Chapter - 4

General Theory *of* Grams

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If a Gram is defined to be a collection or a group of seven notes in which the notes are placed on different Shruti's how many Grams can be constructed from the Shruti's of Bharat ?
Well, if no restriction whatsoever is prescribed on which of the Shrutis the notes can lie, there can be 22 x 21 x 20 x 19 x 18 x 17 x 16 systems of seven notes.

This can be seen in the following way.
If there is no restriction at all for the location of notes, there are 22 choices for the location of the first note, Shadaj. After it has located in one of the 22 ways, there are 21 choices left for Rishabha. Now, with each of the 22 choices of Shadaj, 21 choices of Rishabha are associated. Hence, the two notes Shadaj and Rishabha can be fixed in 22 x 21 ways.

Next there will be 20 choices for the third note Gandhar and, following a similar argument, the three notes can be fixed in 22 x 21 x 20 ways. Continuing the same line of argument, we conclude that there can be 22 x 21 x 20 x 19 x 18 x 17 x 16 (859541760) ways of locating the seven notes of the 22 Shrutis. Thus there can be as many seven note systems in the same system of 22 Shrutis. But it is not the number of Grams, for it includes seven note systems which though different, are in the same cyclic order and which are, therefore, different Murchhanas of the same Gram. As a matter of fact, the above number is the total number of Murchhanas. Then remembering that a Gram has seven Murchhanas obtained by the treating each of the seven notes as the starting note, the number of Grams will be 22 x 21 x 20 x 19 x 18 x 17 x 16 / 7, *(=122791680)*.

A fantastic number ! But, most of these Grams are of practically no musical importance. For example, a Gram in which all the seven notes are situated on the first seven Shrutis, leaving a wide gap from 7th. to 22nd. Shruti is also included in the above number. One cannot have all the notes so closely packed in a small interval and leaving a wide gap between the last note and the first note of the next Saptak. It appears that we must prescribe some restrictions on how far or close two consecutive notes could be.

The interval of 4 Shrutis and 3 Shrutis has been always regarded as desirable for consecutive notes *(with a rearragement of Shrutis, if necessary, the 4 Shruti interval can be a ratio of 9/8 and a 3 Shruti interval, a ratio of 10/9)*. A five Shruti interval is not musical whatever may be the arrangement of Bharat's Shrutis. A six-Shruti interval is certainly desirable *(since it can be 6/5 with proper arrangement of Shrutis)* but if that is admitted for two consecutive notes, the remaining 16 Shrutis will have to be distributed among six intervals. This cannot be done without including a one-Shruti interval or two many two-Shruti intervals.

It is not difficult to see why one-Shruti or two Shruti intervals between two notes are not considered ideal. When two notes very close to each other, their simultaneous production produces beats which create a very uneasy effect on the ears. One Shruti interval *(whether it is 81/80, 256/243 or 25/24)* is so small that if two notes separated by one Shruti were included in a Gram, the beats produced *(when the two are produced simultaneously)* would be very unpleasant. Even the two-Shruti intervals are not considered very desirable and are to be avoided as much as possible. One reason why Bharat did not include the fifth harmonic Gandhar (5/4) in his Shadaj Gram was that it was at an interval of two-Shrutis from Madhyam. Bharat gave a lot of importance to Madhyam and avoided any note in his Shadaj-Gram which did not have a desirable musical relationship with it. Hence he used the Gandhar which was 9/8 away from it. A frequency-ratio of less than 16/15 *(two Shrutis 256/243 x 81/80)* is to be completely avoided between tow consecutive notes of a Gram. Too many ratios equal to 16/15 are also to be avoided. As far as possible, the ratio 16/15 should come a long with a ratio 9/8 so that the two intervals continue to give a relationship of 6/5 *(16/15 x 9/8 = 6/5)*.

Hence, we may stipulate that the frequency ratio between two consecutive notes must be one of the following

1). 9/8 *(four-shruti-interval)*

2). 10/9 *(three-shruti-interval)*

3). 9/8 *(two-shruti-interval)*

It can be seen that if we have 3 intervals like 9/8, 2 like 10/9 and 2 like 16/15, a Saptak can be constructed

since (9/8 x 9/8 x 9/8) x (10/9 x 10/9) x (16/15 x 16/15) = 2 = the frequency ratio of the

