Chapter 2

Voltage Stability With Wind Power

2.1 Introduction

The stability of power system has been defined by various CIGRE and IEEE Task Force reports. However, these completely neither meet the industry needs nor matches the experiences and understandings. The Joint Task Force, set up by CIGRE study committee 38 and IEEE Power System Dynamic Performance Committee, has addressed the stability definition from fundamental and also closely examined from practical point of view. Reference [281] can be referred for detail classification and definition of stability. This chapter is dedicated to the voltage stability, so the voltage stability definition is reproduced here for ready reference.

Stability is the ability of a power system to maintain specified parameters at across the system under normal and abnormal condition. If specified parameter is voltage, it is known as a voltage stability. Power system may tend to become unstable when voltage varies beyond desired limit. Further, voltage stability can be classified into two subclasses, large disturbance voltage stability and small disturbance voltage stability.

Large Disturbance Voltage Stability: It is mainly concerned with the system′s ability to control and restore voltage after the large disturbances like fault, loss of large generator, loss of large load, outage of part of the network and such many more.

Small Disturbance Voltage Stability: It is mainly related with the system's ability to maintain and control voltage following small changes like load change, generation change. Such kind of phenomena are very slow in nature and it slowly and slowly move towards the instability.

Voltage Collapse: Voltage collapse is one of the instability which occurs due to various reasons like heavy load on system, persistent fault and reactive power deficit. Generally, voltage collapse starts with low voltages in some parts of the system and then it slowly progresses towards voltage collapse.

Another kind of voltage collapse is fast occuring where voltage falls to very low value due to fault or network failure.

In this work, the steady state voltage assessment of power system, considering wind power variability, is done using combination of deterministic and stochastic methods.

2.2 Voltage Stability Analysis

An operating point of a power system is stable, if following any small or large disturbance, the power system state returns close to the pre-disturbance operating condition. Thus, a power system is voltage stable, if the load voltage approaches post disturbance steady state value following a given disturbance. If the change in load or generation is gradual, the operating point changes gradually. However, the power system can lose stability following a large change in load or generation. This may occur with wind power. Wind Power is volatile in nature and varies with time. If wind power share is considerable in the system, the change in wind power may disturb the balance between load-generation, hence, the voltage at different nodes may vary and affect the current taken by the loads, which in turn leads to voltage change. It is a cumulative process and leads to the ultimate voltage reductions beyond the controllability range. In a system with significant wind power injection, it is required to monitor wind power injection and voltage at different buses to avoid the system failure. The value of significant wind power depends on various factors like system strength, load level, provision of ancillary power services, availability of conventional generation with high ramp rate, number of hot wheeling generation versus cold generation etc. Apart from these, there are several other factors, which are enlisted here.

1. Real and Reactive power injection in the system

2. Action and Co-ordination of voltage control devices

3. Generator reactive power limit

4. Load characteristics

5. Transmission line and transformer parameters.

Voltage Stability with wind power can be ensured up to certain extent with reactive power compensation. Two possible reactive power compensations are commonly implemented in a WPP, one is reactive compensation at the Wind Turbine level (mostly in Type-1 and Type-2 Wind Turbine generators); the other is at the plant level (usually at the low voltage side of the substation transformer). Add-on to this, Plant-level compensation is usually added when the WPP is connected to a weak grid. Additional compensation in Type-3 and Type-4 is generally not required, as Wind Turbines are equipped with power converters that can provide controllable reactive power.

Consider Figure (2.1), which shows the two bus system, with generation on Bus-1 and load on Bus-2. The two buses are connected through transmission line. The voltage at Bus 1 and Bus 2 is maintained constant, and there is equal magnitude of 1.00 p.u on both the buses. The reactive power is supplied by both Bus-1 and Bus-2 to maintain the voltage. The voltage is to be constant; thus, as the output power from Bus-1 fluctuates, the reactive power must also be adjusted to follow the output power generated by Bus-1. In this case, the load angle varies to accommodate the active power flow. To maintain the voltage at two buses, reactive power is controlled.

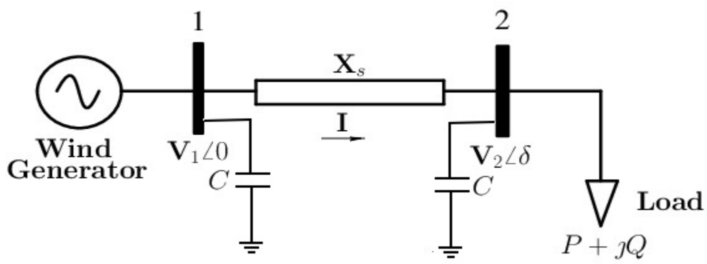

Figure 2.1: Two Bus System

$$\phi_1 = \phi_2 = \frac{\delta}{2} \tag{2.1}$$

Thus, the reactive power generated by Bus-1 is given by,

$$Q_1 = V_1 I \sin\left(\frac{\delta}{2}\right) \tag{2.2}$$

The reactive power is controlled with active power by following relation,

$$Q_1 = P_1 \tan\left(\frac{\delta}{2}\right) \tag{2.3}$$

As, the resistance of transmission line is neglected and power factor is assumed unity, so the real power loss is zero and real power at sending end and receiving end is equal. It is given by,

$$P_1 = P_2 = \frac{V^2}{X_s} \sin \delta \tag{2.4}$$

With reactive power control at Bus-1 and Bus-2, the voltage level can be kept constant with variable wind power. The power angle δ (phase angle between sending-end voltage and receiving-end voltage) is not affected with constant power delivery, but it will fluctuate with fluctuation in real power injection. The reason for controlling reactive power, at both end of network, is to maintain equal contribution of reactive power from both sides of network.

The power transfer capacity can also be increased by using the Flexible Alternating Current Transmission System (FACTS). It can be accomplished by Thyristor Controlled Series Capacitor as shown in figure (2.2). As, the real power injection varies, the transmission line reactance can also be kept constant by varying capacitive element. The advantage of this is that, the voltage and current rating of the series compensation is relatively small, as it is intended to compensate for the voltage drop of the line impedance. Series compensation is commonly used to compensate long transmission lines; however, care must be taken while selection of compensation level. As it may cause sub-synchronous resonance (SSR). By adjusting $X_c = f(X_s)$, it is possible to generate an improved power factor at Bus 1 and reduce the power angle δ between Bus-1 and Bus-2 for the same power delivery. It will contribute to stability of the system.

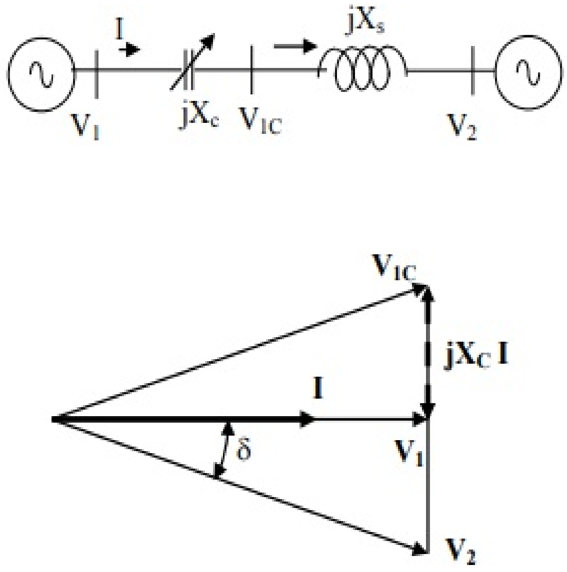

Figure 2.2: Series Compensation of Line

The reactive power compensation can also be accomplished by other FACTS devices. The devices in FACTS family is mentioned here [282].

- Static Synchronous Series Compensator (SSSC)
- Thyristor-Controlled Series Capacitor (TCSC)
- Thyristor-Controlled Reactor (TCR)
- Thyristor-Switched Capacitor (TSC)
- Static Synchronous Compensator (STATCOM)
- Static VAR Compensator (SVC) (TCR+TSC)

2.2.1 Methods of Voltage Stability Analysis

Voltage stability is a highly non-linear phenomena and it is not directly related to any parameter, but it depends on various parameters and relation between them is very non-linear. There are various non-linear dynamics associated with the problem hence various aspects of voltage stability problem can be effectively analysed using various static and dynamic techniques. Few such techniques are listed here.

1. Static Analysis
    - PV or QV Curve [283]
    - Load Flow Study [284]
    - Eigenvalue Analysis [285] [286] [287] [288]
    - Bifurcation Analysis [289]
    - Minimum Singular Value Method [290]
    - Point of Collapse Method [291]
    - Optimization Method [292]
    - Continuation Load Flow Method [293]
2. Dynamic Analysis
    - Time Domain Analysis [294]
    - Transient Energy Function Analysis [295]

It is worth to note that, all methods are ultimately related to the load flow study [294]. Either implicitly or explicitly, all methods are depend on the load flow. For example, $P-V$ curve and $Q-V$ curve can be traced either by load flow or continuation power flow method, which in turn modified form of load flow equations [293]. In this work, the load flow study, continuation power flow method, and $P-V$ curve method have been used along with statistical methods to analyse the voltage stability of power system with wind power. Load flow method has been very popular and is used widely in many literatures, so it is not described here. The cited references may be referred for detail study of each method.

2.3 Statistical Estimation Methods for Voltage Stability Evaluation

The voltage analysis methods, discussed in section (2.2.1), are deterministic in nature. These methods can be used alone for deterministic solution for steady state operating condition. But, when it comes to the dynamic variable, stochastic methods should be incorporated along with the above mentioned methods to determine the probabilistic output. To carry out probabilistic analysis using deterministic methods, voltage analysis method is encapsulated in statistical form. It is done by solving many deterministic problems covering all the possibilities of parameter variation. Statistical methods basically decides the different combination of variable parameter using predetermined rule. Various probabilistic methods considered in this work are given here.

1. Monte Carlo Simulation [296]
2. Point Estimation Method (Two Point Estimation, Three Point Estimation, Multi-Point Estimation) [297] [298] [67] [299] [300]
3. Latin Hypercube Sampling Method (LHS) [301]
4. PDF Estimation using method of Moments [302] [303]
5. PDF Estimation using method of Cummulants [302] [304] [31]
6. Series Expansion Method [305]

2.3.1 Monte Carlo Simulation

It is based on the principle of repeated random sampling to obtain numerical results [296]. It follows the process of generating independent and random samples, having different distribution. Monte Carlo simulation is a technique used to understand the impact of risk and uncertainty in random variable in various models. A Monte Carlo simulator helps to visualize most or all of the potential outcomes to have a better idea regarding the uncertainty of a decision. In this method, the stochastic problem is converted in to the many deterministic sub-problem and then each sub-problem is solved with predetermined method. In the assessment of impact of wind power on power system stability, Monte Carlo Simulation (MCS) is used to generate random wind speed in order to simulate realistic power system condition. The generated wind speed is taken as an input and wind power is calculated. The calculated wind power is then fed to the load flow problem and the voltage and angle at various buses as well as other parameters (like line loss, slack bus power, surplus or deficit reactive power etc...) can be calculated. This approach is though time consuming, but it is simplified in nature and effective in studying probabilistic power flow.

2.3.2 Point Estimation Method

Point estimation Method is based on the first few central moments of random input variables on K points for each variable known as a concentration [300]. These variables are then transferred in to output variable using non-linear function F, in our case it will be a non-linear load flow. The non-linear function relates inputs to outputs. By making use of these points and non-linear function, the uncertainty associated with output variable is captured. The k^{th} concentration of a random variable p_l is defined by location $p_{l,k}$ and weight $w_{l,k}$. The $p_{l,k}$ is the k^{th} value of random variable P_l. At P_l, the non-linear function F is evaluated. The $w_{l,k}$ is a weighting factor which gives relative importance of output in calculation of output moments, i.e. mean, variance etc...

By using point estimate method, the function F is evaluated K times for each input random variable p_l. Each random input vector consists of k^{th} location $p_{l,k}$ of the input random variable p_l and the mean (μ) of $n-1$ remaining input variables, i.e. $(\mu_{p1}, \mu_{p2}, ...p_{l,k}, ...\mu_{pn})$. In other words, the stochastic problem is fragmented into $K \times n$ deterministic problems. Therefore, the total number of evaluation of F is $K \times n$.

The point estimation is applied in different way. One of them is as explained above. The alternative way is to evaluate non-linear function F at $K \times n + 1$ times. Additional point is the average of all random input variables $(\mu_{p1}, \mu_{p2}, ...\mu_{p,l}, ...\mu_{pn})$.

The K concentrations $(p_{l,k}, w_{l,k})$ of n input random variables p_l are obtained from statistical input data (i.e. the probability distribution function f_{p_l} given in figure 2.3).

The location $p_{l,k}$ is determined by,

$$p_{l,k} = \mu_{pl} + \xi_{l,k}\sigma_{pl} \tag{2.5}$$

Where, $\xi_{l,k}$ is the standard location, μ_{p1} is mean, and σ_{pl} is standard deviation of the input random variable p_l.

The standard location $\xi_{l,k}$ and weight $w_{l,k}$ is obtained by solving following two equations (2.6 - 2.7).

$$\sum_{k=1}^{K} w_{l,k} = \frac{1}{n} \tag{2.6}$$

$$\sum_{k=1}^{K} w_{l,k}(\xi_{l,k})^j = \lambda_{l,j}, j = 1, 2, ...2K - 1 \tag{2.7}$$

Where, $\lambda_{l,j}$ denote the j^{th} standard central moment of the random variable p_l with probability density function f_{p_l}.

$$\lambda_{i,j} = \frac{M_j(p_l)}{(\sigma_{p_l})^j} \tag{2.8}$$

$$M_j(p_l) = \int_{-\infty}^{\infty} (p_l - \mu_{p_l})^j f_{p_l} dp_l \tag{2.9}$$

It is important to note that, $\lambda_{l,1}$ equal to zero, $\lambda_{l,2}$ equal to one, and $\lambda_{l,3}$ and $\lambda_{l,4}$ are the skewness and kurtosis of p_l respectively. The equations (2.6) and (2.7) are solved by method described in [306].

First, all concentrations $(p_{l,k}, w_{l,k})$ are obtained. Then, load flow function F is evaluated at the points $(\mu_{p1}, \mu_{p2}, ...p_{l,k}, ...\mu_{pn})$ to obtain the $Z(l,k)$. Where $Z(l,k)$ is the vector of output random variables. Finally, the j^{th} raw moment of output random variable is obtained by using the weighting factor $w_{l,k}$ and the $Z(l,k)$ values as given in equation (2.10).

$$\mu_j = E[Z^j] \cong \sum_{l=1}^{n} \sum_{k=1}^{K} w_{l,k}(Z(l,k))^j \tag{2.10}$$

Based on the number of points K and number of input random variables n, the point estimation scheme $K \times n$ is worked out using above discussed formulas (2.5) to (2.10).

2.3.2.1 Two Point Estimation Method

Two Point Estimation Method is a variant of original point estimate method. The method was proposed by Rosenblueth in 1970's [307]. The Point Estimation Method (PEM) is simple to use numerical method for calculating moment of non-linear problem, having one or several random variables. Using these methods, output variable are calculated without having information of Probability Distribution Function (PDF) of associated input variable. Generally, this method has been widely used for normally distributed data, but in this work it is applied to non-normal, weibull distribution, data. Two PEM, takes only two deterministic value of each uncertain variable, one value below the mean and one value above the mean, and other variables are kept at their mean value.

The point estimate method for probabilistic voltage stability analysis has been described as follows. Let $p_{l,k}$ be the wind power with probability density function f_{pl}. The two point estimation method uses two points of parameter p_l, p_{l1} and p_{l2}. The two points are calculated using following formula.

$$p_{l,k} = \mu_{pl} + \xi_{l,k}\sigma_{pl} \tag{2.11}$$

where μ_{pl} and σ_{pl} are the mean and standard deviation of parameter p_l. The $\xi_{l,k}$ is given by following equation.

$$\xi_{l,k} = \frac{\lambda_{l,3}}{2} + (-1)^{3-k} \times \sqrt{n + (\frac{\lambda_{l,3}}{2})^2}, k = 1, 2 \tag{2.12}$$

The $\lambda_{l,3}$ denotes the skewness of parameter p_l. It is given by,

$$\lambda_{l,3} = \frac{E(p_l - \mu_{pl})^3}{(\sigma_{pl})^3} \tag{2.13}$$

$$E(p_l - \mu_{pl})^3 = \sum_{t=1}^{N} (p_{l,t} - \mu pl)^3 \times Prob(p_{l,t}) \tag{2.14}$$

Where, N is the number of observations of variable p_l and $Prob(p_{l,t})$ is the probability of each observations $p_{l,t}$, which is equal to $\frac{1}{n}$. The datasets prepared using variable $p_{l,1}$ and $p_{l,2}$ are, $X = [\mu_1, \mu_2, \mu_3, ...p_{l,1}, ...\mu_n]$ and $X = [\mu_1, \mu_2, \mu_3, ...p_{l,2}, ...\mu_n]$. Such $2n$ datasets are prepared for n random variables. The factor $w_{l,k}$ is used to give weightage to the dataset $X = [\mu_1, \mu_2, \mu_3, ...p_{l,k}, ...\mu_n]$. It is then used to scale estimates based on given dataset. It is given by,

$$w_{l,k} = \frac{1}{n}(-1)^k \frac{\xi_{l,3-k}}{\zeta_l} \tag{2.15}$$

The value of $w_{l,k}$ ranges from 0 to 1 and sum of all $w_{l,k}$ is unity.

$$\zeta_l = 2\sqrt{m + (\frac{\lambda_{l,3}}{2})^2} \tag{2.16}$$

The j^{th} moment of Z_i is calculated by using following equation.

$$E(Z_i^j) \cong \sum_{l=1}^{n} \sum_{k=1}^{2} w_{l,k} \times [Z_i(l,k)]^j \tag{2.17}$$

$$E(Z_i^j) \cong \sum_{l=1}^{n} \sum_{k=1}^{2} w_{l,k} \times F_i[\mu_1, \mu_2, \mu_3, ...p_{l,k}, ...\mu_n]^j \tag{2.18}$$

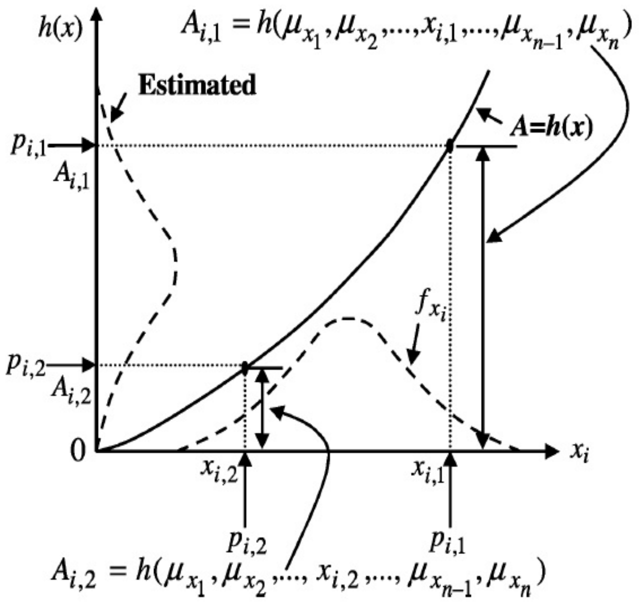

Figure 2.3: Point Estimation Method

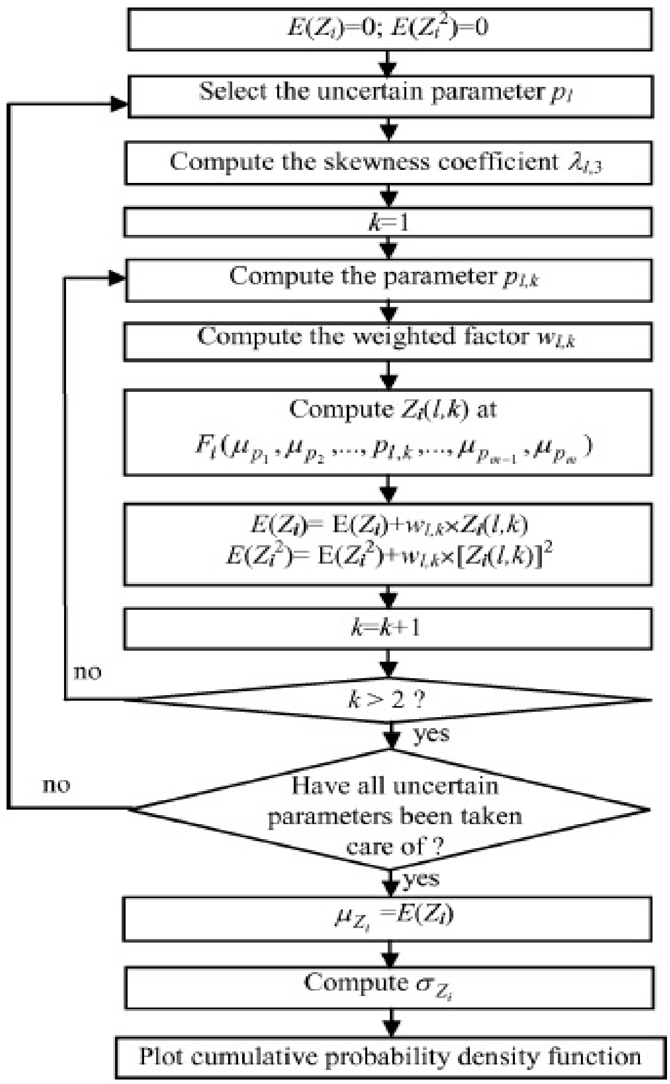

Figure 2.4: Flow Diagram of Two Point Estimation

Figure (2.3) shows the transformation of input to output using Two Point Estimation method. The flow diagram of Voltage Analysis with Two Point Estimation is shown in figure (2.4). First, the datasets are prepared and then non-linear power flow equation is solved using Newton-Raphon method. For n random variables, here wind speed and hence wind power, $2n$ calculations are required. So, the calculation burden is low as compared to the Monte Carlo Simulation method. The point estimation method is originally developed for uncorrelated data, however, using rotational transformation, it can be applied to correlated data also.

