Chapter 7

TOPSIS: Multi-criteria decision making technique

This chapter describes application of Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) a multi-criteria decision making technique on Pareto solutions obtained using NSGA-II, NSGA-III, and MOPSO algorithms in which following topics are covered: Implementation steps of TOPSIS algorithm, application of TOPSIS and comparison of results.

7.1 Introduction

Evolutionary (NSGA-II and NSGA-III) and swarm intelligence (MOPSO) based algorithms are employed to optimize five parameters of two degree of freedom (2DOF) controller for the problem of shell and tube heat exchanger. The result gives Pareto set of solutions under each evaluation criteria. Practically, user needs only one solution from the set of Pareto optimal solutions for particular problem. Generally, user is not aware of exact trade-off among objective functions. Hence, it is desirable to first obtain maximum possible Pareto optimal solutions and select best one using multi-criteria decision making technique. The various multi-criteria decision making techniques are MAXMIN, MAXMAX, SAW (Simple Additive Weighting), AHP (Analytical Hierarchy Process), TOPSIS, SMART (Simple Multi Attribute Rating Technique), ELECTRE (Elimination and Choice Expressing Reality) and many more [16],[17]. The major advantage of TOPSIS method is its rational, easy to implement, and good computational efficiency. Hence, TOPSIS is proposed as a decision support tool to rank the optimal solutions and select the best rank optimal solution [18].

7.2 Implementation steps of TOPSIS algorithm

The Multi-criteria decision making tool is used to select best solution among a finite set of solutions available for multiobjective optimization problems. TOPSIS was implemented by Hwang and Yoon [16]. TOPSIS works based on calculating the euclidean distance from each alternative to a best performing attribute called Positive Ideal Solution (PIS) and a poorest performing attribute called Negative Ideal Solution (NIS) that are defined in n-dimensional space [18]. It consists of two criteria positive and negative. Positive criterion needs to be increased and negative criterion needs to be decreased. This process is implemented by taking the below steps:

Step1: Specify alternative and criteria for nondominated set of solutions of the 2DOF controller parameters. Assume that there are m possible alternatives called A=$[A_1, A_2, ., A_m]$ which are evaluated against criteria C=$[C_1, C_2, ., C_c]$.

Step2: Assign ratings to criteria and alternatives using matrix X shown below where, x_{ij} indicates the value of alternative A_i for criterion C_g.

$$X_{(mxc)} = \begin{matrix} \\ A_1 \\ A_i \\ \vdots \\ A_m \end{matrix} \begin{matrix} C_1 & C_2 & C_g & C_c \\ \end{matrix} \begin{pmatrix} x_{11} & x_{12} & x_{1g} & x_{1c} \\ x_{i1} & x_{i2} & x_{ig} & x_{ic} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & x_{mg} & x_{mc} \end{pmatrix} \tag{7.1}$$

Step3: Calculate weight of criteria by entropy technique to normalize decision matrix $X_{(mxc)}$ using following formula.

$$q_{ig} = \frac{x_{ig}}{(x_{1g} + x_{2g} + x_{mg})}; \forall g \epsilon (1, 2, .., c) \tag{7.2}$$

The information entropy of criterion g is represented as under.

$$\triangle_g = -k \sum_{i=1}^{m} q_{ig}. \ln q_{ig}; \forall g \epsilon (1, 2, .., c) \tag{7.3}$$

Where, $0 \leq \triangle_g \leq 1$ is assured with $k = 1/\ln(m)$
The entropy method for measuring weights of criteria is an objective weight technique determined by data statistical properties. Here, the index with higher information entropy $\triangle_g$ has greater variation hence; weight is calculated based on deviation degree is

$$d_g = 1 - \triangle_g \forall g \epsilon (1, 2, .., c) \tag{7.4}$$

The weight for criteria by the entropy method is calculated as under

$$w_g = \frac{d_g}{(d_1 + d_2 + ... + d_c)} \tag{7.5}$$

Let λ_g be weight vector used to obtain the aggregated weight $w_g^{'}$ shown in following equation

$$w_g^{'} = \frac{\lambda_g w_g}{(\lambda_1 w_1 + \lambda_2 w_2 + ... + \lambda_c w_c)} \tag{7.6}$$

$$w^{'} = (w_1^{'}, w_2^{'}, ..., w_c^{'}) \tag{7.7}$$

Step4: Construct a normalized decision matrix using the vector normalization method, calculate normalized value r_{ig} by (7.8) and construct matrix $N_{(mxc)}$ given by (7.9).