Sa of Tar Saptak to the Sa of Madhyam Saptak

Now the question is how many different arrangements are possible with three 9/8's two 10/9's and two 16/15's ? Let us first treat all the frequency ratios as different. Then there are seven choices for the first note, 6 for the second note, 5 for the third note and so on. Since, with each of the 7 choices of the first note, 6 choices of the second note are associated, the first two notes can be selected in 7 x 6 ways. Similarly, the seven notes can be selected in 7 x 6 x 5 x 4 x 3 x 2 x 1 ways. But, all the frequency ratios are not different. If we rearrange the three 9/8 ratios we do not get different but the same arrangements. This rearrangement can be done in 3 x 2 x 1 ways. Similarly the two 10/9 notes can be exchanged *(two possible arrangements)* without leaving the Gram altered in any way. Thus the different ways in which three 9/8 notes, two 10/9 notes and two 16/15 notes can be arranged, come to

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 2 \times 2} = 210$$

This is the total number of Murchhanas. The total number of Grams will be 210/7 = 30

More liberally, we can accept frequency ratios like 27/25 *(which appears in Gandhar Gram)* and 75/64 *(equivalent to that between modern Komal Re and Shuddha Ga)*, then we have the following frequency ratios to rearrange *(27/25 x 27/25)* x *(10/9 x 10/9 x 10/9 x 10/9)* x 9/8 = 2

in the first case *and*

(75/64 x 75/64) x 9/8 x *(16/15 x 16/15 x 16/15 x 16/15)* = 2
in the second case.

The number of Murchhana in these cases will be

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 4 \times 3 \times 2 \times 1} = 105$$

in earlier case.

Which gives 210 more Murchhanas or 30 more Grams. Hence including the above cases, the number of possible Grams come to 60 restricting the consecutive frequency ratios to value 9/8, 10/9, 16/15,27/25 and 75/64.

So far, we have been regarding a Gram as any arrangement of seven notes composed of desirable frequency ratios. Let us now consider only those Grams which are rich - as rich as possible - in musical relationships.

Considering Shadaj - Pancham Bhav first, let us try to construct a Gram which is rich in Shadaj - Pancham relationships. To be more precise, let us prescribe that the pairs Sa-Pa, Re - Ma Ga - Ni and Ma - Sa exhibit Shadaj - Pancham relationship, the remaining pairs Pa Re and Dha - Ga may or may not do so. We start with Sa. Its third harmonic Pa (3/2) must be included in the Gram. similarly, Ma (4/3) which has as its third harmonic as Sa of Tar Saptak (2) must be included. Now we have fixed the following notes.

Sa	Ma	Pa	$\overline{\text{Sa}}$
(1)	(4/3)	(3/2)	(2)

We have to find four more notes, two between Sa and Ma and the remaining two

between Pa and $\overline{\text{Sa}}$ such that there are two pairs exhibiting third harmonic relationship, there are three possibilities for the note Re viz. 9/8, 10/9 and 16/15. Corresponding to these the frequency of Dha will come to

9/8 x 3/2 - 27/16
10/9 x 3/2 - 5/3
16/15 x 3/2 - 8/5

So, now we have the following three arrangements

(a)	Sa	Re	Ma	Pa	Dha	$\overline{\text{Sa}}$
	1	9/8	4/3	3/2	27/16	2
(b)	Sa	Re	Ma	Pa	Dha	$\overline{\text{Sa}}$
	1	10/9	4/3	3/2	5/3	2
(c)	Pa	Sa	Re	Ma	Dha	$\overline{\text{Sa}}$
	1	16/15	4/3	3/2	8/5	2

For the note Ga, in the first arrangement its separation from Re could be 10/9 or 16/15 but not 9/8 since then it would be too close *(only 246/243 or a Shruti a part)* to Ma. Accordingly, we have the frequency of Ga in the first case *(Re=9/8)* either 5/4 or 6/5. Accordingly, the frequency of Ni will be 5/4 x 3/2 = 15/8 and 6/5 x 3/2 = 9/5 respectively. Hence, corresponding to Re = 9/8, we have the following two systems.

(1)	Sa	Re	Ga	Ma	Pa	Dha	Ni	$\overline{\text{Sa}}$
	1	9/8	5/4	4/3	3/2	27/16	15/8	2
(2)	Sa	Re	Ga	Ma	Pa	Dha	Ni	$\overline{\text{Sa}}$
	1	9/8	6/5	4/3	3/2	27/16	9/5	2

In the case (b) (Re = 10/9), Ga can be 10/9, 9/8 or 16/15 away from Re without it coming two close to Ma *(when Re-G = 9/8, Ga - Ma is 27/25).* The absolute value of the frequency of Ga in these cases come to 5/4, 100/81 and 32/27, and the corresponding frequency ratios of Ni come to $\frac{15}{8}$ $\frac{50}{27}$ and $\frac{16}{9}$ respectively.

