Chapter 4

MOOP and 2DOF Controller Parameter Optimization Method

This chapter covers difference between single objective and multiobjective optimization, definition used in multiobjective optimization, theory of multiobjective optimization, and ideal approach for multiobjective optimization problem solution. It also covers 2DOF controller parameter optimization method in which following topics are covered: Formation of three objective functions for set point tracking and disturbance rejections, discussion on criteria for evaluation of objective functions, and process applying objective functions for multiobjective optimization problem.

4.1 Introduction

An optimization is a process to find one or more suitable solutions which is the maximum or minimum value of one or more objectives. The need for optimization in a problem arises due to ultimate goal of either maximizing or minimizing objective functions. When a problem of an optimization involves only one objective function, the task of finding the optimal solution is called as single objective optimization. The process of optimizing simultaneously collection of objective functions is called multiobjective optimization. Previously, only gradient and heuristic based search techniques were used for the solution of single objective optimization. In order to increase applicability of optimization algorithms to different problem domains various evolutionary and swarm based optimization

algorithms were developed. Evolutionary and swarm based optimization algorithm mimics natural and physical principles for the solution of optimization problems.
Most of the real world problem comprises multiple objectives and hence, extremist principle can't be applied to only one objective as other objectives are also significant. The presence of multiple objectives in a problem give rise to a set of trade-off optimal solutions known as Pareto-optimal solutions instead of single optimal solution [34]. The result of multiobjective optimization algorithm is a set of Pareto optimal solutions so, it is not possible to find out a unique solution which minimizes or maximizes all objectives simultaneously. Hence, user has to select only one solution based on his/her preference [63].The choice of preferences for the solution is either maximization or minimization of objective function. In recent years, many evolutionary and other swarm intelligence based controller tuning is attracted researchers due to following reasons.
1. Traditional optimization algorithms work on single point while, evolutionary and swarm intelligence based algorithms work with population of points hence, it reduces an effort to run same algorithm many times for obtaining multiple optimal solutions.
2. Evolutionary and swarm intelligence based algorithm processes more than one string simultaneously hence, it is expected that output of this algorithm is global solution instead of local solution. There are some traditional algorithms which are population based (i.e. Box evolutionary optimization & complex search methods) those algorithms do not use previously obtained information efficiently as evolutionary and swarm intelligence.
3. Evolutionary and swarm intelligence do not require any auxiliary information except the fitness value.
4. The traditional algorithms has rigid rules and hence, there is no escape from its local optima.

4.2 Difference between single and multiobjective optimization

Consider a problem of multiobjective optimization has two conflicting objectives. Here, each objective corresponds to a different optimal solution. If we visualize a set of optimal solutions, then a gain in one objective calls for suffer in other objective. Hence, it's

difficult to determine which solution is the best with respect to both objectives. The fact is none of these trade-off solutions is the best solution with respect to both objectives, due to two conflicting objectives. Therefore, a problem consisting of more than one conflicting objectives does not give single optimum solution. There exist a number of solutions which are all optimal. This is the fundamental difference between single and multiobjective optimization.
Following definitions are used in the solution of multiobjective optimization problems.

4.3 Definition: Multiobjective optimization problem formulation

A general multiobjective optimization problem consists of many objective functions to be minimized (or maximized) under constraints. First of all, consider a multiobjective optimization problem consist of optimizing a vector of functions $f_m(X)$. Where, m= 1,2,,M, is the M objective functions can be either maximized or minimized subject to following constraints. Inequality constraints, $g_j(X) \geq 0$. Where, j= 1, 2,,J (It may be $\leq$ type). Equality constraints, $h_k(X) = 0$. Where, k= 1, 2,K Here, X is a solution vector of n decision variable space $(x_1., x_2, .x_n)^T$. The decision variable may be bound within lower and upper bound as $X_i^L \leq X_i \leq X_i^U$.

4.4 Definition: Domination

A solution x^1 is said to be dominated by other solution x^2, if following conditions are satisfied. The solution x^1 is no worse than x^2 in all objectives, or $f_j(x^1) > f_j(x^2)$ for all j= 1, 2, ,M. The solution x^1 is strictly better than x^2 in at least one objective, or $f_j(x^1) > f_j(x^2)$ for at least one j 1, 2,..,M. If any of the above two conditions is violated, solution x^1 does not dominate solution x^2.