$$r_{ig} = \frac{x_{ig}}{\sqrt{x_{1g}^2 + x_{2g}^2 + ... + x_{mg}^2}} \tag{7.8}$$

$$N_{mxc} = [r_{ig}]_{mxc} \quad (i = 1, 2, .., m; g = 1, 2, .., c) \tag{7.9}$$

Step5: Construct the weighted normalized decision matrix by building the diagonal matrix $w_{(cxc)}^{'}$ with element $w_g^{'}$ in (7.6) to reach the V matrix:

$$V = N_{mxc}.w_{(cxc)}^{'} = (v_{ig})_{mxc} \quad (i = 1, 2, .., m; g = 1, 2, .., c) \tag{7.10}$$

Step6: Compute the positive ideal solution (PIS) A^+ and the negative ideal solution (NIS) A^- of the alternatives:

$$A^+ = (\max v_{ig} | g \epsilon G)(\min v_{ig} | g \epsilon G^{'}) = (v_1^+, v_2^+, ..., v_c^+) \tag{7.11}$$

$$A^- = (\min v_{ig} | g \epsilon G)(\max v_{ig} | g \epsilon G') = (v_1^-, v_2^-, ..., v_c^-) \tag{7.12}$$

Where, G and G' are the subsets of positive and negative criteria.

Step7: Compute the distance of each alternative from PIS (d_i^+) and NIS (d_i^-):

$$d_i^+ = \sqrt{\sum_{g=1}^{c} (v_{ig} - v_g^+)^2} \tag{7.13}$$

$$d_i^- = \sqrt{\sum_{g=1}^{c} (v_{ig} - v_g^-)^2} \tag{7.14}$$

Step8: Compute the closeness coefficient of each alternative:

$$CC_i^+ = \frac{d_i^-}{(d_i^- + d_i^+)} \quad (i = 1, 2, .., m) \tag{7.15}$$

Step9: Rank the alternatives.

$$v = v_i | \max_{1 \le i \le m} CC_i^+ \tag{7.16}$$

MATLAB software tool is used to implement above steps.

7.3 Application of TOPSIS on NSGA-II, NSGA-III, and MOPSO

The number of nondominated set of solutions obtained for 2DOF controller parameters optimization using NSGA-II, NSGA-III, and MOPSO algorithms under three test criteria IAE, ISE, and ITAE are shown in Table 7.1.

Table 7.1: Pareto set of solutions using NSGA-II,NSGA-III, and MOPSO

Type of Algorithm	Number of non-dominated set of solutions under three test criteria		
	IAE	ISE	ITAE
NSGA-II	27	27	27
NSGA-III	80	80	80

MOPSO	95	26	9
Combined non-dominated solutions under each criteria.	202	133	116

TOPSIS algorithm is applied to prioritize the Pareto set of solutions shown in Table 7.1. Here, minimization of peak overshoot, flow disturbance rejection, and temperature disturbance rejection are considered as three criteria C_1,C_2,and C_3 for TOPSIS. All three criteria are negative as it requires to be minimized. The weights of criteria assumed to be identical (w=1). After applying TOPSIS, rank of each nondominated solution along with closeness coefficient is obtained shown in following Table 7.2,7.3, and 7.4.