This schemes only uses two concentrations for each input random variable. As per the equation (2.6) and (2.7), for two points ($K = 2$), the skewness $\lambda_{l,3}$ is sufficient to reach anaylitical solution for the standard locations $\xi_{l,1}$ and $\xi_{l,2}$ and weights $w_{l,1}$ and $\xi_{l,2}$ of random variables p_l. Then, from mean μ_{pl} and standard deviation σ_{pl} of input random variable, the standard locations $p_{l,1}$ and $p_{l,2}$ can be worked out using equation (2.5).

It is worth noting that from equation (2.12) that, the standard location $\xi_{l,1}$ and $\xi_{l,2}$ are function of number of input random variables n. As n increases, the locations $p_{l,1}$ and $p_{l,2}$ move away from mean (μ_{pl}) according to $\sqrt{n}$. Then, locations may be the points outside the region of definition, where probability distribution of p_l is not well known. This drawback is not limited to only two point schemes, but all (K) and ($K \times n$) schemes. Despite all this drawbacks ($2n$) scheme is very user friendly and advantageous due to its simplicity in application and low computation burden. Also, it mostly gives real value solutions for the concentrations.

2.3.2.2 Two Point Plus One Sampling $(2n + 1)$ Method

This method of sampling is same as the Two Point Sampling with one additional sample at the mean of all random variables [297]. This is achieved by making $\xi_{l,3}$=0. This will create points at the location $p_{l,k} = \mu_{p,l}$. It gives the data points $(\mu_{p,1}, \mu_{p,2}...\mu_{p,n})$. In this method only one additional location is to be evaluated as compared to Two Point method. The related weights w_0 are evaluated as follows.

$$w_0 = \sum_{l=1}^{n} w_{l,3} \tag{2.19}$$

and,

$$w_3 = \frac{1}{n} - \frac{1}{\lambda_{l,4} - \lambda_{l,3}^2} \tag{2.20}$$

The $2n + 1$ scheme yields higher accuracy as compared to Two Point ($2n$) scheme. This is because consideration of kurtosis $\lambda_{l,4}$ of random variables. Also, the calculation burden is not increased much, as only one additional evaluation is required.

2.3.2.3 Three Point $(3n)$ Method

In this method, the location and weights of three distinct points are calculated. However, the method of calculation is different than Two Point method. The method explained in

[297] is not stable and produce negative location points with positive data samples. So the method is modified. The modification is done in calculation of coefficients C_n, using relation $[A][C_n] = [\lambda_{l,k}]$. The necessary condition to acertain the stability of this method is to ensure positive definiteness of matrix $[A]$, i.e.the eigenvalues of matrix should be real. As the standard method of calculation of location points and weights are not possible, it is solved by different method as suggested in [306]. The standard location $\xi_{l,k}$ are the roots of the third order polynomial as given below.

$$G(\xi) = \xi^3 + C_2\xi^2 + C_1\xi + C_0 \tag{2.21}$$

The coefficients of equation (2.21) is calculated as following.

$$\begin{bmatrix} n & \lambda_{l,1} & \lambda_{l,2} \\ \lambda_{l,1} & \lambda_{l,2} & \lambda_{l,3} \\ \lambda_{l,2} & \lambda_{l,3} & \lambda_{l,4} \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix} = - \begin{bmatrix} \lambda_{l,3} \\ \lambda_{l,4} \\ \lambda_{l,5} \end{bmatrix} \tag{2.22}$$

where, $\lambda_{l,1}$=0 and $\lambda_{l,1}$=1.

The weights are then calculated using following formula.

$$w_{l,k} = \frac{1 + \frac{1}{n}\prod_{j \neq k} \xi_{l,j}}{\prod_{j \neq k}(\xi_{l,j} - \xi_{l,k})} \tag{2.23}$$

This method is theoretically more accurate as compared to $2n$ and $2n + 1$. This is because it takes more statistical information of random variable in to consideration (i.e.higher order moments) and also three points of each variable. This additional points requires additional $2n$ evaluations, so computational burden is slightly greater than the $2n$ and $2n + 1$ method.

2.3.2.4 Four Point Plus One Method ($4n + 1$)

This scheme is basically derived from $5m$ scheme by making $\xi_{l,5}$ equal to zero [297]. The other four standard locations $\xi_{l,k}$ are the roots of the following fourth order polynomial equation.

$$G(\xi) = \xi^4 + C_3\xi^3 + C_2\xi^2 + C_1\xi + C_0 \tag{2.24}$$

The coefficients of equation (2.24) is calculated as following.

$$\begin{bmatrix} \lambda_{l,1} & \lambda_{l,2} & \lambda_{l,3} & \lambda_{l,4} \\ \lambda_{l,2} & \lambda_{l,3} & \lambda_{l,4} & \lambda_{l,5} \\ \lambda_{l,3} & \lambda_{l,4} & \lambda_{l,5} & \lambda_{l,6} \\ \lambda_{l,4} & \lambda_{l,5} & \lambda_{l,6} & \lambda_{l,7} \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} = - \begin{bmatrix} \lambda_{l,5} \\ \lambda_{l,6} \\ \lambda_{l,7} \\ \lambda_{l,8} \end{bmatrix} \tag{2.25}$$

where, $\lambda_{l,1}$=0 and $\lambda_{l,1}$=1.

After finding location points, the weights are determined by,

$$\begin{bmatrix} \xi_{l,1} & \xi_{l,2} & \xi_{l,3} & \xi_{l,4} \\ \xi_{l,1}^2 & \xi_{l,2}^2 & \xi_{l,3}^2 & \xi_{l,4}^2 \\ \xi_{l,1}^3 & \xi_{l,2}^3 & \xi_{l,3}^3 & \xi_{l,4}^3 \\ \xi_{l,1}^4 & \xi_{l,2}^4 & \xi_{l,3}^4 & \xi_{l,4}^4 \end{bmatrix} \begin{bmatrix} w_{l,1} \\ w_{l,2} \\ w_{l,3} \\ w_{l,4} \end{bmatrix} = - \begin{bmatrix} \lambda_{l,1} \\ \lambda_{l,2} \\ \lambda_{l,3} \\ \lambda_{l,4} \end{bmatrix} \tag{2.26}$$

where, $\lambda_{l,1}$=0 and $\lambda_{l,1}$=1.

The additional one sample point is created by making $\xi_{l,5}$=0. This will create points at the location $p_{l,k} = \mu_{p,l}$. It gives the data points $(\mu_{p,1}, \mu_{p,2} ... \mu_{p,n})$. The weight of additional point w_0 has been calculated as below.

$$w_0 = \sum_{l=1}^{n} w_{l,5} \tag{2.27}$$

and,

$$w_{l,5} = \frac{1}{n} - \sum_{l=1}^{4} w_{l,k} \tag{2.28}$$

The $4n + 1$ scheme is same as the $5n$ scheme with $n - 1$ less computation iterations as compared to $5n$ schemes.

The higher the data points, higher is the accuracy in estimation of statistical moments. But, as the data point increase, more information of random variable is required, hence greater the computation burden. Also, it increases the probability of non-real solutions.

2.3.3 Latin Hypercube Sampling (LHS)

Latin Hypercube Sampling is one of the approach to improve the accuracy of sampling of random variable. It has been very widely used in the area of design with multi-simulation approach [308]. Another area of application is uncertainty analysis [309]. The standard deviation in results due to uncertainty can be improved by using effective method like LHS. There are upcoming areas like adaptive modelling, where LHS can be applied to achieve better results [310]. The reliability analysis was the first domain where LHS effective applied [311]. In this work, the LHS is applied for power system voltage analysis [312]. Compared to other random sampling methods, LHS has many advantages like good space filling, fine convergence characteristics, and of-course the robustness. LHS has been considered in two cases. First, where the time of simulation is important. And second, the repetitive or sequential sampling. The LHS can extended to higher number of samples, but it is difficult to keep the stratification properties of original samples. But in reference [313], extension of LHS in multiple of integer is proposed for stratification sampling. Multiple extension of LHS algorithm for correlated variable is nicely presented in reference [314]. The nested LHS and LHS based on nested orthogonal array is proposed in reference [315] and [316] respectively. A special integral extension of LHS is explained in reference [317]. The general philosophy of working of LHS is explained here. As far as the various variants concerned, are not given in detail here to maintain the brevity of the work.

Latin Hypercube Sampling is a statistical method for generating a near random sample of parameter values from a multidimensional distribution. Random Sampling method (like Monte Carlo Simulations) is based on randomness and it can make it inefficient. It might end up with some points clustered closely, while other interval space get no samples. Latin Hypercube Sampling (LHS) aims to spread the sample more evenly across all possible values. It partitions each input distribution into N intervals of equal probability, and select sample from each interval. It shuffles the sample for each input so that there is no correlation between the inputs, unless a correlation is needed. There are two variants of Latin Hypercube Sampling, Median Latin Hypercube (MLHS) uses the median value of each equi-probable interval. Random Latin Hypercube (RLHS) selects a random point within each interval. The impact of LHS is visually evident in the following Probability Density Graphs (PDFs) that are created using Analyticas built-in Kernel Density

Smoothing method for graphing, using samples generated from MC, RLHS and MLHS. Monte Carlo (MC) and Latin Hypercube Sampling (LHS) both are random sampling techniques, but MC requires more sample size to reduce computational errors. Also, it is difficult to answer, what sample size is required to achieve required level of precision. The concept of convergence rate partially answers this question. Alternatively, the term variance reduction is also used. To check the efficiency of LHS method as compared to MC Simulation, a sample of N=100 points from a standard Normal (0,1) distribution using each method has been generated and the sample mean is computed. RLHS and MC each end up with a non-zero value as a result of randomness in sampling, where MLHS estimates the mean as exactly zero.

Latin Hypercube Sampling is typically used to save computer processing time when running Monte Carlo simulations. Studies have shown that, a well-performed LHS can cut down the processing time by up to 50 % (versus a standard Monte Carlo sampling). Particularly, LHS is more important when working with slow operating systems and software than when doing analysis on faster devices. It is general perception that, the modern computers available to almost any researcher have made LHS obsolete, but it is still widely used. Although, it does not make big difference in analysis as it did in the days of slowly working computer systems. But, it still leads to marginally more accurate results (in terms of true variability) for any given amount of processing time

Table 2.1: Comparison of Sampling Error with LHS and MC, Source: Analytica [1]

Sampling Method	**Sampling Error**
Monte Carlo	0.100
Random Latin Hypercube Sampling	0.005
Median Latin Hypercube Sampling	0.000

Here, the sampling error with MC is 20 times larger than error with RLHS (Random Latin Hypercube Sampling). Published theoretical results shows that, the sampling error of MC goes down as $O(\frac{1}{\sqrt{N}})$, whereas the sampling error for LHS is $O(\frac{1}{N})$, which is quadratically faster. In other words, if N samples are needed for desired accuracy using LHS, N^2 samples are required for MCS for the same accuracy. The above results holds only for the univariate case, when model has a single uncertain input variable. The convergence rate for LHS starts looking like MCS with multiple probabilistic inputs. This is because LHS shuffles each univariate sample, so that, the pairing of samples across input remain random. With multiple variables, randomness from shuffling becomes dominant source of randomness. According to results reported in some publications LHS has a measurable advantage up to three uncertain inputs. However, against this, there are numerous publications, which are in not agreement with this. Majority of conclusions are in favour of LHS with multiple variables.

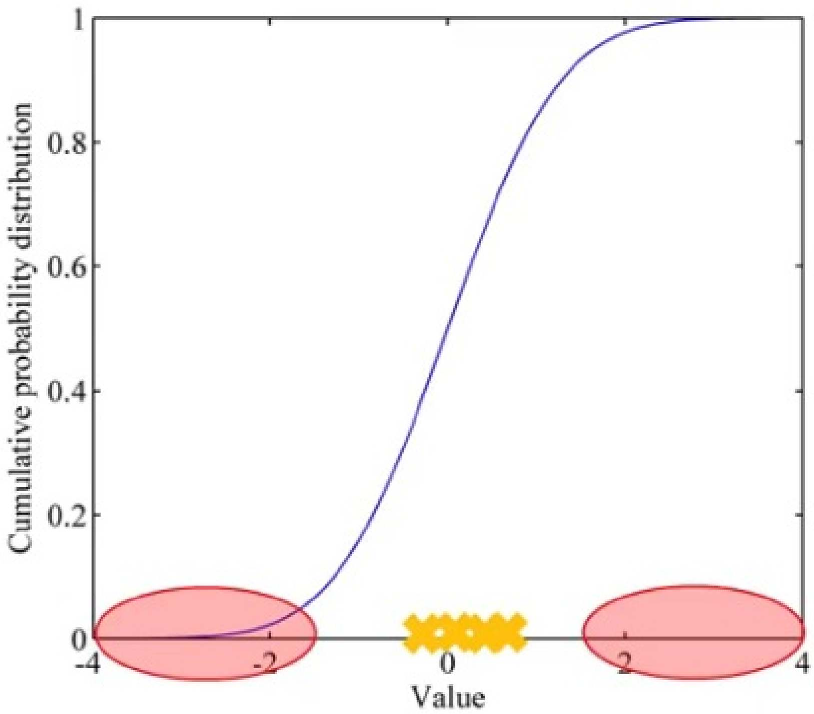

Figure 2.5: Problem with Random Sampling

The deficiencies of Random Sampling are,

1. The values in outer ranges of the distribution are not represented in the samples and thus their impact on the results is not in the simulation output
2. A large number of samples are needed
3. New samples are generated without taking into account the previously generated sample points

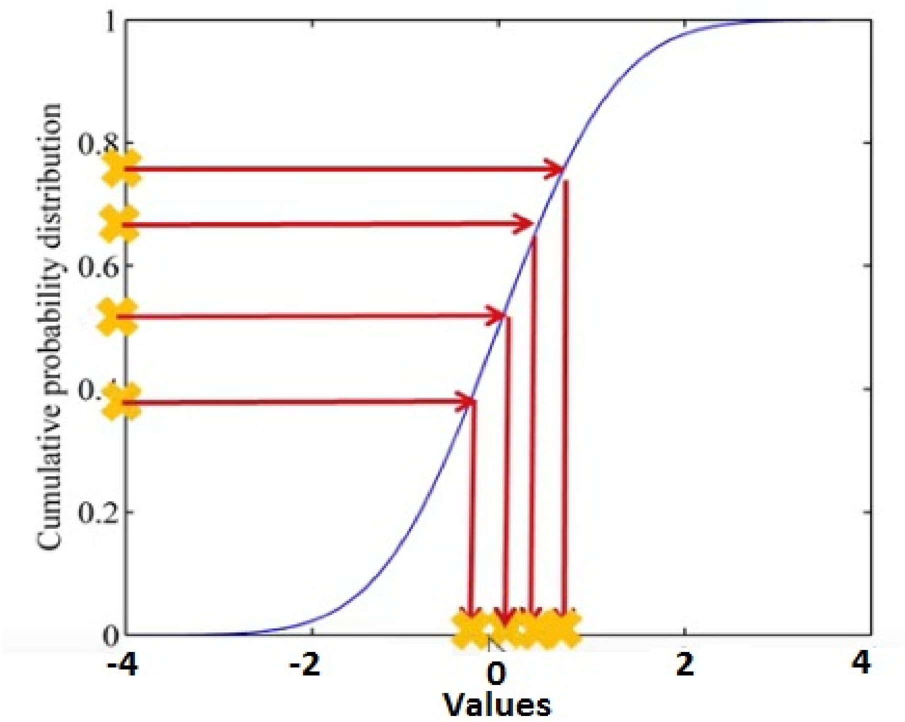

Figure 2.6: Latin Hypercube Sampling

The method of Lating Hypercube Sampling are,

1. For each sample, a random number between 0 and 1 is generated
2. The number will be assigned to a value using cumulative probability distribution
3. Each Sample is entirely random. Any given sample may fall anywhere within the range of the input distribution.
4. The outcomes are more likely in the range where the cumulative curve is the steepest.

For sampling N variables, the range of each variable is divided in to M equal probable interval. M sample points are then placed to meet the Latin Hypercube Sampling (LHS) requirements. LHS requires that, the intervals for each variable has to be same i.e.M. This independence is one of the main advantages of this sampling scheme. Another advantage is, it remembers which sample is taken so far, i.e. with memory sampling. In random sampling new sample points are generated without taking into account the previously generated sample points. It is not required to know beforehand how many sample points are needed. In LHS, one must first decide how many sample points to use. It also remembers in which row and column the sample point is taken. It is explained with the help of two dimension sample space.

