Chapter 3

2DOF controller and Heat Exchanger System Description

This chapter covers theory of conventional 1DOF control system in detail, drawback of 1DOF controller, 2DOF control structure, its design, and equivalent transformation of 2DOF control structure. The next section contains heat exchanger system description of bench mark problem selected for the 2DOF controller parameter optimization. The types of heat exchangers available in the market, basic functional block diagram with 2DOF control structure, and derivation of transfer function of each block is provided in the description. Based on this, transfer functions using superposition principle is derived which is used for optimization of 2DOF control parameters in the programming.

3.1 Introduction

In control system, the number of variables required to describe the process is called the design degrees of freedom [41] and the number of closed-loop transfer functions that can be adjusted independently is called as control degree of freedom [42]. The drawback of 1DOF (One Degree of Freedom) control structure is that, it can't optimize both the objective functions i.e. set point tracking and disturbance rejections simultaneously. Due to this reason two separate optimal tuning parameters for set point tracking and disturbance rejections are proposed in the literature. If we try to control simultaneously two control system objectives i.e. set point tracking and disturbance rejection then it results in a structure of 2DOF (Two-Degree-of-Freedom) control system [43]. Though, this fact was not attracted researchers for a long time and two decades after Horowitz's work in 1984 researchers started to exploit advantages of 2DOF control structure over conventional PID controller [44].

3.2 Conventional 1DOF feedback control system

The conventional feedback control system having 1DOF structure is shown in following Figure 3.1. Where 'r' is set point, 'e' is error between set point and process variable, 'u' is controller output, 'd' is disturbance input, 'y' is process variable, C(s) is controller , P(s) is process or plant and H(s) feedback gain. Following two assumptions are introduced for simplicity, which is appropriate for many practical design problems with some exceptions [3].

Assumption 1: The detector is accurate, quick in response and noise free, i.e. H(s) = 1.

Assumption 2: The main disturbance enters at the manipulating point, i.e. $P_d(s) =$ P(s).

The responses of the controlled variable 'y' to the unit change of the set point variable 'r', and to the unit step disturbance 'd' are called 'set-point response' and 'disturbance response' respectively. They are used as measures of the performance in tuning the PID controller. The closed-loop transfer functions of this control system from the set-point variable 'r', to the controlled variable 'y' and that from the disturbance 'd' to 'y' are $G_{(yr1)}(s)$ and $G_{(yd1)}(s)$ respectively. The subscript '1' represents that it's of 1DOF control structure. Transfer functions $G_{(yr1)}(s)$, and $G_{(yd1)}(s)$ of 1DOF structure is derived considering following two cases [3].

Case 1: Transfer function $G_{yr1}(s)$, assuming zero disturbance d= 0.

Set point response transfer function derived from Figure 3.1, assuming zero disturbance input is given by following equation.

$$G_{yr1}(s) = \frac{y}{r} \tag{3.1}$$

from above Figure 3.1,

$$y = (r - y * H(s)) * C(s) * P(s) \tag{3.2}$$

$$y[1 + C(s) * P(s) * H(s)] = r * P(s) * C(s) \tag{3.3}$$

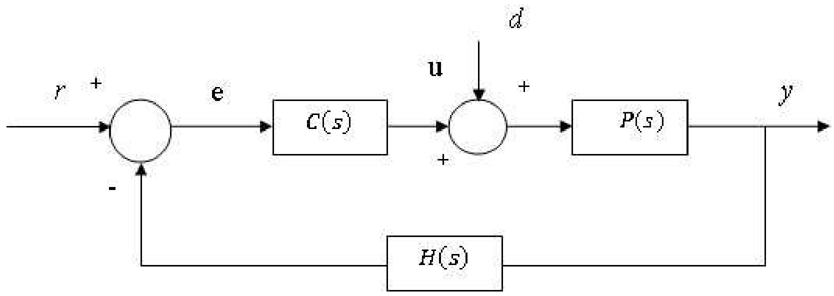

Figure 3.1: Conventional 1DOF control system

Rearranging above equations 3.2 & 3.3 to derive transfer function $G_{yr1}(s)$ shown in following equation 3.4.