Table 7.2: Rank of 2DOF controller parameters using TOPSIS for NSGA-II under IAE, ISE, and ITAE

	Rank of nondominated set of solution under IAE			Rank of nondominated set of solution under ISE			Rank of nondominated set of solution under ITAE	
Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
1	0.139499983	25	1	0.916156397	2	1	0.736904849	4
2	0.999790511	1	2	0.969106095	1	2	0.568957309	14
3	0.450991856	21	3	0.111113239	27	3	0.599328282	11
4	0.612548684	15	4	0.124500851	26	4	0.398997174	24
5	0.000873994	26	5	0.874856154	5	5	0.568651174	16
6	0.500815242	18	6	0.28202096	23	6	0.743282348	3
7	0.245774548	23	7	0.245031117	24	7	0.63607864	7
8	0.716252347	8	8	0.206231281	25	8	0.444637078	21
9	0.834714135	4	9	0.89583837	3	9	0.637014324	5
10	0.713062952	9	10	0.780720049	11	10	0.521594285	19

Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
11	0.176315852	24	11	0.564060093	17	11	0.615701589	9
12	0.523272748	17	12	0.414971725	18	12	0.636593288	6
13	0.838580319	3	13	0.386584924	20	13	0.576239573	13
14	0.619401354	14	14	0.570190088	16	14	0.487027384	20
15	0.999670866	2	15	0.323650959	22	15	0.96421824	1
16	0.602867228	16	16	0.407643506	19	16	0.385520134	26
17	0.707633622	10	17	0.77134111	12	17	0.416764044	22
18	0.620119539	13	18	0.799426983	10	18	0.52914569	17
19	0.470478286	20	19	0.848795482	8	19	0.170838835	27
20	0.717695573	7	20	0.865987774	6	20	0.407484677	23
21	0.730277754	6	21	0.839758921	9	21	0.578041421	12
22	0.65026265	12	22	0.601596904	15	22	0.947696444	2
23	0.76601709	5	23	0.86343887	7	23	0.605737	10
24	0.000873994	26	24	0.732148897	14	24	0.619770205	8
25	0.660481014	11	25	0.335439278	21	25	0.396063663	25
26	0.476964994	19	26	0.884289536	4	26	0.525753039	18
27	0.346500567	22	27	0.751702822	13	27	0.568957309	14

Table 7.3: Rank of 2DOF controller parameters using TOPSIS for NSGA-III under IAE, ISE, and ITAE

Rank of nondominated set of solution under IAE			Rank of nondominated set of solution under ISE			Rank of nondominated set of solution under ITAE		
Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
1	0.439553488	41	1	0.317155401	33	1	0.659965912	9
2	0.210662124	64	2	0.28830177	58	2	0.359385135	62
3	0.348456745	49	3	0.405056451	12	3	0.447136428	46
4	0.971350992	1	4	0.251428197	67	4	0.661714626	8
5	0.28768881	57	5	0.267608639	61	5	0.415263455	55
6	0.545861924	28	6	0.360409747	20	6	0.407534457	56
7	0.322304835	52	7	0.302603379	48	7	0.669751308	6
8	0.677678826	14	8	0.301588821	49	8	0.545724189	25
9	0.128213252	70	9	0.252740132	64	9	0.346273572	64
10	0.279717395	59	10	0.330748807	26	10	0.257145617	76
11	0.617646691	19	11	0.231743511	72	11	0.54913437	24
12	0.682728137	10	12	0.762647655	2	12	0.416525188	54
13	0.679302693	13	13	0.302847732	47	13	0.556628155	21
14	0.774979908	6	14	0.307250078	41	14	0.432646626	50
15	0.62145335	17	15	0.266339439	63	15	0.302926938	69
16	0.676856737	15	16	0.303263049	46	16	0.421798675	53
17	0.537558448	29	17	0.304146372	44	17	0.352838672	63
18	0.125314425	71	18	0.227281613	74	18	0.539847249	28
19	0.438500882	42	19	0.319061088	31	19	0.52008084	31
20	0.216546243	63	20	0.375424795	17	20	0.600674629	17
21	0.043885699	79	21	0.249535522	69	21	0.292728238	72

Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
22	0.526132336	34	22	0.251778613	66	22	0.44545096	47
23	0.311369363	54	23	0.305825024	42	23	0.308414571	68
24	0.682728137	10	24	0.314250724	38	24	0.373881496	61
25	0.440849961	40	25	0.377903814	15	25	0.431643379	51
26	0.116707029	73	26	0.317903252	32	26	0.44350692	49
27	0.497799896	36	27	0.371390856	19	27	0.500983321	34
28	0.80613819	5	28	0.451705244	11	28	0.339246264	65
29	0.820746197	4	29	0.387367782	14	29	0.467652281	43
30	0.830172755	3	30	0.356523335	21	30	0.541719928	27
31	0.141246346	69	31	0.333724201	25	31	0.407310364	57
32	0.429036049	44	32	0.845854233	1	32	0.299969542	70
33	0.451551397	38	33	0.546454591	8	33	0.468112783	42
34	0.668844559	16	34	0.326851505	29	34	0.596182603	18
35	0.116542605	74	35	0.459874427	9	35	0.611988201	15
36	0.451943192	37	36	0.311084497	40	36	0.297375514	71
37	0.345296355	50	37	0.28994159	57	37	0.318827228	67
38	0.022754558	80	38	0.327410882	28	38	0.337132887	66
39	0.185450287	65	39	0.233241297	70	39	0.664750022	7
40	0.060141258	78	40	0.328769444	27	40	0.000953462	80
41	0.371430373	47	41	0.32490235	30	41	0.78279646	2
42	0.297161091	55	42	0.268815717	60	42	0.278149462	73
43	0.682728137	10	43	0.352404163	23	43	0.698814442	4
44	0.100956242	75	44	0.3047964	43	44	0.544724408	26
45	0.356684181	48	45	0.458483638	10	45	0.472524781	41
46	0.570876027	24	46	0.232892679	71	46	0.426289835	52
47	0.583773938	23	47	0.209475178	77	47	0.389772951	59
48	0.184572682	66	48	0.295368743	53	48	0.671815216	5
49	0.529087805	33	49	0.398952889	13	49	0.592719166	19
50	0.331470436	51	50	0.29171257	56	50	0.399685147	58
51	0.597368105	20	51	0.316013762	35	51	0.154826454	78

Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
52	0.555316163	27	52	0.226845923	75	52	0.638828441	12
53	0.535023021	31	53	0.303662915	45	53	0.63286338	13
54	0.559730437	25	54	0.372804087	18	54	0.278083112	74
55	0.312915395	53	55	0.349561114	24	55	0.501480823	33
56	0.082540688	76	56	0.377702607	16	56	0.465948466	44
57	0.279300305	60	57	0.316151501	34	57	0.500214902	35
58	0.29060933	56	58	0.295719881	52	58	0.609017615	16
59	0.532056477	32	59	0.295243241	54	59	0.380915994	60
60	0.726377935	8	60	0.698258206	3	60	0.534416068	30
61	0.232050518	62	61	0.266474158	62	61	0.49156618	39
62	0.772625012	7	62	0.137002069	80	62	0.27260422	75
63	0.384688528	46	63	0.315243759	37	63	0.537982991	29
64	0.695940402	9	64	0.284592551	59	64	0.49740558	36
65	0.286374769	58	65	0.227692657	73	65	0.553231672	23
66	0.517743307	35	66	0.225047538	76	66	0.589466716	20
67	0.596985476	21	67	0.674972666	5	67	0.496807092	37
68	0.448623576	39	68	0.658994266	6	68	0.778863734	3
69	0.423921555	45	69	0.599598393	7	69	0.062615749	79
70	0.067369531	77	70	0.300966448	50	70	0.473048576	40
71	0.618913651	18	71	0.205619461	78	71	0.465774347	45
72	0.555501545	26	72	0.698258206	3	72	0.510743801	32
73	0.866894865	2	73	0.354380485	22	73	0.624099411	14
74	0.536261437	30	74	0.250648242	68	74	0.24223292	77
75	0.116868948	72	75	0.197234242	79	75	0.443820198	48
76	0.432585716	43	76	0.298779771	51	76	0.651996723	11
77	0.177506132	67	77	0.313631301	39	77	0.656274124	10
78	0.270286649	61	78	0.252374139	65	78	0.493524766	38
79	0.590010232	22	79	0.315782746	36	79	0.976027897	1
80	0.158613181	68	80	0.293856154	55	80	0.556425336	22

Table 7.4: Rank of 2DOF controller parameters using TOPSIS for MOPSO under IAE, ISE, AND ITAE

