Chapter 6

Small Signal Stability of Doubly Fed Induction Generator

6.1 Introduction

Stability is defined as "the state of being stable" in a simple words. It can be elaborated as the ability of a system, component or substance to remain unchanged over time under stated or reasonably expected conditions of storage and use. Power system stability definition is similar to any other dynamic system. Small Signal Stability is related with the ability of system with small change in its associated parameters. Small disturbance such as load changes, voltage changes, network configuration changes shall affect stability of power systems.

Small signal stability problem inherently refers to the stability problems caused by small disturbances. Small disturbances in power system cause oscillations. The question of "How much small is small?" depends on the concerned system. Oscillation's property is the key to analyse the stability of power system. This definition is equally valid for any part of the whole system or any single equipment. Small Signal Stability is necessary but not sufficient condition for stability of power system. Any disturbance in parameter of system may cause transient condition. The sustainability of transient depends on the system's capacity to absorb the disturbace. Faster the decaying of transient, better is the stability of system or equipment. Small-Signal Stability is assessed before assessment of transient stability. A system remain stable under small disturbance, may or may not remain stable under large disturbance. So, large system stability assessment is inevitable. But, a system which do not survive under small disturbance is definitely unstable under large disturbance. So, Small-Signal Stability assessment is done before the transient stability analysis. It is important to note that the Small-Signal Stability is not replacement or alternative of transient stability.

The stability of system is related with the damping of transients or oscillations. The damping is defined as "A reduction in the amplitude of an oscillation as a result of energy being drained from the system to overcome frictional or other resistive forces". In a simple words, "Damping is a reduction in vibration over a period of time". Another definition found in literature is "The gradual reduction of excessive oscillation, vibration, or signal intensity, and therefore of instability in a mechanical or electrical device, by a substance or some aspect of the device". The undamped power oscillation may cause the blackout. The example of such event is the blackout of North American Western Interconnected (WSCC) System happened in 1996 and failing of northern grid of India in 2014.

In order to examine the oscillation's mode, two different methods are applied to analyze the oscillation caused by small disturbances. The first method is referred to as model-based analysis for small signal stability problem. It uses mathematical model to assess whether the system is stable based on the eigenvalue calculation for the mathematical model. The other method is the measurement-based analysis for power system. This method is known as WAM (Wide Area Measurement). It is gaining momentum now a days for power system analysis. It uses real-time synchro-phasor measurements, collected from Phasor Measurement Units (PMUs), installed at various buses to estimate the mode of oscillations based on prony method or uniformly sampled signals. This method requires the system to be visible using PMUs. In addition, the mode of oscillation for every bus can be detected with the time stamp.

This chapter studies small signal stability of Doubly Fed Induction Generator (DFIG) using Modal (i.e. Eigenvalue) based method. Definitely, it is advantageous to use measurement-based analysis. Also, as the power system is highly dynamic, model-based analysis cannot provide accurate real-time information. However, the mathematical model based analysis should not be under-estimated. First, it is useful at the design stage, when the real time data is not available. Another point that goes in favour of mathematical model base analysis is the challenge of applying measurement-based analysis. This challenge is to find the optimum locations for installing PMUs, so that, non-observable area does not exist in the whole system. Also, the distrubance in measurement of parameters may lead to a false result.

In this chapter the probabilistic small signal stability of DFIG has been given.

6.2 Modelling of DFIG

There are many literatures are available on DFIG modelling. Various approach can be found on the modelling of DFIG [267, 270, 271, 272, 274, 277, 280]. In this work, the focus is maintained on the statistical part of the analysis. So, the base of modelling part is prepared from [270]. However, the modelling given in [270] is not accurate due to use of certain simplified relations. To use this model, the detailing part is improved by using exact relations instead of simplified relations. It is explained here, part by part.

6.2.1 DFIG Equations in a-b-c Form

The convention of the positive direction of rate of change of flux and voltage given in the figure 6.1, which is further shown in expanded form in 6.2.

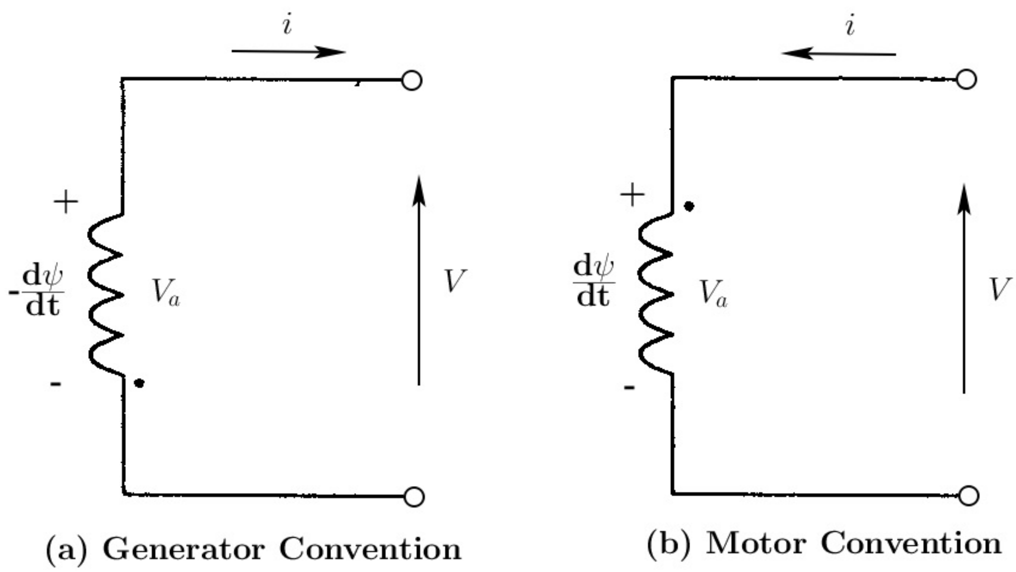

Figure 6.1: Generator - Motor Convention

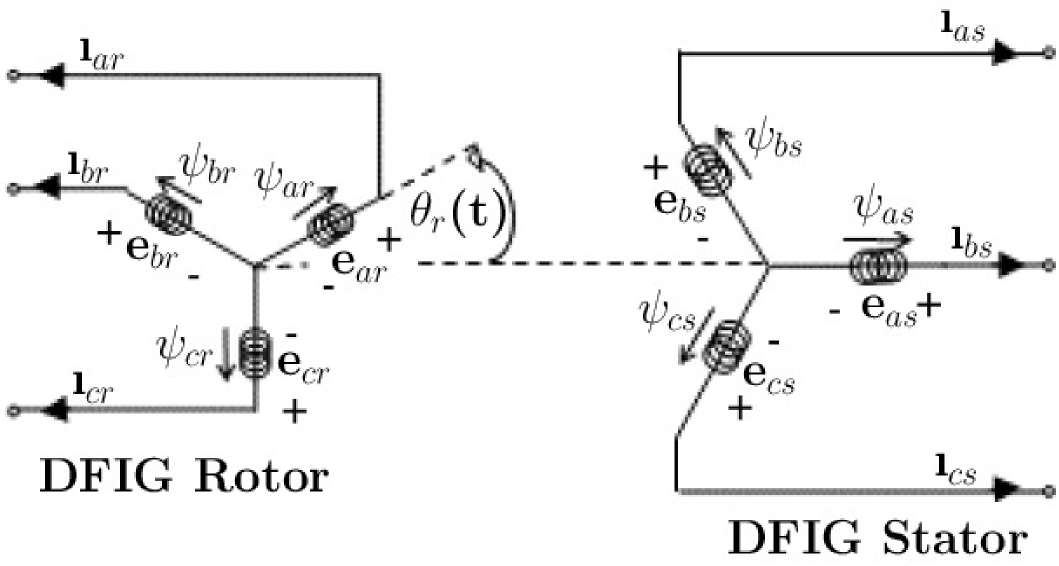

Figure 6.2: Convention of Positive Direction of Current, Voltage and Flux

The flux induced in the stator coil makes current to flow in the circuit, so the direction of flux and current is the same. According to Lenz's Law, the voltage induced in the coil is in the direction such that it opposes the flux which produces it, so the polarity of voltage and flux are opposite. Applying the Kirchhoff's Voltage Law to the circuit given in figure 6.2 will give following equations.