$$\frac{y}{r} = \frac{P(s) * C(s)}{1 + P(s) * C(s) * H(s)} \tag{3.4}$$

Case 2: Transfer function $G_{yd1}(s)$, assuming zero reference input r= 0. Here, disturbance response transfer function derived from Figure 3.1, assuming zero reference input is given by following equation.

$$G_{yd1}(s) = \frac{y}{d} \tag{3.5}$$

from above Figure 3.1,

$$y = (d - y * H(s) * C(s)) * P(s) \tag{3.6}$$

$$y[1 + P(s) * C(s) * H(s)] = d * P(s) \tag{3.7}$$

Rearranging above equations 3.6 & 3.7 to derive transfer function $G_{yd1}(s)$ shown in following equation 3.8.

$$\frac{y}{d} = \frac{P(s)}{1 + P(s) * C(s) * H(s)} \tag{3.8}$$

Now, multiplying $G_{yr1}(s)$ by P(s) and adding with $G_{yd1}(s)$ and H(s)=1.

$$G_{yr1}(s) * P(s) + G_{yd1}(s) = \frac{P(s) * P(s) * C(s)}{1 + P(s) * C(s) * H(s)} + \frac{P(s)}{1 + P(s) * C(s) * H(s)} \tag{3.9}$$

$$G_{yr1}(s)*P(s)+G_{yd1}(s) = P(s)*[\frac{P(s) * C(s)}{1 + P(s) * C(s) * H(s)}+\frac{1}{1 + P(s) * C(s) * H(s)}] \tag{3.10}$$

$$G_{yr1}(s) * P(s) + G_{yd1}(s) = P(s) \tag{3.11}$$

The above equation 3.11 shows that for a given P(s), $G_{yr1}(s)$ is determined if $G_{yd1}(s)$ is chosen, and vice versa. This causes the difficulty that, if the set-point response is optimized, the disturbance response is often found to be poor, and vice versa. Due to this, the classical researches on the optimal tuning of PID controllers gave two separate tables i.e. the "disturbance optimal" parameters, and the "set point optimal "parameters [45], [46].

3.3 Two degree of freedom(2DOF) control system

The Figure 3.2 is a general form of the 2DOF control system. Here, controller consists of two compensator C(s) and $C_f(s)$. The transfer function from the disturbance 'd' to the controlled variable 'y' is $P_d(s)$, this is different from the transfer function P(s) from the manipulated variable 'u' to 'y' . Where, C(s) is called the serial (or main) compensator and $C_f(s)$ the feed forward compensator. The closed-loop transfer functions from 'r' to 'y' and 'd' to 'y' are $G_{yr2}(s)$ and $G_{yd2}(s)$ respectively. The subscript "2" means that the quantities are of the 2DOF control system. Consider following two cases for structure of 2DOF controller to derive transfer function for $G_{yr2}(s)$ and $G_{yd2}(s)$ [3].

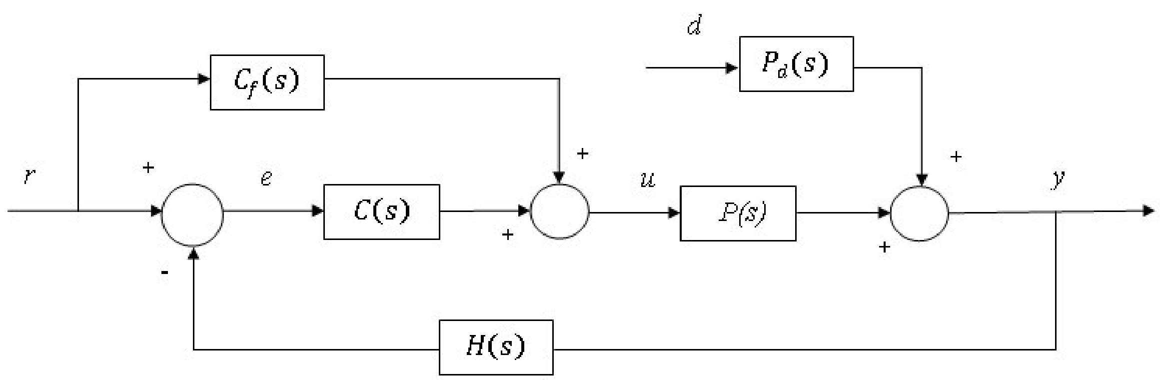

Figure 3.2: Conventional 2DOF control system

Case 1: Transfer function $G_{yr2}(s)$, assuming d = 0.