4.5 Definition: Nondominated set

Among set of solution P, the nondominated set of solution P are those that are not dominated by any member of the set P.

4.6 Definition: Pareto optimal set

When the set P is entire search space S, or P=S, the resulting nondominated set P' is called Pareto optimal set.

4.7 Definition: Pareto front

Multiobjective optimization problem consists of optimizing a vector of functions $f_m(X)$. Where, m= 1,2,.M, has Pareto optimal set P' then Pareto front is defined as:
$(f_1(x), .f_k(x)|x\epsilon P')$. A representation of Pareto front of two objective functions is shown in Figure 4.1. An objective in multiobjective optimization algorithm is to find a set of solutions near to Pareto optimal front.

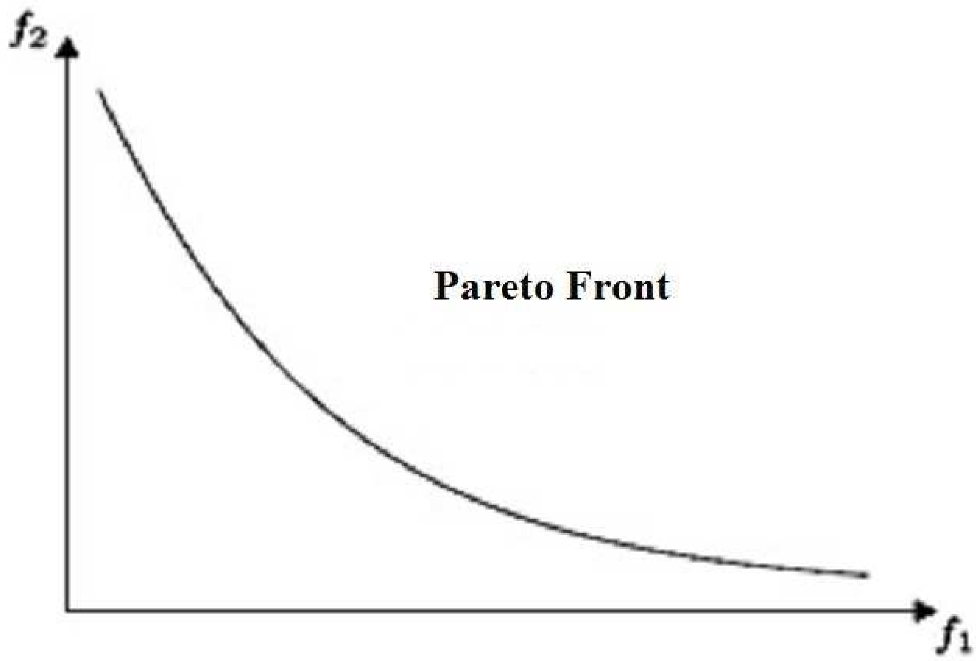

Figure 4.1: Pareto front of two objective functions [62].

4.8 Solution of multiobjective optimization problem

Irrespective of the fact that whether optimization problem is single objective or multiobjective user needs only one optimal solution. In the problem of multiobjective optimization user is now in dilemma that, which one of these solutions to choose out of set of available many trade-off solutions? If set of trade-off solutions are available then user has to select one considering merits and demerits of each of these available solutions. The decision for selection of best solutions from the available set of trade-off solutions required much higher level, qualitative and non-technical information . The main objective in multiobjective optimization is to find a set of trade-off solutions considering all the objectives to be significant. Once, a set of trade-off solutions are obtained, user required to use higher level information's to select best solution. Considering above points, following ideal multiobjective optimization procedure is proposed.

Step1: Obtain multiple trade-off solutions with vast range of values for objectives.

Step 2: Select the best solution considering higher level information.

Here, higher level information is a qualitative information of problem objectives ex. Minimum overshoot, Maximum disturbance rejection. The above procedure is also represented schematically in following Figure 4.2. As shown in above Figure 4.2, step 1 obtains mul-