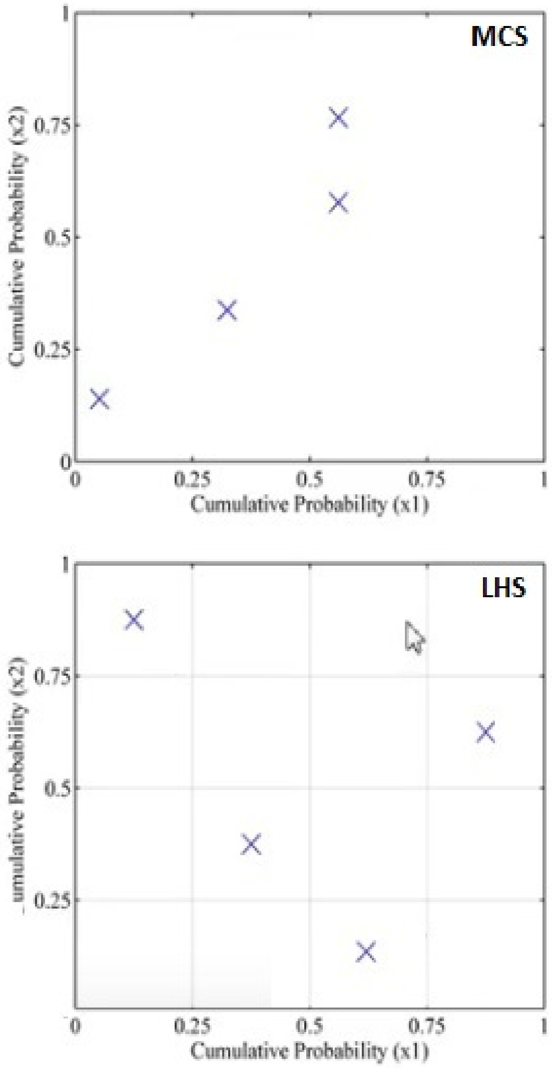

Figure 2.7: Comparison Between MCS and LHS

2.3.4 Method of Series Expansion

Any unknown "Gaussian" or "Non-Gaussian" probability distribution can be approximated by known probability distribution, known as a kernel. There are numerous ways to expand series. Among different series, the most popular are Gram - Charlier expansion and Edgeworth series. The purpose of Gram-Charlier and Edgeworth series is to express unknown distribution in terms of Gaussian distribution by correcting Gaussian distribution by using additional statistical information such as moments and cummulants. These methods relate an unknown probability distribution to a Gaussian by way of an operator transformation that is a function of the differentiator operator. The method of Phase-Space formulation of quantum mechanics has been used to generalize these methods. It has been generalized in two ways. First, by relating any two probability functions by way of differentiator function. Second, by relating any two probability function by Hermitian operator. The generalization results in unified approach for transformation of probability densities. It is worth noticing that, when these series are truncated, the resulting approximation does generally not positive. But, the series given here mostly remain positive. To understand the method of expressing PDF using series expansion, it is imperative to understand the Moments and Cummulants and relationship between them.

2.3.4.1 Moments and Cummulants

$$\mu'_t = E(x^t) = \int_{-\infty}^{\infty} x^t f(x) dx \tag{2.29}$$

Random variable x has a finite t^{th} moment, if the integral converges. It can be shown that, if x has a finite t^{th} moment, then it has a finite moment for each order less than t^{th}. In statistics, the moment μ'_t about a constant a are defined by $\int_{-\infty}^{\infty}(x-a)^t f(x)d(x)$. If a is equal to the mean μ'_1, they are called the *central moments* or moments about the mean

$$\mu_t = E((x-\mu_1)^t) = \int_{-\infty}^{\infty} (x-\mu_1)^t f(x) dx \tag{2.30}$$

The prime is used to differentiate the *moments* and *central moments*. Then, moment generating function $M_X(t)$ of the random variable x is calculated as,

$$M_X(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x) dx \tag{2.31}$$

The moment generating function $M_X(t)$ can also be expressed in infinite series form using Taylor expansion

$$M_X(t) = 1 + t\mu_1 + \frac{1}{2!}t^2\mu_2 + \frac{1}{3!}t^3\mu_4 + \frac{1}{4!}t^4\mu_4 + ... \tag{2.32}$$

In some cases theoretical treatments of problems in terms of Cummulants are simpler than those using moments. In this work, Cummulants are used rather than moments. Cummulants are most easily defined through the "Cummulant Generating Function", which is the logarithm of the moment generating function,

$$K_X(t) = logM_X(t) \tag{2.33}$$

In analogy with equation 2.32 , the expansion of the Cummulant generating function has the set of Cummulants as coefficients,

$$K_X(t) = 1 + t\kappa_1 + \frac{1}{2!}t^2\kappa_2 + \frac{1}{3!}t^3\kappa_4 + \frac{1}{4!}t^4\kappa_4 + \ldots\ldots\ldots \tag{2.34}$$

The coefficients κ_t are called Cummulants of order t of random variable x.

The moments have following desired properties, which make them more useful than moments.

- Most statistical calculations using Cummulants are simpler than the corresponding calculations using moments.
- The Cummulant of sum of n independent random variable y_k,

  $$X = \sum_{k=1}^{n} Y_k$$

  is equal to the sum of cummulants of each variable

  $$\kappa_j(X) = \kappa_j(\sum_{k=1}^{n} Y_k) = \sum_{k=1}^{n} \kappa_j(Y_k)$$

- Where two variables in the case of multivariate distributions are statistically independent, the cross cummulants involving those variables are zero. If X and Y are independent, then $COV(X, Y) = E(X, Y) - E(X)E(Y) = 0$
- Unlike moments, cummulants behave very simply under affine transformations from X to a new variable Y.

2.3.4.2 Relation between Cummulants and Moments

The logarithmic function of Moments is the Cummulants (equation 2.33) . For small values of t, moment generating function is given by

$$K_x(t) = logM_x(t) = \sum_{j=1}^{\infty} \frac{t_j}{j!}\kappa_j + 0(t_k) \tag{2.35}$$

The coefficient κ_j is called Cummulant of order j of random variable x. Practically, κ_j and μ_j are difficult to calculate than μ'. But,it can be resolved with the fact that κ_j can be expressed in terms of $\mu'_1....\mu'_j$ and μ_j can be expressed in terms of $\kappa_1....\kappa_j$

$$\begin{aligned}
&\kappa_1 = \mu'_1 \\
&\kappa_2 = \mu'_2 - {\mu'_1}^2 \\
&\kappa_3 = \mu'_3 - 3\mu'_1\mu'_2 + 2{\mu'_1}^3 \\
&\kappa_4 = \mu'_4 - 3{\mu'_2}^2 - 4\mu'_1\mu'_3 + 12{\mu'_1}^2\mu'_2 - 6{\mu'_1}^4 \\
&\kappa_5 = \mu'_5 - 5\mu'_4\mu'_1 - 10\mu'_3\mu'_2 + 20\mu'_3{\mu'_1}^2 + 30{\mu'_2}^2\mu'_1 - 60\mu'_2{\mu'_1}^3 + 21{\mu'_1}^5 \\
&\kappa_6 = \mu'_6 - 6\mu'_5\mu'_1 - 15\mu'_4\mu'_2 + 30\mu'_4{\mu'_1}^2 - 10{\mu'_3}^2 + 120\mu'_3\mu'_2\mu'_1 - 120\mu'_3{\mu'_1}^3 + 30{\mu'_2}^3 - 270{\mu'_1}^2{\mu'_1}^2 \\
&+ 360\mu'_2{\mu'_1}^4 - 120{\mu'_1}^6 \\
&...
\end{aligned} \tag{2.36}$$

$$\begin{aligned}
&\mu_1 = 0 \\
&\mu_2 = \kappa_2 \\
&\mu_3 = \kappa_3 \\
&\mu_4 = \kappa_4 + 3{\kappa_2}^2 \\
&\mu_5 = \kappa_5 + 10\kappa_3\kappa_2 \\
&\mu_6 = \kappa_6 + 15\kappa_4\kappa_2 + 10{\kappa_3}^2 + 15{\kappa_2}^3 \\
&\mu_7 = \kappa_7 + 21\kappa_5\kappa_2 + 35\kappa_4\kappa_3 + 105\kappa_3{\kappa_2}^2 \\
&...
\end{aligned} \tag{2.37}$$

The cummlants also can be represented in terms of moments.

$$
\begin{aligned}
&\kappa_1 = \mu_1 = 0 \\
&\kappa_2 = \mu_2 \\
&\kappa_3 = \mu_3 \\
&\kappa_4 = \mu_4 - 3{\mu_2}^2 \\
&\kappa_5 = \mu_5 = \kappa_5 - 10\mu_3\mu_2 \\
&\kappa_6 = \mu_6 - 15\mu_4 - 10{\mu_2}^2 + 30{\mu_2}^3 \\
&...
\end{aligned}
\tag{2.38}
$$

The cummulats can also be expressed to the moments by the following recursion formula:

$$\kappa_n = \mu_n^{'} - \sum_{m=1}^{n-1} \binom{n-1}{m-1} \kappa_m \mu_{n-m}^{'} \tag{2.39}$$

2.3.4.3 PDF Approximation in terms of Reference PDF

The two probability densities $P_1(x)$ and $P_2(x)$ may be related by,

$$P_2(x) = \Omega(iD)P_1(x) \tag{2.40}$$

Where, $\Omega(iD)$ is function of the operator iD that is given below. Suppose $M_1(x)$ and $M_2(x)$ are the corresponding characteristics functions to the two densities.

$$\begin{cases} M_1(\theta) = \int e^{ix\theta} P_1(x)dx \\ P_1(x) = \frac{1}{2\pi} \int e^{-ix\theta} M_1(\theta)d\theta \end{cases} \tag{2.41}$$

$$\begin{cases} M_2(\theta) = \int e^{ix\theta} P_2(x)dx \\ P_2(x) = \frac{1}{2\pi} \int e^{-ix\theta} M_2(\theta)d\theta \end{cases} \tag{2.42}$$

It can be written as,

$$M_2(\theta) = \frac{M_2(\theta)}{M_1(\theta)} M_1(\theta) = \Omega(\theta)M_1(\theta) \tag{2.43}$$

Where,

$$\Omega(\theta) = \frac{M_2(\theta)}{M_1(\theta)} \tag{2.44}$$

The probability distribution $P_2(x)$, is given by

$$\begin{cases} P_2(x) & = \frac{1}{2\pi}\int e^{-ix\theta}M_2(\theta)d\theta \\ & = \frac{1}{2\pi}\int e^{-ix\theta}\frac{M_2(\theta)}{M_1(\theta)}M_1(\theta)d\theta \\ & = \frac{1}{2\pi}\int e^{-ix\theta}\Omega(\theta)M_1(\theta)d\theta \end{cases} \tag{2.45}$$

after using following relation

$$\Omega(\theta)e^{-ix\theta} = \Omega(iD)e^{-ix\theta} \tag{2.46}$$

We have,

$$\begin{cases} P_2(x) & = \frac{1}{2\pi}\int \Omega(iD)e^{-ix\theta}M_1(\theta)d\theta \\ & = \Omega(iD)\frac{1}{2\pi}\int e^{-ix\theta}M_1(\theta)d\theta \\ & = \Omega(\theta)P_1(x) \end{cases} \tag{2.47}$$

Two densities $P_1(x)$ and $P_2(x)$ can be represented in corresponding Cummulant Generating Characteristics function,

$$M_1(\theta) = \exp\left[\sum_{n=1}^{\infty}\kappa_n^{(1)}\frac{i^n}{n!}\theta^n\right] \tag{2.48}$$

$$M_2(\theta) = \exp\left[\sum_{n=1}^{\infty}\kappa_n^{(2)}\frac{i^n}{n!}\theta^n\right] \tag{2.49}$$

where $\kappa_(1)$ and $\kappa_(2)$ are the relative cummulants of $P_1(x)$ and $P_2(x)$.

$$\Omega(\theta) = \frac{M_2(\theta)}{M_1(\theta)} = \frac{\exp\left[\sum_{n=1}^{\infty}\kappa_n^{(1)}\frac{i^n}{n!}\theta^n\right]}{\exp\left[\sum_{n=1}^{\infty}\kappa_n^{(2)}\frac{i^n}{n!}\theta^n\right]} \tag{2.50}$$

These gives,

$$P_2(x) = \exp\left[\sum_{n=1}^{\infty}(\kappa_n^{(2)} - \kappa_n^{(1)})\frac{i^n}{n!}(iD)^n\right] P_1(x) \tag{2.51}$$

or

$$P_2(x) = \exp\left[\sum_{n=1}^{\infty}(\kappa_n^{(2)} - \kappa_n^{(1)})\frac{(-1)^n}{n!}(D)^n\right] P_1(x) \tag{2.52}$$

For convenience, it is defined that

$$\eta_n = \kappa_n^{(2)} - \kappa_n^{(1)} \tag{2.53}$$

$$P_2(x) = \exp\left[\sum_{n=1}^{\infty}(\eta_n)\frac{(-1)^n}{n!}(D)^n\right] P_1(x) \tag{2.54}$$

Equation (2.54) is called as generalized Edgeworth series. It relates any two probability densities.

2.3.4.3.1 Exact Solution The exponential function can be expanded in series form as given below.

$$\exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + ... \tag{2.55}$$

Expanding equation (2.54) as given in equation (2.55),

$$\left\{\begin{aligned} P_2(x) = \quad & [1 + \frac{[\frac{-\eta_1}{1!}(D) + \frac{\eta_2}{2!}(D)^2 + \frac{-\eta_3}{3!}(D)^3 + \frac{\eta_4}{4!}(D)^4 + ...]}{1!} \\ & + \frac{[\frac{-\eta_1}{1!}(D) + \frac{\eta_2}{2!}(D)^2 + \frac{-\eta_3}{3!}(D)^3 + \frac{\eta_4}{4!}(D)^4 + ...]^2}{2!} \\ & + \frac{[\frac{-\eta_1}{1!}(D) + \frac{\eta_2}{2!}(D)^2 + \frac{-\eta_3}{3!}(D)^3 + \frac{\eta_4}{4!}(D)^4 + ...]^3}{3!} + ...]P_1(x) \end{aligned}\right. \tag{2.56}$$

Equation (2.56) is exact solution. The accuracy of PDF prediction depends on the truncation of series. Another factor which impacts the accuracy is the selection of reference PDF. The detail of which is discussed ahead in this chapter.

2.3.4.3.2 First Approximation Equation (2.52) can be expanded in a power series

$$\exp\left[\sum_{n=1}^{\infty}(\kappa_n^{(2)} - \kappa_n^{(1)})\frac{(-1)^n}{n!}(D)^n\right] = 1 - a_1 D + \frac{1}{2}a_2 D^2 - \frac{1}{6}a_3 D^3 + \frac{1}{24}a_4 D^4 - \frac{1}{120}a_5 D^5 + \frac{1}{720}a_6 D^6 + ... \quad (2.57)$$

$$P_2(x) \sim [1 - a_1 D + \frac{1}{2}a_2 D^2 - \frac{1}{6}a_3 D^3 + \frac{1}{24}a_4 D^4 - \frac{1}{120}a_5 D^5 + \frac{1}{720}a_6 D^6 + ...]P_1(x) \quad (2.58)$$

$$\eta_n = \kappa_n^{(2)} - \kappa_n^{(1)} \quad (2.59)$$

The coefficients of equation (2.54) are given by,

$$\begin{cases} a_1 = \eta_1 \\ a_2 = \eta_2 + \eta_1^2 \\ a_3 = \eta_3 + 3\eta_2\eta_1 + \eta_1^3 \\ a_4 = \eta_4 + 4\eta_3\eta_1 + 3\eta_2^2 + 6\eta_2\eta_1^2 + \eta_1^4 \\ a_5 = \eta_5 + 5\eta_4\eta_1 + 10\eta_3\eta_2 + 10\eta_3\eta_1^2 + 15\eta_2^2\eta_1 + 10\eta_2\eta_1^3 + \eta_1^5 \\ a_6 = \eta_6 + 6\eta_5\eta_1 + 15\eta_4\eta_2 + 15\eta_4\eta_1^2 + 10\eta_3^2 + 60\eta_3\eta_2\eta_1 + 20\eta_3\eta_1^3 \\ +15\eta_2^3 + 45\eta_2^2\eta_1^2 + 15\eta_2\eta_1^4 + \eta_1^6 \end{cases} \quad (2.60)$$

If we assume that P_1 is standardize, that is

$$\begin{cases} \kappa_1^{(1)} = 0 \\ \kappa_2^{(1)} = 1 \end{cases} \quad (2.61)$$

then,

$$\begin{cases} a_1 = \kappa_1^{(2)} \\ a_2 = (\kappa_2^{(2)} - 1) + (\kappa_1^{(2)})^2 \\ a_3 = \eta_3 + 3(\kappa_2^{(2)} - 1)\kappa_1^{(2)} + (\kappa_1^{(2)})^3 \\ a_4 = \eta_4 + 4\eta_3\kappa_1^{(2)} + 3(\kappa_2^{(2)} - 1)^2 + 6(\kappa_2^{(2)} - 1) + (\kappa_1^{(2)})^4 \\ a_5 = \eta_5 + 5\eta_4\kappa_1^{(2)} + 10\eta_3(\kappa_2^{(2)} - 1) + 10\eta_3(\kappa_1^{(2)})^2 + 15(\kappa_2^{(2)} - 1)^2\kappa_1^{(2)} \\ +10(\kappa_2^{(2)} - 1)(\kappa_1^{(2)})^3 + (\kappa_1^{(2)})^5 \\ a_6 = \eta_6 + 6\eta_5\kappa_1^{(2)} + 15\eta_4(\kappa_2^{(2)} - 1) + 15\eta_4(\kappa_1^{(2)})^2 + 10\eta_3^2 + 60\eta_3(\kappa_2^{(2)} - 1)\kappa_1^{(2)} \\ +20\eta_3(\kappa_1^{(2)})^3 + 15(\kappa_2^{(2)} - 1)^3 + 45(\kappa_2^{(2)} - 1)^2(\kappa_1^{(2)})^2 + 15(\kappa_2^{(2)} - 1)(\kappa_1^{(2)})^4 + (\kappa_1^{(2)})^6 \end{cases} \tag{2.62}$$

If we further standardize P_2,

$$\begin{cases} \kappa_1^{(2)} = 0 \\ \kappa_2^{(2)} = 1 \end{cases} \tag{2.63}$$

then

$$\begin{cases} a_1 = 0 \\ a_2 = 0 \\ a_3 = \eta_3 \\ a_4 = \eta_4 \\ a_5 = \eta_5 \\ a_6 = \eta_6 + 10\eta_3^2 \end{cases} \tag{2.64}$$

substituting above in equation (2.58) gives,

$$P_2(x) \sim [1 - \frac{\eta_3}{6}D^3 + \frac{\eta_4}{24}D^4 - \frac{\eta_5}{120}D^5 + \frac{\eta_6 + 10\eta_3^2}{720}D^6 + ...]P_1(x) \tag{2.65}$$

2.3.4.3.3 Second Approximation The second approximation is given by replacing differential operator by Hermitian operator. Also, the normal PDF is taken as reference. So, the $\kappa_1^{(1)}$ equals to 0 and $\kappa_2^{(1)}$ equal to 1. This simplifies the the equation (2.56) to equation (2.66) as given below.

$$P_2(x) \sim [1+\frac{\kappa_3^{(2)}}{6}H_{e3}(x)+\frac{\kappa_4^{(2)}}{24}H_{e4}(x)+\frac{\kappa_5^{(2)}}{120}H_{e5}(x)+\frac{\kappa_6^{(2)}+10(\kappa_3^{(2)})^2}{720}H_{e6}(x)+....]N(0,1) \tag{2.66}$$

Where $H_{en}(x)$ is the Hermite polynomial.

2.3.4.4 Gram-Charlier Expansion

This method was proposed by the mathematician Jorgen Pedersen Gram and Carl Charlier. Different variants of this method can be found in different literature, where primarily moments of data are used to approximate the PDF. The recent development in this method is to use both cummulant and moments [302]. This method is used here to compare its performance vis-a-vis other methods. Consider variable x, that has a distribution of continuous type with a mean m and and standard deviation δ. A standardized variable ξ is expressed as $\xi = \frac{(x-m)}{\delta}$. According to Gram-Charlier expansion, its CDF ($F(\xi)$) and PDF ($f(\xi)$) are expressed as

$$\begin{aligned} F(\xi) &= c_0\Phi(\xi) + c_1\Phi'(\xi) + c_2\Phi''(\xi) + ... \\ f(\xi) &= c_0\phi(\xi) + c_1\phi'(\xi) + c_2\phi''(\xi) + ... \end{aligned} \tag{2.67}$$

where $\Phi(\xi)$ and $\phi(\xi)$ represent the CDF and PDF of the normal distribution, respectively.

