9
GENETIC ALGORITHM ASSISTED M.U.D. FOR S-CDMA

9.1 INTRODUCTION

In this chapter, we will provide a simple model for investigating the feasibility of applying GAs in CDMA multi-user detection (MUD) as well as for determining the GAs configuration, in order to obtain a satisfactory performance. Here we will apply a GA assisted scheme as a suboptimal multi-user detection technique in CDMA system over single path Rayleigh fading channels.

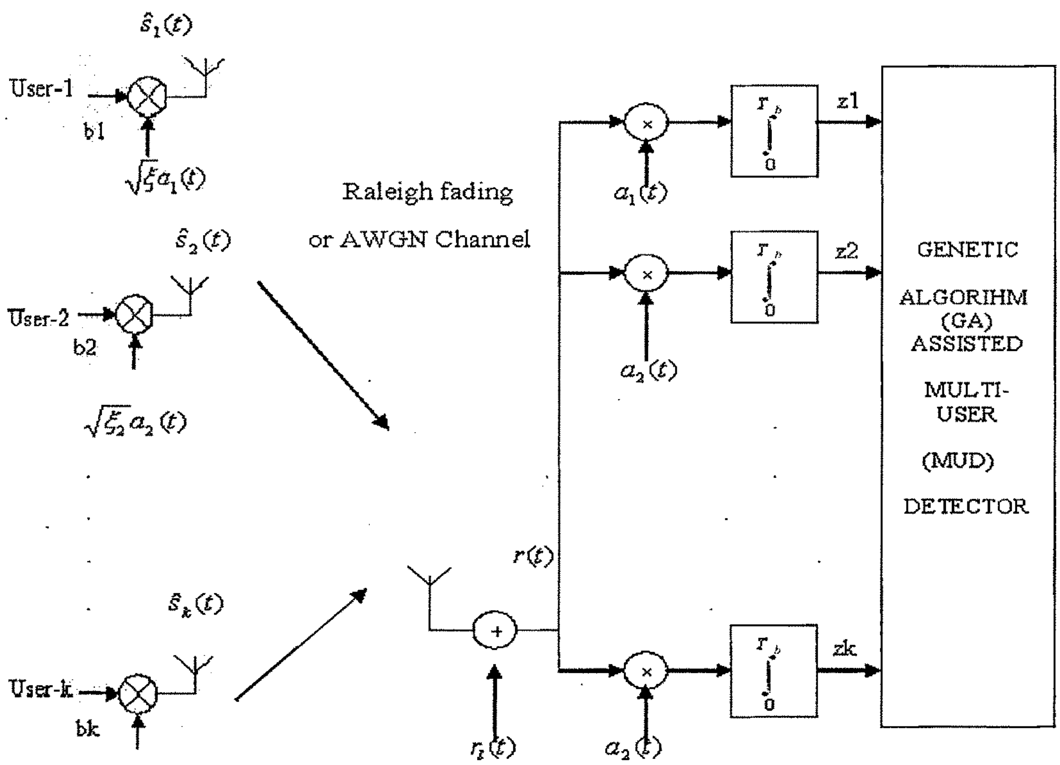

Figure 9.1 Block diagram of the k-user synchronous CDMA system model in a flat Rayleigh fading channel

9.2 S-CDMA SYSTEM MODEL

We consider a bit synchronous CDMA system as illustrated in figure 9.1, where K users simultaneously transmit data packets of equal length to a single receiver. In this chapter we will adopt the binary phase shift keying (BPSK) modulation technique for all the transmissions. The transmitted signal of the K^{th} user can be expressed in an equivalent low pass representation as:

$$S_k(t) = \sqrt{\xi_k} \sum_{m=0}^{M-1} b_k^{(m)} a_k(t - mT_b), \qquad \forall k = 1, \ldots, K \tag{1}$$

Where,

ξ_k is the k^{th} user's signal energy per bit,

$b_k^{(m)} \in \{+1, 1\}$ denotes the m^{th} data bit of the k^{th} user,

$a_k(t)$ is the k^{th} user's signature sequence,

T_b is the data bit duration and

M is the number of data bits transmitted in packet.

When considering a synchronous system experiencing no multi-path interference, it is sufficient to observe the signal bit duration T_b, since there is no interference inflicted by symbols outside this duration. Hence without loss of generality, we can omit the superscript (m) from all our equations.