Rank of nondominated set of solution under IAE			Rank of nondominated set of solution under ISE			Rank of nondominated set of solution under ITAE		
Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
1	0.23019673	43	1	0.47290316	25	1	0.158978146	8
2	0.23019673	43	2	0.51852310	23	2	0.482927944	4
3	0.312720045	34	3	0.47834572	24	3	0.292044621	7
4	0.243222672	42	4	0.82930288	7	4	0.650128628	3
5	0.050949174	87	5	0.30867298	26	5	0.369120678	6
6	0.122341775	71	6	0.62800031	21	6	0.400733273	5
7	0.118887117	74	7	0.68586489	18	7	0.002442736	9
8	0.195762628	55	8	0.78152446	13	8	0.986438065	1
9	0.180892406	60	9	0.82644290	8	9	0.668850975	2
10	0.068071498	83	10	0.85468947	3			
11	0.368637245	23	11	0.84726258	6			
12	0.438498015	15	12	0.86652768	2			
13	0.255182165	39	13	0.73133140	17			
14	0.220514515	47	14	0.82581318	9			
15	0.21453224	52	15	0.84855227	5			
16	0.247074255	40	16	0.66800027	20			
17	0.261211809	37	17	0.77747811	14			

Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
18	0.217878313	48	18	0.78789276	12			
19	0.007278731	94	19	0.74207063	15			
20	0.046581497	88	20	0.68338236	19			
21	0.244337626	41	21	0.84896959	4			
22	0.337775271	32	22	0.80180545	11			
23	0.003006195	95	23	0.80710761	10			
24	0.376072103	22	24	0.61412107	22			
25	0.355282527	27	25	0.74139962	16			
26	0.056130698	85	26	0.88194695	1			
27	0.163327677	65						
28	0.143472658	68						
29	0.126395308	70						
30	0.120716072	73						
31	0.107963358	75						
32	0.344657339	30						
33	0.045972154	89						
34	0.171095354	63						
35	0.538133766	11						
36	0.021098635	93						
37	0.04404276	90						
38	0.189419127	58						
39	0.13554931	69						
40	0.169115449	64						
41	0.055317839	86						

Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
42	0.29813782	35						
43	0.354978976	28						
44	0.09546303	76						
45	0.17216143	62						
46	0.292668621	36						
47	0.364034251	25						
48	0.36039207	26						
49	0.227814308	45						
50	0.227003254	46						
51	0.353056452	29						
52	0.654700592	7						
53	0.367726791	24						
54	0.753944692	5						
55	0.99148973	1						
56	0.566617107	9						
57	0.260907806	38						
58	0.214582939	51						
59	0.183988557	59						
60	0.070307479	81						
61	0.058823445	84						
62	0.080369764	80						
63	0.091854976	77						
64	0.069951752	82						
65	0.429908931	16						

Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
66	0.401808726	19						
67	0.481777661	13						
68	0.033793742	91						
69	0.120970091	72						
70	0.425816038	17						
71	0.161906037	66						
72	0.401404308	20						
73	0.846366476	4						
74	0.752990374	6						
75	0.420206792	18						
76	0.633409989	8						
77	0.513723146	12						
78	0.191753614	56						
79	0.99148973	1						
80	0.379157043	21						
81	0.030601199	92						
82	0.540654964	10						
83	0.467590372	14						
84	0.314745086	33						
85	0.190332633	57						
86	0.343038099	31						
87	0.161042883	67						
88	0.99148973	1						
89	0.208701266	53						

Solution	Closeness	Rank	Solution	Closeness	Rank	Solution	Closeness	Rank
90	0.089212406	78						
91	0.080776326	79						
92	0.180689375	61						
93	0.20032953	54						
94	0.216205294	49						
95	0.216103897	50						

In order to obtain the comparative analysis of optimization algorithms (NSGA-II, NSGA-III, and MOPSO) all the Pareto optimal solutions are combined under three separate evaluation criteria IAE, ISE, and ITAE. The combined nondominated set of solutions are 202, 133, and 116 under evaluation criteria IAE, ISE, and ITAE shown in Table 7.1. TOPSIS is used to obtain top 10 high rank individual solution from combined set of solution shown in Table 7.5 to Table 7.7.

