all the notes are lowered by the same ratio 81/80. This is done so that the entire scale is depressed by a frequency ratio 81/80. This is stage 1 of the experiment shown in the *Diagram No.13.* ?

It is clear from the first stage of the experiment that the following Shruti intervals are equal
4-3
7-6
9-8
13-12 *(each of these intervals being equal to 81/80)*
17-16
20-19
22-21

In the second stage of the experiment, all the note are lowered by the same *(but a new)* frequency ratio until the Gandhar and Nishad of the variable Veena coincide with Rishabha and Dhaivat respectively of the fixed Veena. Since all the notes have been lowered by the same frequency ratio in the second stage also, relatively the original pattern of Shadaj Gram is maintained. It will be found that in the second stage while the Gandhar and Nishad coincide with Rishabha and Dhaivat respectively of the fixed Veena, other notes still remain higher than those corresponding preceding notes on the fixed Veena which shows that in the fixed Veena, the ratio Ni-Sa, Sa-Re, Ga-Ma, Ma-Pa, Pa-Dha a bigger than the ratio's Re-Ga and Dha-Ni while the two latter ratios are equal.

We have already assumed the value of Re-Ga and Dha-Ni to be equal to 16/15 *(seem tone of Western diatonic scale)*. Hence, it follows that 81/80 X = 16/15
(the ratio by which the pitch of every note is lowered in the second stae).

(Remembering that in the first stage the notes were lowered by a ratio 81/80) which shows that the value of the frequency ratio by which all notes of the variable Veena have been lowered in the second stage - 16/15 X 80/81 = 256/243.

Thus the second state of the experiment shows that the following Shruti - ratios are equal to 256/243

3-2, 6-5, 8-7, 12-11, 16-15, 19-18 *and* 21-20

(2,5,11,15 and 18 Shrutis are defined by the second stage of the experiment).

Diagram 12

Sr.No.of Shruti	Initial arrangement of notes in Shadaj-Gram *(Fixed Veena)*	1st. Stage of variable Veena	2nd. Stage of variable Veena	3rd. Stage of variable Veena	4th. Stage of variable Veena
22	Nishad				Shadaj
1				Shadaj	
2			Shadaj		
3		Shadaj			
4	Shadaj			Rishabha	
5			Rishabha		
6		Rishabha			
7	Rishabha		Gandhar		
8		Gandhar			
9	Gandhar				Madhyam
10				Madhyam	
11			Madhyam		
12		Madhyam			
13	Madhyam				Pancham
14				Pancham	
15			Pancham		
16		Pancham			
17	Pancham			Dhaivat	
18			Dhaivat		
19		Dhaivat			
20	Dhaivat		Nishad		
21		Nishad			
22	Nishad				

(Shruti No.22 shown at the top is an octave lower than the same shruti list at last).

In the third stage, the notes Shadaj, Rishabha, Madhyam, Pancham and Dhaivat are lowered by another Shruti *(it is note necessary to touch Gandhar and Nishad because their ratio with the corresponding Rishabha and Dhaivat has already been estatablised)* in such a way that Rishabha and Dhaivat coincide with Shadaj and Pancham respectively of the fixed Veena other notes are also depressed by the same ratio. This operation defines new Shrutis 1st., 10th., 19th. and shows that following ratios are equal.

2-1, 5-4, 11-10, 15-14, 18-17

To calculate the value of this ratio,
it should be noted that the Sa-Re is equal to 7-6 x 6-5 x 5-4 =10/9 *(minor tone).*

Now 7-6 in the ratio 81/80 corresponding to the first stage and 6-5 is 256/243,
corresponding to the second stage.

Hence, 5-4 = 10/9 x 80/81 x 243/256 = 25/24 and the same ratio is between Shrutis 2-1, 11-10, 15-14 and 18-17. The third stage of the experiment shows that the ratios Sa-Re and Pa-Dha are equal.. Sa, Ma and Pa still remains higher than Ni *(of a lower octave),* Ga and Ma respectively of the fixed Veena showing that the intervals Ni-Sa, Ga-Ma and Ma-Pa are higher.