The Figure 3.3 is a general form of the 2DOF control system with zero disturbance input (only reference input is present). Set point response transfer function $G_{yr2}(s)$ is

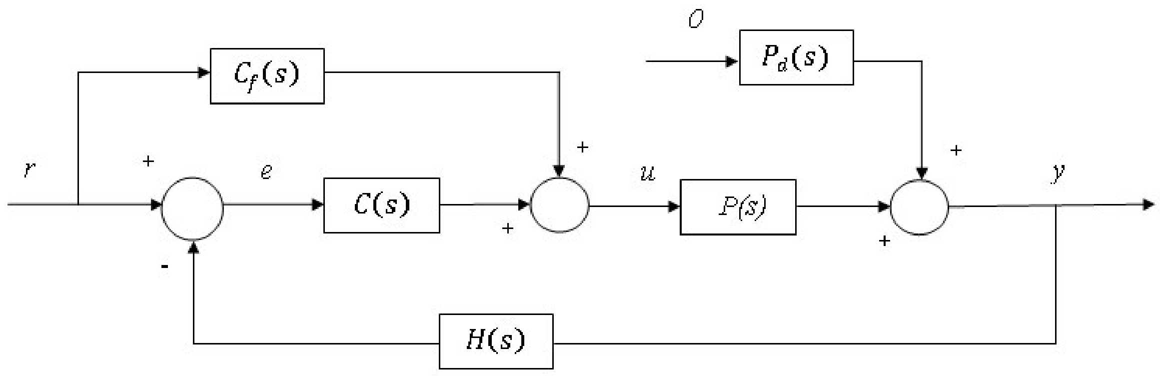

Figure 3.3: 2DOF Control System with reference input only

derived as under.

$$e = r - y * H(s) \tag{3.12}$$

$$U = e * C(s) + r * C_f(s) \tag{3.13}$$

$$y = U * P(s) \tag{3.14}$$

Now, Substituting values of equations 3.12 & 3.13 in 3.14 and then manipulating equations as under.

$$y = [((r - y * H(s)) * C(s)) + r * C_f(s)] * P(s) \tag{3.15}$$

$$y = [(r * C(s) - y * H(s) * C(s)) + r * C_f(s)] * P(s) \tag{3.16}$$

$$y = [((C(s) + C_f(s)) * r) - y * H(s) * C(s)] * P(s) \tag{3.17}$$

$$y + y * H(s) * C(s) * P(s) = r * P(s)[C(s) + C_f(s)] \tag{3.18}$$

Rearranging above equation 3.18, set point response transfer function of 2DOF controller is derived shown in equation 3.19.

$$G_{yr2}(s) = \frac{y}{r} = \frac{P(s)[C(s) + C_f(s)]}{1 + P(s) * C(s) * H(s)} \tag{3.19}$$

Derived steady state error for unit step input assuming zero disturbance for set point response transfer function, $G_{yr2}(s)$ as under.

$$e(s) = r(s) - y(s) \tag{3.20}$$

Substituting value of y(s) in equation 3.20 in terms of r(s).

$$e(s) = r(s) - \frac{P(s)[C(s) + C_f(s)]}{1 + P(s) * C(s) * H(s)} * r(s) \tag{3.21}$$

$$e(s) = r(s)[1 - \frac{P(s)[C(s) + C_f(s)]}{1 + P(s) * C(s) * H(s)}] \tag{3.22}$$

For, step input r(s)=1/s substituting value of r(s) in equation 3.22.

$$e(s) = \frac{1}{s} * [1 - \frac{P(s)[C(s) + C_f(s)]}{1 + P(s) * C(s) * H(s)}] \tag{3.23}$$

Where, steady state error is [3].