Hence, corresponding to Re = 10/9, we have the following systems

(3)	Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
	1	10/9	5/4	4/3	3/2	5/3	15/8	2

(4)	1	10/9	100/81	4/3	3/2	5/3	50/27	2
(5)	1	10/9	32/27	4/3	3/2	5/3	16/9	2

In the case (C) (Re-16/15), Ga can be taken to be 9/8 10/9, 16/15 or 75/64 from Re without bringing it too close to Ma. The corresponding frequency ratios from Sa come to 6/5, 32/27, 256/225 and the corresponding frequency ratios of Ni comes to :

6/5x3/2 = 9/5, 32/27x3/2 = 16/9, 256/225x3/2 = 128/75

5/4x3/2 = 15/8

Corresponding to Re = 16/15, we then have

(6) Sa	Re	Ga	Ma	Pa	Dha	Ni	$\overline{Sa}$
1	16/15	6/5	4/3	3/2	8/5	9/5	2

(7) Sa	Re	Ga	Ma	Pa	Dha	Ni	$\overline{Sa}$
1	16/15	32/27	4/3	3/2	8/5	16/9	2

(8) Sa	Re	Ga	Ma	Pa	Dha	Ni	$\overline{Sa}$
1	16/15	256/225	4/3	3/2	8/5	128/75	2

(9) Sa	Re	Ga	Ma	Pa	Dha	Ni	$\overline{Sa}$
1	16/15	5/4	4/3	3/2	8/5	15/8	2

All the systems (1) to (9) exhibit Shadaj - Pancham Bhav between the following pairs :

Sa - Pa

Re - Dha

Ga - Ni

Ma - $\overline{Sa}$

No. (5) is the Shadaj Gram of Bharat. No. (1) is the Murchhana of Shadaj Gram starting with Ni while no. (6) *(identical with what is known as Bhairavi Thata of today)* is the Murchhana of Madhyam Gram starting with Re. No. (7) is a Murchhana of No. (2) starting with Re. The remaining arrangements viz. (2) (3)(4) (8) and (9) are new Grams. Particularly rich in Shadaj Pancham relationships are (1), (2) (3) (5) (6) and (7) which exhibit Shadaj-Pancham relationship. If two notes are related by Shadaj Pancham Bhav, by raising the higher note or lowering the higher note by an octave, the relationship between the tow notes would be transferred to Shadaj Madhyam Bhav. Similarly we can pass from Shadaj Madhyam Bhav to Shadaj Pancham Bhav by raising or lowering one of the notes by an octave.

One important Gram remains to be discussed from the point of view of Shadaj-Pancham relationship in which six pairs of notes *(out of seven)* exhibit Shadaj-Pancham relationship. This Gram is obtained as follows :

From Shadaj, we obtain Pancham (3/2) by the usual third-harmonic method. Its third harmonic brought to Madhya Saptak is Re (9/8). Its third harmonic is Dha (27/16). The third harmonic brought to Madhya Saptak is Ga (27/16x3/2x1/2=81/64). Third harmonic of this Ga is Ni (81/64x3/2=243/12).

Now we have the following Gram

Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
1	9/8	81/64	4/3	3/2	27/16	243/128	2

in which the following pairs exhibit Shadaj-Pancham relationship

Sa	-	Pa
Re	-	Dha
Ga	-	Ni
Ma	-	Sa
Pa	-	Re
Dha	-	Ga

Ni - Ma is the only pair which is not musically related in this way.
This gram is the scale used by the Greeks and is called the Pythagorean Scale.

Hence, the conclusion is that, including the Pythagorean scale there are 8 Grams *(including Shadaj and Madhyam Grams)* such that in some Murchhana or the other, at least four pairs Sa-Pa, Re-Dha, Ga-Ni, Ma-Sa exhibit Shadaj - Pancham relationship.

Out of these Grams 5 *(including the Pythagorean scale)* are such that in some Murchhana or the other at least five pairs *(out of Sa-Pa, Re-Dha, Ga-Ni, Ma-Sa, Pa-Re, Dha-Ga, Ni-Ma exhibit Shadaj-Pancham Bhav)*.

We cannot find a scale in which all the seven pairs exhibit this relationship. We know form the latest chapter that it is impossible : the cycle of successive third harmonic never closes, much less on the eight note.