$$v_{as} = -\frac{1}{\omega_B}\frac{d\psi_{as}}{dt} - R_s i_{as} \tag{6.1}$$

$$v_{bs} = -\frac{1}{\omega_B}\frac{d\psi_{bs}}{dt} - R_s i_{bs} \tag{6.2}$$

$$v_{cs} = -\frac{1}{\omega_B}\frac{d\psi_{cs}}{dt} - R_s i_{cs} \tag{6.3}$$

In a similar way, the rotor equations can be given as follows.

$$v_{ar} = -\frac{1}{\omega_B}\frac{d\psi_{ar}}{dt} - R_r i_{ar} \tag{6.4}$$

$$v_{br} = -\frac{1}{\omega_B}\frac{d\psi_{br}}{dt} - R_r i_{br} \tag{6.5}$$

$$v_{cr} = -\frac{1}{\omega_B}\frac{d\psi_{cr}}{dt} - R_r i_{cr} \tag{6.6}$$

The values of resistance R, voltages V, currents i and fluxes ψ are given in per unit [pu]. Whereas, the ω_B is the base frequency in rad/sec. It is given by $2\pi f$. The time t is given in seconds. The flux linked in circuit is given as,

$$\begin{aligned}\psi_{as} =& (L_{ss} + L_{ls})i_{as} + L_m(i_{bs} + i_{cs}) + ...\\ & L_{sr}(\cos\theta_r i_{ar} + \cos(\theta_r + \frac{2\pi}{3})i_{br} + \cos(\theta_r - \frac{2\pi}{3}i_{cr})\end{aligned} \tag{6.7}$$

$$\begin{aligned}\psi_{bs} =& (L_{ss} + L_{ls})i_{bs} + L_m(i_{cs} + i_{as}) + ...\\ & L_{sr}(\cos\theta_r i_{br} + \cos(\theta_r + \frac{2\pi}{3})i_{cr} + \cos(\theta_r - \frac{2\pi}{3}i_{ar})\end{aligned} \tag{6.8}$$

$$\begin{aligned}\psi_{cs} =& (L_{ss} + L_{ls})i_{cs} + L_m(i_{as} + i_{bs}) + ...\\ & L_{sr}(\cos\theta_r i_{cr} + \cos(\theta_r + \frac{2\pi}{3})i_{ar} + \cos(\theta_r - \frac{2\pi}{3}i_{br})\end{aligned} \tag{6.9}$$

Simillarly, for rotor circuit, the flux equation can be given as follows.

$$\begin{aligned}\psi_{ar} =& (L_{sr} + L_{lr})i_{ar} + L_m(i_{br} + i_{cr}) + ...\\ & L_{rs}(\cos\theta_r i_{as} + \cos(\theta_r + \frac{2\pi}{3})i_{bs} + \cos(\theta_r - \frac{2\pi}{3}i_{cs})\end{aligned} \tag{6.10}$$

$$\begin{aligned}\psi_{br} =& (L_{sr} + L_{lr})i_{br} + L_m(i_{cr} + i_{ar}) + ...\\ & L_{rs}(\cos\theta_r i_{bs} + \cos(\theta_r + \frac{2\pi}{3})i_{cs} + \cos(\theta_r - \frac{2\pi}{3}i_{as})\end{aligned} \tag{6.11}$$

$$\begin{aligned}\psi_{cr} =& (L_{sr} + L_{lr})i_{cr} + L_m(i_{ar} + i_{br}) + ...\\ & L_{sr}(\cos\theta_r i_{cs} + \cos(\theta_r + \frac{2\pi}{3})i_{as} + \cos(\theta_r - \frac{2\pi}{3}i_{bs})\end{aligned} \tag{6.12}$$

Where,
L_{ss} = Self Inductance of Stator
L_{rr} = Self Inductance of Rotor
L_{ls} = Leakage Inductance of Stator
L_{lr} = Leakage Inductance of Rotor
$L_{rs} = L_{rs}$ = Mutual Inductance of Stator to Rotor
L_m= Mutual Inductance of Rotor to Rotor and Stator to Stator
$\theta_r(t)$=Angle between Stator - a axis (Stationary Axis) and Rotor - a axis (Rotationary Axis)

6.2.2 $a-b-c$ to $q-d-0$ Transformation

6.2.2.1 Transformation Matrix

It is difficult and laborious to work with equations in $a-b-c$ form. So, to ease out the modelling work, all the equations are transformed from $a-b-c$ to $q-d-0$ frame (i.e. Rotational to Stationary frame). In matrix form we can write as,

$$v_{qd0} = T_\theta v_{abc} \tag{6.13}$$

Where,
$v_{qd0} = [v_q \; v_d \; v_0]$'
$v_{abc} = [v_a \; v_b \; v_c]$'
T_θ = $a-b-c$ to $d-q-0$ transformation martix

In this work power invariant transformation is used. The d-axis is leading the q-axis as per the recommendation for modelling of Synchronous Machine [9]. Figure 6.3 shows the respective position of $d-q$ frame and $a-b-c$ frame. The transformation matrix T_θ is given in equation 6.14.

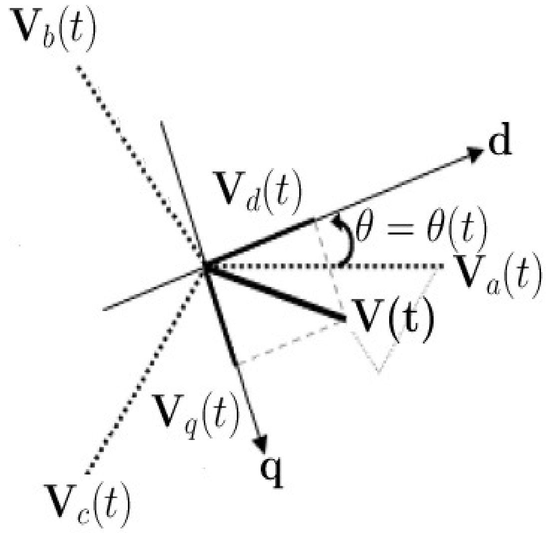

Figure 6.3: Position of $d-q$ frame with respect to abc frame

$$T_\theta = \sqrt{\frac{2}{3}} \begin{pmatrix} \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{6.14}$$

Tranformation matrix T_θ is a orthogonal matrix, so the inverse transformation is equal to transpose of matrix itself, i.e $T_\theta^{-1} = T_\theta'$.

6.2.2.2 DFIG Equations in $d-q-0$ Frame

Applying transformation given in equation 6.14 to equations 6.1-6.12. The resulting equations in $d-q-0$ form is given as below in equations 6.15-6.22.

$$v_{qs} = -\frac{1}{\omega_B}\frac{d\psi_{qs}}{dt} - R_s I_{qs} + \omega\psi_{ds} \tag{6.15}$$

$$v_{ds} = -\frac{1}{\omega_B}\frac{d\psi_{ds}}{dt} - R_s I_{ds} - \omega\psi_{qs} \tag{6.16}$$

$$v_{qr} = -\frac{1}{\omega_B}\frac{d\psi_{qr}}{dt} - R_r I_{qr} + (\omega - \omega_r)\psi_{dr} \tag{6.17}$$

$$v_{dr} = -\frac{1}{\omega_B}\frac{d\psi_{dr}}{dt} - R_r I_r - (\omega - \omega_r)\psi_{qr} \tag{6.18}$$

$$\psi_{qs} = L_{ss} i_{qs} + L_m i_{qr} \tag{6.19}$$

$$\psi_{ds} = L_{ss} i_{ds} + L_m i_{dr} \tag{6.20}$$

$$\psi_{qr} = L_{ss} i_{qr} + L_m i_{qs} \tag{6.21}$$

$$\psi_{dr} = L_{ss} i_{dr} + L_m i_{dr} \tag{6.22}$$

Where,
ω = Rotational speed of dq frame, i.e $\omega = \frac{d\theta}{dt}$
θ = Angle between d axis and a axis
ω_r=Rotational speed of Rotor, i.e $\frac{d\theta_r}{dt}$
ω=ω_s=1 [pu] for synchronously rotating reference frame
s = slip of rotor =$\frac{(\omega_s - \omega_r)}{\omega_s}$
$(\omega - \omega_r)$=$s\omega_s$
$L_{ss} = L_{ss} + L_{ls} - L_{sr}$
$L_{rr} = L_{sr} + L_{lr} - L_{rs}$
$L_{sr} = L_{rs} = L_m$