$$ess = \lim_{s->0} s * e(s) \tag{3.24}$$

$$Assumethat, \lim_{s->0} H(s) = 1 \tag{3.25}$$

$$ess = \lim_{s->0} s * \frac{1}{s} * [1 - \frac{P(s)[C(s) + C_f(s)]}{1 + P(s) * C(s) * H(s)}] \tag{3.26}$$

By, mathematically manipulating right hand side of equation 3.26 we get steady state error as in the form of equation 3.27.

$$ess = \lim_{s->0} s * \frac{1}{s} * \frac{1 - P(s) * C_f(s)}{1 + P(s) * C(s)} \tag{3.27}$$

Taking, $C_f(s)$ and C(s) common from numerator and denominator part and canceling common terms, we obtain equation 3.27 in the form of 3.28.

$$ess = \lim_{s->0} s * \frac{C_f(s)}{C(s)} * \frac{(1/C_f(s)) - P(s)}{(1/C(s)) + P(s)} \tag{3.28}$$

$\lim_{s->0}(C(s)) = \infty$, $\lim_{s->0}(P(s))$ is not zero, $\lim_{s->0} \frac{C_f(s)}{C(s)} = 0$, Above conditions imposes a constraints on the design of controller and process. The cases that satisfy above conditions are that C(s) include an integral and $C_f(s)$ does not include an integral term. If the detector is not accurate i.e. $\lim_{s->0}(H(s))$ is not equal to 1, then the steady-state error is given by following equations [3].

$$e_{rstep} = ess = \lim_{s->0} s * \frac{1}{s} * [1 - \frac{P(s)[C(s) + C_f(s)]}{1 + P(s) * C(s) * H(s)}] \tag{3.29}$$

$$ess = \lim_{s->0} s * \frac{1}{s} * \frac{1 + P(s) * C(s) * H(s) - P(s) * C(s) - P(s) * C_f(s)}{1 + P(s) * C(s) * H(s)} \tag{3.30}$$

$$ess = \lim_{s->0}[\frac{(1/C(s)) + P(s) * H(s) - P(s) - P(s) * C_f(s)/C(s)}{(1/C(s)) + P(s) * H(s)}] \tag{3.31}$$

$$ess = \lim_{s->0}[\frac{H(0)-1}{H(0)}] \tag{3.32}$$

The above equation gives value of steady state error for unit step input when H(s) not equal to 1.

Case 2: Transfer function, $G_{yd2}(s)$ assuming r = 0.

The Figure 3.4 is a general form of the 2DOF control system with zero reference input (only disturbance input is present). Disturbance response transfer function $G_{yd2}(s)$ is derived as under.

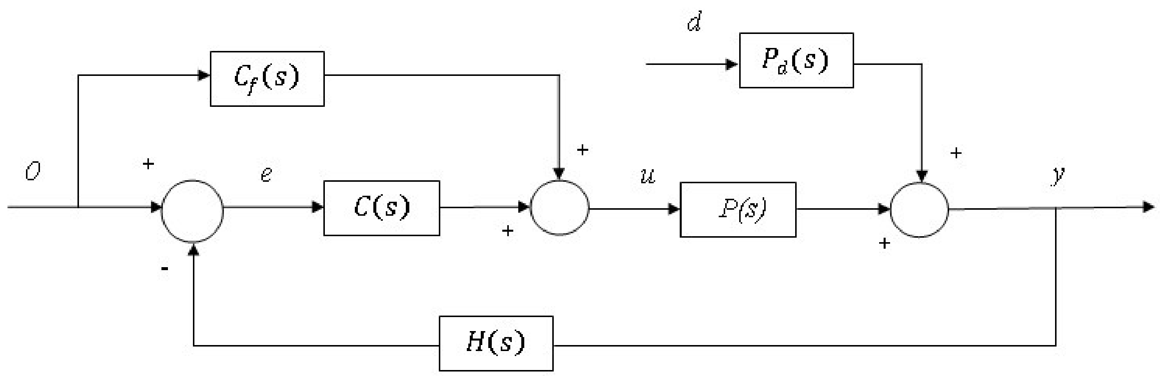

Figure 3.4: 2DOF Control System with disturbance input only

$$y = d * P_d(s) - y * C(s) * H(s) * P(s) \tag{3.33}$$

$$y[1 + C(s) * P(s) * H(s)] = d * P_d(s) \tag{3.34}$$

Rearranging above equation 3.34 to derive 2DOF control disturbance response transfer function shown in equation 3.35.