In the fourth stage, the notes Sa, Ma and Pa of the variable Veena are lowered by the same frequency ratio until they coincide with Ni, Ga and Ma respectively of the fixed Veena. This process does not define any new Shrutis but shows that the following ratios are equal.

1-22, 109, 10-9, 14-13

(Here "Shruti 22" means the shruti No.22 of Madhya Saptak lowered by an Octave).

To calculate the value of one of these ratios,
let us take the ratio Ga-Ma which we know to be equal to 9/8 *(Major tone).*
This ratio is composed of four Shruti ratios 10 - 9 x 11 - 10 x 12 - 11 x 13 - 12.

But 13-12 is the ratio defined by 1st. stage = 81/40

12-11 is the ratio defined by 2nd.Stage = 256/243
13-12 is the ratio defined by 3rd.Stage = 25/24

Hence Ga - Ma = 81/80 x 256/243x25/24x10-9=9/8

Hence the ratio 10 - 9 = 9/8 x 80/81 x 243/256 x 24/25 = 81/80

Thus the ratio 10-9 = 14-13 = 22-1 = 81/80
which is equal to the ratio by which the notes were lowered at the first stage.

According to these interpretations, we have been able to arrive at the exact frequencies of all the 22 Shrutis which is given below -

Table No.12

S. No. of Shruti	**Successive frequency ratio** ***(with respect to the preceding Shruti)***	**Name of the note** ***(Shadaj Gram)***	**Actual frequency adopting the frequency of Shadaj as 240**
22 *(of lower octave)*			
1	81/80		216
2	25/24		225
3	256/243		237 *1/2*
4	81/80	Shadaj	240
5	25/24		250
6	256/243		263 *91/293*
7	81/80	Rishabha	286 *2/3*
8	256/243		279 *509/729*
9	81/80	Gandhar	284 *4/9*
10	81/80		288
11	25/24		300
12	256/243		316 *4/81*
13	81/80	Madhyam	320
14	81/80		324
15	25/24		337 *1/2*
16	256/243		355 *5/9*
17	81/80	Pancham	360
18	25/24		337 *1/2*
19	256/243		395 *51/81*
20	81/80	Dhaivat	400
21	256/243		421 *97/243*
22	81/80	Nishad	426 *2/3*
1	81/80		432
2	25/24		450
3	256/243		470 *2/3*
4 *(of a higher octave)*	81/80	Shadaj	480

It is observed by surveying Table-12 that :

1). **There are three kinds of successive frequency - ratios**
81/80 *(which is the ratio by which the notes are lowered in the first and fourth stage),*
256/243 *(being the ratio by which the notes are depressed in the second stage), and*
25/24 *(by which the notes are depressed in the third stage).*

Any frequency ratio between two consecutive Shrutis is one of these three ratios.

2) **There are three kinds of ratios between consecutive notes**
   a) Ni-Sa, Ga-Ma, Ma-Pa which is equal to 9/8 ***(Major tone)***
   and is composed of 4-Shruti ratios :
   81/80 *x* 25/24 *x* 256/243 *x* 81/80 = *9/8*
   b) Sa-Re, Pa Dha which equals 10/9 ***(Minor tone)***
   and is composed of 3 Shrutis ratios :
   25/24 *x* 256/243 *x* 81/80 *(=10/9)*
   c) Re-Ga and Dha-Ni which is equal to 16/15 ***(Semi tone)***
   and is composed of 2 Shruti ratios
   256/243 *x* 81/80 *(=16/15).*

Following pairs of notes separated by 13 Shrutis
have third harmonic relationship with each other

1-14
2-15
3-16
4-17
5-18
6-19
7-20
8-21
9-22
10-1*
11-2*
12-3*
13-4*
14-1*

* *of a higher octave*

Thus, corresponding to each Shruti, there is another Shruti *(in the same Saptak or in a higher Saptak)* thirteen Shrutis higher or thirteen Shrutis lower which exhibits a third harmonic relationship with it.

This can be verified by multiplying the corresponding frequencies given in column 4, the product will be 3/2 in each pair.

The last property is of great significance and establishes how important the third harmonic relationship was regarded by Bharat. Not only the notes but the 22 Shrutis are also bound by this relationship in such a way that each Shruti has a corresponding Shruti related to it by a third harmonic relationship.