The derivative of order k of $\phi(\xi)$ is calculated as

$$\phi^k(\xi) = (\frac{d}{d\xi})^k\phi(\xi) = (-1)^k H_k(\xi)\phi(\xi) \tag{2.68}$$

where $H_k(\xi)$ is the Hermite polynomial. The zero and first order polynomials are given as

$$
\begin{aligned}
H_0(\xi) &= 1 \\
H_1(\xi) &= \xi
\end{aligned}
$$

The higher order Hermite Polynomials can be found out using recursive relationship [64]. In addition, a recursive relationship is available to determine the second and higher order polynomials.

$$H_n(\xi) = \xi H_{n-1}(\xi) - (n-1)H_{n-2}(\xi) \tag{2.69}$$

Constant coefficients of Gram-Charlier expansion can be expressed in terms of central moments μ_j of variable ξ with continuous type distribution.

$$
\begin{aligned}
C_0 &= 1 \\
C_1 &= C_2 = 0 \\
C_3 &= \frac{1}{3!}(-\frac{\mu_3}{\delta^3}) \\
C_4 &= \frac{1}{4!}(\frac{\mu_4}{\delta^4} - 3) \\
C_5 &= \frac{1}{5!}(-\frac{\mu_5}{\delta^5} - 10\frac{\mu_3}{\delta^3}) \\
C_6 &= \frac{1}{6!}(\frac{\mu_6}{\delta^6} - 15\frac{\mu_4}{\delta^4} + 30) \\
&\ldots
\end{aligned} \tag{2.70}
$$

2.3.4.4.1 Selection of Reference PDF Effective estimation of unknown distribution $g(x)$ depends on the selection of reference PDF. Most of the published literatures have adopted the Gaussian reference PDF. There is one obvious question arises, how results will be with non-Gaussian PDF? It has already been discussed that, it is possible to use non-Gaussian PDFs as reference. The complication arising from the choice of Non-Gaussian PDFs are possibly worth it, if the resulting series expansion is more accurate and converges more quickly than the equivalent Gaussian reference. The next question is to which Non-Gaussian PDF to choose as a reference. This answer depends on the data being handled. The reference PDF should be near to the data being handled. Here, the wind power variation effect is being studied, so Weibull PDF would be obvious second choice after Gaussian PDF.

The reference PDF $f(x)$ must have following properties.

- Moments ξ_j of all orders of $f(x)$ must be finite
- Derivatives of any required order must exist
- The relationship of $f(x)$ and $g(x)$ is assumed to be compact. There is no need for the supports of two PDF to be the same, but it can be useful.
- If $x = a$ or $x = b$, and the support of $f(x) \in (a, b)$, there exist a high order "contact"

2.3.4.4.2 Selection Weibull Parameter for Reference PDF To use Weibull as a reference PDF, its parameters (Scale Parameter c and Shape Parameter k) should be selected carefully. Here one of the approximation method is given. The empirical method has a practical and straight forward, requires only the average wind speed $\bar{v}$, and the standard deviation of the wind speed data, σ. The Weibull Parameter estimated as given below.

$$k = (\frac{\sigma}{\bar{v}})^{-1.086} \tag{2.71}$$

$$c = \frac{\bar{v}}{\Gamma(1 + \frac{1}{k})} \tag{2.72}$$

Where, $'k'$ is Shape Parameter, $'c'$ is Scale Parameter and $'\Gamma'$ is the Gamma Function

There are other methods also available to estimate the Weibull Parameter. For more detail, reference [318] can be referred.

2.4 Correlated Wind Power Sources

Wind speed at any location is correlated (either positive or negative) with wind speed of other locations. If correlation is positive, the wind speed of two distinct site varies linearly in direct proportion. Whereas, for negative correlation, the wind speed of two distinct site varies linearly inversely. There are several methods to correlate data of different sites. The latest development in this regard is cupola theory. However, the simple yet effective method is direct conversion using Cholesky Decomposition. The main disadvantage of direct method is large error in correlated data, i.e.there is large difference between reference correlation factor and actual correlation factor of processed data. Due to this limitation, here the cupola method is used. Generation of correlated variable from uncorrelated variable using two different method is given here.

1. Form the Correlation Matrix $'C'$
2. Generated Matrix of Uncorrelated Variable (X)
3. Decompose Correlation Matrix in to Lower and Upper Triangular Matrix using Cholesky Decomposition $(LL^T = C)$
4. The Correlated Variable Matrix Y is derived from Uncorrelated Variable Matrix X using relation $LX = Y$.
5. For example, for two variable with correlation ρ is given by

$$[Y] = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix} [X]$$

6. From here there are two ways to get the correlated data. First method is to select values of uncorrelated data vector X based on the actual distribution of data. This method is known as Direct Method.
7. The second method is Cupola Method. In this method, uncorrelated data vector X is taken. This vector contains the normal random data with zero mean value for every variable. Then, the correlated normal random value is obtained by Cholesky Decomposition, using above equation $Y = LX$. Now, from this correlated value, the probability of each value of every variable is found out using normal CDF. Thus obtained value of probability is put in to the actual inverse function to get the correlated value with actual distribution. The actual implementation of this logic is given in section (2.5.2.3). This method is also known as Cupola Method.

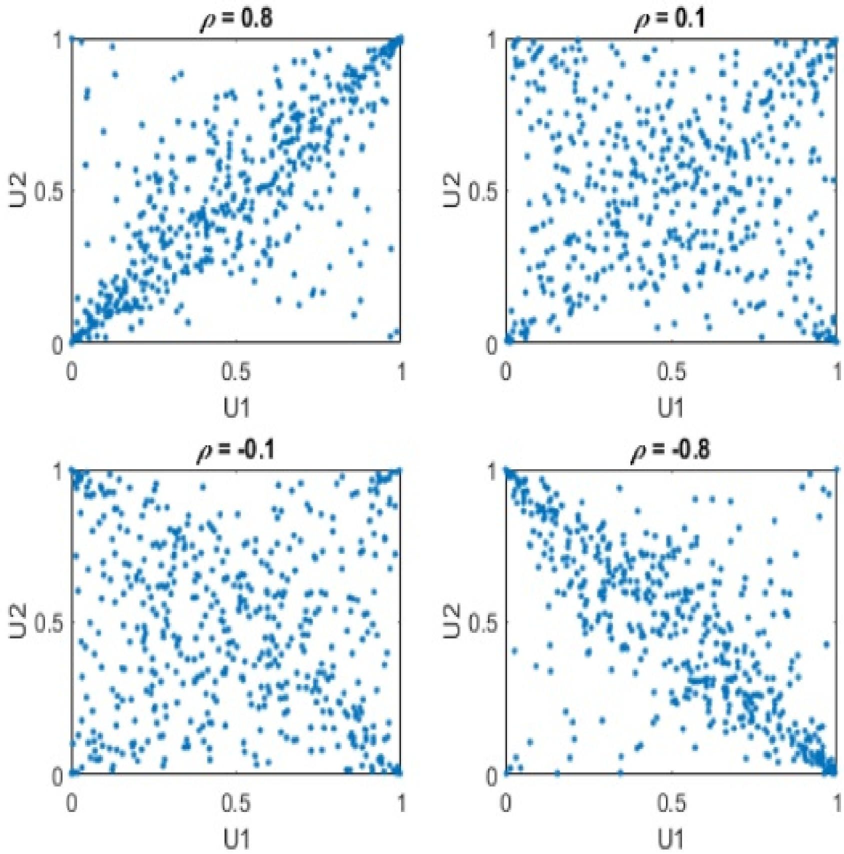

Figure 2.8: Relation of Variables with different correlation

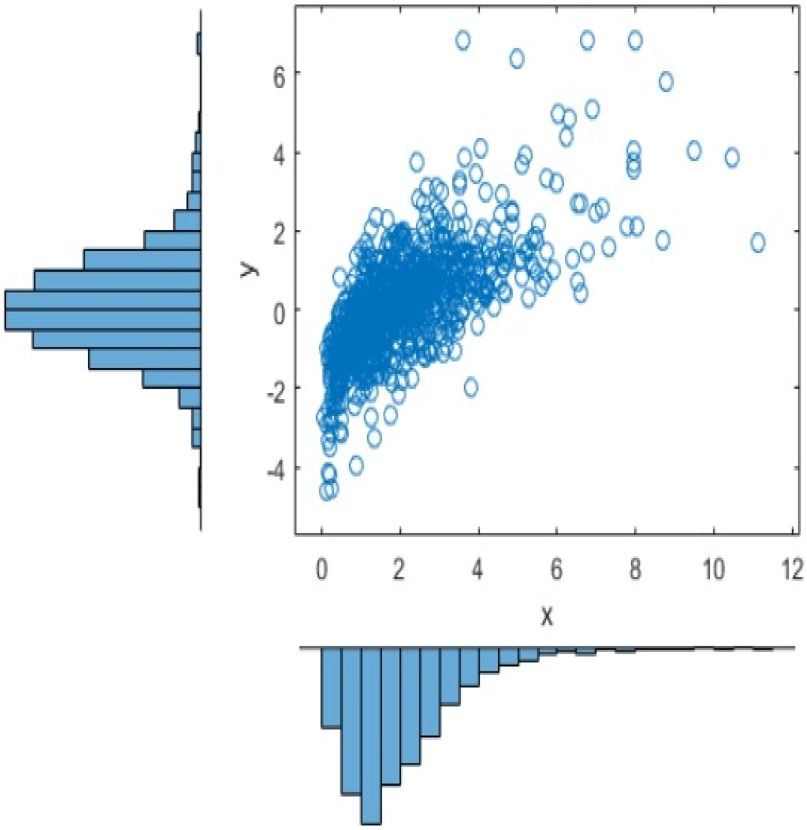

Figure 2.9: Transformation of Variable using Correlation

2.4.1 Relationship Between Correlation Matrix and Covariance Matrix

The correlation matrix is derived from covariance matrix. The covariance matrix is derived as given below.

$$E(X) = \frac{1}{N}\sum_{i=1}^{N} x(i) \tag{2.73}$$

$$VAR(X) = \frac{1}{N-1}\sum_{i=1}^{N}(x(i) - E(X))^2 \tag{2.74}$$

$$COV(X,Y) = \frac{1}{N-1}\sum_{i=1}^{N}(x(i) - E(X))(y(i) - E(Y)) \tag{2.75}$$

$$CORR(X,Y) = \frac{COV(X,Y)}{\sigma_X \sigma_Y} \tag{2.76}$$

Where,
$\sigma_X = \sqrt{VAR(X)}$
$\sigma_Y = \sqrt{VAR(Y)}$

2.4.1.1 Conversion of Covariance Matrix to Correlation Matrix

Here the covariance matrix is converted in to correlation matrix.

1. Form Covariance Matrix S
2. Find Out Standard Deviation of Each Variable D=Diagonal(S)
3. Find Inverse of Standard Deviation Matrix $D^{-1} = \frac{1}{D}$
4. Correlation Matrix R is given by, $R = D^{-1} \times S \times D^{-1}$

2.4.1.2 Conversion of Correlation Matrix to Covariance Matrix

Here the correlation matrix is converted in to covariance matrix.

1. Form Correlation Matrix R
2. Define Standard Deviation of Each Variable C
3. Form Standard Deviation Matrix D = Diagonal(C)
4. Covariance Matrix S is given by, $S = D \times R \times D$

The above mentioned method can be verified taking sample data as given below.
The Covariance Matrix S.

$$[S] = \begin{bmatrix} 1.0 & 1.0 & 8.1 \\ 1.0 & 16.0 & 18.0 \\ 8.1 & 18.0 & 81.0 \end{bmatrix}$$

The Variance Matrix C.

$$[C] = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 16.0 & 0.0 \\ 0.0 & 0.0 & 81.0 \end{bmatrix}$$

The Standard Deviation Matrix D.

$$[D] = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 4.0 & 0.0 \\ 0.0 & 0.0 & 9.0 \end{bmatrix}$$

The Correlation Matrix R.

$$[R] = \begin{bmatrix} 1.0 & 0.25 & 0.9 \\ 0.25 & 1.0 & 0.5 \\ 0.9 & 0.5 & 1.0 \end{bmatrix}$$

2.5 Results and Discussion

2.5.1 Simple Method of Voltage Stability with Wind Power Plant

Here the voltage assessment is done with the help of continuation power flow method given in [293]. This method is applied on two different configurations. For analysis purpose, a simple two bus system is taken and two different cases are studied. In the first case, the Wind Farm is connected to Bus-1 of two bus system as a single source, and the load is connected to Bus-2 as shown. In the second case, The Wind Farm along with Infinite Bus is considered as an additional source of supply. The wind power and Load are connected at Bus-2. Whereas, the Bus-1 is considered as an Infinite Bus. The results of two cases are given and discussed here.

The Wind Farm is consisting of 50 Wind Turbines of 2.5 MW each. The transmission line having impedance of j0.1 PU. The Base Power is taken as 100 MVA at a Base Voltage of 132 kV. The impedance of connected transmission line between Bus-1 and Bus-2 is considered as $j0.1$ p.u.

2.5.1.1 Case-1: Wind Farm Supplying the Load

In this case, the wind farm is the only source of supply and is connected to the Bus-1, the supply end of the system. And, the load is connected to the Bus-2, the receiving end of the system as shown in figure (2.5). In this case, it is assumed that the power generated by Wind Farm is completely consumed by load at Bus-2. So, the load level is following the Wind Power Generation. The voltage at receiving end (Bus-2) is worked out using standard load flow for different value of scale parameter (c), shape parameter (k) and load power factor ($\cos\phi$). The scale factor is varied in two discrete steps, whereas the three different values of shape factor is considered. Finally, the average of shape factor and scale factor (c =7.3 and k = 2.0) is used to carry out the voltage analysis. The step-wise procedure is given here.

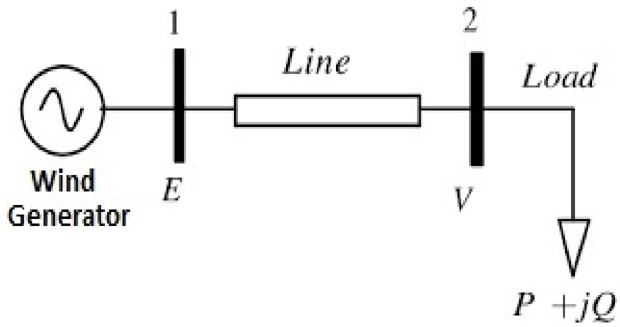

Figure 2.10: Two Bus System with Wind Farm Supplying Load

1. Set the Shape Parameter (k) and Scale Parameter (c) for Wind Farm
2. Fix the load power factor ($\cos\phi$)
3. Set the Wind Power Coefficient (C_p)
4. Calculate the Wind Power for different Wind Speed
5. Find the Wind Power PDF/CDF from Wind Speed PDF/CDF
6. For a given power factor work out the Voltage Stability Margin and using Predictor - Corrector Method [293] i.e. P-V Curve
7. From P-V curve find the voltage at receiving end for a given wind power
8. Determine the Voltage PDF and CDF from Wind power PDF/CDF

Table 2.2: Voltage Stability Probability with Scale Factor of 8.6 m/s and $\cos\phi = 0.95$ (lag)

c=8.6, $cos\phi = 0.95$ (lag)							
		k=1.5		k=2.0		k=2.5	
V (PU)	Power (PU)	PDF (%)	CDF (%)	PDF (%)	CDF (%)	PDF (%)	CDF (%)
0.95	1.2	4.3	79.04	5.15	83.69	5.6	87.80
0.90	2.1	3.0	87.34	2.72	92.81	2.11	96.51
0.85	2.66	2.27	90.23	1.88	95.42	1.15	98.32
0.80	3.0	2.0	91.55	1.5	96.46	0.82	98.91

Table 2.3: Voltage Stability Probability with Scale Factor of 8.6 m/s and $\cos\phi = 0.83$ (lag)

c=8.6, $cos\phi = 0.83$ (lag)							
		k=1.5		k=2.0		k=2.5	
V (PU)	Power (PU)	PDF (%)	CDF (%)	PDF (%)	CDF (%)	PDF (%)	CDF (%)
0.95	0.675	5.71	69.02	7.53	70.93	9.3	72.82
0.90	1.22	4.22	79.31	5.05	84.01	5.5	88.15
0.85	1.665	3.41	84.31	3.67	89.52	3.42	93.70
0.80	2.015	2.93	86.80	2.90	92.28	2.25	96.09

Table 2.4: Voltage Stability Probability with Scale Factor of 8.6 m/s and $\cos\phi = 1$ ($unity$)

c=8.6, $cos\phi = 1$ ($unity$)							
		k=1.5		k=2.0		k=2.5	
V (PU)	Power (PU)	PDF (%)	CDF (%)	PDF (%)	CDF (%)	PDF (%)	CDF (%)
0.95	2.966	2.03	91.43	1.57	96.37	0.8536	98.56
0.90	3.922	1.47	94.07	0.87	98.16	0.302	99.56
0.85	4.47	1.24	95.10	0.625	98.72	0.165	99.82
0.80	4.80	1.13	95.16	0.528	98.86	0.123	99.87

Table 2.5: Voltage Stability Probability with Scale Factor of 6 m/s and $\cos\phi = 0.95$

c=6.0, $cos\phi = 0.95$							
		k=1.5		k=2.0		k=2.5	
V (PU)	Power (PU)	PDF (%)	CDF (%)	PDF (%)	CDF (%)	PDF (%)	CDF (%)
0.95	1.2	2.4	93.15	1.62	97.59	0.71	99.43
0.90	2.1	1.10	97.12	0.35	99.55	0.041	99.97
0.85	2.66	0.735	98.15	0.15	99.82	0	1.0
0.80	3.0	0.594	98.86	0.01	99.90	0	1.0

Table 2.6: Voltage Stability Probability with Scale Factor of 6 m/s and $\cos\phi = 0.83$

c=6.0, $cos\phi = 0.83$							
		k=1.5		k=2.0		k=2.5	
V (PU)	Power (PU)	PDF (%)	CDF (%)	PDF (%)	CDF (%)	PDF (%)	CDF (%)
0.95	0.675	4.3	86.61	4.35	92.10	3.7	95.94
0.90	1.22	2.4	93.30	1.56	97.69	0.67	99.47
0.85	1.665	1.57	95.75	0.72	99.03	0.1615	99.89
0.80	2.015	1.2	96.90	0.417	99.48	0.065	99.97

Table 2.7: Voltage Stability Probability with Scale Factor of 6.0 m/s and $\cos\phi = 1(unity)$

c=6.0, $cos\phi = 1(unity)$							
		k=1.5		k=2.0		k=2.5	
V (PU)	Power (PU)	PDF (%)	CDF (%)	PDF (%)	CDF (%)	PDF (%)	CDF (%)
0.95	2.966	0.613	98.52	0.1041	99.89	0	1.0
0.90	3.922	0.337	99.22	0	99.97	0	1.0
0.85	4.47	0.245	99.43	0	99.99	0	1.0
0.80	4.80	0.2	99.53	0	99.99	0	1.0

Table 2.8: Voltage Stability Probability with Scale Factor of 7.3 m/s, Shape Factor of 2.0 and $\cos\phi = 0.95$

c=7.3, k=2.0, $\cos\phi = 0.95$			
V (PU)	Power (PU)	PDF (%)	CDF (%)
0.95	1.2	3.57	91.93
0.90	2.1	1.36	97.41
0.85	2.66	0.8	98.61
0.80	3.0	0.59	99.03

Table 2.9: Voltage Stability Probability with Scale Factor of 7.3 m/s, Shape Factor of 2.0 and $\cos\phi = 0.83$ (lag)

c=7.3, k=2.0, $\cos\phi = 0.83$ (lag)			
V (PU)	Power (PU)	PDF (%)	CDF (%)
0.95	0.675	6.55	82.00
0.90	1.22	3.48	92.15
0.85	1.665	2.14	95.63
0.80	2.015	1.45	97.14

Table 2.10: Voltage Stability Probability with Scale Factor of 7.3 m/s, Shape Factor of 2.0 and $\cos\phi = 1$ $(unity)$

c=7.3, k=2.0, $\cos\phi = 1$ $(unity)$			
V (PU)	Power (PU)	PDF (%)	CDF (%)
0.95	0.966	0.615	99.00
0.90	3.92	0.26	99.61
0.85	4.47	0.16	99.76
0.80	4.80	0.127	99.82

From the results of above analysis, given in the tables (2.2) to (2.10), the following key points can be deduced.