$$G_{yd2}(s) = \frac{y}{d} = \frac{P_d(s)}{1 + P(s) * C(s) * H(s)} \tag{3.35}$$

Derived Steady state error for unit step disturbance input assuming zero reference input, $G_{yd2}(s)$ as under.

$$e(s) = d(s) - y(s) \tag{3.36}$$

$$e(s) = d(s) - \frac{P_d(s)}{1 + P(s) * C(s) * H(s)} * d(s) \tag{3.37}$$

$$e(s) = d(s)[1 - \frac{P_d(s)}{1 + P(s) * C(s) * H(s)}] \tag{3.38}$$

For, step disturbance input d(s)=1/s substituting value of d(s) in above equation 3.38.

$$e(s) = \frac{1}{s} * [1 - \frac{P_d(s)}{1 + P(s) * C(s) * H(s)}] \tag{3.39}$$

$$e_{dstep} = e(s) = \lim_{s->0} s * e(s) \tag{3.40}$$

$$\lim_{s->0} H(s) = 1 \tag{3.41}$$

$$e_{dstep} = e(s) = \lim_{s->0} s * \frac{1}{s} * [1 - \frac{P_d(s)}{1 + P(s) * C(s) * H(s)}] \tag{3.42}$$

$$\lim_{s->0} \frac{P_d(s)}{P(s)} < \infty \tag{3.43}$$

The above equations put conditions on the plant, the denominator of equation 3.43 requires that P(s) is not of differentiating and the numerator of equation 3.43 requires that the disturbance is not integrated more times than the manipulated variable in order to have $P_d(s) < \infty$. From the mathematical stand point, conditions are nothing but sufficient conditions that make the steady-state errors zero. Considering above conditions C(s) and $C_f(s)$ are derived as under [3].

$$C(s) = [K_p + K_p/(T_i S) + K_p * T_D * D(s)] \tag{3.44}$$

$$C_f(s) = -K_p[\alpha + \beta * T_D * D(s)] \tag{3.45}$$

Where, D(s) is the derivative term. Three parameters of C(s) i.e., the proportional gain K_p , the integral time T_i , and the derivative time T_D , are referred to as basic parameters, and two parameters of $C_f(s)$ i.e., α and β are referred to as "2DOF parameters"[3]. Transforming this controller part of, Figure 3.4 it can be changed equivalently in following four different forms and some of the special cases of PID controller [3].

1. Feed forward type (FF type) of 2DOF Controller.
2. Feedback type (FB type) of 2DOF Controller.
3. Set point filter type of 2DOF Controller.
4. Filter with preceded-derivative type expression of 2DOF Controller.

Various 2DOF PID controllers are proposed for industrial use, its equivalent transformations, special cases of PID controller, and list of 2DOF controller optimal parameters are discussed in [44], [47], [48]. As a consequence, the results obtained were adopted by vendors [49], [50], [51]. Advance studies were done about optimal tuning of 2DOF controller are discussed in [52], [53], [54]. The methods for digital implementation with magnitude and/or slope limiters [54], an anti-reset-windup method in [55], and other industrial applications [56], [57], [47] are also proposed by researchers. Here, feed forward type (FF type) of 2DOF Controller optimization for the problem of shell and tube heat exchanger system is selected because; it is easy to convert the conventional PID controller structure already built in to the FF type of 2DOF structure [3].

3.4 Heat exchanger system description

Heat exchangers are the devices that are used at different temperatures and thermal contact to transfer thermal energy between two or more fluids, a solid surface and a fluid, or between solid particles and a fluid [1]. The varieties of heat exchangers are available in the market like, shell and tube heat exchanger, plate heat exchanger, plate and shell heat exchanger, adiabatic wheel type heat exchanger, and plate fin heat exchanger. Shell and tube type of heat exchanger can withstand higher pressure compared to other types of heat exchangers hence; it is widely used in industry [2], [56]. Shell and tube type of heat exchanger has two predominant disturbances; one is due to flow variation of input fluid and second is due to temperature variation of input fluid. Increase in flow variation of process fluid result in increase in mass flow rate of the fluid causes reduction in mean exit temperature of process fluid. On the contrary, increase in temperature variation of process fluid causes increase in mean exit temperature process fluid. The step input is applied to both the disturbances which are in conflict [58]. The prime goal in the process of heat exchanger is to keep outlet temperature of process fluid flowing through it at desire value in the presence of two major conflicting disturbances. Hence, the problem of shell and tube heat exchanger system is selected as a test bench for multiobjective optimization problem of 2DOF control parameters [59].