There is only one Gram in which six pairs out of seven exhibit the Shadaj-Pancham Bhav-the Pythagorean scale. Its method of construction makes it clear that there cannot be any other Gram with this property. One can start with any note and continue till the seventh note is derived. By starting with different notes, we can get different Murchhanas but there will be only one Gram.

How many Grams are possible such that five, out of a possible of seven are related by Shadaj-Pancham Bhav ? We have seen that out of (1) to (9) arrangements derived above, (1) (2) (3) (5)(6) and (7) have this property. But, the Grams are only four since (1) and (5) are different Murchhanas of the same Gram and (2) and (7) are also from the same Gram. However, this problem can be solved in another way which is instructive.

The seven possible pairs of notes which can be expected to be connected by Shadaj-Pancham Bhav are the following :

Sa	-	Pa
Re	-	Dha
Ga	-	Ni
Ma	-	$\overline{\text{Sa}}$
Pa	-	$\overline{\text{Re}}$
Dha	-	$\overline{\text{Ga}}$
Ni	-	$\overline{\text{Ma}}$

For the sake of convenience, we shall denote any pair by its first note, i.e. the pair-Sa-Pa will be denoted by *"Sa"* and the Pair Pa-Re will be denoted by *"Pa"*. Now if five pairs out of them have to be related by Shadaj-Pancham Bhav, then only two pairs will fail to exhibit them.

There are only three following possibilities :

(1) The pairs failing to exhibit Shadaj-Pancham Bhav may be "Sa", "Re" type i.e. such that two consecutive notes fail to exhibit the relationship with their fifth note.

(2) The pair failing to exhibit Shadaj-Pancham Bhav may be "Sa" "Ga" type - i.e. such that the two notes failing to exhibit this relationship with their respective fifth notes are separated by one note.

(3) The pairs failing to exhibit Shadaj-Pancham Bhav may be "Sa" "Ma" type i.e. such that the two notes failing to exhibit Shadaj - Pancham Bhav with their respective fifth notes are separated by two notes.

It is not difficult to see that if the two notes failing to exhibit the above relationship with respective fifth notes are separated by more than two notes, it is always possible to find a Murchhana starting with a suitable note in which the case falls within one of the three cases cited above.

Taking case (1) first, let us consider the Murchhana in which all pairs except "Dha" and "Ni" are related by Shadaj Pancham Bhav, Starting with Sa, then, let us construct the successive third harmonics as follows

Sa Pa *(3/2)*

Pa *(3/2)* Re *(9/4) (Re=9/8)*

Re *(9/8)* Dha *(27/16)*

We stop here, for Dha-Ga pair is not to exhibit the Shadaj Pancham Bhav. If Ga is not to be found by the third harmonic method, we can take this opportunity to prescribe some other musical relationship for Ga. Two musical alternatives are

Sa - Ga = 5/4 or

Sa - Ga = 6/5

If Ga is taken to be (5/4) correspondingly, Ni comes to (15/8) (so that Ga-Ni = 3/2) Ni (15/8) and Ma do not show the Shadaj Pancham Bhav, which is as expected.

Hence, corresponding to Ga (5/4), the Gram comes to :

(d)	Sa	Re	Ga	Ma	Pa	Dha	Ni
	1	9/8	5/4	4/3	3/2	27/16	15/8

Which is the arrangement (1) above and the Nishad Murchhana of the Shadaj-Gram of Bharat.

Corresponding to Ga (6/5), Ni comes to 6/5 x 3/2 = 9/5 and the complete Gram is

(e)	Sa	Re	Ga	Ma	Pa	Dha	Ni
	1	9/8	6/5	4/3	3/2	27/16	9/5

Which is the arrangement (2) derived earlier.

Considering the case (2), in which the notes failing to exhibit Shadaj-Pancham Bhav with their respective fifth notes are separated by a note let us take "Re" and "Ni" to be the pairs which fail to exhibit the above relationship *(Re and Ni are separated by one note only when we consider the seven notes in a cyclic order. Change of octave is immaterial.)* Starting from Ma (4/3) *(which exhibits Shadaj-Pancham Bhav with Sa)*, we get Sa *(in Madhyam Saptak)*.