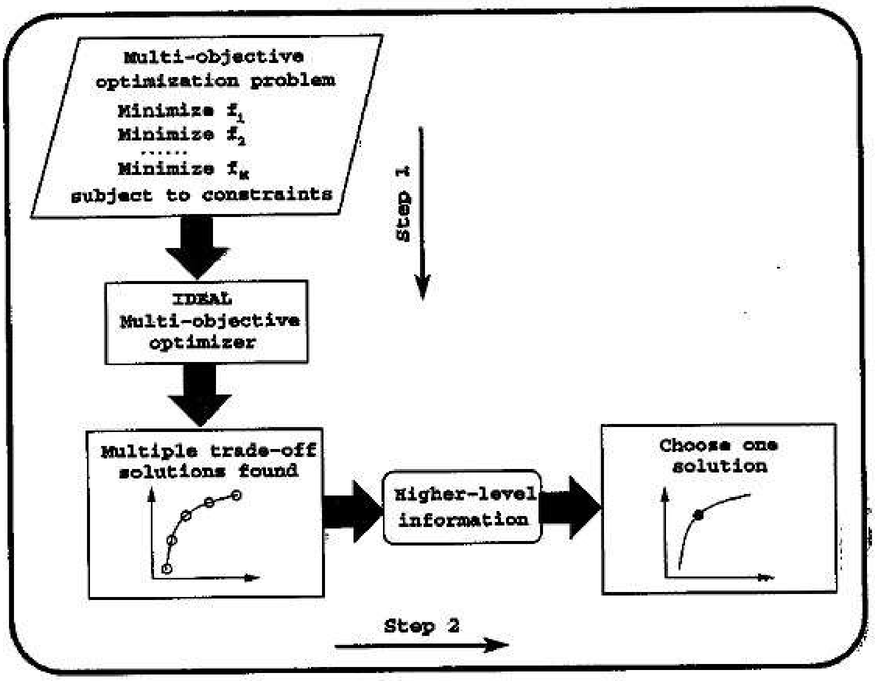

Figure 4.2: Procedure ideal multiobjective optimization [36].

tiple trade-off solutions. Higher level information is applied to select one of the trade-off solutions in step 2. From this procedure, it can be concluded that single objective optimization is decayed case of multiobjective optimization. If we consider the case of single objective optimization having only one global optimal solution then step 1 is sufficient to find solution while step 2 is not required. On the contrary, if single objective optimization having multiple global optima both step 1 & 2 are necessary to find global optima. Here, each trade-off solution corresponds to definite order of relevance of the objectives. If for a given specific problem, corresponding preference factors among the objectives are known then there is no need to follow method for solving multiobjective optimization shown in above Figure 4.2. Another straightforward method is to form a composite objective function comprising of weighted sum of the objectives. In this case weight is proportional to the preference assigned to that individual objective. This approach of scalarizing an objective vector using weighted sum method converts multiobjective optimization problem into single objective optimization. This procedure is represented schematically in following Figure 4.3, which is known as preference based or weighted sum approach of multiobjective optimization. As shown in above Figure 4.3, preference vector is selected