The basic block diagram shown in Figure 3.5 consists of shell and tube heat exchanger system with boiler, storage tank, and controller. A boiler is used to generate super heated steam for heating process fluid. The process fluid which is required to be heated is stored in storage tank. Pump and non returning valve are connected with storage tank to supply process fluid to heat exchanger. The fluid in heat exchanger system heats up to a desired temperature using steam supplied from the boiler [60]. Here, a process of heat exchanger system is approximated as First Order Plus Time Delay (FOPDT) system.

The Figure 3.5 shows proposed 2DOF feedback control scheme for the shell and tube type

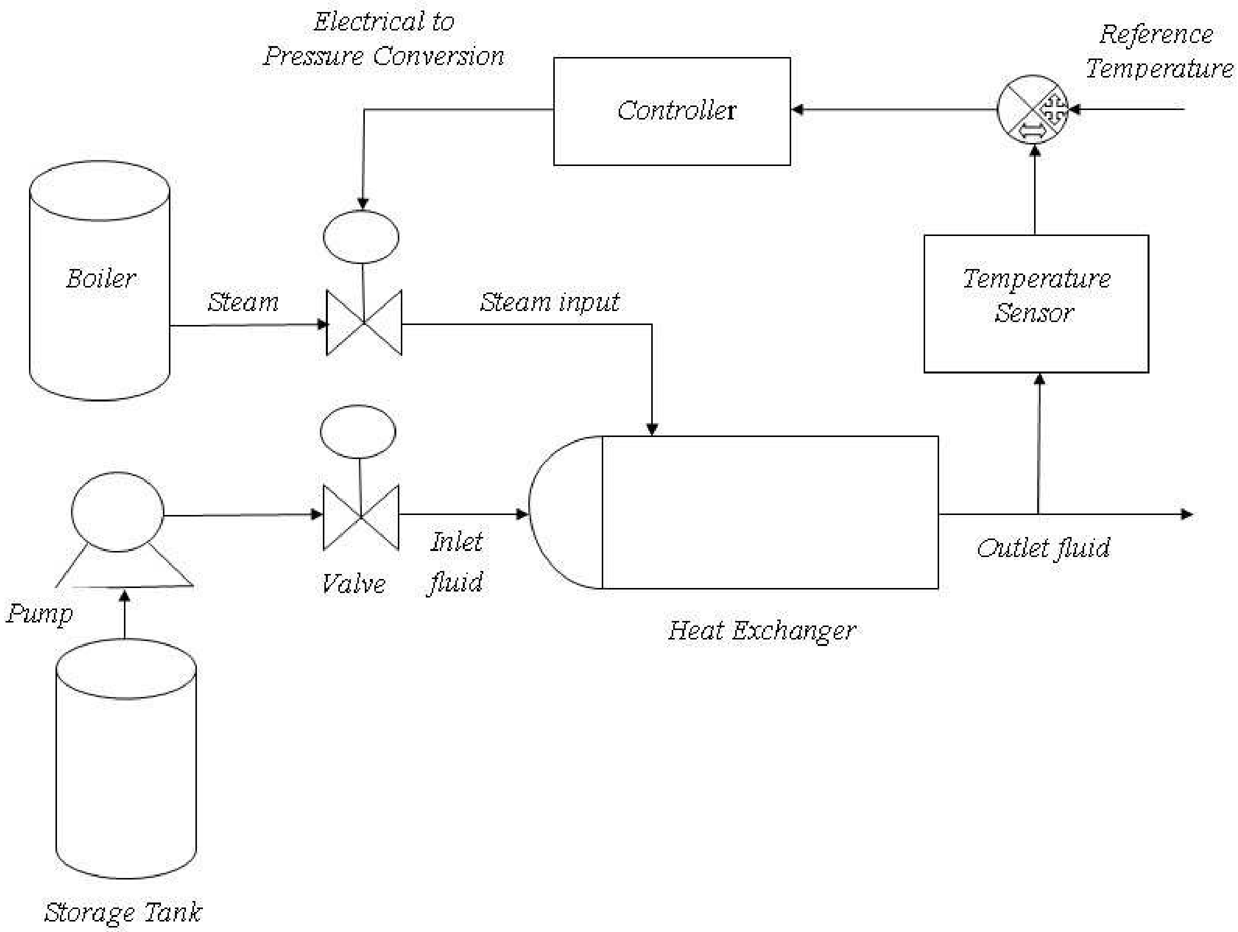

Figure 3.5: Block diagram representation of heat exchanger control system

heat exchanger system. It works as follows: the measured controlled variable (temperature of outlet fluid) is compared with the set point temperature and error difference between two is calculated, which is given as input to controller. On the basis of error input signal of controller, it generates electrical control output signal in the range of 4-20 mA. The electrical control output signal is converted in to pressure signal in the range of 3-15 psig, through an electronumetic device (current to pressure transducer). The output of this pressure signal is connected with the valve actuator whose function is to position a valve in proportion to control signal. Flow of the steam depends on valve position. The temperature of outlet fluid is sensed by temperature sensor as shown in Figure 3.5. Following two types of disturbances are predominant in this process: (1) Flow variation of input fluid. (2) Temperature variation of input fluid. The Flow variation of input fluid is considered to be more predominant disturbance compared to temperature variation in input fluid [59].

Following assumptions have been considered in the system description of heat exchanger system [61].

1. An equal inflow and out flow rate of fluid (kg/sec) is maintained in order to have constant fluid level in heat exchanger system.