There appears to be no doubt that the Shrutis defined by Bharat were unequal and their exact frequencies are as given in the above table. It has been shown that they follow quite logically from the experiment described by Bharat once the 4 Shruti, 3 Shruti and 2 Shruti intervals are presumed to be the ratios 9/8, 10/9 and 16/15 respectively.

Coming now to the Madhyam Gram described by Bharat, it was remarked while describing the first stage of Bharat's experiment that when Pancham is depressed from 17th. to 16th. shruti, such that it has a Shadaj-Madhyam relationship with Rishabha, this arrangement has been called Madhyam Gram.

Thus, in Madhyam Gram
Shadaj is on 4th.Shruti,
Rishabha on 7th.,
Gandhar on 9th.,
Madhyam on 13th.,
Pancham on 16th.,
Dhaivat on 20th. *and*
Nishad on 22nd.

Except Pancham, all the other notes are on the original Shrutis according to the straight forward interpretation of Bharat's description. Pandit Omkarnath Thakur has not taken the starting shruti of Shadaj Gram as the starting point of Madhyam Gram. By locating the Shadaj of Madhyam Gram on 17th. Shruti *(Alapani)* where Pancham of the Shadaj Gram is located, he has given the following table for the Shrutis and notes of Madhyam Gram.

It will be observed that the successive frequency ratios of the Shrutis remain the same, but the absolute frequency appears changed on account of the fact that the frequency of 17th. Shruti *(Alapini)* has been adopted as 240.

Table No.13

Sr.No.of Shruti	Sucessive frequency ratio *(with respect to the preceding Shruti)*	Name of the Note *(Madhyam Gram)*	Actual frequency *(adopting the frequency of Shadaj as 240)*
14	81/80		216
15	25/24		225
16	256/243		237 1/2
17	81/80	Shadaj	240
18	25/24		250
19	256/243		263 91/243
20	81/80	Rishabha	266 2/3
21	256/243		280 680/729
22	81/80	Gandhar	284 4/9
1	81/80		288
2	25/24		300
3	256/243		316 4/81
4	81/80	Madhyam	320
5	825/24		333 1/3
6	256/243		351 121/929
7	81/80	Pancham	355 5/9
8	256/243		374 1262/2181
9	81/80		379 7/81
10	81/80		384
11	25/24	Dhaivat	400
12	256/243		421 97/243
13	81/80	Nishad	426 2/3
14	81/80		432
15	25/24		450
16	256/243		474 2/27
17	81/80	Shadaj	480

Following observations can be made from Table-13.

1) Although the successive frequency ratios of the Shrutis are the same as in the Shadaj Gram *(In fact, a Gram is constructed by locating the notes on different Shrutis) the starting note Shadaj is situated on 17th. Shruti in Shadaj Gram it is on 4th. Shruti).*

2) There are again three kinds of freqency ratios between consecutive notes
viz. 9/8, 10/9 and 16/15,
but now Ma-Pa = 10/9 *(it was 9/8 in Shadaj Gram)*
while Pa-Dha = 9/8 which was 10/9 in Shadaj-Gram.

The successive frequency ratios of the notes of Madhyam Gram are as follows :

Ni --	Sa	Re	Ga	Ma	Pa	Dha	Ni
$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$	

3) There are two kinds of 4-Shrutis intervals which have the same frequency ratio but they differ in the order of successive frequency-ratios.

First kind is Ni - Sa and Ga - Ma
in which the order of successive Shruti - ratios is
81/80 x 25/24 x 256/243 x 81/80 *(=9/8).*

Second kind is Pa-Dha
in which the order is $256/243 \times \frac{81}{80} \times \frac{81}{80} \times 25/24$ *(=9/8)*

The difference arises out of the fact that
a didfferent starting point has been chosen for Madhyam Gram
than was chosen for Shadaj Gram.

The cyclic order of the Shrutis is of course, the same.

4) The following pairs of Shrutis *(separated by 19 Shrutis)* exhibit the Shadaj-Madhyam relationship.