Table 7.5: Top 10 optimal solutions obtained using TOPSIS from NSGA-II, NSGA-III, and MOPSO under IAE

Top 10 solution	from the combined	Set of solution	under IAE
Algorithm	Solution Number	Closeness Coefficient CC	Rank
NSGA-II	177	0.999790511	1
NSGA-II	190	0.999670866	2
MOPSO	55	0.99148973	3
MOPSO	79	0.99148973	4
MOPSO	88	0.99148973	5
NSGA-III	99	0.971350992	6
NSGA-III	168	0.866894865	7
MOPSO	73	0.846366476	8
NSGA-II	188	0.838580319	9
NSGA-II	184	0.834714135	10

Table 7.6: Top 10 optimal solutions obtained using TOPSIS from NSGA-II, NSGA-III, and MOPSO under ISE

Top 10 solution	from the combined	set of solution	under ISE
Algorithm	Solution Number	Closeness Coefficient CC	Rank
NSGA-II	82	0.969106	1
NSGA-II	81	0.916156	2
NSGA-II	89	0.895838	3
NSGA-II	106	0.88429	4
MOPSO	133	0.881947	5
NSGA-II	85	0.874856	6
NSGA-II	119	0.866528	7
NSGA-II	100	0.865988	8
NSGA-II	103	0.863439	9
MOPSO	117	0.854689	10

Table 7.7: Top 10 optimal solutions obtained using TOPSIS from NSGA-II, NSGA-III, and MOPSO under ITAE

Top 10 solution	from the combined	set of solution	under ISE
Algorithm	Solution Number	Closeness Coefficient CC	Rank
MOPSO	115	0.986438	1
NSGA-II	79	0.976028	2
NSGA-II	95	0.964218	3
NSGA-II	102	0.947696	4
NSGA-III	41	0.782796	5
NSGA-III	68	0.778864	6
NSGA-II	86	0.743282	7
NSGA-II	81	0.736905	8
NSGA-III	43	0.698814	9
NSGA-III	48	0.671815	10

As shown in Table 7.5 to Table 7.7 after merging the solutions of algorithms, the percentage of solutions from NSGA-II is greater than NSGA-III and MOPSO under all three evaluation criteria IAE, ISE, and ITAE. Also, the best rank solution is obtained from NSGA-II (under IAE and ISE) and MOPSO (under ITAE). Here, NSGA-II algorithm outperforms NSGA-III and MOPSO algorithms. Following Figure 7.1 to Figure 7.3 are plots of set point tracking, flow disturbance rejection, and temperature disturbance rejection using the best rank result obtained after applying TOPSIS for optimization of 2DOF controller parameters under each evaluation criteria IAE, ISE , and ITAE.

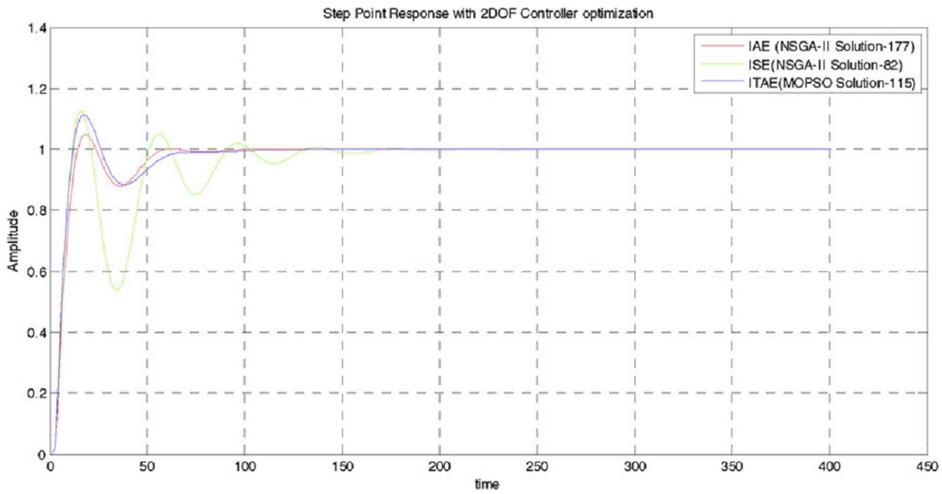

Figure 7.1: Set point response with the best rank result from TOPSIS for 2DOF controller optimization.