Then the following method is followed :

Sa - Pa (3/2)

Pa - Re (9/8)

We stop here, for Re-Dha pair may not exhibit the Shadaj-Pancham Bhav. We can choose Dha to be such that it has earlier 5/4 or 6/5 relationship with Sa *(in Tar Saptak)* which Dha-Sa = 6/5, we have Dha = 5/3. Then Ga is obtained by third harmonic method, which comes to 5/4. Then Ni is its third harmonic or 15/8.

The complete Gram is

(f)	Sa	Re	Ga	Ma	Pa	Dha	Ni
	1	9/8	5/4	4/3	3/2	5/3	15/8

Then Ga is $\frac{6}{5}$ away form Tar Sa or 6/5 Ni is its third harmonic *i.e.* 9/5.

(g)	Sa	Re	Ga	Ma	Pa	Dha	Ni
	1	9/8	6/5	4/3	3/2	8/5	9/5

(f) is the Nishad Murchhana of the Madhyam Gram of Bharat (incidentally the scale which is considered to be the Shuddha Bilaval Thata by Pandit Omkarnath Thakur) (g) is the "Dha" - Murchhana of the scale (3) derived above.

Taking the last case in which the defaulting pairs are separated by two notes, let us assume that the pairs *"Ga"* and *"Dha"* fail to exhibit the Shadaj-Pancham relationship.

Then, we first fix Ma *(4/3)* so as to have third harmonic relationship with Sa. Then we start with Sa to find its third harmonic Pa *(3/2)*. The third harmonic of Pa, brought to Madhya Saptak is Re *(9/8)*. The third harmonic of Re is Dha *(27/16)*.

We have so far fixed the following notes

Sa	Re	Ma	Pa	Dha	Sa
1	9/8	4/3	3/2	27/16	2

It remains to fix Ga and Ni. Ni can also be fixed since Ni-Ma = 3/2. Hence Ni = 4/3x4/3 = 16/9. Now Ga remains to be fixed. Relating it musically with Sa, Sa-Ga can be fixed either 6/5 or 5/4. Accordingly, we have the following Grams :

(h)	Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
	1	9/8	6/5	4/3	3/2	27/16	16/9	2
(i)	Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
	1	9/8	5/4	4/3	3/2	27/16	16/9	2

But these Grams cannot be accepted in music. Dha and Ni are two close, their ratio being 16/9, 16/27=256/243 which is just one Shruti. Hence, we conclude that (d), (e), (f) and (g) are the only Grams in which five pairs of notes out of possible seven exhibit musical relationship.

It may be wondered it sufficient to consider only one case in each of the above categories. For example, in case (2), we considered only the Grams in which the defaulting pairs of notes where *"Ni"* and *"Re"* which have we not considered other pairs separated by one note i.e. *"Sa"* and *"Ga"* ? The answer is that we are considering Grams in which the cyclic order of the notes is immaterial. If there is a Gram in which the pairs of notes failing to exhibit the Shadaj Pancham Bhav are *"Sa"* and *"Ga"*, then by taking an appropriates Murchhana *(in this case the one starting with Ni)* it is always possible to adjust in such a way that in the new Murchhana the defaulting pairs become *"Ni"* and *"Re"*.

Let us, now consider the Grams rich in musical relationships from the point of view of Shadaj-Gandhar Bhav *(5/4 or 6/5)*. The relationship 6/5 and 5/4 are really different aspects of the same relationship in one sense; if two notes are related by the 5/4 *(or fifth harmonic)* relationship, raising the lower note to its third harmonic or lowering the higher note by its third harmonic *(3/2)*, the relationship 5/4 are related by fifth harmonic *(5/4)*. If we raise Sa to its third harmonic, it becomes Pa *(3/2)* which is related to Ga *(5/4)* by a 6/5 relationship *(3/2-54/=6/5)*. Similarly, lowering Ga by third harmonic, we get Dha (5/6) which is again related to Sa by a 6/5 relationship. Hence we can say that the relationship 5/4 and 6/5 are the images of each other in third harmonic relationship, much as the relationships 4/3 and 3/2 are the image of each other in a second harmonic relationship *(2:1)*.