16-3 *(of a higher octave)*	6-19 *(of a lower octave)*
17-4 *(of a higher octave)*	7-16
18-5 *(of a higher octave)*	8-21 *(of a lower octave)*
19-6 *(of a higher octave)*	9-22
20-7 *(of a higher octave)*	10-1 *(of a higher octave)*
21-8 *(of a higher octave)*	11-2 *(of a higher octave)*
22-9 *(of a higher octave)*	12-3 *(of a higher octave)*
1-10	13-4 *(of a higher octave)*
2-11	14-1 *(of a higher octave)*
3-12	15-2 *(of a higher octave)*
4-13	15-2
5-18 *(of a lower octave)*	

Their relationships can be verified by multiplying the corresponding frequencies in the table to yiedl 4/3 in each case.

It will be observed that the pairs 5-14, 6-15 and 8-17 of the same octave do not have Shadaj - Madhyam relationship although these are separated by 9 Shrutis.

The last property of the Madhyam Gram is the most significatn. Starting from the 17th Shruti which has Shadaj-Madhyam Bhav with its 9th Shruti *(starting from 17th Shruti, the Shruti of the higher octave becomes the nineth Shruti)*, each consecutive Shruti has Shadaj-Madhyam Bhav with its nineth Shruti, except Shrutis at S. No.5,6 and 8 which do not have this relationship with their nineth Shrutis. However, these Shrutis have Shadaj-Madhyam relationship with the Shrutis 18, 19, 21 *(of a lower octave)* respectively which are nine Shruti below the Shrutis 5, 6 and 8.

If we confine ourselves to one Spatak only, from 17th Shruti to 17th Shruti of a higher Saptak, all Shrutis except 5th, 6th, 8th, 14th and 15th have a corresponding Shruti related by Shadaj-Madhyam Bhav. If the starting note of Madhyam Gram is taken to be 13th Shruti, it can be verified that all the Shrutis in a Saptak *(13th Shruti to 13th Shruti of a higher Saptak)* have a corresponding Shruti in the same Saptak related by Shadaj-Madhyam relationship.

Shadaj-Pancham Bhav is the underlying feature of the Shadaj Gram while Shadaj-Madhyam Bhav is the main feature of Madhyam Gram.

Shadaj Gram and Madhyam are different arrangements in which notes are situated on different Shrutis such that in Shadaj Gram, Shadaj Pancham Bhav is emphasised while in Madhyam Gram Shadaj Madhyam Bhav is emphasised.

Another significance of the two Grams will be evident by the following consideration. Following the successive third harmonic method of constructing notes described in the earlier chapter, we may start from Sa and follow the sequence Sa-Pa-Re-Dha-Ga-Ni-Ma*-Re'-Dha'-Ga'-Ni-Ma-Sa, but we have seen that this Sa is not equal to the starting Sa, but somewhat higher. The ratio between the two Sa's comes out to be approximately equal to 81/80 which is nothing but the freuency ratio between the Pancham of the two Grams. Thus if we start the process of construction of notes by the successive third harmonic method of last chapter, starting from the Panchama of Madhyam Gram, we will, following the cycle of twelve notes arrive at the Pancham of Shadaj Gram which is 81/80 times higher *(of course this will be so if the Shadaj of both the Grams is treated on the same shruti No 4; for, then, all the six notes, except Pancham, are the same in both the Grams)*.

The Shadaj of both the grams mentioned by Bharat can be taken to be situated on the same Shruti so that except Pancham, all the notes of both the Grams are coincident. Pandit Omkarnath Thakur has assumed 17th Shruti to be the starting point of Madhyam Gram having regard to the system in which the chords are tuned on a Veena. If, in a Veena tunned to Shadaj Gram, Pancham is lowered by a ratio 81/80, even though the notes of Madhyam Gram are established, the Veena may appear out of tune. While, if 17th Shruti *(or Pancham of Shadaj Gram)* is taken as the starting note *(Shadaj)* of Madhyam Gram the other notes of Madhyam Gram come naturally without apprearing out of tune, Otherwise, there is not harm in saying that notes of Madhyam are located exactly where the notes of Shadaj Gram are located except that Pancham of Madhyam Gram is 81/80 times lower than that of Shadaj Gram. Bharat perhaps know that by construction of notes by the successive third harmonic method, we can never come back to the same note, but we certainly come to the Pancham of Shadaj Gram starting from the Pancham of Madhyam Gram. Thus the two Grams complete the otherwise endless cycel of the successive third harmonic chain of notes. But this also is only approximate, for the ratio of the final note on which we come back at the end of twelve note cycle is only approximately 81/80.