6.2.3 State Space Equation Formation

DFIG is represented as a voltage source behind transient impedance for Small-Signal Stability study. The same model is can also be applied for Transient Stability as well. The difference between two studies is that, for Small-Signal Stability, the state equations are linearized, whereas for Transient Stability, the differential equations are used directly. For formulation of state equations, equations 6.15 - 6.18 are substituted with relations defined in 6.19 - 6.22 and new variables are defined as given in 6.23 - 6.26.

$$v_q^{'} = \frac{L_m}{L_{rr}}\omega_s\psi_{dr} \tag{6.23}$$

$$v_d^{'} = -\frac{L_m}{L_{rr}}\omega_s\psi_{qr} \tag{6.24}$$

$$X_s^{'} = \omega_s\left(L_{ss}L_{rr} - L_m^2\right)/L_{rr} \tag{6.25}$$

$$T_0^{'} = \omega_s L_{rr}/R_r \tag{6.26}$$

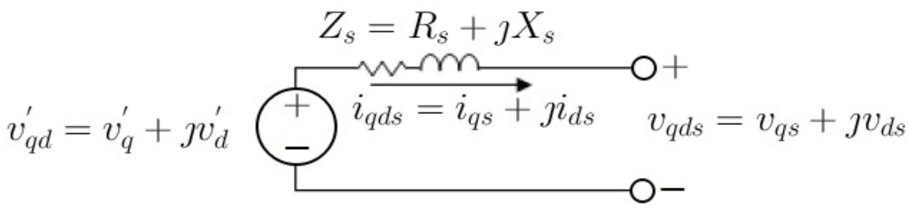

Figure 6.4: DFIG Model For Stability Study

After substituting of newly defined variables a given in 6.23 - 6.26, the DFIG states equations 6.15 - 6.18 in [pu] can be re-written as given below.

$$\begin{aligned}\frac{X_s^{'}}{\omega_s\omega_B}\frac{di_{qs}}{dt} = & -\left(R_s + \frac{X_s - X_s^{'}}{T_0^{'}}\right)i_{qs} + X_s^{'}i_{ds}... \\ & + (1-s)v_q^{'} - \frac{1}{T_0^{'}}v_d^{'} - v_{qs} + \frac{L_m}{L_{rr}}v_{qr}\end{aligned} \tag{6.27}$$

$$\begin{aligned}\frac{X_s^{'}}{\omega_s\omega_B}\frac{di_{ds}}{dt} = & -X_s^{'}i_{qs} - \left(R_s + \frac{X_s - X_s^{'}}{T_0^{'}}\right)i_{ds}... \\ & + \frac{1}{T_0^{'}}v_q^{'} + (1-s)v_d^{'} - v_{ds} + \frac{L_m}{L_{rr}}v_{dr}\end{aligned} \tag{6.28}$$

$$\frac{1}{\omega_s\omega_B}\frac{dv_q^{'}}{dt} = \left(\frac{X_s - X_s^{'}}{T_0^{'}}\right) i_{ds} - \frac{1}{T_0^{'}}v_q^{'} + sv_d^{'} - \frac{L_m}{L_{rr}}v_{dr} \tag{6.29}$$

$$\frac{1}{\omega_s\omega_B}\frac{dv_d^{'}}{dt} = -\left(\frac{X_s - X_s^{'}}{T_0^{'}}\right) i_{qs} - sv_q^{'} - \frac{1}{T_0^{'}}v_d^{'} + \frac{L_m}{L_{rr}}v_{qr} \tag{6.30}$$

$$i_{qr} = -\left(\frac{1}{\omega_s L_m}\right) v_d^{'} - \left(\frac{L_m}{L_{rr}}\right) i_{qs} \tag{6.31}$$

$$i_{dr} = \left(\frac{1}{\omega_s L_m}\right) v_q^{'} - \left(\frac{L_m}{L_{rr}}\right) i_{ds} \tag{6.32}$$

$$\psi_{qs} = -\left(\frac{1}{\omega_s}\right) v_d^{'} + \left(\frac{X_s^{'}}{\omega_s}\right) i_{qs} \tag{6.33}$$

$$\psi_{ds} = \left(\frac{1}{\omega_s}\right) v_q^{'} + \left(\frac{X_s^{'}}{\omega_s}\right) i_{ds} \tag{6.34}$$

6.2.4 Electromagnetic Torque

The power of DFIG at any istance is algebraic sum of Rotor Active Power, Stator Active Power and losses (negative).

$$P_{DFIG} = P_s + P_g - Losses\ in\ GSC\ transformer \tag{6.35}$$

$$P_{DFIG} = P_s + P_g - (\Delta v_{dg} i_{dg} + \Delta v_{qg} i_{qg}) \tag{6.36}$$

$$P_s = v_{qs} i_{qs} + v_{ds} i_{ds} \tag{6.37}$$

$$
\begin{aligned}
P_r &= v_{qr} i_{qr} + v_{dr} i_{dr} \\
P_g &= v_{qg} i_{qg} + v_{dg} i_{dg} \\
P_g &= v'_{qg} i'_{qg} + v'_{dg} i'_{dg}
\end{aligned} \tag{6.38}
$$

Where,

$S_s = v_{qds} i^*_{qds} = P_s + \jmath Q_s$

$S_r = v_{qdr} i^*_{qdr} = P_r + \jmath Q_r$

$S_g = v_{qdg} i^*_{qdg} = P_g + \jmath Q_g$

$S'_g = v'_{qdg} i'^*_{qdg} = P_g + \jmath Q_g$

P_g=Grid Side Converter Power

v'_g = GSC Voltage on Primary Side of Transformer

i'_g = GSC Current on Primary Side of Transformer

v_g = GSC Voltage on Secondary Side of Transformer

i_g = GSC Current on Secondary Side of Transformer

$v_{qds} = v_{qs} + \jmath v_{ds}$

$v_{qdr} = v_{qr} + \jmath v_{dr}$

$i_{qds} = i_{qs} + \jmath i_{ds}$

$i_{qdr} = i_{qr} + \jmath i_{dr}$

$\Delta v_{dg} = v_{dg} - v_{ds} = i_{qg} R_T - i_{dg} X_T$

$\Delta v_{qg} = v_{qg} - v_{qs} = i_{dg} R_T + i_{qg} X_T$

$i_{qdg} = i_{qg} + \jmath i_{dg}$

$\Delta v_{qdg} = \Delta v_{qg} + \jmath \Delta v_{dg}$

Neglecting losses in the converters and DC link capacitor.

$$
P_g = P_r \quad (i.e.\ P_{dc} \approx 0) \tag{6.39}
$$

Substituting equations (6.15) - (6.18) to equations (6.37) - (6.38).

$$\begin{aligned} P_{DFIG} = & - R_s(i_{qs}^2 + i_{ds}^2) - R_r(i_{qr}^2 + i_{dr}^2)... \\ & - i_{qs}\frac{1}{\omega_B}\frac{d\psi_{qs}}{dt} - -i_{ds}\frac{1}{\omega_B}\frac{d\psi_{ds}}{dt}... \\ & - i_{qr}\frac{1}{\omega_B}\frac{d\psi_{qr}}{dt} - -i_{dr}\frac{1}{\omega_B}\frac{d\psi_{dr}}{dt}... \\ & + \omega_s\psi_{ds}i_{qs} - \omega_s\psi_{qs}i_{ds} + s\omega_s\psi_{dr}i_{qr} - s\omega_s\psi_{qr}i_{dr}... \\ & - \Delta v_{dg}i_{dg} - \Delta v_{qg}i_{qg} \end{aligned} \tag{6.40}$$

The first two terms represent the machine losses, the second four terms are the power associated with flux variation, the third four terms are related to the air-gap power and the last four terms are losses in GSC transformer. The electromagnetic torque T_e is obtained by dividing the air-gap power by the mechanical speed of DFIG rotor. To achieve electromagnetic, using equations (6.19) - (6.22), we get following equation.