The k^{th} user's signature sequence $a_k(t)$ may be written as,

$$a_k(t) = \sum_{h=0}^{Nc-1} a_k^{(h)} \; \Gamma \; Tc\,(t - hTc), \quad 0 \le t < T_b, \quad \forall k = 1 \ldots\ldots\ldots, k \tag{2}$$

Where Tc is the chip duration, $a_k^{(h)} \in \{+1, 1\}$ denotes the h^{th} chip, Nc is the spreading factor, which refers to the number of chips per data bit duration T_b such that Nc $= Tb/_{Tc}$ and $\Gamma_{Tc}(t)$ is the chip pulse shape and is defined as

$$\Gamma_r(t) = \begin{cases} 1 & t \le 0 < T \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

Assuming that the signature sequence $a_k(t)$ of all K user's has unit energy, as given by

$$\int_{0}^{Tb} a_k^2(t)\,dt = 1, \qquad \forall k = 1.........,k \qquad (4)$$

Each user's signal $\hat{s}_k$(t) is assumed to propagate over a single path frequency nonselective slowly Rayleigh fading channel as shown in figure 9.1 and the fading of each path is statistically independent for all users. The complex low pass channel impulse response (CIR) for the link between the Kth user's transmitter and receiver, as shown in figure 9.1, can be written as

$$h_k(t) = a_k(t) e^{j\Phi_k(t)} \delta(t), \qquad \forall k = 1.........,k \qquad (5)$$

Where the amplitude $a_k(t)$ is a Rayleigh distributed random variable and the phase $\Phi_k(t)$ is uniformly distributed between [0 , 2π].

Hence, when the Kth user's spread spectrum signal $\hat{s}_k$(t) given by equation (1) propagates through a slowly Rayleigh fading channel having an impulse response given by equation (5), the resulting output signal $\hat{s}_k$(t) over a single bit duration can be written as

$$\hat{s}_k(t) = \sqrt{\xi_k}\; a_k b_k a_k(t) e^{j\Phi_k}, \qquad \forall k = 1.........,k \qquad (6)$$

Upon combining Equation (6) for all k users, the received signal at the receiver, which is denoted by r(t) in figure 9.1., can be written as

$$r(t) = \sum_{k=1}^{k} s_k(t) + n(t), \qquad (7)$$

where n(t) is the zero-mean complex Additive White Gaussian Noise (AWGN) with independent real and imaginary components, each having a double sided spectral density of $\sigma^2 = N_0/2$ W/Hz.

At the receiver, the output of a bank of filters matched to the corresponding set of the user's signature is sampled at the end of bit interval. The output of the l^{th} user's matched filter, denoted as z_l in figure 9.1, can be written as,

$$z_l = \int_0^{T_b} r(t)a_l(t)dt$$

$$= \int_0^{T_b}\sum_{k=1}^{k}\sqrt{\xi_k}\,a_k b_k a_k(t)e^{j\Phi k}a_l(t)dt \;+\; \int_0^{T_b} n(t)a_l(t)dt$$

$$= \sqrt{\xi_l}\,a_l b_l e^{j\Phi_l} \quad + \quad \sum_{\substack{k=1\\k\neq 1}}\sqrt{\xi_k}\,a_k b_k \rho_{lk} e^{j\Phi_k} \;+\; n_l \tag{8}$$

(Desired signal) (M. A. I.) (Noise)

Where ρ_{lk} is the cross-correlation of the l^{th} user's and the k^{th} user's signature sequence, as given by,

$$\rho_{lk} = \int_0^{T_b} a_l(t)a_k(t)dt, \tag{9}$$

and

$$n_l = \int_0^{T_b} n(t)a_l(t)dt, \tag{10}$$

As seen in Equation (8), apart from the Gaussian noise n_l, the desired signal is interfered by signals transmitted by the other users. This interference due to the other user's signals is also known as Multiple Access Interference (MAI).

Probability (BEP) of the desired user can be shown to be given by

$$p_l = \frac{1}{2}\left(1 - \sqrt{\frac{\xi_l}{N_0/2 + \sum_k \xi_k \rho_{lk}}}\right) \tag{11}$$

Hence from equation (11), we can see that unless the signature sequences of the interfering users are orthogonal to that of the desired user, the BEP performance of the desired user will be inferior to that achieved to that achieved in a single user environment in conjunction with a single user matched filter.

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9.3 DISCRETE TIME S-CDMA MODEL

For our application, it is more convenient to express the associated signals indiscrete time format. The sum of the transmitted signals of all users can be expressed in vector notations as

$$\text{s(t)} = \sum_{k=1}^{k} \hat{s}_k(t)$$

$$= aC\xi b, \qquad (12)$$

Where,

$a = [\, a_1(t), \ldots\ldots\ldots\ldots\ldots, a_k(t)\,]$

$c = \text{diag}\, [\, \alpha_1 e^{j\Phi_1}, \ldots\ldots\ldots, \alpha_k e^{j\Phi_k}\,]$

$\xi = \text{diag}\, [\, \sqrt{\xi_1}, \ldots\ldots\ldots\ldots, \sqrt{\xi_k}\,]$