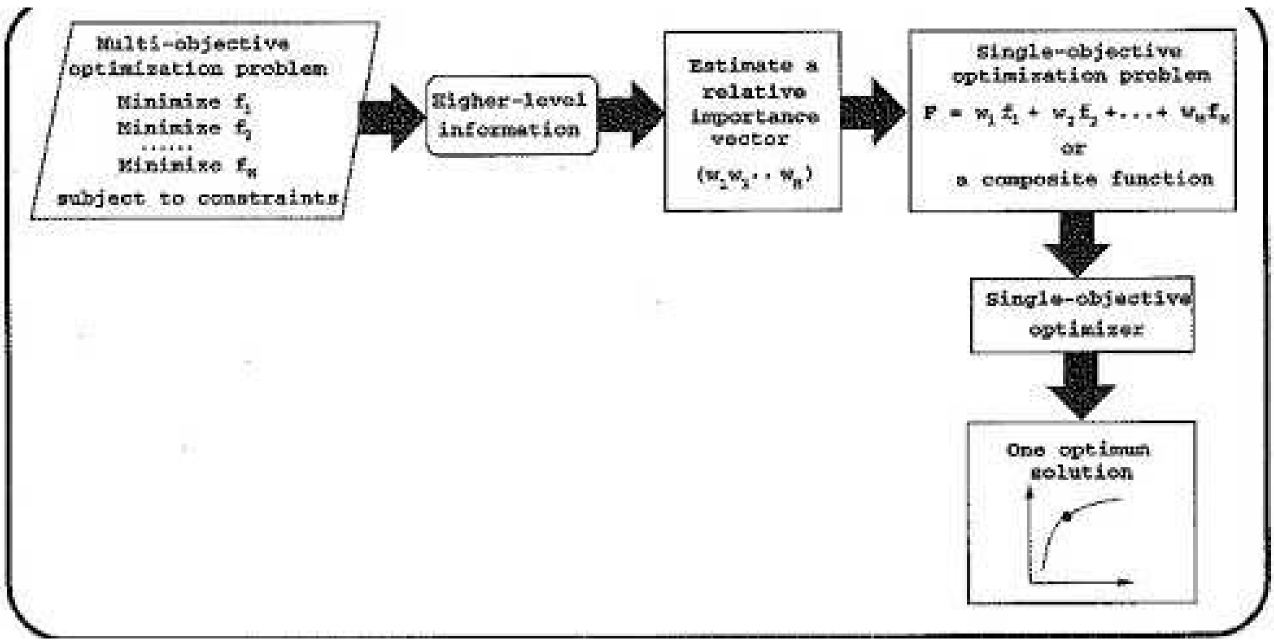

Figure 4.3: Preference based multiobjective optimization [36].

using higher level qualitative information. After that, composite function is built using preference based vector which is optimized to find single trade-off solution. This method can be repeated to find multiple trade-off solutions using different preference vector.

From above explanation following points can be concluded:

1. Trade-off solution obtained using preference based method is sensitive to the preference vector used in forming composite objective. Hence, small change in preference vector results in change in trade-off solution.

2. In order to find appropriate preference vector user has to apply higher level information means finding preference vector itself is challenging task.

3. Solution of multiobjective optimization problem obtained using preference vector based method is highly subjective to the individual user.

4. In the case of ideal multiobjective optimization method shown in Figure 4.2, objective is to find as many different trade-off solutions as possible and once these solutions are obtained, task is to select one solution based on given problem information.

5. In the case of ideal multiobjective optimization, problem information is used to search for the best solution from the available set of solutions. While, in the case of single objective optimization, problem information is used to search for new solution.

6. Ideal approach seems to be more feasible, more systematic, and less subjective. On the contrary, if trusty preference vector is known to the user, preference based vector approach is competent.

Nowadays, a study in the area of search and optimization has switched by varieties of non classical and stochastic search and optimization algorithm. Of these, evolutionary and swarm based algorithms have attracted researchers due to its advantage to optimize parameters based on cost function, without any knowledge about process. GA is an evolutionary and PSO is swarm based optimization algorithm. Hence, GA and PSO are selected for multiobjective optimization of 2DOF controller parameter for the process of heat exchanger consist of two conflicting objectives i.e. set point tracking and disturbance rejections. TOPSIS a multiple criteria decision making method is used to rank the set of Pareto optimal solutions for reducing number of Pareto optimal solutions to a single solution. The method of optimizations using GA and PSO is discussed in the subsequent chapters

4.9 Discussion on criteria for evaluation of objective functions

The general form of the 2DOF control structure consists of two compensators serial (or main) $C_s(s)$ and feed forward $C_f(s)$. The main objective in the proposed multiobjective optimization problem is to optimize five parameters of 2DOF controller. Three objective functions set point tracking, flow disturbance rejection, and temperature disturbance rejection are formed for the shell and tube heat exchanger system. Here, fitness is same as objective functions. Three criteria IAE, ISE and ITAE are used for evaluation of objective functions. Vector of objective functions are deployed for obtaining Pareto set of optimal solutions. Evolutionary and swarm optimization algorithms are called one at time to optimize five parameters of 2DOF controller for the shell and tube heat exchanger system. Here, five parameters of 2DOF controller are required to be optimized by considering set of performance indices which are function of error and time. This will help investigators to observe performances when such kind of optimizations problem is analyzed. The prime objective in any feedback control scheme is to reduce the value of error zero as quickly as possible. Therefore, any criterion applied to evaluate the quality of system response must take into account the variation of error over the entire range of time. The performance indices considered here for evaluation of objective functions are IAE, ISE and ITAE described as under.

1. Integral of Absolute Error (IAE) $= \int_0^\infty |e(t)| * d(t)$
2. Integral Squared of Error (ISE) $= \int_0^\infty e(t)^2 * d(t)$
3. Integral of Time-weighted Absolute Error (ITAE) $= \int_0^\infty t * |e(t)| * d(t)$

ISE integrates square of the error over time and it penalizes large errors more than smaller ones. If control objective is to minimize ISE will tend to eliminate large errors quickly, but will tolerate small errors persisting for a long period of time. This results in fast response, but exhibits low amplitude oscillations. IAE integrates the absolute error over time. As there is no any weight is being added to any error in the system, it produces slower response compared to ISE but has low amplitude oscillations. ITAE integrates the absolute error multiplied by the time over time. This adds weight to error in the system with time. Hence, weight errors exist after a long time much more heavily than those at the start of the response. ITAE tuning produces systems which settle much more quickly than the IAE and ISE [9].