2. Heat exchanger systems insulating wall does not store any heat.

Heat exchanger system's response to the steam flow has a gain of $50^0C/(kg/sec)$, time constant of 30 sec, and transport delay of 2 sec. Hence, system plant model is defined as, $G(s) = \frac{50*e^{-2s}}{(30*S+1)}$. Response to the variation of process fluid flow has a gain of $3^0C/(kg/sec)$, and time constant of 30 sec. A gain of $1^0C/(kg/sec)$, and time constant of 3 sec, is considered for heat exchanger response to the variation of process temperature. Therefore, transfer function of flow disturbance of input fluid is derived as $F(s) = \frac{3}{(30*S+1)}$ and transfer function of temperature disturbance of input fluid is $T(s) = \frac{1}{(3*S+1)}$. Control valve transfer function is calculated assuming, maximum capacity of valve 1.6 kg/sec, time constant of 3 sec, and linear characteristics of steam input control valve has the nominal pressure range is 3 to 15 psig . Hence, valve gain is(1.6 kg/sec)/((15-3)psig), this results in control valve transfer function $A(s) = \frac{0.1}{(3*S+1)}$. Considering, sensor has a gain of 1 and time constant of 10 sec, which results in sensor transfer function as,$H(s) = \frac{1}{(10*S+1)}$. The resultant system consisting of heat exchanger with controller and disturbances is shown in following Figure 3.6.

Here, feed forward type 2DOF controller comprising of serial compensator $C_s(s)$ and feed forward compensator $C_f(s)$ are used.

Where, $C_s(s)$ and $C_f(s)$ are represented as under.

$$C_s(s) = [K_p + K_p/(T_iS) + K_p * T_D * D(s)] \tag{3.46}$$

$$C_f(s) = -K_(p)[\alpha + \beta * T_D * D(s)] \tag{3.47}$$

The parameters of serial compensator $C_s(s)$ are known as proportional gain K_p , integral time T_i , and derivative time T_D, they are called as "basic parameters". The parameters of feed forward compensator $C_f(s)$ i.e., α and β are called as "2DOF parameters". Where, D(s) = s/(1+τ s) is approximate derivative [3]. Assume, $D_T(s)$ and $D_f(s)$ are temperature and flow disturbance step inputs respectively. Derived transfer function based on