The third Gram called the Gandhar Gram, has not been explored by Bharat or any musicians of a later era at all. It has been defined in such a way that the ratio between any two consecutive notes is three Shrutis interval except in the case of Sa-Ni where it is four Shruti interval.

Usually the notes of Gandhar Gram have been assumed to be on the following Shrutis :

Sa	-	4th
Re	-	7th
Ga	-	10th
Ma	-	13th
Pa	-	16th
Dha	-	19th
Ni	-	1st

(Edition of Ratnakar published by Adyan Library - first part appendix 4)

But, according to this interpretation, the Gandhar Gram does not seem to have any worthwhile properly. In fact Ga and Dha appear to be rather ackwardly placed.

However, if 2oth Shruti is taken to be the starting point, Sa,
the arrangement comes to the following

Sa	-	20th
Re	-	1st
Ga	-	4th
Ma	-	7th
Pa	-	10th
Dha	-	13th
Ni	-	17th
Sa	-	20th

(Since all note intevals are of three Shruti except Dha-Ni which has four Shrutis).

Accoring to this interpretation,
the successive ratios of the frequencies of the notes come to

Ni	*Sa*	*Re*	*Ga*	*Ma*	*Pa*		*Dha*	*Ni*
10	10	27	10	10	10	_	1.107	1685 $\left(\frac{256 \times (81)^3}{243 \quad 83}\right)$
9	9	25	9	9	9			

This can be easily verified from *Table 12.*
This brings in a very complicated ratio between Dha-Ni.
Moreover, the ratios Ni-Sa, Ma-Dha, Pa-Ni are are not musical.
(Ideally, they should be eigher 6/5 or 5/4 since 5/4 is the fifth harmonic relationship and 6/5 is the relationship of the fifth harmonic to the third harmonic. These relationships will be discussed more later).

If the Gandhar Gram is defined as in *Table 14,* it is seen to contain very interesting properties. It may be sen that the order of the Shrutis has been changed a bit as compared to the order of Bharat's Shrutis *(Table 12).*

First of all

Sa-Ga = Re-Ma = Ga-Pa = Ma-Dha = 6/5 and Pa-Ni = $\overline{\text{Dha}}$-Sa = 5/4.

Also, beteen the Madhyam Sa and Tar Sa, each Shruti has the relationship 6/5 with a Shruti within six Shrutis lower or higher in the same Saptak.

For example.

	4	-	10	-6/5	2-8	- 6/5
	5	-	11	-6/5	3-9	- 6/5
	6	-	12	6/5		
	7	-	13	6/5		
	8	-	14	6/5		
	9	-	15	6/5		
	10	-	16	6/5		
	11	-	17	6/5		
	12	-	18	6/5		
	13	-	19	6/5		
	14	-	20	6/5		
	15	-	21	6/5		
	16	-	22	6/5		
	17	-	11	6/5		
12 - 18 =	18	-	2	6/5		
13 - 19 =	19	-	3	6/5		
	20	-	14	6/5		
	21	-	15	6/5		
	22	-	16	6/5		

Shruti No. 1 is the only exception to this rule
since both the relations 1-7 and 17-1 do not come to 6/5.

But, Shruti No. 1 exhibits the 5/4 relationship with the Shruti No. 8 *(Sapta Shruti Bhav)*.