Starting with Sa then, we construct the following possible chains :

(A)	Sa		Ga		Pa		Ni	Sa
		5/4		6/5		5/4	16/15	
(B)	Sa		Ga		Pa		Ni	Sa
		5/4		6/5		6/5	10/9	
(C)	Sa		Ga		Pa		Ni	Sa
		6/5		5/4		5/4	16/15	
(D)	Sa		Ga		Pa		Ni	Sa
		6/5		5/4		6/5	10/9	
(E)	Sa		Ga		Pa		Ni	Sa
		6/5		6/5		5/4	10/9	
(F)	Sa		Ga		Pa		Ni	Sa
		5/4		5/4		6/5	16/15	

Similarly, chains Re-Ma Dha can be constructed as follows :

(G)	Re		Ma		Dha		Sa
		5/4		5/4		6/5	
(H)	Re		Ma		Dha		Sa
		6/5		5/4		6/5	
(I)	Re		Ma		Dha		Sa
		5/4		6/5		5/4	
(J)	Re		Ma		Dha		Sa
		6/5		6/5		5/4	

To relate the two chains, we can fix Sa-Ma=4/3. Then with each of A, B, C, D, E, F we can associate G, H, I, J chains. Thus we have 6 x 4 = 24 arrangements. However, I and J chains cannot be combine with F, since the Pa and Dha come too close. The Pa in (F) is 5/4 x 5/4 = 25/16 from Sa and Dha in I and J is 6/5 of Ma *i.e.* 4/3x6/5=8/5: the ratio of Pa and Dha = 8/5x16/25 = 128/125. Excluding the combinations of I and J chains with F, there remain 22 arrangements of notes which are given below :

I.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa *(A-G)***
	1	16/15	5/4	4/3	3/2	5/3	15/8	2 (4)
II.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa *(A-H)***
	1	10/9	5/4	4/3	3/2	5/3	15/8	2 (4)
III.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa *(A-I)***
	1	1615	5/4	4/3	3/2	8/5	15/8	2 (4)
IV.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa *(A-J)***
	1	10/9	5/4	4/3	3/2	8/5	15/8	2 (3)
V.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa** (3)
	1	10/9	5/4	4/3	3/2	5/3	15/8	2 ***(B-G)***
VI.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa** (3)
	1	10/9	5/4	4/3	3/2	5/3	9/5	2 ***(B-H)***

VII.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (-I)**
	1	16/15	5/4	4/3	3/2	8/5	9/5	2 (3)
VIII.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (B-J)**
	1	10/9	5/4	4/3	3/2	8/5	9/5	2 (2)
IX.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (C-G)**
	1	16/15	6/5	4/3	3/2	5/3	15/8	2 (2)
X.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (C-H)**
	1	10/9	6/5	4/3	3/2	5/3	15/8	2 (3)
XI.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (C-I)**
	1	16/15	6/5	4/3	3/2	8/5	15/8	2 (4)
XII.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (C-J)**
	1	16/15	6/5	4/3	3/2	8/5	15/8	2 (3)
XIII.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (D-G)**
	1	16/15	6/5	4/3	3/2	5/3	9/5	2 (3)
XIV.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (D-H)**
	1	10/9	6/5	4/3	3/2	5/3	9/5	2 (4)
XV.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (D-J)**
	1	16/15	6/5	4/3	3/2	8/5	9/5	2 (5)
XVI.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (D-J)**
	1	10/9	6/5	4/3	3/2	8/5	9/5	2 (3)
XVII.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (E-G)**
	1	10/9	6/5	4/3	36/25	5/3	9/5	2 (3)
XVIII.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (E-H)**
	1	10/9	6/5	4/3	36/25	5/3	9/5	2 (3)

XIX.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (E-I)**
	1	16/15	6/5	4/3	36/25	8/5	9/5	2 (4)
XX.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (E-J)**
	1	10/9	6/5	4/3	36/25	8/5	9/5	2 Gandhar Gram
XXI.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (F-G)**
	1	16/15	5/4	4/3	25/16	5/3	15/8	2 (3)
XXII.	**Sa**	**Re**	**Ga**	**Ma**	**Pa**	**Dha**	**Ni**	**Sa (F_H)**
	1	10/9	5/4	4/3	25/16	5/3	15/8	2 (4)

All these Murchhanas are rich in Shadaj-Gandhar relationship *(5/4 or 6/5)* and, in each of these, all possible pairs except *(Ni, Re)* exhibit this relationship. This is as far as one can go to in a musical scale, since these can be not scale in which all possible pairs are connected by this relationship. It should be noted that the Gandhar Gram of the ancients is included in these scales at XX.

But most of these scales are poor in Shadaj-Pancham *(for Shadaj-Madhyam)* relationships. Except the scales at S. No.I. II, III, VI, XI, XIV, XV, XIX, and XXII all others show this relationship only is less than 4 pairs. S. No. II and XV exhibit it in 5 pairs *(out of possible 7)* while the remaining scales exhibit it in 4 pairs. As a matter of fact the scales II, III and XV are none other than the scales (3), (9) and (6) derived earlier during the discussion of scales rich in Shadaj-Pancham relationships.