1. The degradation in power factor deteriorates the voltage variation specturm and hence the stability of voltage

2. By improving power factor, the wind power evacuation can be increased without making changes in the power system.

3. The shape factor and voltage variation are related inversely. As the shape factor decreases, the Cumulative Distribution Factor (CDF) of voltage is improved. For

example, the change is scale factor from 8.6 m/s to 6.0 m/s improves the probability of voltage remains above 0.95 p.u from 79.04 % to 93.15 %. It is a significant improvement as 16 % reduction in shape factor improves the probability of voltage remains above 0.95 p.u by nearly 18 %.

4. Similarly, the relationship between shape factor of wind power distribution and voltage stability can be established from the derived data. The increase in shape factor improves the voltage profile. The probability of voltage remains 0.95 p.u and above is increased from 79.04 % to 87.80 % when shape factor k varies from 1.5 to 2.5. There is an improvement of 11 % in voltage CDF with 67 % increase in shape factor.

5. This inference can be generalised up to certain extent for the prediction of voltage behaviour with wind power.

6. The Wind Farm location should be selected based on the scale factor '**c**' and shape factor '**k**', besides other important factors like system strength, nearby another sources of supply, nearby load centers, load variation pattern etc...

2.5.1.2 Case-2: Wind Farm and Grid Supplying the Load

In this case, the two bus system is taken. At Bus-2 the load and wind farm are connected. The Bus-2 is connected to infinite bus (Bus-1) through transmission line having impedance of j0.1 PU. The step by step procedure for analysis is given here.

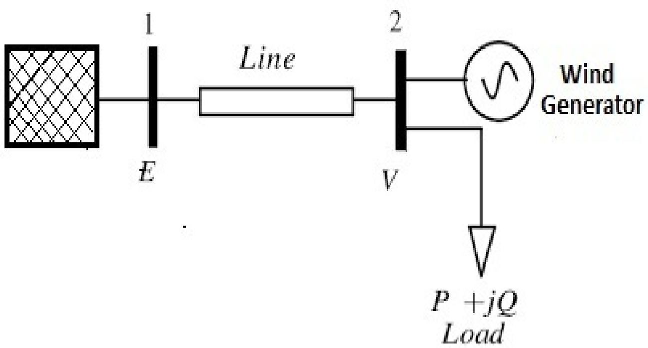

Figure 2.11: Two Bus System with Wind Farm and Infinite Bus Supplying Load

1. Fix the load Power Factor

2. Fix the Infinite bus Voltage (1.04 PU)

3. Work out the P-V curve using Predictor - Corrector Method

4. Find the critical power from P-V Curve

5. Fix the load equal to critical power

6. Vary wind Power according to its characteristics (for given scale factor (c) and shape factor (k))

7. Subtract wind power from load power (Power Difference). The rest of the power flowing through line.

8. Work out P-V curve using Predictor - Corrector method [293]

9. Find the wind power PDF/CDF from the Weibull distribution

10. Find Voltage PDF/CDF from Power PDF/CDF and P-V Curve

11. Go to step -1, change the power factor and repeat the procedure

Table 2.11: Voltage Stability Probability with $\cos\phi = 0.95$ (lag)

$\cos\phi = 0.95$ (lag) Critical Power = 3.5 PU					
		c=8.6, k=2.0		c=6.0, k=2.0	
V (PU)	Power (PU)	PDF (%)	CDF (%)	PDF (%)	CDF (%)
0.95	3.5 - 1.2 =2.3	2.5	94.00	0.2715	99.68
0.90	3.5 - 2.0 = 1.5	4.15	87.80	0.966	98.67
0.85	3.5 - 2.6 = 0.9	6.4	77.61	2.79	95.38
0.80	3.5 - 3.0 = 0.5	8.5	63.63	6.11	87.48

Table 2.12: Voltage Stability Probability with $\cos\phi = 0.83$ (*lag*)

$\cos\phi = 0.83$ (*lag*) Critical Power = 2.6 PU					
		c=8.6, k=2.0		c=6.0, k=2.0	
V (PU)	Power (PU)	PDF (%)	CDF (%)	PDF (%)	CDF (%)
0.95	2.6 - 0.675 = 1.925	3.09	91.66	0.486	99.39
0.90	2.6 - 1.225 = 1.375	4.5	86.27	1.16	98.31
0.85	2.6 - 1.65 = 0.95	6.16	78.81	2.53	95.87
0.80	2.6 - 2.01 = 0.59	7.94	68.09	4.95	90.43

Table 2.13: Voltage Stability Probability with $\cos\phi = 1.0$ (*unity*)

$\cos\phi = 1.0$ (*unity*) Critical Power = 5.0 PU					
		c=8.6, k=2.0		c=6.0, k=2.0	
V (PU)	Power (PU)	PDF (%)	CDF (%)	PDF (%)	CDF (%)
0.95	5 - 2.97 = 2.03	2.86	92.38	0.40	99.50
0.90	5 - 3.923 = 1.077	5.61	81.49	1.98	96.88
0.85	5 - 4.48 = 0.52	8.39	64.59	5.85	88.15
0.80	5 - 4.80 = 0.20	10.0	42.25	11.50	67.64

In this case, the effect of wind power on voltage variation in presence of grid supply is evaluated. From the results of above analysis, given in the tables (2.11) to (2.13), the following key points can be inferred.

1. The presence of grid supply, the voltage variation is reduced. So, it is in the favour of voltage stability. So, it is not exaggeration to say that the presence grid supply augment the voltage stability.

2. It is also obvious from the above that the nearness of conventional power sources is also important. Nearer the conventional source to Wind Source, better is the voltage regulation.

3. The comparison of Case-1 and Case-2 shows the improvement in voltage profile with for respective values of scale factor '**c**' , shape factor '**k**' and power factor '**cos**ϕ'

2.5.2 Voltage Estimation with Wind Power Using Statistical Methods

The method used in Case-1 and Case-2 of section (2.5.1) is simple and easy to apply. However, it becomes laborious when multiple parameter variation needs to be taken into consideration. So, the suitable and effective statistical estimation method is required for analysis of larger system with many variables. In this section, Voltage Magnitude and Angle is evaluated with integration of wind power using different statistical methods as discussed in section (2.3). For study purpose IEEE standard 12 Bus Radial Network is selected. The reason behind selection of this network is to assess the effect of wind power with distributed load. Also, radial network gives the liberty to judge the relative distance between conventional generation and wind generation. The method of study is discussed here. The purpose of this study is twofold. First is the evaluation of voltage variation with wind power under different condition. And second is, assessment of effectiveness of various statistical methods. Total seven methods, including Monte Carlo Simulation (MCS), Point Estimation Method (Four Variants) and Latin Hypercube Sampling (Two Variants) are used. For Monte Carlo Simulation (MCS) and Point Estimation Method, 1000 data points are used, whereas for Latin Hypercube Sampling only 10 data points are used. In this analysis, the Monte Carlo Method is taken as pivotal or reference method and the results of other methods are compared with MCS for error analysis.

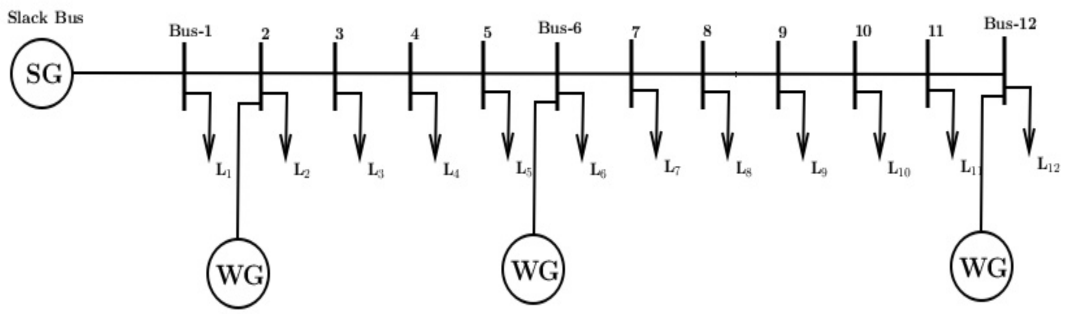

Figure 2.12: IEEE 12-Bus Radial Network

First, the location of wind power is fixed. Here, three different locations are used, first, Bus-2, which is nearest to the Bus-1. Next, Bus-6, which is the approximately middle point of network. Finally, Bus-12 which is at the far end of the network. Then, the penetration of wind power is adjusted. In the first part, it is adjusted to approximately 8% Peak. Also, the impedance of the line segment between Bus-1 and Bus-2 is increased to 5 times of normal value to make the moderate strength of supply system. The next part is the selection of wind speed for different Wind Turbine Generator. Then, Wind Power is calculated using following equation (2.77)

$$P_w = \frac{1}{2}\rho C_p (2\pi R^2)(V_w^3) \tag{2.77}$$

Where,
C_p= Co-efficient of Performance = (0.32 to 0.42). Here it is taken as 0.32.
A=Blade Swept Area = $2\pi R^2$
R= Radius of Blade = 50 meters
V_w=Wind Speed in m/s

Then, this power is normalised using base of 100 MVA to get power in p.u (per unit). In this study, three different WTG are considered which are located at three different places and power is evacuated from same bus. The impedance between WTG and evacuation bus is neglected and assumed that the WTG bus is same as evacuation bus. The scale parameter (c) and shape parameter (k) is given here.

- $WTG_1, c = 8.6,\ k = 2.4$
- $WTG_2, c = 6,\ k = 1.8$
- $WTG_3, c = 7.2,\ k = 2$

2.5.2.1 Analysis of Voltage Variation Estimation of IEEE-12 Bus Radial Network with Wind Power using Statistical Estimation Methods

In this section three different cases are studied using IEEE 12 Bus Radial network to assess the various aspects of voltage stability. In each case, wind power is assume connected to one bus and voltage variation is calculated using different statistical load flow methods.

2.5.2.1.1 Voltage Variation Estimation with Wind Power at Bus-12 In this case, the Bus-1 is considered as a slack bus and wind power is connected to Bus-2, Bus-6 and Bus-6 each time and the mean value of voltage variation and standard deviation is calculated using different methods. The results are given here in table (2.14) to (2.37). The results of analysis with 8% peak wind power connected at Bus-12 is given here.

Table 2.14: Voltage Magnitude Estimation with Wind Power at Bus-12

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	1.06	1.06	1.06	1.06	1.06	1.06	1.06
2	1.0105	1.0114	1.0114	1.012	1.0109	1.0099	1.0104
3	1.0059	1.007	1.0069	1.0077	1.0065	1.0053	1.0059
4	0.9989	1.0001	1	1.0009	0.9995	0.9981	0.9988
5	0.9902	0.9917	0.9916	0.9927	0.9909	0.9892	0.9901
6	0.9876	0.9892	0.9891	0.9902	0.9884	0.9866	0.9875
7	0.9789	0.9806	0.9805	0.9817	0.9797	0.9778	0.9788
8	0.9731	0.9752	0.975	0.9765	0.9741	0.9718	0.973
9	0.9684	0.971	0.9708	0.9727	0.9697	0.9668	0.9683
10	0.9673	0.9701	0.9699	0.9719	0.9687	0.9656	0.9671
11	0.9673	0.9702	0.97	0.9721	0.9687	0.9655	0.9671
12	0.9677	0.9708	0.9706	0.9728	0.9692	0.9659	0.9676

Table 2.15: Error(%) in Voltage Magnitude Estimation with Wind Power at Bus-12

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	-0.09	-0.09	-0.15	-0.04	0.06	0.01
3	0.00	-0.11	-0.10	-0.18	-0.06	0.06	0.00
4	0.00	-0.12	-0.11	-0.20	-0.06	0.08	0.01
5	0.00	-0.15	-0.14	-0.25	-0.07	0.10	0.01
6	0.00	-0.16	-0.15	-0.26	-0.08	0.10	0.01
7	0.00	-0.17	-0.16	-0.29	-0.08	0.11	0.01
8	0.00	-0.22	-0.20	-0.35	-0.10	0.13	0.01
9	0.00	-0.27	-0.25	-0.44	-0.13	0.17	0.01
10	0.00	-0.29	-0.27	-0.48	-0.14	0.18	0.02
11	0.00	-0.30	-0.28	-0.50	-0.14	0.19	0.02
12	0.00	-0.32	-0.30	-0.53	-0.16	0.19	0.01

Table 2.16: Voltage Magnitude Standard Deviation Estimation with Wind Power at Bus-12

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.0086	0.0088	0.0092	0.0141	0.0071	0.0048	0.0076
3	0.0095	0.0098	0.0102	0.0156	0.0079	0.0054	0.0084
4	0.0112	0.0115	0.012	0.0183	0.0093	0.0063	0.0099
5	0.0137	0.014	0.0147	0.0224	0.0114	0.0077	0.0121
6	0.0145	0.0149	0.0156	0.0238	0.0121	0.0082	0.0128
7	0.0154	0.0158	0.0165	0.0252	0.0128	0.0087	0.0136
8	0.0188	0.0193	0.0202	0.0308	0.0157	0.0106	0.0166
9	0.0232	0.0237	0.0248	0.0379	0.0193	0.013	0.0204
10	0.0254	0.0259	0.0271	0.0415	0.0211	0.0142	0.0223
11	0.0266	0.0271	0.0283	0.0434	0.022	0.0148	0.0233
12	0.0275	0.028	0.0293	0.0449	0.0228	0.0153	0.0241

Table 2.17: Error(%) in Voltage Magnitude Standard Deviation Estimation with Wind Power at Bus-12

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	-2.33	-6.98	-63.95	17.44	44.19	11.63
3	0.00	-3.16	-7.37	-64.21	16.84	43.16	11.58
4	0.00	-2.68	-7.14	-63.39	16.96	43.75	11.61
5	0.00	-2.19	-7.30	-63.50	16.79	43.80	11.68
6	0.00	-2.76	-7.59	-64.14	16.55	43.45	11.72
7	0.00	-2.60	-7.14	-63.64	16.88	43.51	11.69
8	0.00	-2.66	-7.45	-63.83	16.49	43.62	11.70
9	0.00	-2.16	-6.90	-63.36	16.81	43.97	12.07
10	0.00	-1.97	-6.69	-63.39	16.93	44.09	12.20
11	0.00	-1.88	-6.39	-63.16	17.29	44.36	12.41
12	0.00	-1.82	-6.55	-63.27	17.09	44.36	12.36

Table 2.18: Voltage Angle Estimation with Wind Power at Bus-12

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	1.1732	1.1918	1.1912	1.2102	1.1805	1.1595	1.1716
3	1.2875	1.308	1.3074	1.3282	1.2956	1.2724	1.2857
4	1.4789	1.503	1.5022	1.5261	1.4885	1.4614	1.4769
5	1.7255	1.7552	1.7541	1.7824	1.7369	1.7046	1.7232
6	1.8018	1.8334	1.8322	1.8619	1.814	1.7797	1.7994
7	1.5542	1.6064	1.6039	1.6455	1.5771	1.5207	1.5511
8	1.8144	1.8715	1.8687	1.9134	1.8396	1.7781	1.8112
9	2.0571	2.121	2.1177	2.1659	2.0858	2.0174	2.0538
10	2.1401	2.2075	2.204	2.2538	2.1707	2.0986	2.1368
11	2.1681	2.2374	2.2338	2.2844	2.1997	2.1258	2.1648
12	2.1793	2.2503	2.2465	2.2977	2.2118	2.1362	2.176

Table 2.19: Error(%) in Voltage Angle Estimation with Wind Power at Bus-12

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.00	-1.59	-1.53	-3.15	-0.62	1.17	0.14
3	0.00	-1.59	-1.55	-3.16	-0.63	1.17	0.14
4	0.00	-1.63	-1.58	-3.19	-0.65	1.18	0.14
5	0.00	-1.72	-1.66	-3.30	-0.66	1.21	0.13
6	0.00	-1.75	-1.69	-3.34	-0.68	1.23	0.13
7	0.00	-3.36	-3.20	-5.87	-1.47	2.16	0.20
8	0.00	-3.15	-2.99	-5.46	-1.39	2.00	0.18
9	0.00	-3.11	-2.95	-5.29	-1.40	1.93	0.16
10	0.00	-3.15	-2.99	-5.31	-1.43	1.94	0.15
11	0.00	-3.20	-3.03	-5.36	-1.46	1.95	0.15
12	0.00	-3.26	-3.08	-5.43	-1.49	1.98	0.15

Table 2.20: Voltage Angle Standard Deviation Estimation with Wind Power at Bus-12

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.1826	0.1745	0.1845	0.2987	0.1029	0.0916	0.1528
3	0.2014	0.1932	0.2041	0.3298	0.1246	0.1015	0.169
4	0.2343	0.226	0.2387	0.3843	0.1622	0.119	0.1975
5	0.2837	0.2763	0.2916	0.4665	0.2184	0.146	0.241
6	0.3005	0.2936	0.3097	0.4945	0.2373	0.1553	0.2559
7	0.4768	0.4815	0.5052	0.785	0.4210	0.2597	0.4158
8	0.5184	0.5259	0.5516	0.8546	0.4687	0.2839	0.4538
9	0.5729	0.5855	0.6137	0.9457	0.5299	0.317	0.5043
10	0.6012	0.6169	0.6463	0.9929	0.5611	0.3346	0.5309
11	0.6164	0.6339	0.664	1.0181	0.5676	0.3442	0.5453
12	0.6293	0.6484	0.6789	1.0394	0.5915	0.3524	0.5574

Table 2.21: Error(%) in Voltage Angle Standard Deviation Estimation with Wind Power at Bus-12

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.00	4.44	-1.04	-63.58	43.65	49.84	16.32
3	0.00	4.07	-1.34	-63.75	38.13	49.60	16.09
4	0.00	3.54	-1.88	-64.02	30.77	49.21	15.71
5	0.00	2.61	-2.78	-64.43	23.02	48.54	15.05
6	0.00	2.30	-3.06	-64.56	21.03	48.32	14.84
7	0.00	-0.99	-5.96	-64.64	11.70	45.53	12.79
8	0.00	-1.45	-6.40	-64.85	09.59	45.24	12.46
9	0.00	-2.20	-7.12	-65.07	07.51	44.67	11.97
10	0.00	-2.61	-7.50	-65.15	06.67	44.34	11.69
11	0.00	-2.84	-7.72	-65.17	06.29	44.16	11.53
12	0.00	-3.04	-7.88	-65.17	06.01	44.00	11.43

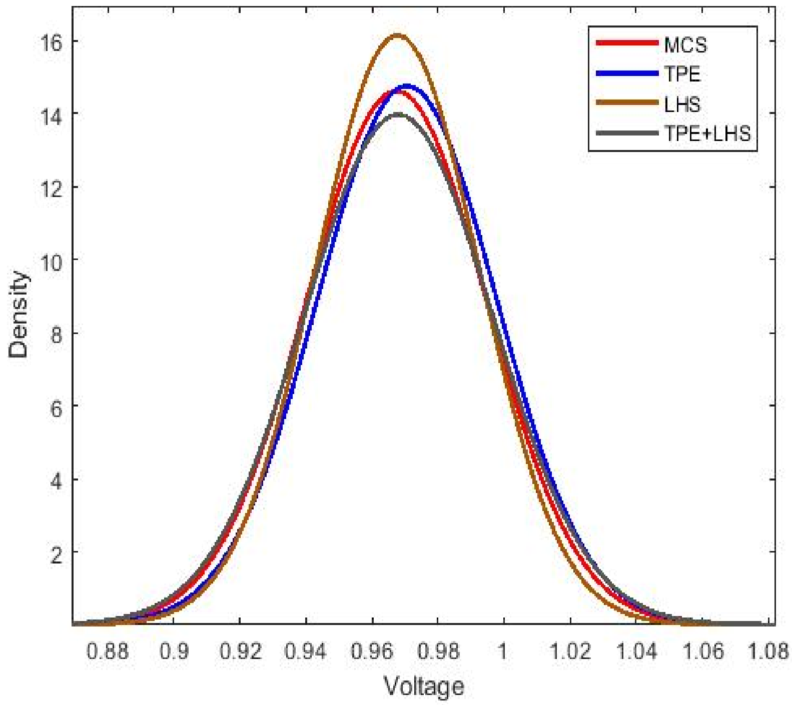

Figure 2.13: Comparison of Voltage Magnitude at Bus-12 with Wind Power at Bus-12 using MCS, TPE, LHS MEADIAN and TPE+LHS MEADIAN

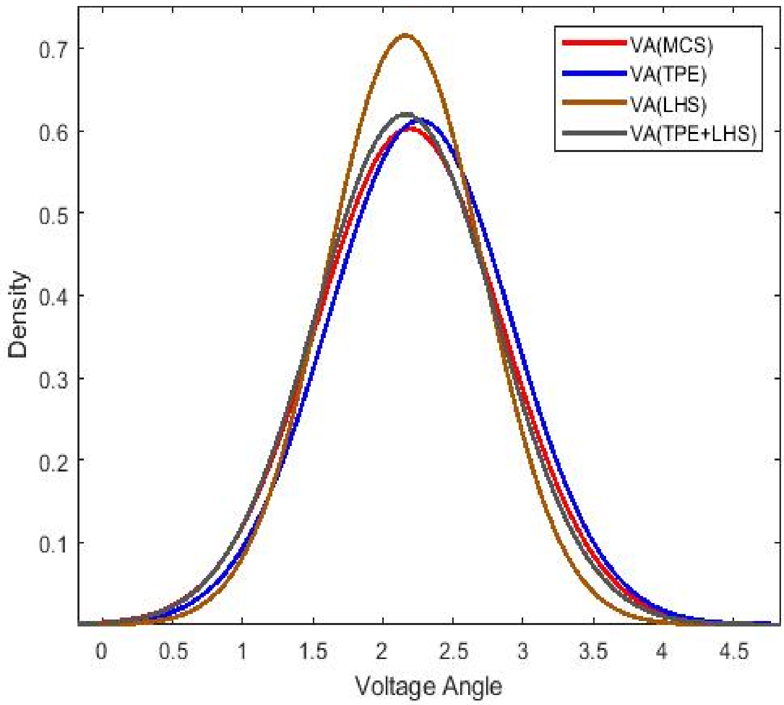

Figure 2.14: Comparison of Voltage Angle at Bus-12 with Wind Power at Bus-12 using MCS, TPE, LHS MEADIAN and TPE+LHS MEADIAN

2.5.2.1.2 Voltage Variation Estimation with Wind Power at Bus-6 In this section, the results of voltage analysis with 8% peak wind power connected at Bus-6 is given.