4.10 Formation of objective functions

The objective in the problem of shell and tube heat exchanger system is to keep outlet temperature of process fluid at desired value and eliminate two predominant disturbances. Hence, problem consists of minimization of three objectives: 1. Error between set point value and process variable. 2. Reduce disturbance in temperature variation of process fluid. 3. Reduce disturbance in flow variation of process fluid. Mathematically, this results in minimization of error between SP- y(s), eliminating output temperature variation of process fluid $y_{temp}(s)$, and eliminating flow variation of process fluid $y_{flow}(s)$. Three criteria for evaluation of objective functions are used: Integral of Absolute Error (IAE), Integral Squared of Error (ISE) and Integral of Time-weighted Absolute Error (ITAE) [46], [57]. Three objective functions are formed considering above these criteria are as under.

Criterion 1: Integral of absolute value of error IAE

$$f(K_P, K_I, K_D, \alpha, \beta) = J(\sum_{k=0}^{n}[|SP - y(k)|], \sum_{k=0}^{n}[|y_{flow}(k)|], \sum_{k=0}^{n}[|y_{temp}(k)|]) \tag{4.1}$$

Criterion 2: Integral of Squared Error ISE

$$f(K_P, K_I, K_D, \alpha, \beta) = J(\sum_{k=0}^{n}[SP - y(k)]^2, \sum_{k=0}^{n}[y_{flow}(k)]^2, \sum_{k=0}^{n}[y_{temp}(k)]^2) \tag{4.2}$$

Criterion 3: Integral of Time-weighted Absolute Error ITAE

$$f(K_P, K_I, K_D, \alpha, \beta) = J(\sum_{k=0}^{n}[t*(SP-y(k))], \sum_{k=0}^{n}[t*(y_{flow}(k))], \sum_{k=0}^{n}[t*(y_{temp}(k))]) \tag{4.3}$$

Where,

SP = Set point or reference input.

$y(k) = \frac{(C_f(k)+C(k))}{C(k)} * \frac{(C(k)*A(k)*G(k))}{(1+C(k)*A(k)*G(k)*H(k))} * r(k)$ from equation 3.48 is process value output at k^{th} interval is a function of 2DOF controller parameters.

$y_{flow}(k) = \frac{F(k)}{(1+C(k)*A(k)*G(k)*H(k))} * D_f(k)$ from equation 3.49 is flow disturbance output at k^{th} interval is a function of 2DOF controller parameters.

$y_{temp}(k) = \frac{T(k)}{(1+C(k)*A(k)*G(k)*H(k))} * D_T(k)$ from equation 3.50 is temperature disturbance output at k^{th} interval is a function of 2DOF controller parameters.

In the multiobjective optimization problem vector of objective functions is required to be supplied for optimization. Vector of three objective functions are supplied for optimization of 2DOF controller parameters as under.

$$f(K_P, K_I, K_D, \alpha, \beta) = ([J_{setpoint}, J_{flow}, J_{temp}]) \tag{4.4}$$

Where,
$J_{setpoint}$=Function of set point tracking considering any of the above three criteria for evaluation one at a time.
J_{flow}=Function of flow disturbance rejection considering any of the above three criteria for evaluation one at a time.
J_{temp}=Function of temperature disturbance rejection considering any of the above three criteria for evaluation one at a time.
Vector of objective functions is optimized using evolutionary and swarm based algorithms and Pareto set of solutions are obtained. The method of 2DOF controller parameter optimization using evolutionary and swarm algorithms are provided in the subsequent chapters.

4.11 Conclusion

Theory of single and multiobjective optimization is studied. From this, it is concluded that procedure of ideal multiobjective optimization will be applied for the solution of 2DOF controller parameter optimization problem of shell and tube heat exchanger systems. Three evaluation criteria IAE, ISE, and ITAE will be used for the evaluation of objective functions, justifications given in section 4.9. Objective functions are formed considering above three evaluation criteria to optimize five parameters of 2DOF controller, discussed in section 4.10.