Now that we have derived scales rich in Shadaj-Pancham relationship and rich in Shadaj-Gandhar relationship separately, it is upto us to prescribe a standard about when a scale or Gram should be considered rich in musical relationships taking into account both Shadaj-Pancham as well as Shadaj-Gandhar relationships. Naturally, we would not like to neglect one kind of relationship altogether. For instance, we should discard Grams like Pythagorean scale in which 6 pairs are related by Shadaj-Pancham relationship but not a single pair exhibits Shadaj-Gandhar relationship. This is precisely why this Gram was never considered for any musical system by the ancient musicians of India. They accepted scales or Grams which were rich when both kinds of musical relationships were taken into consideration.

To be precise let us prescribe that the total number of pairs exhibiting one kind of relationship or the other way be at least 9 *(out of a possible 14 pairs)*. At the same time we may also lay down that the Gram should not be very poor in one of the relationships of the Shadaj-Pancham-Bhav at the cost of other *(Shadaj Gandhar Bhav)*, thus Shadaj-Pancham Bhav must be exhibited by at least 4 pairs of notes. Similarly, Shadaj-Gandhar relationship (5/4 *or* 6/5) must be exhibited by at least 4 pairs. Then, we find that the following Grams fit into these criteria.

I, II, III, IV, V, VI, VII, X, XI, XII, XII, XIV, XV, XVIII, IX, XX, XXI, XXII, Shadaj Gram(5), 2, 4. Hence there are 22 Grams which are as musically rich as the Shadaj-Gram of Bharat. Out of two are the richest showing musical relationships of one kind or other in 11 pairs out of 14. There are XV *(Madhyam Gram)* and II.

Next, are seven Grams in which 10 pairs out of 14 exhibit musical relationship, these are I, II, VI, XI, XIV, XIX and XXII.

The remaining 13 Grams exhibit musical relationships in 9 pairs only. These include Shadaj Gram and Gandhar Gram *(II is the same as Kafi Thata of today while III is the same as Bhairavi Thata of today as we shall see later).*

The yardstick regarding the richness in musical relationships adopted above is quite arbitrary. If a more liberal yardstick is prescribed for instance that all corresponding pairs except six should be related by Shadaj-Pancham or Shadaj_Gandhar relationship without there being any other restriction we shall obtain a larger number of Gram qualifying as such in musical relationships.

To sum up the general discussion on Grams so far, a fantastic number (122791680) of Grams can be obtained by locating the seven notes on the 22 Shrutis, but most of these Grams are useless in music. If a condition is imposed that the interval between two consecutive notes will be 9/8, 10/9 or 16/15 *(as it is in the Shadaj of Madhyam Gram of Bharat)* the number comes down to 30. When we include 27/25 and 75/64 also as possible between consecutive notes, another 30 Grams get included in the stock. These 60 Grams are more like the ancient three Grams at least in terms of consecutive net ratios.

Coming to musical relationships among the corresponding pairs, we have observed that there are basically two types of musical relationships as follows :

(1) Shadaj-Pancham or third harmonic relationship equivalent to a frequency ratio 3/2.

(2) Shadaj-Gandhar of fifth harmonic relationship equivalent to a frequency ratio 5/4.

However, there are two more relationships which are actually the manifestations of the above relationships when one of the notes is raised or lowered by an octave or by a ratio 3/2 *(Shadaj-Pancham Bhav)*. These are as follows :

(3) Shadaj-Madhyam Bhav equivalent to a frequency ratio of 4/3. If two notes are related by Shadaj-Pancham Bhav and the lower note is raised by an octave or the higher note is lowered by an octave the Shadaj-Pancham relationship transforms into Shadaj-Madhyam relationship. Thus Shadaj Madhyam Bhav is the image of Shadaj Pancham Bhav in a lower or higher octave.

(4) Shadaj-Gandhar relationship equivalent to a frequency ratio of 6/5. When two notes are related by the relationship (2) above *(ratio 5/4)*, raising the lower note to its third harmonic (ratio 3/2) or lowering the higher note by the same ratio, transforms the relationship (2) (5/4) to the relationship (4) (6/5). Thus this relationship is the image of Shadaj-Shuddha Gandhar Bhav (5/4) in Shadaj Pancham Bhav.

The relationship (1) to (4) above have been known as Samvadas since the time so the ancients in Indian classical music. They have been given considerable importance.