Table 2.22: Voltage Magnitude Estimation with Wind Power at Bus-6

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	1.06	1.06	1.06	1.06	1.06	1.06	1.06
2	1.0103	1.0113	1.0112	1.0121	1.0107	1.0093	1.0104
3	1.0058	1.0068	1.0068	1.0077	1.0062	1.0047	1.0058
4	0.9987	0.9999	0.9998	1.0009	0.9992	0.9974	0.9988
5	0.9899	0.9914	0.9914	0.9927	0.9906	0.9884	0.99
6	0.9873	0.9889	0.9888	0.9903	0.988	0.9857	0.9874
7	0.9777	0.9793	0.9792	0.9807	0.9784	0.976	0.9778
8	0.9694	0.971	0.9709	0.9724	0.9701	0.9677	0.9695
9	0.9615	0.9631	0.963	0.9645	0.9622	0.9598	0.9616
10	0.9587	0.9603	0.9603	0.9617	0.9594	0.957	0.9588
11	0.9578	0.9595	0.9594	0.9608	0.9585	0.9561	0.9579
12	0.9576	0.9592	0.9592	0.9606	0.9583	0.9559	0.9577

Table 2.23: Error(%) in Voltage Magnitude Estimation with Wind Power at Bus-6

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	-0.10	-0.09	-0.18	-0.04	0.10	-0.01
3	0.00	-0.10	-0.10	-0.19	-0.04	0.11	0.00
4	0.00	-0.12	-0.11	-0.22	-0.05	0.13	-0.01
5	0.00	-0.15	-0.15	-0.28	-0.07	0.15	-0.01
6	0.00	-0.16	-0.15	-0.30	-0.07	0.16	-0.01
7	0.00	-0.16	-0.15	-0.31	-0.07	0.17	-0.01
8	0.00	-0.17	-0.15	-0.31	-0.07	0.18	-0.01
9	0.00	-0.17	-0.16	-0.31	-0.07	0.18	-0.01
10	0.00	-0.17	-0.17	-0.31	-0.07	0.18	-0.01
11	0.00	-0.18	-0.17	-0.31	-0.07	0.18	-0.01
12	0.00	-0.17	-0.17	-0.31	-0.07	0.18	-0.01

Table 2.24: Voltage Magnitude Standard Deviation Estimation with Wind Power at Bus-6

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.009	0.0089	0.0094	0.0147	0.0074	0.005	0.0081
3	0.01	0.0099	0.0104	0.0164	0.0082	0.0055	0.009
4	0.0118	0.0116	0.0122	0.0192	0.0096	0.0065	0.0106
5	0.0144	0.0142	0.0149	0.0235	0.0118	0.0079	0.013
6	0.0153	0.015	0.0158	0.0249	0.0125	0.0084	0.0138
7	0.0154	0.0152	0.016	0.0252	0.0126	0.0085	0.0139
8	0.0155	0.0153	0.0161	0.0254	0.0127	0.0086	0.014
9	0.0157	0.0154	0.0162	0.0256	0.0128	0.0087	0.0141
10	0.0157	0.0155	0.0163	0.0256	0.0129	0.0087	0.0142
11	0.0157	0.0155	0.0163	0.0257	0.0129	0.0087	0.0142
12	0.0157	0.0155	0.0163	0.0257	0.0129	0.0087	0.0142

Table 2.25: Error(%) in Voltage Magnitude Standard Deviation Estimation with Wind Power at Bus-6

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.00	1.11	-4.44	-63.33	17.78	44.44	10.00
3	0.00	1.00	-4.00	-64.00	18.00	45.00	10.00
4	0.00	1.69	-3.39	-62.71	18.64	44.92	10.17
5	0.00	1.39	-3.47	-63.19	18.06	45.14	9.72
6	0.00	1.96	-3.27	-62.75	18.30	45.10	9.80
7	0.00	1.30	-3.90	-63.64	18.18	44.81	9.74
8	0.00	1.29	-3.87	-63.87	18.06	44.52	9.68
9	0.00	1.91	-3.18	-63.06	18.47	44.59	10.19
10	0.00	1.27	-3.82	-63.06	17.83	44.59	9.55
11	0.00	1.27	-3.82	-63.69	17.83	44.59	9.55
12	0.00	1.27	-3.82	-63.69	17.83	44.59	9.55

Table 2.26: Voltage Angle Estimation with Wind Power at Bus-6

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	1.1761	1.1949	1.1943	1.2129	1.184	1.1565	1.1774
3	1.2907	1.3116	1.3108	1.3312	1.2994	1.2691	1.2921
4	1.4827	1.5072	1.5063	1.5296	1.4931	1.4576	1.4844
5	1.7303	1.7604	1.7592	1.7867	1.7433	1.6999	1.7326
6	1.8069	1.8389	1.8376	1.8664	1.8209	1.7748	1.8094
7	1.434	1.4672	1.4658	1.4953	1.4485	1.4007	1.4365
8	1.6799	1.7123	1.711	1.74	1.694	1.6473	1.6824
9	1.9042	1.9359	1.9346	1.9632	1.918	1.8723	1.9066
10	1.9778	2.0092	2.0079	2.0364	1.9914	1.9461	1.9802
11	2.0008	2.0322	2.0309	2.0593	2.0144	1.9692	2.0032
12	2.0077	2.039	2.0377	2.0662	2.0213	1.9761	2.01

Table 2.27: Error(%) in Voltage Angle Estimation with Wind Power at Bus-6

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.00	-1.60	-1.55	-3.13	-0.67	1.67	-0.11
3	0.00	-1.62	-1.56	-3.14	-0.67	1.67	-0.11
4	0.00	-1.65	-1.59	-3.16	-0.70	1.69	-0.11
5	0.00	-1.74	-1.67	-3.26	-0.75	1.76	-0.13
6	0.00	-1.77	-1.70	-3.29	-0.77	1.78	-0.14
7	0.00	-2.32	-2.22	-4.27	-1.01	2.32	-0.17
8	0.00	-1.93	-1.85	-3.58	-0.84	1.94	-0.15
9	0.00	-1.66	-1.60	-3.10	-0.72	1.68	-0.13
10	0.00	-1.59	-1.52	-2.96	-0.69	1.60	-0.12
11	0.00	-1.57	-1.50	-2.92	-0.68	1.58	-0.12
12	0.00	-1.56	-1.49	-2.91	-0.68	1.57	-0.11

Table 2.28: Voltage Angle Standard Deviation Estimation with Wind Power at Bus-6

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.1815	0.177	0.1867	0.3007	0.1483	0.098	0.1632
3	0.2001	0.1959	0.2066	0.3318	0.1637	0.1086	0.1805
4	0.2327	0.2292	0.2416	0.3863	0.1905	0.1274	0.2109
5	0.2817	0.2802	0.295	0.4686	0.2312	0.1564	0.2571
6	0.2984	0.2977	0.3133	0.4965	0.245	0.1664	0.2729
7	0.3088	0.3089	0.325	0.5137	0.2537	0.1729	0.2829
8	0.3019	0.3015	0.3173	0.5024	0.248	0.1687	0.2764
9	0.2956	0.2947	0.3102	0.4919	0.2427	0.1647	0.2703
10	0.2935	0.2925	0.3079	0.4884	0.2409	0.1634	0.2683
11	0.2928	0.2918	0.3071	0.4873	0.2404	0.163	0.2676
12	0.2926	0.2915	0.3069	0.487	0.2402	0.1628	0.2674

Table 2.29: Error(%) in Voltage Angle Standard Deviation Estimation with Wind Power at Bus-6

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.00	2.48	-2.87	-65.67	18.29	46.01	10.08
3	0.00	2.10	-3.25	-65.82	18.19	45.73	9.80
4	0.00	1.50	-3.82	-66.01	18.13	45.25	9.37
5	0.00	0.53	-4.72	-66.35	17.93	44.48	8.73
6	0.00	0.23	-4.99	-66.39	17.90	44.24	8.55
7	0.00	-0.03	-5.25	-66.35	17.84	44.01	8.39
8	0.00	0.13	-5.10	-66.41	17.85	44.12	8.45
9	0.00	0.30	-4.94	-66.41	17.90	44.28	8.56
10	0.00	0.34	-4.91	-66.41	17.92	44.33	8.59
11	0.00	0.34	-4.88	-66.43	17.90	44.33	8.61
12	0.00	0.38	-4.89	-66.44	17.91	44.36	8.61

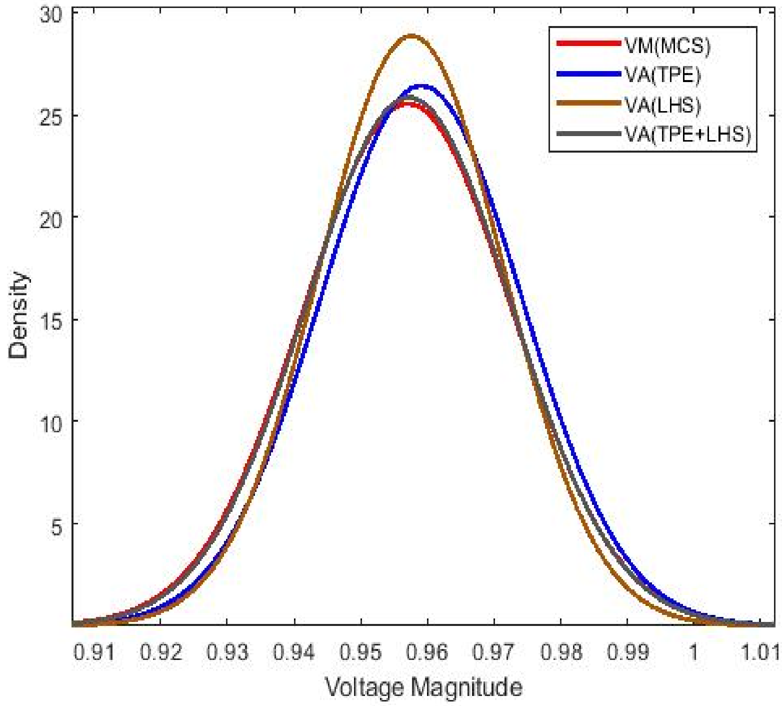

Figure 2.15: Comparison of Voltage Magnitude at Bus-12 with Wind Power at Bus-06 using MCS, TPE, LHS MEADIAN and TPE+LHS MEADIAN

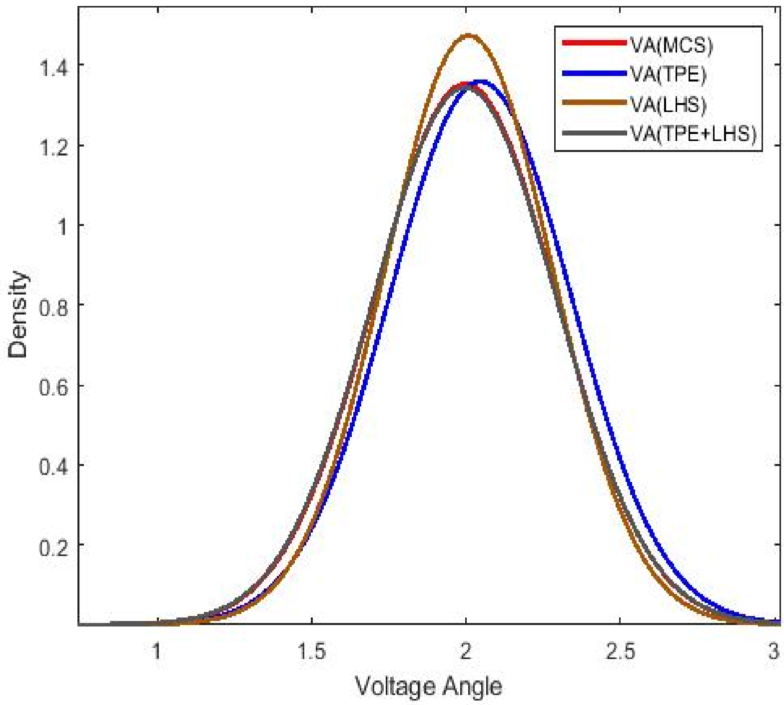

Figure 2.16: Comparison of Voltage Angle at Bus-12 with Wind Power at Bus-06 using MCS, TPE, LHS MEADIAN and TPE+LHS MEADIAN

2.5.2.1.3 Voltage Variation Estimation with Wind Power at Bus-2 The results of analysis with 8% peak wind power connected at Bus-2 is given here.

Table 2.30: Voltage Magnitude Estimation with Wind Power at Bus-2

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	1.06	1.06	1.06	1.06	1.06	1.06	1.06
2	1.0101	1.0109	1.0109	1.0118	1.0104	1.0089	1.0101
3	1.0048	1.0057	1.0057	1.0066	1.0052	1.0037	1.0049
4	0.9965	0.9974	0.9974	0.9983	0.9969	0.9953	0.9966
5	0.9859	0.9868	0.9868	0.9877	0.9863	0.9847	0.986
6	0.9826	0.9835	0.9835	0.9845	0.983	0.9815	0.9827
7	0.973	0.9739	0.9739	0.9748	0.9734	0.9718	0.973
8	0.9646	0.9655	0.9655	0.9665	0.965	0.9634	0.9647
9	0.9566	0.9576	0.9576	0.9585	0.9571	0.9554	0.9567
10	0.9539	0.9548	0.9548	0.9558	0.9543	0.9527	0.9539
11	0.953	0.9539	0.9539	0.9549	0.9534	0.9518	0.953
12	0.9528	0.9537	0.9537	0.9547	0.9532	0.9516	0.9528

Table 2.31: Error(%) in Voltage Magnitude Estimation with Wind Power at Bus-2

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.00	-0.08	-0.08	-0.17	-0.03	0.12	0.00
3	0.00	-0.09	-0.09	-0.18	-0.04	0.11	-0.01
4	0.00	-0.09	-0.09	-0.18	-0.04	0.12	-0.01
5	0.00	-0.09	-0.09	-0.18	-0.04	0.12	-0.01
6	0.00	-0.09	-0.09	-0.19	-0.04	0.11	-0.01
7	0.00	-0.09	-0.09	-0.18	-0.04	0.12	0.00
8	0.00	-0.09	-0.09	-0.20	-0.04	0.12	-0.01
9	0.00	-0.10	-0.10	-0.20	-0.05	0.13	-0.01
10	0.00	-0.09	-0.09	-0.20	-0.04	0.13	0.00
11	0.00	-0.09	-0.09	-0.20	-0.04	0.13	0.00
12	0.00	-0.09	-0.09	-0.20	-0.04	0.13	0.00

Table 2.32: Voltage Magnitude Standard Deviation Estimation with Wind Power at Bus-2

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.0087	0.0087	0.0092	0.0148	0.0074	0.0037	0.0087
3	0.0087	0.0088	0.0092	0.0148	0.0074	0.0037	0.0087
4	0.0088	0.0088	0.0093	0.015	0.0075	0.0038	0.0088
5	0.0089	0.0089	0.0094	0.0151	0.0076	0.0038	0.0089
6	0.009	0.009	0.0095	0.0152	0.0076	0.0038	0.009
7	0.009	0.0091	0.0095	0.0153	0.0077	0.0039	0.009
8	0.0091	0.0091	0.0096	0.0155	0.0077	0.0039	0.0091
9	0.0092	0.0092	0.0097	0.0156	0.0078	0.0039	0.0092
10	0.0092	0.0092	0.0097	0.0156	0.0078	0.0039	0.0092
11	0.0092	0.0092	0.0097	0.0156	0.0078	0.0039	0.0092
12	0.0092	0.0092	0.0098	0.0156	0.0078	0.0039	0.0092

Table 2.33: Error(%) in Voltage Magnitude Standard Deviation Estimation with Wind Power at Bus-2

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.00	0.00	-5.75	-70.11	14.94	57.47	0.00
3	0.00	1.15	-5.75	-70.11	14.94	57.47	0.00
4	0.00	0.00	-5.68	-70.45	14.77	56.82	0.00
5	0.00	0.00	-5.62	-69.66	14.61	57.30	0.00
6	0.00	0.00	-5.56	-68.89	15.56	57.78	0.00
7	0.00	-1.11	-5.56	-70.00	14.44	56.67	0.00
8	0.00	0.00	-5.49	-70.33	15.38	57.14	0.00
9	0.00	0.00	-5.43	-69.57	15.22	57.61	0.00
10	0.00	0.00	-5.43	-69.57	15.22	57.61	0.00
11	0.00	0.00	-5.43	-69.57	15.22	57.61	0.00
12	0.00	0.00	-6.52	-69.57	15.22	57.61	0.00

Table 2.34: Voltage Angle Estimation with Wind Power at Bus-2

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	1.1776	1.1963	1.1956	1.2145	1.1853	1.1542	1.1791
3	1.2833	1.3017	1.3011	1.3199	1.2909	1.2601	1.2847
4	1.4598	1.478	1.4773	1.4958	1.4673	1.437	1.4613
5	1.6843	1.702	1.7013	1.7195	1.6916	1.6619	1.6857
6	1.753	1.7706	1.77	1.7881	1.7603	1.7308	1.7544
7	1.3765	1.3948	1.3942	1.4128	1.3841	1.3535	1.378
8	1.6248	1.6426	1.642	1.6603	1.6321	1.6023	1.6262
9	1.8513	1.8686	1.868	1.8859	1.8584	1.8293	1.8526
10	1.9256	1.9428	1.9422	1.96	1.9326	1.9037	1.9269
11	1.9488	1.9659	1.9654	1.9831	1.9558	1.927	1.9501
12	1.9557	1.9729	1.9723	1.99	1.9628	1.934	1.9571