$$T_e = L_m(i_{qs}i_{dr} - i_{ds}i_{qr}) \tag{6.41}$$

Adding and subtracting the $(L_m L_{rr}/L_{rr})i_{qs}i_{ds}$ gives the expression of T_e in different form.

$$T_e = (L_m/L_{rr})(i_{qs}\psi_{dr} - i_{ds}\psi_{qr}) \tag{6.42}$$

Substitution of equation (6.23) - (6.24) gives following equation of T_e to be used in DFIG model represented by equation (6.27) - (6.34)

$$T_e = \left(v_d^{'}/\omega_s\right) i_{ds} + \left(v_q^{'}/\omega_s\right) i_{qs} \tag{6.43}$$

6.2.5 Drive Train Model

6.2.5.1 Two Mass Model

The turbine, generator, gearbox, shaft and other transmission components are having certain mass. It is represented by lumped Inertia. For accuracy purpose, each and every component can be modelled individually. But, for ease in analysis, two mass model is given here. In a Two Mass Model, the generator and turbine are considered using moment of inertia H_t and H_g.

$$2H_t\frac{d\omega_t}{dt} = T_m - T_{sh} \tag{6.44}$$

$$2H_g\frac{d\omega_r}{dt} = T_{sh} - T_e \tag{6.45}$$

$$\frac{d\theta_{tw}}{dt} = (\omega_t - \omega_r)\omega_B \tag{6.46}$$

Where,
H_t=Inertia of Turbine in second [s]
H_g=Inertia of Generator in second [s]
ω_t=Turbine Speed in per unit [pu]
ω_r=DFIG Rotor Speed in per unit [pu]
T_m=Mechanical Torque in per unit [pu]
T_{sh}=Shaft Torque in per unit [pu]

The Shaft Torque in [pu] is given by,

$$T_{sh} = K\theta_{tw} + D\frac{d\theta_{tw}}{dt} \tag{6.47}$$

Where,
θ_{tw} = Shaft Twist Angle in [rad]
K= Shaft Stiffness in per unit/ radian [pu/rad]
D=Damping Coefficient in per unit-second / rad [pu-s/rad]

Substituting equation (6.47) in equation (6.44) and (6.45) and utilizing the relation $\omega_r = (1 - s)\omega_s$ and equation (6.46), gives the equations for Two Mass Model.

$$2H_t \frac{d\omega_t}{dt} = T_m - (K\theta_{tw} + D(\omega_t - (1-s)\omega_s)\omega_B) \tag{6.48}$$

$$-2H_g\omega_s \frac{ds}{dt} = (K\theta_{tw} + D(\omega_t - (1-s)\omega_s)\omega_B) - T_e \tag{6.49}$$

$$\frac{d\theta_{tw}}{dt} = (\omega_t - (1-s)\omega_s)\omega_B \tag{6.50}$$

Where, T_e is an electromagnetic torque given in equation (6.43).

6.2.5.2 One Mass Model

One Mass Model is further simplification of Two Mass Model. In One Mass Model, the mass of turbine H_t and generator H_g are combined and given in term of total inertia H_{tot} in seconds [s]. Where H_{tot}=$H_t + H_g$. Using this relation, the expression for One Mass Model is given by equation (6.51). It is also known as swing equation.

$$-2H_{tot}\omega_s \frac{ds}{dt} = T_m - T_e \tag{6.51}$$

Where, T_e is an electromagnetic torque given in equation (6.43).

6.2.6 Converter Model

DFIG includes two converters. One converter is Rotor Side Converter (RSC) and another is Grid Side Converter (GSC) as shown in figure (6.5). GSC side converter is connected to supply side by transformer with $N_1 : N_2$ ratio. Here, the transformer is represented by ideal transformer with no losses and $N_1 : N_2$ ratio. The losses are represented separately on the secondary side of transformer. The primary side voltage and currents are $v_g^{'}$ and $i_g^{'}$. Whereas, the secondary side of voltage and currents are v_g and i_g respectively. Both converters are PWM based Voltage Source Converter (VSC). The two converters are separated by DC link capacitor. Both converters are working with variable modulation index. The Grid Side Converter varies its modulation index m_{a2} in order to maintain the constant DC link voltage V_{DC}. Generally, it operates at unity power factor and delivers no reactive power to the grid. However, there are literatures available, which claims that variable power factor gives better stability in operation of DFIG. The rotor side converter (RSC) operates at variable modulation index m_{a1}. The rotor side converter injects appropriate voltage, so as to maintain desired Wind Generator's speed. The generator speed is a non-linear function of factors like Blade Pitch Angel (β) , Tip-Speed Ratio (TSR (λ)) and Coefficient of Power (C_p). Normally, the WT are operated with variable TSR and hence variable C_p, so as to maintain the speed of generator rotor. Another strategy is to vary the TSR, to extract the maximum amount of wind power. These are shown in figure (6.6) and (6.7).