Table No.14

S. No. of Shruti	Name of the Note	Successive frequency ratio	
1	Ni	25/24	
2		81/80	
3		256/243	
4	Sa	25/24	10/9
5		81/80	
6		25/24	
7	Re	256/243	
8	Ga	81/80	27/25
9		256/243	
10	Ga	81/80	27/25
11		81/80	
12		25/24	
13	Ma	256/243	10/9
14		81/80	
15		256/243	
16	Pa	81/80	27/25
17		81/80	
18		81/80	
19	Dha	256/243	10/9
20		81/80	
21		256/243	
22		81/80	
1	Ni	25/24	9/8
2		81/80	
3		256/243	
4		25/24	10/9
		Sa - Ga - 6/5	
		Re - Ma - 6/5	
		Ga - Ma - 6/5	
		Ma - Dha - 6/5	
		Pa - Ni - 5/4	
		Dha - Sa - 5/4	

In this interpretation of Gandhar Gram *(due to the author)*, no new successive frequency ratio has been introduced apart from 80/81, 256/243 or 25/24, only the order of the Shrutis has been changed. It is notes and all the Shrutis exhibit one of the relationships 6/5 and 5/4 between corresponding pairs separated by Six or Seven Shrutis. Both these relationships are Gandhar-Shadaj relationships *(6/5 is the relationship of modern Komal Gandhar with Sa, while 5/4 is the relationship of modern Shuddha Gandhar fifth harmonic Gandhar with the Shadaj)*.

The name Gandhar Gram was something like this and it was supposed to be unearthly because its order of Shrutis was different than in Shadaj or Madhyam Gram and also becuase a new, and rather peculiar, frequency ratios 27/25 was introduced between two successive notes *(Re-Ga and Ma-Pa)*.

A few words on the musical ratio 6/5 are in order.
The close musical relationship of 5/4 *(Shuddha Gandhar of today)* is not difficult to see, since it is the fifth harmonic of the fundamental note *(reduced to the Madhya Saptak)*.

The third harmonic Pancham 3/2 is predominantly present in the fundamental note.

Now the note which has fifth harmonic relationship with Pancham is the Komal Gandhar of today having the frequency ratio 6/5 with the fundamental note *(Shadaj)*. It has been explained in the last chapter that along with the fundamental note *(Shadaj of Madhyam Saptak)* second harmonic *(Shadaj of Tar Saptak)*, third harmonic Pancham and fifth harmonic Gandhar are always present. Then Komal Gandhar of modern scale (6/5) appears musical on account of its fifth harmonic relationship with Pancham. Also fifth harmonc Gandhar (5/4) and third harmonic Pancham (3/2) are separated by a frequency ratio 3/2x4/5 = 6/5. This also underlines the musical importance of the ratio 6/5.

Four kinds of relationships have been considered most important for the acient times :

1. Shadaj Pancham Bhav or a separation of thirteeen Shrutis which means a ratio of 3/2.
   (It may be noted that any two Shrutis separated by 13 Shrutis may not exhibit what is called "thirteen-Shruti Samvad").

2. Shadaj Madhyam Bhav or a *"nine-Shruti samved"* which means a ratio of 4/3.
   (It may be seen that any two Shrutis separated by 9 Shrutis may not exhibit nine Shruti samvad).

   As has been remarked earlier,
   Shadaj Madhyam Bhav can be converted into Shadaj Pancham Bhav and vice versa by changing the Saptak of one of the notes.

3. Seven-Shruti samvad or a ratio of 5/4. This is the fifth harmonic relationship.
(Any two notes separated by seven Shrutis do not exhibit this relationship).

4. Six-shruti samvad or a ratio of 6/5.
This is the relationship between modern Shudha Gandhar and Pancham.
(A separation by six Shrutis is not sufficient for this relationship).
This relationship is musical because
a note 6/5 has fifth harmonic relationship with the Pancham.

Out of the above four musical relationships, the first one dominates the Shadaj Gram, the second one the Madhyam Gram and the third and fourth are embodied in the Gandhar Gram.

To sum up the discussionof Bharat's Grams and Shrutis so far, seven notes of a pre-determined frequency-ratios with respect to Shadaj were assumed in Shadaj Gram. Starting with the reasonable presumption that the four-Shruti, three-Shruti and two Shruti intervals described by Bharat were the major lone, minor tone and semitone of the modern scale, following the experiment of Bharat consisting of our stages, its possible to derive the exact frequency of each of the 22 Shrutis involved as a ratio of the fundamental frequency.