Table 2.35: Error(%) in Voltage Angle Estimation with Wind Power at Bus-2

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.00	-1.59	-1.53	-3.13	-0.65	1.99	-0.13
3	0.00	-1.43	-1.39	-2.85	-0.59	1.81	-0.11
4	0.00	-1.25	-1.20	-2.47	-0.51	1.56	-0.10
5	0.00	-1.05	-1.01	-2.09	-0.43	1.33	-0.08
6	0.00	-1.00	-0.97	-2.00	-0.42	1.27	-0.08
7	0.00	-1.33	-1.29	-2.64	-0.55	1.67	-0.11
8	0.00	-1.10	-1.06	-2.18	-0.45	1.38	-0.09
9	0.00	-0.93	-0.90	-1.87	-0.38	1.19	-0.07
10	0.00	-0.89	-0.86	-1.79	-0.36	1.14	-0.07
11	0.00	-0.88	-0.85	-1.76	-0.36	1.12	-0.07
12	0.00	-0.88	-0.85	-1.75	-0.36	1.11	-0.07

Table 2.36: Voltage Angle Standard Deviation Estimation with Wind Power at Bus-2

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.1861	0.1774	0.1872	0.3022	0.1515	0.0752	0.1779
3	0.1844	0.1756	0.1853	0.2993	0.15	0.0744	0.1761
4	0.1814	0.1725	0.1821	0.2943	0.1475	0.0731	0.1731
5	0.1775	0.1685	0.1779	0.2879	0.1443	0.0713	0.1693
6	0.1763	0.1673	0.1766	0.2859	0.1433	0.0708	0.1681
7	0.183	0.1741	0.1838	0.297	0.1489	0.0738	0.1747
8	0.1786	0.1696	0.1791	0.2897	0.1452	0.0718	0.1704
9	0.1745	0.1654	0.1747	0.2829	0.1419	0.07	0.1663
10	0.1732	0.164	0.1732	0.2807	0.1407	0.0693	0.1649
11	0.1727	0.1636	0.1727	0.2799	0.1404	0.0691	0.1645
12	0.1726	0.1635	0.1726	0.2797	0.1403	0.0691	0.1644

Table 2.37: Error(%) in Voltage Angle Standard Deviation Estimation with Wind Power at Bus-2

Bus No	MCS	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
1	0	0	0	0	0	0	0
2	0.00	4.67	-0.59	-62.39	18.59	59.59	4.41
3	0.00	4.77	-0.49	-62.31	18.66	59.65	4.50
4	0.00	4.91	-0.39	-62.24	18.69	59.70	4.58
5	0.00	5.07	-0.23	-62.20	18.70	59.83	4.62
6	0.00	5.10	-0.17	-62.17	18.72	59.84	4.65
7	0.00	4.86	-0.44	-62.30	18.63	59.67	4.54
8	0.00	5.04	-0.28	-62.21	18.70	59.80	4.59
9	0.00	5.21	-0.11	-62.12	18.68	59.89	4.70
10	0.00	5.31	0.00	-62.07	18.76	59.99	4.79
11	0.00	5.27	0.00	-62.07	18.70	59.99	4.75
12	0.00	5.27	0.00	-62.05	18.71	59.97	4.75

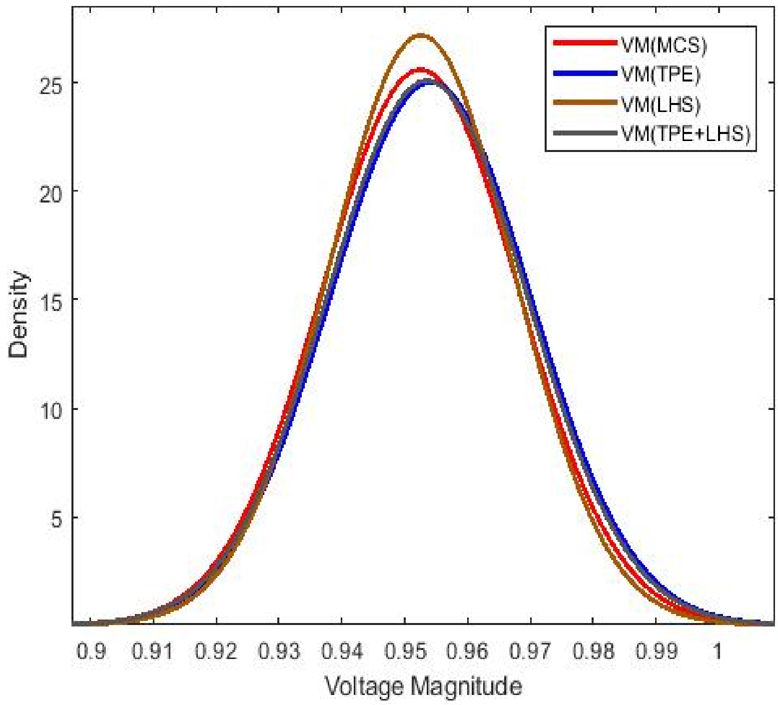

Figure 2.17: Comparison of Voltage Magnitude at Bus-12 with Wind Power at Bus-02 using MCS, TPE, LHS MEADIAN and TPE+LHS MEADIAN

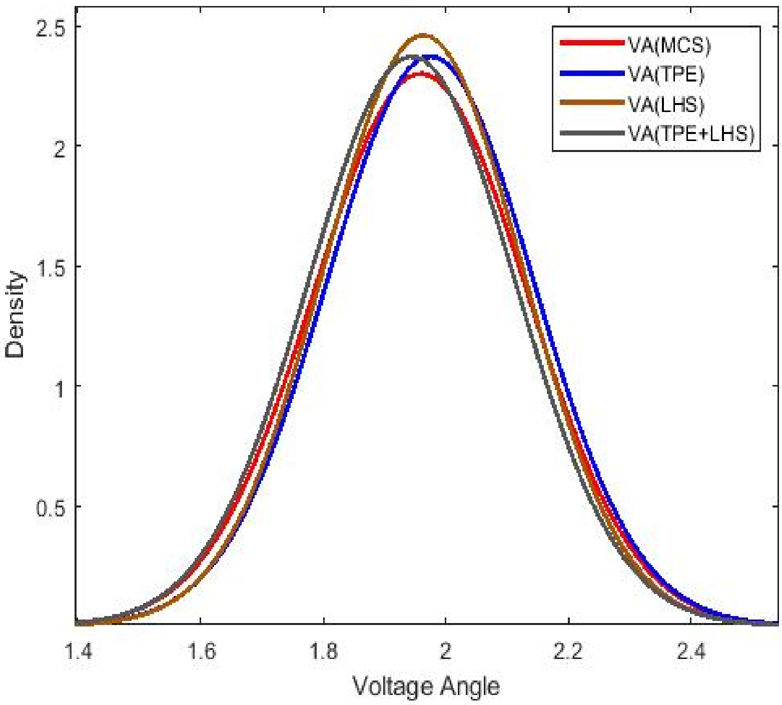

Figure 2.18: Comparison of Voltage Angle at Bus-12 with Wind Power at Bus-02 using MCS, TPE, LHS MEADIAN and TPE+LHS MEADIAN

The root mean square of error is calculated and tabulated here to summarise the performance of different estimation methods. It is given in table (2.38)

Table 2.38: Summary of Estimation Error with Different Statistical Methods

Parameter	TPE	TPE+1	THREE PE	FPE+1	LHS (RAND)	LHS (MEADIAN)
$V_{m(\mu)}$	0.15	0.15	0.27	0.07	0.13	0.01
$V_{m(\sigma)}$	52.50	52.51	70.83	54.65	64.22	53.17
$V_{a(\mu)}$	1.85	1.77	3.36	0.80	1.56	0.12
$V_{a(\sigma)}$	3.28	3.94	61.62	82.27	48.68	9.41

By carefully studying the results given in table (2.14) to (2.38), the following key points can be derived.

1. The point estimation method gives very impressive results, but requires large number of data points to work out evaluation points. While actual computation of voltage (magnitude and angle) is still done at few points, based on variants of method like Two Point, Three Point, the preliminary calculations required many data points. However, this is not the case with Latin Hypercube Sampling (LHS). In LHS only few data points are required.

2. The higher order point estimation are not adding much values in accuracy. On the other side, it is observed that the in higher order point estimation there will be risk on stability (i.e.invalid data points may be generated)

3. The LHS method offers very competitive results with only few data points (10) against large data points (1000) required for Point Estimation (PE).

4. In LHS also, there are two variants. One is LHS Random and second is LHS Median. The LHS Median is very effective method and its results are comparable with that of MCS and Point Estimation Methods (PE).

5. Careful evaluation reveals very important points, that the LHS (Median) gives good estimate of meanwhile PE gives good estimate of standard deviation.

6. The error in estimation of Standard Deviation (σ) using LHS is somewhat higher as compared to PE, but errors are consistent.

7. If LHS and PE is used in combination, it gives overall better results but it doesn′t add much value against the addition of extra computation burden. So, certain questions are raised when this two methods are used in combination.

8. The placement of wind power away from the conventional power supports the grid voltage, if network is structure is predominantly radial.

9. As the wind power increases, the voltage variation is also increases and hits the voltage control limits, which ultimately hampers the large wind power integration in the system.

10. With moderate strength of grid, the peak value of wind power is limited to around 8 %. However, this may be strictly indicative, as this value is depend on the network

strength, relative distance between Wind Farm and Other conventional sources, and the load variation.

11. This study is useful at planning stage to decide the maximum wind power evacuation possible from a give bus. Also, the network changes required to safeguard the system against any voltage violation in future or under different contingencies.

2.5.2.2 Voltage Estimation of IEEE - 12 Bus Radial System with Multiple Parameters Variation

In this section, three parameters Slack Bus Voltage Variation, Impedance Variation between line 1 -2 and Wind Power Variation. The wind power variation is considered having Weibull distribution with scale parameter c equal to 8.6 and shape parameter k equal to 2.4. The penetration of wind power is taken as 15% at rated value of Wind Turbine Generator. The slack bus variation is normally distributed with mean (μ) of 1 and standard deviation (σ) of 0.02. The third parameter, line impedance is increased with a constant factor, with a mean (μ) of 2.5 and standard deviation (σ) of 0.5. Two different cases are studied here, one with wind power at Bus-2 and second with wind power at Bus-12. The results of study is tabulated here.

Table 2.39: Voltage Magnitude Estimation with Wind Power at Bus-2 with Three Parameters Variation

Bus No	MCS	TPE	LHS (MEADIAN)
1	1.0003	1.0006	1.0000
2	0.9874	0.9879	0.9875
3	0.9820	0.9826	0.9822
4	0.9735	0.9740	0.9736
5	0.9626	0.9632	0.9627
6	0.9593	0.9598	0.9594
7	0.9494	0.9499	0.9495
8	0.9408	0.9413	0.9409
9	0.9326	0.9332	0.9328
10	0.9298	0.9303	0.9299
11	0.9288	0.9294	0.9290
12	0.9286	0.9292	0.9288

Table 2.40: Voltage Magnitude Standard Deviation Estimation with Wind Power at Bus-2 with Three Parameters Variation

Bus No	MCS	TPE	LHS (MEADIAN)
1	0.0199	0.0199	0.0196
2	0.0205	0.0205	0.0165
3	0.0206	0.0206	0.0166
4	0.0208	0.0208	0.0168
5	0.0210	0.0210	0.017
6	0.0211	0.0211	0.017
7	0.0213	0.0213	0.0172
8	0.0215	0.0215	0.0174
9	0.0217	0.0217	0.0175
10	0.0218	0.0218	0.0176
11	0.0218	0.0218	0.0176
12	0.0218	0.0218	0.0176

Table 2.41: Voltage Angle Estimation with Wind Power at Bus-2 with Three Parameter Variation

Bus No	MCS	TPE	LHS (MEADIAN)
1	0.0000	0.0000	0.0000
2	0.3253	0.3233	0.3239
3	0.4361	0.4340	0.4346
4	0.6214	0.6191	0.6198
5	0.8572	0.8546	0.8554
6	0.9295	0.9268	0.9276
7	0.5333	0.5311	0.5318
8	0.7945	0.7920	0.7927
9	1.0330	1.0302	1.031
10	1.1113	1.1084	1.1092
11	1.1358	1.1329	1.1337
12	1.1431	1.1402	1.141

Table 2.42: Voltage Angle Standard Deviation Estimation with Wind Power at Bus-2 with Three Parameter Variation

Bus No	MCS	TPE	LHS (MEADIAN)
1	0.0000	0.0000	0.0000
2	0.0819	0.0770	0.0827
3	0.0831	0.0781	0.0795
4	0.0856	0.0805	0.0742
5	0.0899	0.0847	0.0677
6	0.0915	0.0862	0.0659
7	0.0841	0.0791	0.0771
8	0.0886	0.0834	0.0695
9	0.0939	0.0887	0.0632
10	0.0959	0.0907	0.0613
11	0.0966	0.0913	0.0607
12	0.0968	0.0915	0.0606

Table 2.43: Voltage Magnitude Estimation with Wind Power at Bus-12 with Three Parameter Variation

Bus No	MCS	TPE	LHS (MEADIAN)
1	1.0000	0.9993	1.0000
2	0.9874	0.9867	0.9875
3	0.9828	0.9821	0.9828
4	0.9755	0.9749	0.9756
5	0.9666	0.9660	0.9667
6	0.9639	0.9633	0.9641
7	0.9550	0.9544	0.9552
8	0.9491	0.9486	0.9493
9	0.9442	0.9438	0.9445
10	0.9431	0.9427	0.9434
11	0.9431	0.9427	0.9434
12	0.9436	0.9432	0.9439

Table 2.44: Voltage Magnitude Standard Deviation Estimation with Wind Power at Bus-12 with Three Parameter Variation

Bus No	MCS	TPE	LHS (MEADIAN)
1	0.0203	0.0203	0.0201
2	0.0209	0.0209	0.0180
3	0.0211	0.0211	0.0182
4	0.0215	0.0215	0.0187
5	0.0223	0.0224	0.0196
6	0.0226	0.0227	0.0200
7	0.0230	0.0232	0.0205
8	0.0244	0.0247	0.0221
9	0.0265	0.0271	0.0244
10	0.0277	0.0284	0.0257
11	0.0283	0.0291	0.0264
12	0.0288	0.0297	0.0270

Table 2.45: Voltage Angle Estimation with Wind Power at Bus-12 with Three Parameter Variation

Bus No	MCS	TPE	LHS (MEADIAN)
1	0.0000	0.0000	0.0000
2	0.3171	0.3196	0.3172
3	0.4370	0.4401	0.4373
4	0.6381	0.6423	0.6387
5	0.8975	0.9033	0.8987
6	0.9778	0.9841	0.9792
7	0.7185	0.7279	0.7225
8	0.9926	1.0034	0.9971
9	1.2486	1.2610	1.2539
10	1.3362	1.3493	1.3419
11	1.3659	1.3794	1.3718
12	1.3778	1.3916	1.3839

Table 2.46: Voltage Angle Standard Deviation Estimation with Wind Power at Bus-12 with Three Parameter Variation

Bus No	MCS	TPE	LHS (MEADIAN)
1	0.0000	0.0000	0.0000
2	0.0758	0.0773	0.0678
3	0.0871	0.0915	0.0779
4	0.1116	0.1208	0.1013
5	0.1540	0.1695	0.1433
6	0.1692	0.1866	0.1584
7	0.3248	0.3606	0.3253
8	0.3678	0.4082	0.3660
9	0.4243	0.4705	0.4208
10	0.4535	0.5027	0.4497
11	0.4690	0.5199	0.4652
12	0.4820	0.5343	0.4784

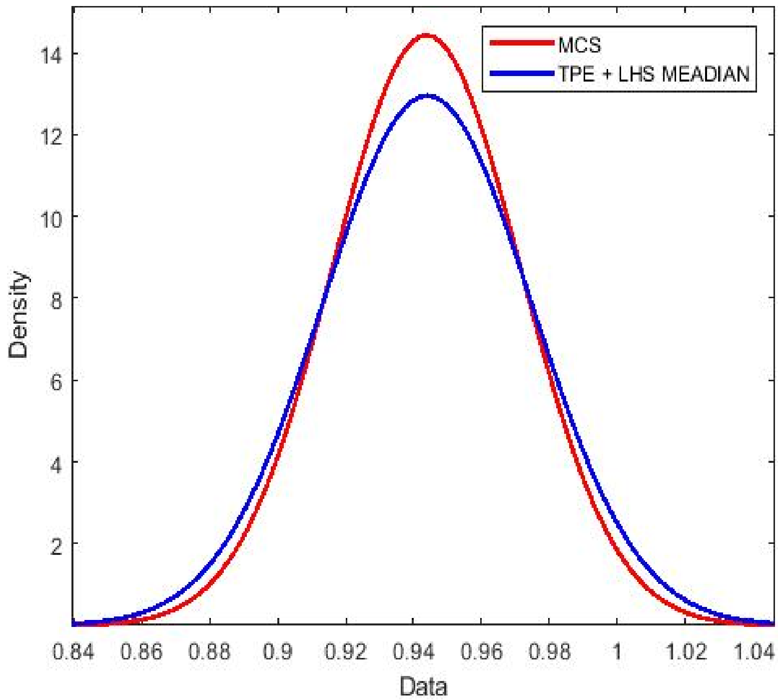

Figure 2.19: Comparison of Voltage Magnitude Variation at Bus-12 with MCS and TPE+LHS MEADIAN

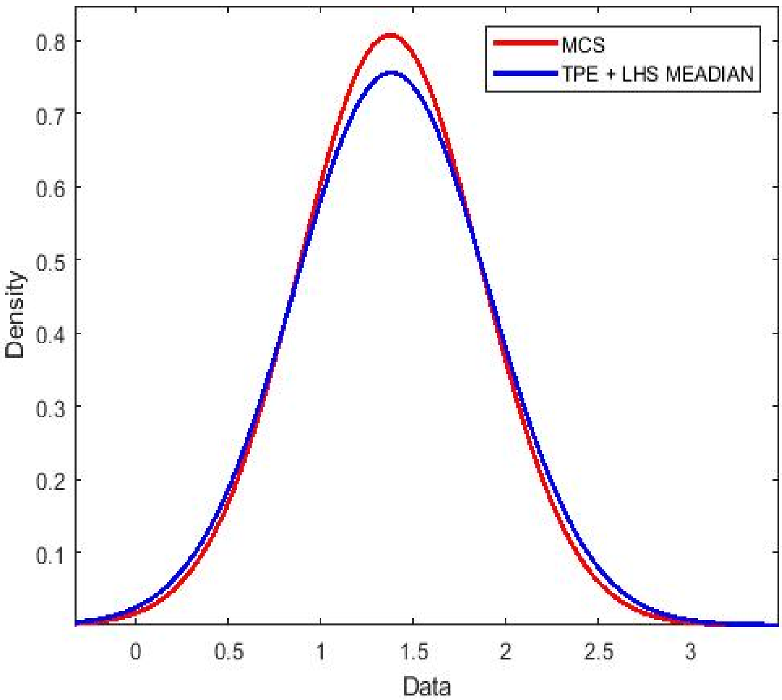

Figure 2.20: Comparison of Voltage Angle Variation at Bus-12 with MCS and TPE+LHS MEADIAN

By carefully studying the results given in table (2.39) to (2.46), the following key points can be derived.

1. This case is further exploration of the case, explained in section (2.5.2.1.1). In this section along with wind power variation, two more parameters variation is taken in to consideration. These parameters are grid impedance multiplier (μ=2.5 ,σ=0.5) and voltage at slack bus (μ=1.0 ,σ=0.02). The grid impedance is made variable by changing impedance between Bus-1 and Bus-2.

2. The variation in parameters alongwith wind power variation deteriorates the voltage stability. When the results of this case is compared with the results of previous case (section -2.5.2.1.1), the difference can be observed.

3. The voltage magnitude and standard deviation is degraded with multiple parameter variation and this can be related with the variation in slack bus voltage and

impedance variation.

4. There is improvement is seen in angle (mangnitude & standard deviation) and this is contrary to our understanding. So, to assess the actual impact of different parameters on system behaviour, it is necessary to carry out the study for concerned network.

5. The generalization of these results is possible up to certain extent and can be said that the wind power variation along with other parameters can affect the steady state voltage stability.