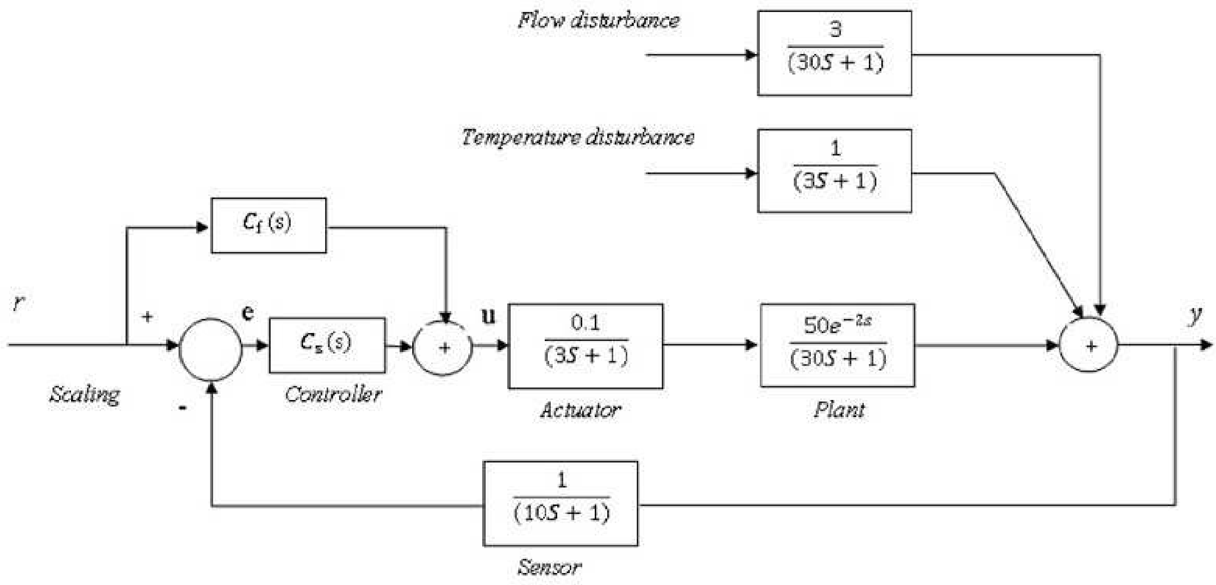

Figure 3.6: Transfer function of heat exchanger with controller and disturbances

superposition principle as under which is used for optimization of 2DOF control parameters in the programming.

Case 1: Reference input r is present and both disturbances flow & temperature are zero.

$$\frac{y(s)}{r(s)} = \frac{C_f(s) + C(s)}{C(s)} * \frac{C(s) * A(s) * G(s)}{1 + C(s) * A(s) * G(s) * H(s)} \tag{3.48}$$

Case 2: Flow disturbance input is present and both temperature disturbance & reference input is zero.

$$\frac{y_{flow}(s)}{D_f(s)} = \frac{F(s)}{1 + C(s) * A(s) * G(s) * H(s)} \tag{3.49}$$

Case 3: Temperature disturbance input is present and both flow disturbance & reference input is zero.

$$\frac{y_{temp}(s)}{D_T(s)} = \frac{T(s)}{1 + C(s) * A(s) * G(s) * H(s)} \tag{3.50}$$

3.5 Conclusion

The present chapter provides detail mathematical analysis starting from conventional 1DOF PID control, 2DOF controller, its equivalent forms, and special cases of 2DOF

Controller, to understand the topic in detail. From the mathematical analysis it has been concluded that naturally 2DOF controller has advantages over the 1DOF controller. Analysis also derives constrains on the design of 2DOF controller, plant and detector. Finally, we conclude that variants of 2DOF controller are nothing but different expressions of the same 2DOF PID controller. The basic block diagram of heat exchanger system with feed forward type 2DOF controller with disturbances are derived. Transfer functions using superposition principle is derived which is used for optimization of 2DOF control parameters in the programming. Theory of multiobjective optimization and method of optimizations using evolutionary and swarm intelligence is discussed in the subsequent chapters.