Lowering the frequency of Pancham by a ratio 81/80 and keeping the other notes as in Shadaj Gram, we get the notes of Madhyam Gram *(although to get these notes in a natural way from Veena, it is advisable to start from the Pancham of Shadaj Gram as will be explained later)*. If the notes Sa, Re, Ga, Ma, Pa, Dha, Ni are situated on Shruti Nos. 4,7,10,13,16,19,1 respectively and the order of Shrutis rearranged, one gets Gandhar Gram in which alternate notes exhibit the Shadaj, Gandhar relationship 6/5 or 5/4. Even the Shrutis exhibit this relationship.

It is worth while explaining how Shadaj-Gram might have originated out of the concept of Shadaj-Pancham Bhav. Starting from Sa, we get its third harmonic Pa. Next, the note Ma is located which has third harmonic relationship with Sa of Tar Saptak, *(the second harmonic)*.

Now, the question is where should Re be located.
If the ratio Sa-Re is kept 9/8 *(as it is in modern scale)*
Re has Shadaj-Madhyam Bhav with Pa,
but Re-Ma becomes non musical *(different from 6/5 slightly)*.

If Sa-Re is fixed as 10/9, its relationship with Pa becomes non-musical *(being different from 4/3)*. Somehow, Bharat attached a lot of importance to Ma. He has often called *"Ma"* as the permanent or unchanging note. Hence, perhaps he chose Re *(10/9)* which had a musical relationship with Ma *(Re-Ma being 6/5)* although Re-Pa is no longer 4/3.

Once Re was chosen, Dha was fixed according to Shadaj-Pancham Bhav. Thus Dha was 10/9 from Pa. Next Ni was chosen so that its relationship with Ma was musical *(Ma-Ni = 4/3)*. This Ni can be easily seen to be 16/15 from Dha. Then Ga had to be 16/15 from Re so that Ga and Ni could be related by Shadaj-Pancham Bhav. Thus, we have all notes of Shadaj Gram in which the following pairs exhibit Shadaj-Pancham *or* Shadaj-Madhyam Bhav.

Sa	-	Pa	=	3/2
Re	-	Dha	=	3/2
Ga	-	Ni	=	3/2
Ma	-	Sa	=	3/2
Ma	-	Ni	=	4/3

The following pairs have the 5/4 relationship :

Ni	-	Re	=	5/4
Ma	-	Dha	=	5/4

The following pairs have the 6/5 relationship :

Re	-	Ma	=	6/5
Dha	-	Sa	=	6/5

The following pairs have no musical relationship :

Re	Pa
Sa	Ga
Pa	Ni
Ga	Dha

It should be remembered that it is impossible for all possible pairs to exhibit relationships. Hence the Shadaj-Gram is as perfect as the system of 7 notes can be.

In Madhyam Gram, only Pancham is different *(being lower than the Pancham of Shadaj-Gram by a ratio of 81/80)* and the ratio Sa-Pa is sacrificed for the sake of making the ratio Re-Pa musical. The alternative was to elevate Re by a ratio 81/80 but that would have meant elevating Dha, Ga and Ni also by the same ratio. This would have spoiled the relationship of Ni and Dha with Ma. Perhaps Bharat was keen that Ma must have musical relationship with all other notes. So, he changed Pa in Madhyam Gram and kept all the other notes intact.

Apart for the 7 notes of Shadaj Gram or Madhyam Gram, Bharat has defined other notes also :

1. **Antar Gandhar** which is on the 11th.Shruti *(Shadaj being taken on the 4th.Shruti)* - midway between a Gandhar and Madhyam.
   This Antar Gandhar is the modern 5th.harmonic Ga, bearing a ratio of 5/4 to the Shadaj.
2. Kakali Nishad - is on 2nd.Shruti - midway Nishad and Tar Shadaj.
   This is the modern Shudha Nishad and has Shadaj-Pancham Bav with Antar Gandhar.
3. Chyut Shadaj - is on 3rd.Shruti, being 81/80 times lower than Sadhaj.
4. Chyut Madhyam - is on 12th.Shruti, being 81/80 times lower than Madhyam.
5. Chyut Pancham - is on 16th.Shruti, which is in fact, the pancham of Madhyam Gram.
6. Keshik Gandhar - is on 12th.Shruti, being 81/80 times higher than Gandhar.
   This is the modern Komal Gandhar which is 6/5 times than the Shadaj.
7. Keshik Nishad - is on 1st.Shruti, being 81/80 times higher than Nishad.
   This is the modern Komal Nishad which is 6/5 times than the Pancham.