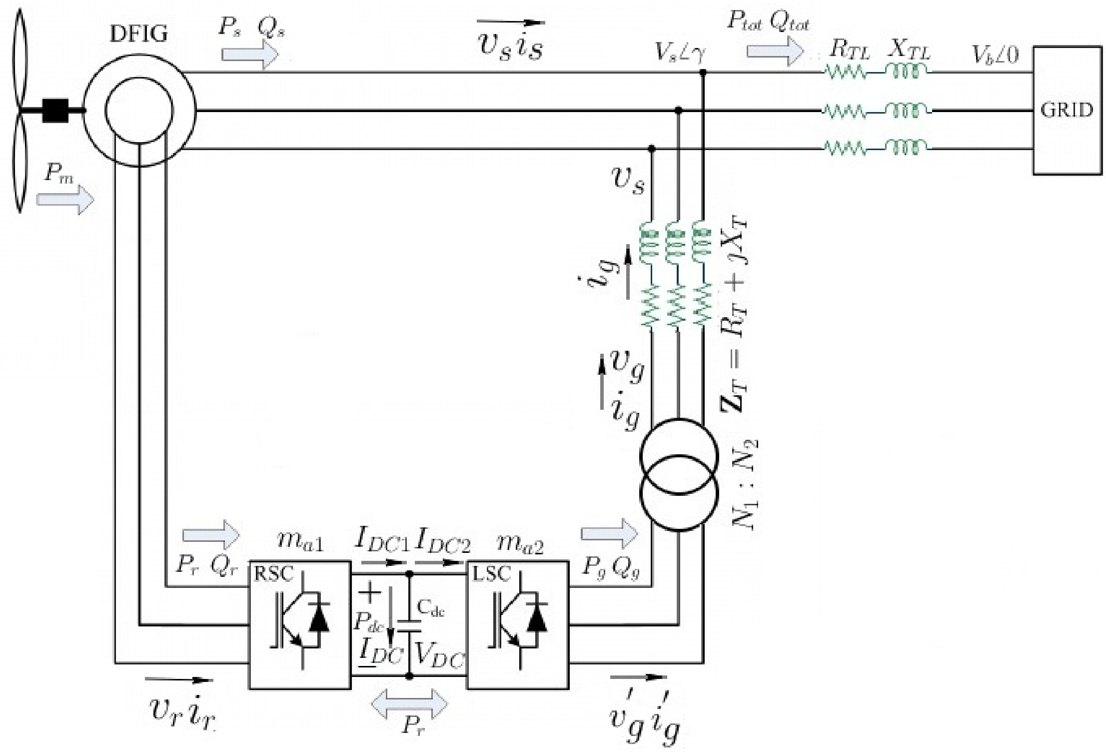

Figure 6.5: DFIG Converter Model

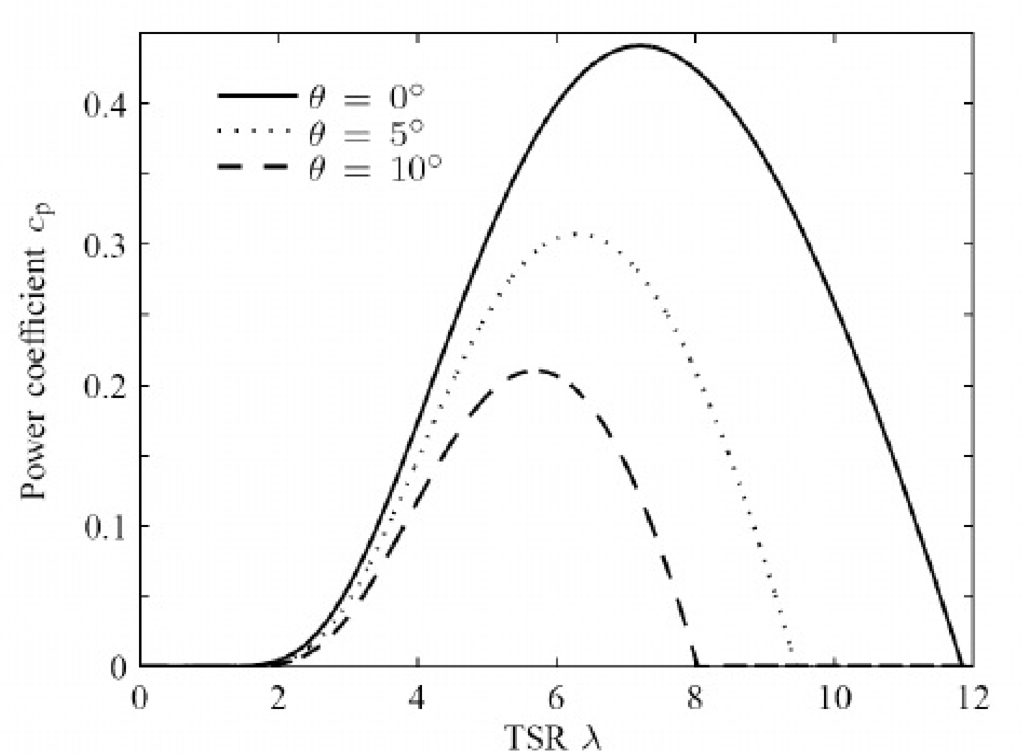

Figure 6.6: Coeffficient (C_p) Versus Lambda (λ)

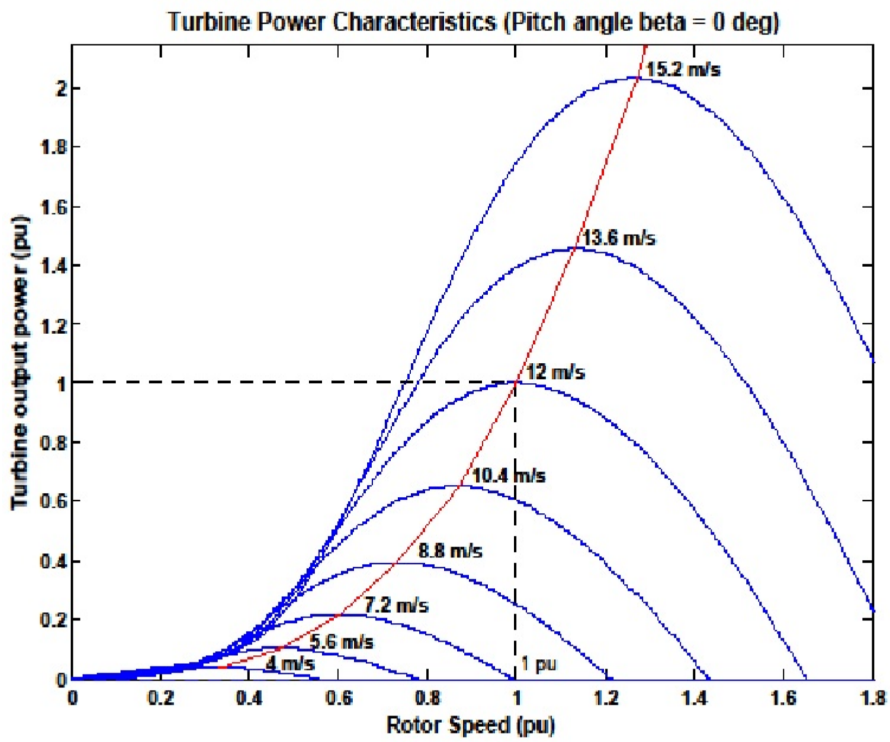

Figure 6.7: Turbine Output Versus Rotor Speed

The converter switching frequency is very high, so it is not considered in converter dynamics. Also, generally the DC link capacitor dynamic is not considered, but here it is considered. To model the converter, first the relationship between AC and DC quantities are established as shown in figure (6.8).These are first presented in actual unit and then converted in to per unit [pu] [365].

$$V_{LN_{RMS}} = \left(\frac{1}{2\sqrt{2}}\right) mV_{DC} \tag{6.52}$$

Where,
$V_{LN_{RMS}}$=Line-to-Neutral RMS AC Voltage in Volts [V]
m_a = Amplitude Modulation Ratio
V_{DC}=DC Voltage in Volts [V]

The power balance in Volt-Ampere [VA] between AC and DC side is given in equation (6.53).

$$3V_{LN_{RMS}} I_{RMS} = V_{DC} I_{DC} \tag{6.53}$$

Where,
I_{RMS}=Line RMS AC Current in Ampere [A]
I_{DC}=DC Current in Ampere [A]

Substitution of equation (6.52) in equation (6.53) gives the following relationship between DC current and AC current as shown in figure (6.8).

$$I_{DC} = \left(\frac{3}{2\sqrt{2}}\right) m_a I_{RMS} \tag{6.54}$$

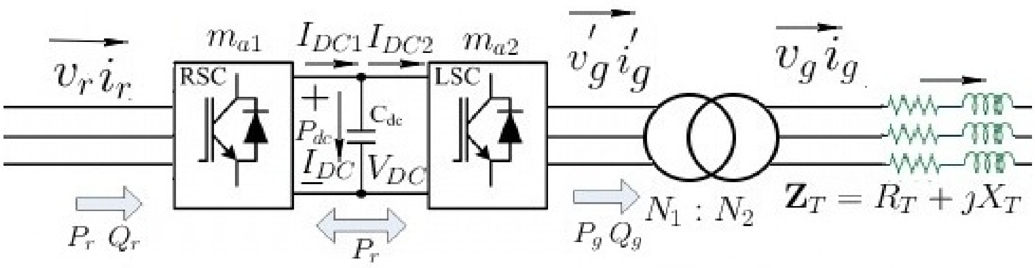

Figure 6.8: AC-DC Relationship of PWM-VSC

All DFIG equations are derived in per unit [pu], so the DC voltage V_{DC} and DC current I_{DC} are expressed in per unit [pu] as given below in equations (6.55) and (6.56).

$$V_{dc} = \frac{V_{DC}}{2\sqrt{2}V_{base_{1Ph}}} \tag{6.55}$$

$$I_{dc} = 3\frac{I_{DC}}{2\sqrt{2}I_{base_{1Ph}}} \tag{6.56}$$

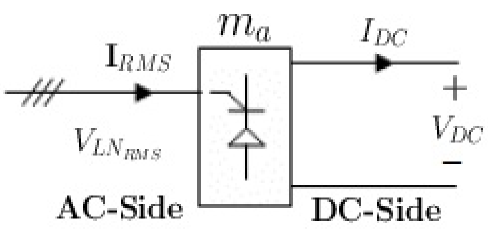

Figure 6.9: AC-DC Relationship of PWM-VSC

Where,
V_{dc}=DC Voltage in per unit [pu]
I_{dc}=DC Current in per unit [pu]

Now all quantities, including DFIG stator, DFIG rotor and DC link, are expressed in per unit [pu]. As shown in figure (6.5), following relations are established.

$$V_r = m_{a1} V_{dc} \tag{6.57}$$

$$V_g = m_{a2} V_{dc} \tag{6.58}$$

$$I_{dc1} = m_{a1} I_r \tag{6.59}$$

$$I_{dc2} = m_{a2} I_g \tag{6.60}$$

The dynamic of DC link capacitor is given by,

$$I_{dc} = I_{dc1} - I_{dc2} = C\frac{dV_{dc}}{dt} \tag{6.61}$$

Equation (6.61) can be modified using equations (6.57) - (6.60). The modified expression is given in equation (6.62).

$$I_{dc} = \frac{1}{V_{dc}}(V_r I_r - V_g I_g) = C\frac{dV_{dc}}{dt} \tag{6.62}$$

The GSC is isolated from grid side by transformer with turns ratio $N_1 : N_2$ and impedance $Z_T = R_T + \jmath X_T$. All quantities are expressed in per unit [pu], so the turns ratio of transformer does not appear in equation. The dynamic of DC power is given by following differential equation.

$$C\frac{dV_{dc}}{dt} = \frac{1}{V_{dc}}\left((v_{qr}i_{qr} + v_{dr}i_{dr}) - (v_{qg}i_{qg} + v_{dg}i_{dg})\right) \tag{6.63}$$

Assumption of constant DC voltage leads $\frac{dV_{dc}}{dt}$ to zero. Under this condition, the active power consumption by DC capacitor is zero and real power of RSC and GSC is equal and equation (6.63) equals to zero.