2.5.2.3 Voltage Estimation of IEEE - 12 Bus Radial System with Correlated Wind Power Sources at Different Locations

The effect of Correlation between different wind sources, located at different places, on voltage stability is carried out in steps here. The first and important steps in this study is to generate the correlated data. It has been already explained in the section (2.4). However, to maintain the flawless explanation, the complete process in steps is given here.

- Based on wind speed distribution of location find out scale parameter 'c' and shape parameter 'k' for each site
- Decide the correlation factor 'ρ' between wind speed of two sites
- Generate correlated normal random variable
- Convert random variable in to cummulative distibution function.
- Generate correlated random weibull data using inverse weibull function.
- The MATLAB -2018b code is given here for better understanding of implementation.
  ρ=-0.5; $\Rightarrow$ The value of Correlation Factor is depends on the wind speed correlation of two sites
  Z = mvnrnd([0 0],[1 ρ; ρ 1],n); $\Rightarrow$ Generation of Correlated Normal Random Variable with Correlation equal to 'ρ'

U = normcdf(Z); ⇒ Probability of Normal Random Variable
X = [wblinv(U(:,1),c_1,k_1) wblinv(U(:,2),c_2,k_2)]; ⇒ Generation of Correlated Wind Speed Data

Based on the method discussed above, the analysis is carried out and results are given here. For analysis purpose, IEEE-12 Bus Radial Network is taken. Two wind source with scale parameter 'c' =8.6 and shape parameter 'k'=2.0 is considered. One wind farm is connected at Bus-12 and second wind source is connected at Bus-2. The peak capacity of each source is fixed at 4%. The correlation between two sources is varied in the range of -1 to 1 and the standard deviation σ of voltage at Bus-12 is calculated.

Table 2.47: Voltage Magnitude Standard Deviation at Bus-12 with Different Correlation of Wind Power

Sr. No	Correlation Factor (ρ)	Standard Deviation (σ)
1	-1.0	0.0130
2	-0.5	0.0145
3	0.0	0.0150
4	0.5	0.0160
5	1.0	0.0180

From tabulated results given in table (2.47), it can be seen that the correlation between different wind power sources will have impact on the voltage deviation. The positive correlation increases the deviation in voltage magnitude and negative correlation decreases the deviation in voltage magnitude. And, this can be seen from the value of standard deviation. This study is helpful at planning stage to decide the location and maximum capacity of wind power to keep the voltage variation under control.

2.5.3 Probability Function Estimation

To explain the estimation of probability function of voltage (magnitude and angle), which varies with wind power variation is demonstrated here using IEEE-39 Bus System. In this analysis, IEEE-39 Bus New England System is taken for study. The total generation of system is 56 p.u at 100 MVA basis. The conventional power generation at Bus- 30 is replace with Wind Generation and the Bus type is changed in to P-Q type from P-V type. The rating of each generator is considered as 1 MW. Such 150 numbers of Wind Turbines are considered. So, the total power generation contribution is 2.5% of the total generation at rated capacity. The Bus-31 is taken as slack bus with mean voltage 'μ' of 0.98 pu and 0.2 standard deviation 'σ'. The wind velocity is assumed to follow Weibull distribution with shape parameter 'c' equal to 8.6 and shape factor 'k' equal to 1.8.The load flow analysis is carried out with 1000 iterations and the Bus voltages with comparatively higher variations is analysed for Probability Distribution Function Estimation. The IEEE-39 Bus system is given in figure (2.23).

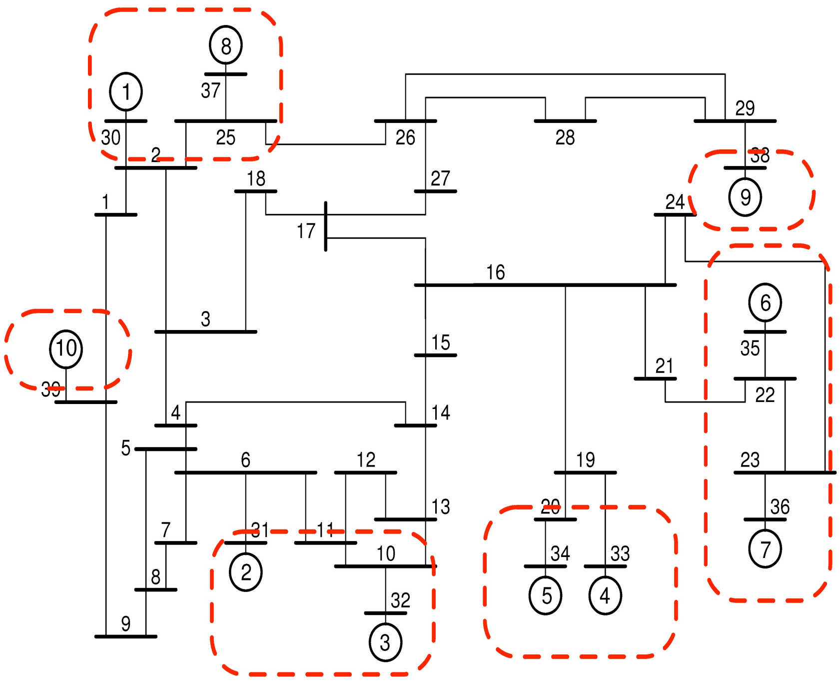

Figure 2.21: IEE-39 Bus New England System

The voltage variations at Bus -4, Bus-14 and Bus-31 is observed slightly higher as compared to other buses. So, this buses are selected for PDF estimation. The best Weibull fit of these voltages gave different Weibull parameters as shown in table (2.48)

Table 2.48: Weibull Parameter of Voltage Distribution at 3 Buses

Sr. No	Bus-No	Scale Factor (c)	Shape Factor (k)
1	Bus-4	0.9875	70.74
2	Bus-14	0.9866	53.1935
3	Bus-31	1.10419	19.7692

The PDF of these voltage is estimated using different methods, as explained in section (2.3.4) is given here in table (2.49)- table (2.51). The name of each method is given in short form. The description of each method is given here.

- FA = First Approximation
- $SA^{(4)}$= Second Approximation with series expansion up to 4^{th} order
- $SA^{(6)}$= Second Approximation with series expansion up to 6^{th} order
- GCE = Gram-Charlier Expansion
- MCS_{fit} = Monte Carlo Simulation Fit
- EE = Exact Expansion

Table 2.49: Bus-4 Voltage PDF Estimation

Sr. No	Voltage	FA	$SA^{(4)}$	$SA^{(6)}$	GCE	MCS_{fit}	EE
1.00	0.90	1.39	0.04	0.20	0.00	0.11	0.12
2.00	0.91	2.19	0.23	0.74	0.01	0.24	0.22
3.00	0.92	3.12	0.91	1.31	0.08	0.51	0.34
4.00	0.93	5.94	2.20	0.12	0.47	1.08	0.52
5.00	0.94	7.85	2.98	-1.88	1.91	2.23	0.85
6.00	0.95	8.02	2.44	4.09	5.69	4.51	1.26
7.00	0.96	10.15	5.23	17.54	12.35	8.73	1.63
8.00	0.97	11.02	16.84	19.82	19.51	15.52	1.35
9.00	0.98	9.96	28.47	14.32	22.45	23.49	0.60
10.00	0.99	3.11	25.59	19.04	18.81	25.84	2.19
11.00	1.00	-6.10	12.23	19.90	11.48	15.09	11.42
12.00	1.01	-15.49	2.59	7.55	5.10	2.51	24.48
13.00	1.02	-11.07	0.03	-1.39	1.65	0.04	19.02
14.00	1.03	7.89	0.05	-1.57	0.39	0.00	0.60
15.00	1.04	30.43	0.11	-0.10	0.07	0.00	17.64
16.00	1.05	23.61	0.04	0.19	0.01	0.00	16.72
17.00	1.06	4.61	0.01	0.07	0.00	0.00	1.03
18.00	1.07	1.01	0.00	0.01	0.00	0.00	-0.01
19.00	1.08	-0.07	0.00	0.00	0.00	0.00	0.00
20.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
21.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00

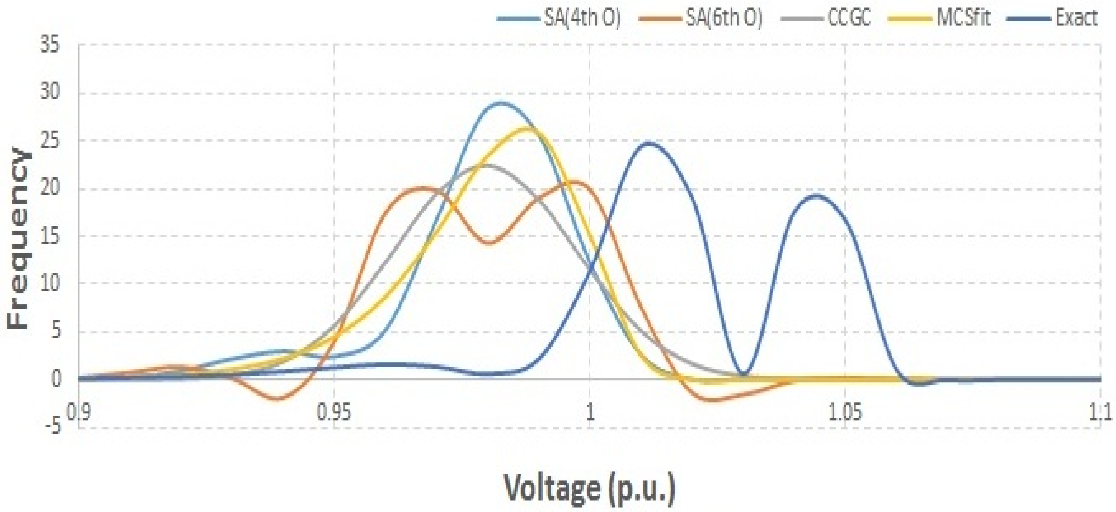

Figure 2.22: Comparision of Bus-4 Voltage PDF Estimation

Table 2.50: Bus-14 Voltage PDF Estimation

Sr. No	Voltage	FA	$SA^{(4)}$	$SA^{(6)}$	GCE	MCS_{fit}	EE
1.00	0.90	10.00	0.09	0.45	0.00	0.11	0.01
2.00	0.91	26.00	0.43	1.21	0.02	0.24	0.05
3.00	0.92	26.00	1.36	1.33	0.14	0.51	0.45
4.00	0.93	-40.00	2.63	-1.33	0.66	1.08	2.96
5.00	0.94	-98.00	2.73	-3.15	2.36	2.23	7.71
6.00	0.95	66.00	1.57	6.41	6.33	4.51	3.34
7.00	0.96	272.00	5.00	21.22	12.70	8.73	5.10
8.00	0.97	22.00	17.41	19.24	19.07	15.52	37.56
9.00	0.98	-347.00	28.66	10.17	21.44	23.49	25.38
10.00	0.99	-156.00	25.33	16.60	18.04	25.84	0.04
11.00	1.00	193.00	12.05	21.44	11.36	15.09	8.93
12.00	1.01	157.00	2.46	9.84	5.36	2.51	7.05
13.00	1.02	-4.00	-0.08	-1.22	1.89	0.04	1.31
14.00	1.03	-31.00	0.06	-2.35	0.50	0.00	0.08
15.00	1.04	-2.00	0.17	-0.41	0.10	0.00	0.01
16.00	1.05	7.00	0.08	0.25	0.01	0.00	0.01
17.00	1.06	4.00	0.02	0.14	0.00	0.00	0.01
18.00	1.07	1.00	0.00	0.04	0.00	0.00	0.01
19.00	1.08	1.00	0.00	0.01	0.00	0.00	0.01
20.00	1.09	1.00	0.00	0.00	0.00	0.00	0.01
21.00	1.10	1.00	0.00	0.00	0.00	0.00	0.01

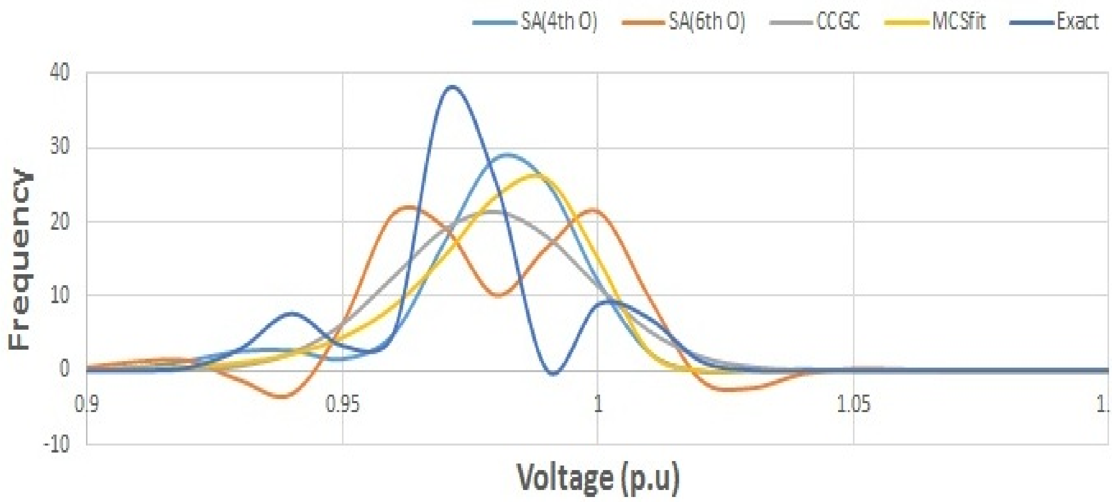

Figure 2.23: Comparision of Bus-14 Voltage PDF Estimation

Table 2.51: Bus-31 Voltage PDF Estimation

Sr. No	Voltage	FA	$SA^{(4)}$	$SA^{(6)}$	GCE	MCS_{fit}	EE
1.00	0.90	2.00	1.77	-5.91	0.21	0.11	0.28
2.00	0.91	3.00	1.51	-16.68	0.54	0.24	0.99
3.00	0.92	4.00	-0.16	-21.21	1.25	0.51	2.67
4.00	0.93	5.00	-2.80	-6.08	2.53	1.08	4.95
5.00	0.94	7.00	-3.97	29.07	4.54	2.23	5.33
6.00	0.95	8.00	-0.24	59.51	7.22	4.51	1.97
7.00	0.96	10.00	9.56	52.39	10.15	8.73	0.35
8.00	0.97	11.00	21.61	5.17	12.63	15.52	9.18
9.00	0.98	10.00	28.60	-41.57	13.90	23.49	22.81
10.00	0.99	5.00	25.64	-43.89	13.53	25.84	23.79
11.00	1.00	-4.00	14.73	-3.05	11.65	15.10	10.74
12.00	1.01	-13.00	2.95	38.32	8.88	2.51	0.74
13.00	1.02	-10.00	-3.63	45.48	5.99	0.04	1.50
14.00	1.03	9.00	-4.02	22.77	3.57	0.00	5.09
15.00	1.04	29.00	-1.27	-2.61	1.88	0.00	5.12
16.00	1.05	24.00	1.26	-12.91	0.88	0.00	2.91
17.00	1.06	6.00	2.17	-9.82	0.36	0.00	1.11
18.00	1.07	0.00	1.85	-3.13	0.13	0.00	0.33
19.00	1.08	0.00	1.13	0.96	0.04	0.00	0.09
20.00	1.09	0.00	0.54	1.87	0.01	0.00	0.03
21.00	1.10	0.00	0.22	1.33	0.00	0.00	0.02

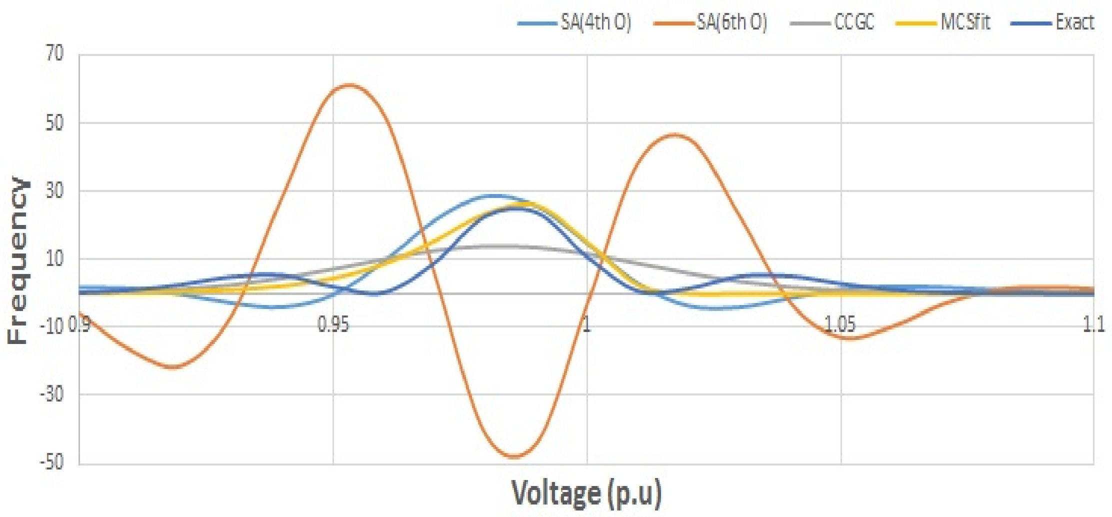

Figure 2.24: Comparision of Bus-31 Voltage PDF Estimation

The comparison of results of Voltage PDF estimation using different series expansion methods are carries out. The comparison reveals interesting facts, which are discussed here.

1. The voltage at three different buses are estimated using various series expansion methods as discussed in section (2.3.4). The comparison of each series expansion method is done with best fit curve obtained by Monte Carlo Simulation with 1000 Samples.

2. Bus voltages are following Weibull distribution with different charcteristic parameters (c and k). The variation in shape paramter (k) is observed in the range of 20 to 75. The higher value of shape parameter k indicates that the stadard deviation or variance is low. This can be related with the fact that only small amount of wind power is intergrated in the system (2.5% of total generation). As higher and higher wind power is integrated in to the system, there will be large voltage variations.

3. The Second Approximation Method (4^{th} order) and Gram Charlier Expansion are two methods which are giving relatively good results when compared with the best fit PDF obtained from Monte Carlo Simulation Data. However, both method works in different way. On one side, the Gram Charlier method is a very conservative approach. This method ascertains that the PDF remains positive and follow the Gaussian distribution and neglects the error part completely.

4. The Second Approximation Method (4^{th} order) is radical and tries to correct the error very fast and this cause the oscillation in calculated values for low and very low value of probability. This can be seen from the results given in the form of chart.

5. The Second Approximation Method (6^{th} order) shows very oscillatory behaviour and this may be because of the consideration of higher order terms. The simlilar behaviour is observed with Exact Expansion method, which is also considers the higher order terms.

6. The moment based Gram Charlier Expansion method is more suitable for normally distributed data, whereas the cummulant based Second Approximation Method (4^{th} order) gives better results with non-normal data.

2.6 Conclusion

In this chapter, various statistical methods have been demonstrated for voltage analysis with non conventional energy sources like wind. Starting from simple method, which is suited for priliminary analysis or for small system study, the other complex and computationally intensive methods are discussed and applied to the test system to see there accuracy and consistency with different data distributions. There are family of Point Estimation methods, which are suitable to expedite the analysis processs. The Point Estimation methods are applied before the data simulation to cut short the analysis time and still achieve better results. So, it is a pre-opreative method. On the other side, the Series Expansion methods are post operative methods which are used to obtain data distribution from simulated data. The exact comparison of both the methods are not possible directly as the objective of both the method is different. In Point Estimation methods, the Two Point Estimation is found very promising. The similar results can be obtained with less data using Latin Hypercube Sampling method. In series expansion method, the Second Approximation Method (4^{th} order) is found very suitable for non-normal data (with different skewness and kurtosis). In this chapter, it is also investigated the effect of different correlated wind farms on the voltage variations. It is found that the negatively correlated wind power sources are complementing other wind plants and gives better results (i.e. lower voltage variations). Finally, this chapter is concluded with remark that by making use of various statistical techniques the system performance can be estimated with different penetration level of wind power. These methods can equally be applied to cross check the performance of existing system.