$b = [b_1, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, b_2]$

Hence the received signal of equation (7) can be written as

$$\text{r(t)} = \text{s(t)} + \text{n(t)} \qquad (13)$$

Based on equation (13), the output vector Z of the bank of matched filters portrayed in figure 9.1 can be formulated as

$$Z = [\, z_1, \ldots\ldots\ldots\ldots\ldots\ldots, z_2\,]^T$$

$$= RC\xi b + \text{n} \qquad (14)$$

Where

$$R = \begin{matrix} 1 & \rho_{12} & \cdots\cdots & \rho_{12} \\ \rho_{12} & 1 & \cdots\cdots & \rho_{12} \\ \cdot & & & \\ \rho_{12} & \rho_{12} & \cdots\cdots & 1 \end{matrix}$$

is the K × K dimensional user signature sequence cross correlation matrix elements and

$$\mathrm{n} = [n_1, \ldots\ldots\ldots\ldots\ldots, n_k]^T$$

is the zero mean Gaussian noise vector with a covariance matrix Rn = 0.5 N_0 R.

9.4 OPTIMUM MULTI-USER DETECTOR FOR S-CDMA SYSTEMS

In this section we will derive the joint optimum decision rule for a k-user CDMA System based on the synchronous system model highlighted in section 9.2. Specifically, we want to maximize the probability of jointly correct decisions of the k users supported by the system based on the received signal r (t) of equation (13).

From Equation (12) we note that there are m = 2^k possible combinations of b. we shall denote the i^{th} combination as b_i and the combined transmit signal of all users in equation (12) corresponding to the i^{th} combination as $b_i \leftrightarrow s_i(t)$.

Based on above notations, we can express the joint maximum a*posteriori* probability (MAP) criterion as

$$\hat{b} = \arg\{\max b_i [p(s_i(t)/r(t))]\} \tag{15}$$

Where $\hat{b}$ denotes the detected bit combination.

According to equation (7), the received signal r(t) is a Gaussian distributed random variable having mean equal to that of s(t) given by equation (12). Hence it can be shown that the likelihood function $p(r(t)/s_i(t))$ is given by,

$$\begin{aligned} p(Z/s) &= \exp\left(-\frac{1}{2\sigma^2} \int_0^{T_b} \left| r(t) - s(t) \right|^2 dt \right) \\ &= \exp\left(-\frac{1}{2\sigma^2} \int_0^{T_b} \left| r(t) - \sum_{k=1}^{k} \sqrt{\xi_k}\, \alpha_k b_k a_k(t) e^{j\Phi_k} \right|^2 dt \right) \end{aligned} \tag{16}$$

Taking the natural logarithm of the likelihood function of equation (16), the resulting so called *log likelihood function* (LLF) can be written as

$$\ln p(Z/s) = -\frac{1}{2\sigma^2} \left\{ \int_0^{T_b} |r(t)|^2 dt + \int_0^{T_b} \left| \sqrt{\xi_k}\, \alpha_k b_k a_k(t) e^{j\Phi_k} \right|^2 dt - 2\Re\left[\int_0^{T_b} r(t) \sum_{k=1}^{k} \sqrt{\xi_k}\, \alpha_k b_k a_k(t) e^{-j\Phi_k} dt \right] \right\} \tag{17}$$

The term $|r(t)|^2$ is common to all decision metrics, and hence it can be ignored during the optimization. Similarly, the constant term $1/2\sigma^2$ will not influence the maximization. Thus we can express the log-likelihood function of equation (17) in the form of a correlation metric as,

$$\Omega(b) = 2\Re\left[\int_0^{T_b} r(t)\sum_{k=1}^{K}\sqrt{\xi_k}\alpha_k b_k a_k(t)e^{-j\Phi_k}\,dt\right] - \int_0^{T_b}\left|\sum_{k=1}^{K}\sqrt{\xi_k}\alpha_k b_k a_k(t)e^{j\Phi_k}\right|^2 dt$$

$$= 2\Re\left[\sum_{k=1}^{K}\sqrt{\xi_k}\alpha_k b_k e^{-j\Phi_k} z_k\right] - \sum_{l=1}^{K}\sum_{k=1}^{K}\sqrt{\xi_l\xi_k}\, b_l b_k \alpha_l \alpha_k e^{j\Phi_l} e^{-j\Phi_k}\rho_{lk} \tag{18}$$

Where z_k and ρ_{lk} are given by equation (8) and equation (9), respectively. By expressing in vector notation as,

$$\Omega(b) = 2\Re\left[b^T \xi C^* Z\right] - b^T \xi C R C^* \xi b \tag{19}$$

Hence the decision rule for the optimum CDMA multi-user detection scheme based on the maximum likelihood criterion is to choose the specific bit combination b, which maximizes the correlation metric of equation (19). Hence

$$\hat{b} = \arg\{\max b|\Omega(b)|\} \tag{20}$$

9.5 SUMMARY

In this chapter we described GA assisted MUD for CDMA systems. It also gives details about S-CDMA system model implemented and its performance. It also mentions various modulation scheme and spreading codes used for MATLAB simulation.