6.2.7 Interface of DFIG with Grid

DFIG converts wind power in to electrical power. This is transmitted to grid. Now it is required to establish interface relationship between DFIG and Grid. It is done by including the equations of active power and reactive power between DFIG to grid. The procedure is given below.

6.2.7.1 $d-q$ to $D-Q$ Transformation

All the equations of DFIG are derived in *d-q* axis, which is an internal machine axis. The *d-q* axis is aligned to the terminal bus voltage of phase *a*. The active power and reactive power of DFIG are fed the grid through transmission line. One point worth noting is that, there is angle between grid voltage and terminal voltage. Here, grid voltage is taken as a reference, so the angle of grid voltage is taken as zero. Now it is required to transform either grid reference frame (*D-Q*) to DFIG reference frame (*d-q*) or DFIG reference frame (*d-q*) to grid reference (*D-Q*). So, new reference frame *D-Q* is used. The *Q* axis is aligned with the grid voltage.

The relationship between $D-Q$ and $d-q$ reference frame has been given here. The angle between two reference frames are separated by an angle (δ), which is also known as load angle. The relationship between $D-Q$ and $d-q$ reference frame are given below.

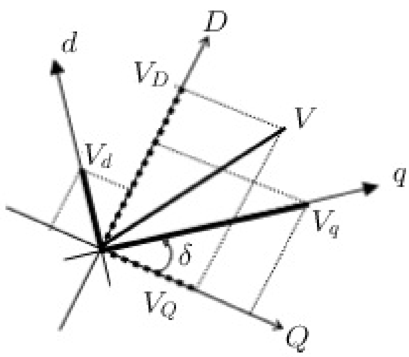

Figure 6.10: Orientation of DFIG Machine $d-q$ Frame and Slack Bus $D-Q$ Frame

$$v_Q = v_q \cos\delta - v_d \sin\delta \tag{6.64}$$

$$v_D = v_q \sin\delta + v_d \cos\delta \tag{6.65}$$

$$v_q = v_Q \cos\delta + v_D \sin\delta \tag{6.66}$$

$$v_d = -v_Q \sin\delta + v_D \cos\delta \tag{6.67}$$

6.2.7.2 Power Flow between DFIG and Grid

As shown in figure (6.5), the power flows from DFIG to grid through transmission line with finite impedance ($Z_{TL} = R_{TL} + \jmath X_{TL}$). The grid is assumed as an infinite bus with constant voltage (1.0 [pu]) and angle equal to zero. The grid is taken as a reference to derive the power flow. For simplified analysis, the transmission line impedance is considered as purely inductive ($Z_{TL} = jX_{TL}$) and resistance is neglected ($R_{TL} = 0$). The strength of transmission line is expressed in ratio of transmission line reactance to magnetization reactance of DFIG (X_{TL}/X_m). The active and reactive power flow in per unit [pu] is given by equations (6.68)-(6.69).

$$P_{tot} = \left(\frac{\sqrt{v_{qs}^2 + v_{ds}^2} \times V_b}{X_{TL}} \right) \times \cos(\gamma) \tag{6.68}$$

$$Q_{tot} = \left(\frac{v_{qs}^2 + v_{ds}^2}{X_{TL}} \right) - \left(\frac{\sqrt{v_{qs}^2 + v_{ds}^2} \times V_b}{X_{TL}} \right) \times \sin(\gamma) \tag{6.69}$$

Where, γ is the angle between DFIG terminal voltage with respect to reference bus voltage, which is grid or infinite bus voltage in our case. For multi-machine system, the slack bus is taken as reference bus and equations (6.68) -(6.69) are replaced by Power Flow Equations at DFIG terminal.

6.2.7.3 DFIG Internal Power Flow

Power flow equations inside DFIG are given by following equations (refer figure (6.5).

$$P_{tot} = P_s + P_g - P_T \tag{6.70}$$

$$P_{tot} = P_s + P_g - ((v_{qg} - v_{qs})i_{qg} + (v_{dg} - v_{ds})i_{sg}) \tag{6.71}$$

$$P_g = v_{qg}i_{qg} + v_{ds}i_{sg} \tag{6.72}$$

$$P_T = ((v_{qg} - v_{qs})i_{qg} + (v_{dg} - v_{ds})i_{sg}) \tag{6.73}$$

$$P_g = P_r - P_{dc} \tag{6.74}$$

$$P_{dc} = P_r - P_g \tag{6.75}$$

$$P_{dc} = (v_{qr}i_{qr} + v_{dr}i_{dr}) - (v_{qg}i_{qg} + v_{dg}i_{dg}) \tag{6.76}$$

$$P_{tot} = P_s + P_r - P_{dc} - ((v_{qg} - v_{qs})i_{qg} + (v_{dg} - v_{ds})i_{sg}) \tag{6.77}$$

$$0 = -P_{tot} - P_s + P_r - P_{dc} - ((v_{qg} - v_{qs})i_{qg} + (v_{dg} - v_{ds})i_{sg}) \tag{6.78}$$

$$Q_{tot} = Q_s + Q_g - Q_T \tag{6.79}$$

$$Q_s = (v_{ds}i_{qs} - v_{qs}i_{ds}) \tag{6.80}$$

$$Q_g = (v_{dg}i_{qg} - v_{qg}i_{dg}) \tag{6.81}$$

$$Q_T = ((v_{dg} - v_{ds})i_{qg} - (v_{qg} - v_{qs})i_{dg}) \tag{6.82}$$

$$Q_{tot} = Q_s + Q_g - ((v_{dg} - v_{ds})i_{qg} - (v_{qg} - v_{qs})i_{dg}) \tag{6.83}$$

$$0 = -Q_{tot} + Q_s + Q_g - ((v_{dg} - v_{ds})i_{qg} - (v_{qg} - v_{qs})i_{dg}) \tag{6.84}$$

The voltages of grid side converter are derived from supply voltage and grid converter current as given below.

$$v_{qdg} = v_{qds} \ + \ i_{qdg} \ Z_T \tag{6.85}$$

The equation (6.85) can be simplified in to q and d axis voltage as given below in equations (6.86) and (6.87).

$$v_{qg} = v_{qs} \ + \ (i_{qg}R_T \ - \ i_{dg}X_T) \tag{6.86}$$

$$v_{dg} = v_{ds} \ + \ (i_{dg}R_T \ + \ i_{qg}X_T) \tag{6.87}$$

Where,
P_{tot} and Q_{tot} are total active and reactive power as given in equations (6.68)-(6.69)
P_s and P_r are stator active power and rotor active power as given in equations (6.37)-(6.38)

P_g is real power of GSC
P_T is real power loss in GSC Transformer
P_{dc} is the power flow in a DC link capacitor during transient condition, is given by equation (6.63) multiplied by V_{dc}
Q_s is the reactive power of stator. It is equal to imaginary part of $v_{qds}i^*_{qds}$
Q_g is reactive power of GSC
Q_T is reactive power loss in GSC Transformer

Total current flow (i_{qdt}) to the grid from DFIG is sum of stator current i_{qds} and GSC current i_{qdg} ($i_{qdt} = i_{qds} + i_{qdg}$). But, these currents are in DFIG machine reference frame. These currents are first transformed in to grid reference frame to get the total current, which can be equated with grid voltages. The transformation is already shown in section (6.2.7.1). The total power flow from DFIG machine to the grid in machine reference frame ($d - q$) and grid reference frame ($D - Q$) is given below by relations given in equations (6.64) to (6.64).