Ratios 9/8 and 10/9 are also regarded as highly desirable between consecutive notes. The importance of these ratios is obvious from the following discussion :

If notes a and B *(B bearing higher than A)* are related by Shadaj-Pancham Bhav *(B/A = 3/2)*, then consider two intermediate notes C and D such that AD and CB are related by Shadaj-Madhyam Bhav. This means that B/C=D/A=4/3. Then, we have C/A=B/D=9/8. Then the ratio 9/8 is the ratio of Shadaj-Pancham Bhav to Shadaj-Madhyam Bhav.

Mathematically $9/8 = \frac{4/3}{5/4}$

Similarly, if two notes A and B are related by Shadaj-Madhyam Bhav, consider two intermediate notes C and D such that and CB are related by the relationship 6/5. Then B/C = D/A = 6/5 and we have C/A=B/D=10/9. Since $10/9 = \frac{4/3}{}$ Thus the ratio 10/9 is the ratio of Shadaj-Madhyam Bhav and Shadaj-Komal Gandhar Bhav.

The ratio 16/15 is considered much less desirable for consecutive note ratios since it is so small that beats are produced when the two notes are played. Simultaneously *(the ancient Gandhar Gram totally avoids this ratio in favour of a slightly larger ratio 27/25)*.

The notes related by a ratio 16/15 are called Vivadi in the classical literature. But, it is difficult to avoid this ratio. In fact, if two notes A and B are related by Shadaj-Madhyam Bhav (4/3) and intermediate note C is chosen so that AC is related by Shadaj-Gandhar Bhav (5/4), the ratio of C to B is $16/15 = \frac{4/3}{5/4}$.

When we consider Grams which are rich in musical relationships, the number of Grams which remain for consideration are much fewer, but we have seen that there are 22 Grams as rich in musical relationships as the Shadaj Gram. Two Grams viz. the Madhyam Gram and the one identical with the Asawari, Thata of today are the richest in musical relationships. It is not possible to find a richer Gram or seven note system than these two Grams.

How Bharat hit upon the Shadaj Gram is not difficult to see. He wanted a Gram which would exhibit Shadaj-Pancham Bhav between the following pairs within the Madhyam Saptak.

Sa	-	Pa
Re	-	Dha
Ga	-	Ni
Ma	-	Sa

At the same time Bharat gave lot of importance to Madhyam (4/3 from Shadaj) and accordingly chose other notes in such a way that all were related to Ma by a musically acceptable ratio (3/2,4/3,5/4,6/5,9/8 or 10/9). He did not allow even a ratio 16/15 between Ma and any other notes *(and of course, the four pairs described above are required to exhibit Shadaj-Pancham Bhav)* we can construct the following two Grams which fulfill these conditions.

Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
1	9/8	5/4	4/3	3/2	27/16	15/8	2 *(i)*

Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
1	9/8	6/5	4/3	3/2	27/16	9/5	2 *(ii)*

The first one is the Murchhana of Shadaj Gram starting from Ni. The second one is the Kafi Thata of today and the scale described by Ahobal as we shall see in the next chapter.

The advantage in (ii) is that both Ma and Pa are kept away from the Vivadi relationship of 16/15.

Later, Bharat must have discovered that by dropping the requirement of Shadaj-Pancham Bhav between the above mentioned four pairs, a much richer Gram can be constructed. This he did by adjusting Pa until it had Shadaj Madhyam Bhav with Re. No richer Gram is possible, though there is another one just as rich.

This completes the discussion of the ancient Shrutis of Bharat and Grams mentioned in the ancient literature as well as those mathematically possible. The ancient literature mentioned as well as those mathematically possible.

The ancient literature mentions only three Grams out of which only two are discussed in detail by Bharat : Shadaj Gram and Madhyam Gram. We have been able to throw some light on Gandhar Gram also and have described 22 Grams which are al least as rich in musical relationship as the Shadaj-Gram.

We shall now pass on to the twelve note systems mentioned by the musicians of later era who have talked litter about the Shrutis. In the system of Bharat, as we have seen, only seven notes have been regarded fundamental although Antar Gandhar and Kakali Nishad have been included as additional notes. In the later era, twelve notes began to be reorganized in an octave approximately consistent with the twelve notes derived in the previous chapter by the third harmonic method. However, the exact frequencies of these notes are slightly different in the systems described by different musicians. Afterwards, we shall come back to the twenty-two Shrutis and the notes and Shrutis actually in use today.