Out of the above notes, however, only Antar Gandhar and Kakali Nishad have been very prevalent.

After Grams, Bharat has mentioned Murchhan as which are *"modes"* of western music in which different notes out of the 7 fundamental notes are taken as starting notes and different scales are constructed. Thus from a Gram of 7 notes, there can be the following 7 Murchhanas.

Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
Re	Ga	Ma	Pa	Dha	Ni	Sa	Re
Ga	Ma	Pa	Dha	Ni	Sa	Re	Ga
Ma	Pa	Dha	Ni	Sa	Re	Ga	Ma
Pa	Dha	Ni	Sa	Re	Ga	Ma	Pa
Dha	Ni	Sa	Re	Ga	Ma	Pa	Dha
Ni	Sa	Re	Ga	Ma	Pa	Dha	Ni

Corresponding to the 3 Grams, there can be 21 Murchhanas. If Anter Gandhar and Kakali Nishad are also included among the notes, two more Murchhanas can be obtained in each Gram. In eac Murchhan, the starting note takes the place of Shadaj or tonic *(Swarit)* and its relationship with the original Shadaj becomes insignificant. This imparts a different colour to each off the Murchhana Scale. It will be shown in later chapters, how regarding the starting note of each Murchhana as Shadaj, we get a resemblance with the modern Thatas.

It should be noted that in the Murchhana of Shadaj Gram in which the starting note is Pa, if Antar Gandhar is used instead of Shuddha Ga, one gets

Pa Dha Ni $\overline{Sa}$ $\overline{Re}$ $\overline{Ga}$ $\overline{Ma}$ $\overline{Pa}$

(Ga - Antar Gandhar)

If Pa is taken to be Sa, this is exactly the scale of Madhyam Gram.
Thus we have the following relationship between the notes of Shadaj Gram and Madhyam Gram.

Notes of **Shadaj Gram**	*Notes of* **Madhyam Gram**
Pa	Sa
Dha	Re
Ni	Ga
Sa	Ma
Re	Pa
(Antar) Ga	Dha
Ma	Ni
Pa	Sa

Similarly, the Murchhana of Madhyam Gram with the starting note as Ni is

Ma Pa Dha Ni $\overline{Sa}$ $\overline{Re}$ $\overline{Ga}$ $\overline{Ma}$

Which is the scale of Shadaj Gram is Ma is taken as Sa according to the following scale

Notes of **Madhyam Gram**	*Notes of* **Shadaj Gram**
Ma	Sa
Pa	Re
Dha	Ga *(Antar Gandhar)*
Ni	Ma
Sa	Pa
Re	Dha
Ga	Ni
Ma	Sa

In Antar Gandhar and Kakali Nishad, if both the Grams are included, we shall have the following comparision table.

Notes of **Shadaj Gram**	*Corresponding Notes of* **Madhyam Gram**
Sa	Ma
Re	Pa
Ga	X
Antar Ga	Dha
Ma	Ni
X	*Kakali* Ni
Pa	Sa
Dha	Re
Ni	Ga
Kakali Ni	*Antar* Ga
Sa	Ma

It could be seen that there is no note in Shadaj Gram corresponding to the Kakali Ni of Madhyam Gram. Similarly, there is no note in Madhyam Gram corresponding to Shuddha Ga of Shadaj Gram. According to the modern nomenclature, the note in Shadaj Gram corresponding to the Kakali Ni of Madhyam Gram may be called Teevra Madhyam. Similarly, the note in Madhyam Gram, corresponding to the Shuddha Ga of Shadaj Gram may be closely called Komal Dha.

We saw how, introduction of two more notes - Antar Gandhar and Kakali Nishad - amalgamates the two Grams, in the sense that by taking the appropriate Murchhana of one of the Grams, it is possible to derive the notes of the other Gram. The Murchhana of Shadaj Gram starting with Pa in which Antar Ga is used instead of Shuddha Ga, produces the notes of Madhyam Gram; while the Murchhana of Madhyam Gram starting with Ma produces the notes of Shadaj Gram but with Antar Ga instead of Shuddha Ga.