$$S_{tot} = P_{tot} + \jmath Q_{tot} = v_{qds}\, i^*_{qdt} = v_{QDs}\, i^*_{QDt} \tag{6.88}$$

$$i_{Qt} = i_{qt}\ \cos\delta - i_{dt}\ \sin\delta \tag{6.89}$$

$$i_{Dt} = i_{dt}\ \cos\delta + i_{qt}\ \sin\delta \tag{6.90}$$

$$i_{qt} = i_{qs} + i_{qg} \tag{6.91}$$

$$i_{dt} = i_{ds} + i_{dg} \tag{6.92}$$

$$i_{Qt} = (i_{qs} + i_{qg})\ \cos\delta - (i_{ds} + i_{dg})\ \sin\delta \tag{6.93}$$

$$i_{Dt} = (i_{ds} + i_{dg})\ \cos\delta\ +\ (i_{qs} + i_{qg})\ \sin\delta \tag{6.94}$$

$$v_{Qs} = v_{qs}\ \cos\delta\ -\ v_{ds}\ \sin\delta \tag{6.95}$$

$$v_{Ds} = v_{ds}\ \cos\delta\ +\ v_{qs}\ \sin\delta \tag{6.96}$$

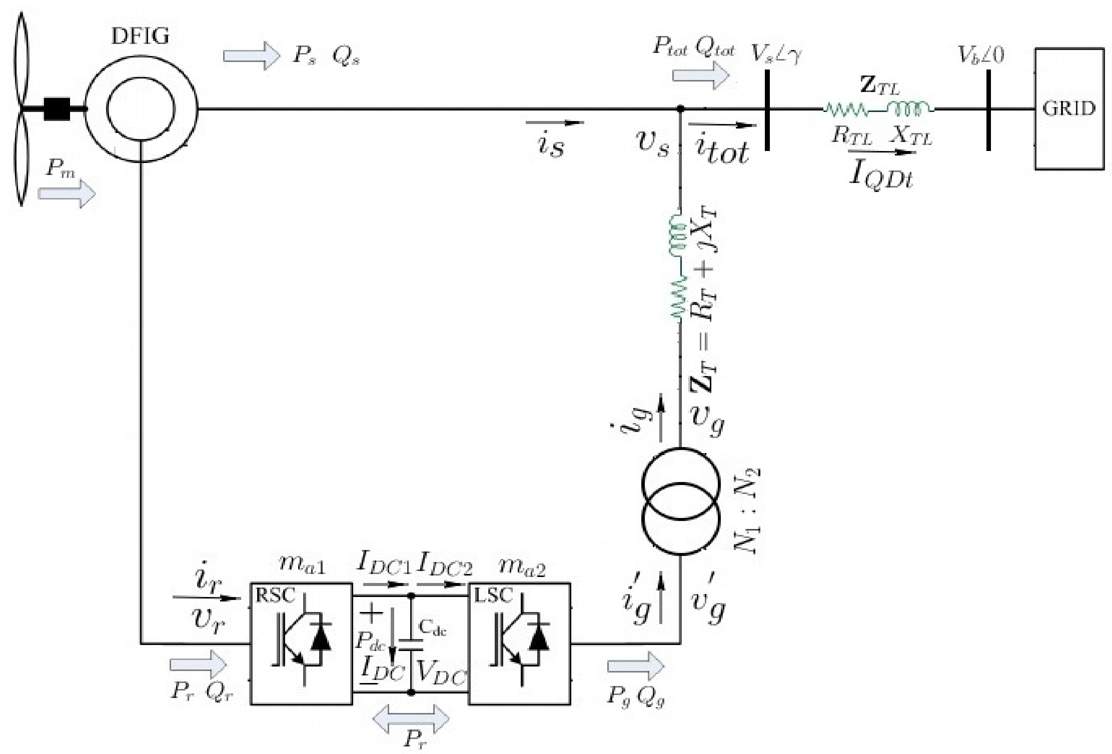

Figure 6.11: Power Flow of Grid Connected DFIG

6.2.8 System Initialization

For system study, first the system parameter initialized to establish the initial condition. System initialization starts with grouping of set of equations in Differential Algebraic Equations (DAE), Algebric Equations (AE) and output variables. The form of DAE, AE and output variable take the following form.

$$\frac{dx}{dt} = f(x, z, u) \tag{6.97}$$

$$0 = g(x, z, u) \tag{6.98}$$

$$y = h(x, z, u) \tag{6.99}$$

Where,
x = State Variables
z = Algebric Variables
u = Control Variables
y = Output Variables
f = State Space Equations
g = Algebric Equations
h = Output Algebric Equations

System initialization is carried out in several steps. First, the load flow study is carried out. To carry out the load flow, the type of DFIG bus is decided. If DFIG is operated in voltage controlled mode, the voltage level is defined in advance and reactive power is calculated, so the DFIG bus is treated PV Bus. If DFIG is operated reactive power control mode, DFIG is treated as PQ bus and voltage at DFIG bus is found out using load flow.

The DFIG machine is connected to grid through transmission line. The grid reference axis (Q - axis) is aligned with the slack bus or infinite bus ($V_b\angle 0$). The machine axis (q - axis) is aligned with the terminal voltage of DFIG ($V_s\angle\gamma$). This will give the following relations of $q - d$ voltages and load angle δ. In case of PV bus, bus voltage is specified. This voltage along with specified power (P_{tot}), gives the reactive power Q_{tot} and angle γ.

$$v_{qs} = V_s \tag{6.100}$$

$$v_{ds} = 0 \tag{6.101}$$

$$\delta = \gamma \tag{6.102}$$

The angle (δ) is the internal angle between d axis and DFIG terminal voltage (V_s). And, the angle (γ) is the angle between terminal voltage of DFIG ($V_s\angle\gamma$) and slack bus or infinite bus ($V_b\angle 0$).

The DFIG is a variable speed Wind Turbine and operates at variable speed, depends on wind power and other variable like blade pitch angle (β), tip to speed ratio (λ) and coefficient of power (C_p). The speed is set with specified speed reference (s^*). The reference speed (s^*) is derived from total power (P_{tot}). If, total power (P_{tot}) is greater than or equal to the nominal power rated power (P_{nom}), the speed is set to nominal speed (ω_{nom}). In case of total power is less than the nominal power rated power, speed is set to optimum value of speed for maximum power extraction by selecting optimal value of C_p , which is a function of blade pitch angle (β) and tip to speed ratio (λ) ($C_p = f(\beta, \lambda)$).

$$s^* = \begin{cases} s_{nom} & \text{if } P_{tot} \geq P_{nom} \\ s(C_p) & \text{if } P_{tot} < P_{nom},\, C_p = f(\beta, \lambda) \end{cases} \tag{6.103}$$

The DC link voltage V_{dc} is an important parameter and is regulated by GSC. The reference for DC link voltage (V_{dc}^*) is set according to one of following methods, either by equation (6.105) or (6.106).

$$V_{dc}^* = \frac{V_{r_{ref}}}{m_{a1_{ref}}} \tag{6.104}$$

$$V_{r_{ref}} = \frac{V_{s_{rated}}}{n} \tag{6.105}$$

$$V_{dc}^* = \frac{V_{DC_{ref}}}{2\sqrt{2}V_{base_{1ph}}} \tag{6.106}$$

Where,

$V_{r_{ref}}$ = DFIG Rotor Reference Voltage in pere unit [pu]

$V_{DC_{ref}}$ = DFIG DC link reference Voltage in Volts [V], generally 1200 - 1400 Volts

$V_{s_{rated}}$ = DFIG Stator rated Voltage in Volts [V], generally 440 - 690 Volts

n = DFIG Stator to Rotor Turns Ratio, generally 1.5

m_{a1}=RSC modulation Index, generally 0.75

The remaining variables are obtained by solving f (equation - 6.97) and g (equation - 6.98) simulataneously.

6.3 Eigenvalue Analysis

The small signal stability analysis is carried out by linearization of state equations 6.27 to 6.30, 6.48 to 6.50 and 6.63. The linearization is achieved by expanding state equations in Taylor's Series form and neglecting higher order terms. The linearized form of state equation is given by,

$$\dot{\Delta x} = A\Delta x + B\Delta u \tag{6.107}$$

$$A = \left[\frac{\partial f}{\partial x} - \frac{\partial f}{\partial z}\left(\frac{\partial g}{\partial z}\right)^{-1}\frac{\partial g}{\partial x}\right]_{(x_0,z_0,g_0)} \tag{6.108}$$

$$B = \left[\frac{\partial f}{\partial u} - \frac{\partial f}{\partial z}\left(\frac{\partial g}{\partial z}\right)^{-1}\frac{\partial g}{\partial u}\right]_{(x_0,z_0,g_0)} \tag{6.109}$$

The A is a state matrix. Eigenvalues of A decides the stability of system. The eigenvalues of matrix may be real or complex conjugate. The real part gives the idea about the stability of system. If real part of eigenvalue is negative, then the system is stable. The postive value indicates the instability. If the real part is zero, then system is said to be on the verge of instability. The damping ratio is an indicator of how well system is damped. Higher the damping ratio, better is the system. It is given by

$$\xi = -\frac{\sigma}{\sqrt{\sigma^2 + \omega^2}} \tag{6.110}$$

Where,
σ = Real part of eigenvalue
ω = Imaginary part of eigenvalue (Rad/Sec)

6.4 Probabilstic Small Signal Stability Analysis

The operating condition of WTG never remains constant in real time scenario. The effect of variation in different parameters on WTG is very necessary to ascertain the stable operation under all conditions. There are many literatures available on the small signal stability analysis of WTG [254, 255, 256, 257, 258, 262, 264, 266, 267, 269]. However, there are only limited literature available on the probabilistic analysis of small signal stability. In [259], the probabilistic small signal stability is carried out using Monte Carlo Simulation (MCS). In [261], the probabilistic small signal analysis is presented with correlated wind sources. To explore the property of correlation, the cupola theory is used. Utlimately MCS is used to perform the small signal stability analysis. In [366], the second order polynomial is presented to understand the relationship between the wind generation and damping of any mode. This method is, however, very complex and also it does only considers only wind power variation and no other variable. All these literatures are time consuming or too computationally intensive. Also, none of the literature have performed the multivariable probabilistic analysis in a very simplified way.

Here, a new approach using Latin Hypercube Sampling (LHS), is presented. It has been already tested in the chapter of Voltage Stability. This is based on the sampling theorem, called Latin Hypercube Sampling (LHS). LHS has already been discussed in the chapter 2 of this thesis, so detail description is omitted here. In the analysis, three parameters variation are considered. These parameters are Wind Speed (i.e. Wind Power), Grid Voltage and Grid Reactance (expressed in term of ratio of grid reactance X_e to magnetization reactance X_m). The distribution and variation of parameters are given as under.

Table 6.1: Paramerter Variation and Distribution

Sr.No	Variable	Distribution	Characteristic Parameter A	Characteristic Parameter B
1	Wind Speed	Weibull	$c = 8.5$ m/s	$k = 2.0$
2	Voltage (V_b)	Normal	$\mu = 1.0$	$\sigma = 0.05$
3	Reactance (X_e/X_m)	Normal	$\mu = 0.055$	$\sigma = 0.03$

The step by step procedure of carrying out analysis is given here.

1. Select variables and define PDF distribution and characteristics parameters for each variable.
2. Decide sample size.
3. Find combination of variables using Latin Hypercube Sampling - Median method (LHS-Median).
4. Decide Tip - Speed Ratio (λ)
5. Find Power Coefficient (C_p) from C_p - λ curve
6. Compute the Wind Power from Wind Speed samples
7. Find the WTG speed from Turbine Output Vs Rotor Speed Curve
8. Find initial conditions of DFIG.
9. Find eigenvalues of state matrix for different combination of variables.
10. Find statistical properties using eigenvalues.

6.5 System Parameters

The DFIG parameters and system parameters, used in this work, are given here.

Table 6.2: Paramerter Variation and Distribution

Sr.No	Parameter	Value	Unit	Description
1	ω_s	1	[pu]	Synchronous Speed
2	Ω_B	2 π 50	[rad/s]	Base Speed
3	V_b	0.9 to 1.1	[pu]	Grid Voltage
4	H_T	3	[sec]	Turbine Inertia
5	H_G	0.5	[sec]	Generator Inertia
6	H_{Tot}	3.5	[sec]	Total Inertia
7	K	10	[pu/rad]	Stiffness Constant
8	D	0.01	[pu-sec/rad]	Damping Constant
9	R_s	0.005	[pu]	Stator Resistance
9	R_r	1.1 $\times$ R_s	[pu]	Rotor Resistance
10	X_m	450 $\times$ R_s	[pu]	Magnetization Reactance
11	L_m	$X_m/\ \omega_s$	[pu]	Magnetization Inductance
12	L_{ss}	1.01 $\times$ L_m	[pu]	Stator Inductance
13	L_{rr}	1.005 $\times$ L_m	[pu]	Rotor Inductance
14	n	1.5	[–]	Stotor to Rotor Turns Ratio
15	X_T	0.5	[pu]	Converter Transformer Reactance
16	C	0.0001	[pu]	DC Link Capacitance
17	X_e/X_m	0.01 to 0.1	[pu]	Grid to Magnetization Inductance ratio
18	V_B	690	[Volt]	Base Voltage
19	S_B	2.0	[MVA]	Base MVA
20	P	4	[no]	Pair of Pole

6.6 Results and Discussions

The result of eigenvalue analysis is given here.

Table 6.3: Statistical Analysis of Eigenvalue

Eigenvalue (mode)	Mean (σ)	Standard Deviation (σ)	Mean (ω)	Standard Deviation (ω)
λ_1	-0.14	0.42	0	0
λ_2	-78.41	40.42	1473.96	630.45
λ_3	-78.41	40.42	-1473.96	630.45
λ_4	-302.56	$5.992\ e^{-14}$	0	0
λ_5	-10.64	8.76	137.78	81.49
λ_6	-10.64	8.76	-137.78	81.49
λ_7	-12.11	$1.87\ e^{-15}$	0	0
λ_8	0	0	0	0

Table 6.4: Statistical Analysis of Eigenvalue

Eigenvalue (mode)	Mean (ξ)	Standard Deviation (ξ)	Participation
λ_1	1	0	V_{DC}
λ_2, λ_3	0.070	0.047	i_{ds}, i_{qs}
λ_4	1	0	ω_t, s, θ_t
λ_5, λ_6	0.159	0.208	v_d', v_q'
λ_7	1	0	ω_t, s, θ_t
λ_8	1	0	ω_t, s

In above results, four modes are found significant, which can affect the system operation. However, these modes are oscillatory but stable. As, the damping of mode-2,3 (λ_2, λ_3) are insignificant, it will take longer time to vanish. Also, the oscillation frequency will be very high (1474 rad/sec, 234.7 Hz). The modes-5,6 (λ_5, λ_6) are also oscillatory, but the damping is very high. The oscillation frequency of these modes is (138 rad/sec, 22 Hz). The other three modes ((λ_1, λ_4 and λ_7) are real in nature and decay exponentially. The mode-1 (λ_1) associated with DC link voltage (V_{DC}) will decay slowly without oscillations.

When the variation in the value of oscillatory modes is checked, it is found that, the possibility of mode-2,3 (λ_2, λ_3) go unstable is 6-7 %. This is calculated from the numbers of positive numbers in randomly generated numbers, with given mean (μ) and standard deviation (λ). The possibility of instability of mode-5,6 (λ_5, λ_6) is 18 - 20 %. It is high and can affect the stability of DFIG. So, its variation should be reduced by designing effective controller.

6.7 Conclusion

The probabilistic small signal stability analysis is essential to ascertain the stability of WTG under uncertain wind speed and other operating conditions. It gives insight in to how WTG will behave under different operating conditions. It is necessarily should be carried out at the planning stage. There are various methods found in literatures, but Latin Hypercube Sampling (LHS) is an easy and less computational intensive method. Also, it should work well with different optimization algorithm due to its simplicity. The controller dynamics is not considered here to see the natural response of DFIG. However, the controller design using proposed method can be carried out to make system operation more stable.