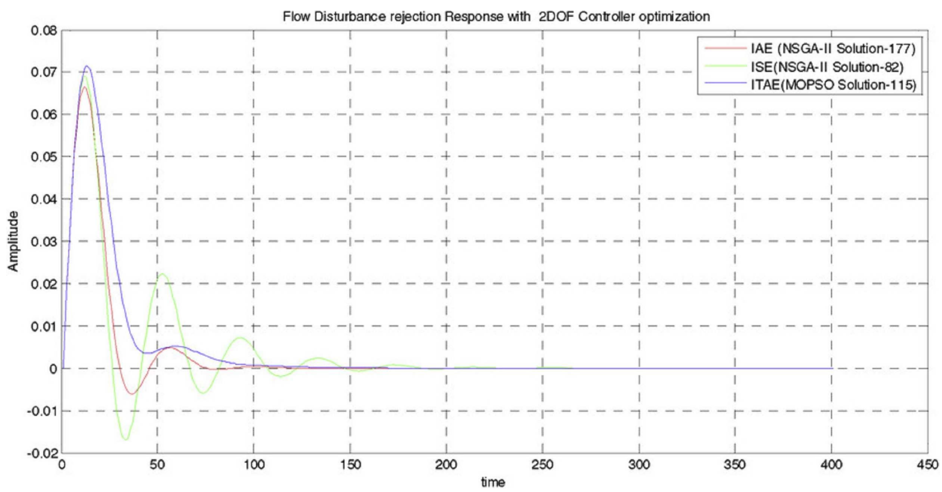

Figure 7.2: Flow disturbance rejection response with the best rank result from TOPSIS for 2DOF controller optimization.

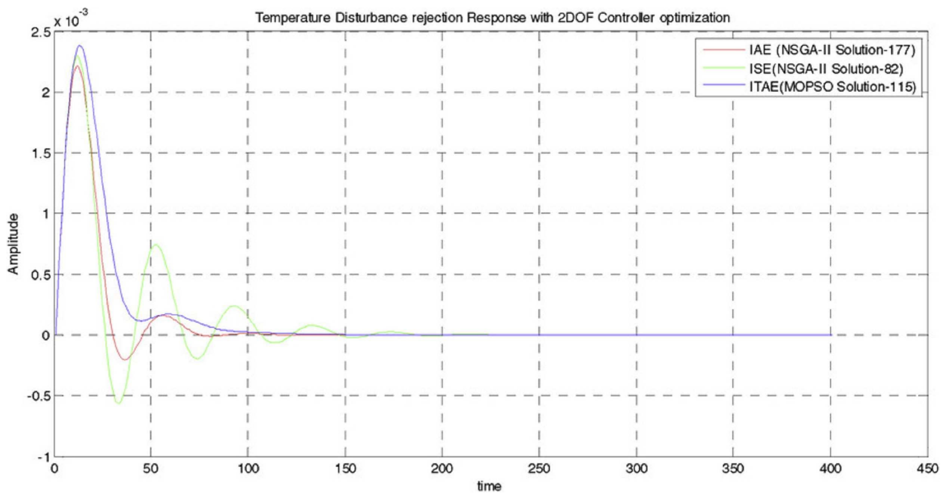

Figure 7.3: Temperature disturbance rejection response with the best rank result from TOPSIS for 2DOF controller optimization.

From the figures (Figure 7.1 to Figure 7.3), it is derived that IAE criterion of NSGA-II (Solution No-177) algorithm has minimum peak overshoot of step response(4.8%), maximum rejection of flow (33.4%), and temperature (78%) disturbances for non-dominated set of solution [1.363,0.052, 6.855,0.601,0.439]. The minimum peak overshoot of step response, maximum rejection of flow, and temperature disturbances are achieved under ISE and ITAE criteria using NSGA-II (Solution No-82) and MOPSO(Solution No-115) algorithms respectively, values are shown in following Table 7.8.

Table 7.8: Parameters of 2DOF controller after applying TOPSIS from NSGA-II, NSGA-III, and MOPSO.

Optimization of 2DOF controller parameters $[K_p, K_i, K_d, \alpha, \beta]$	Peak overshoot of of Step Response In (%)	Reduction Flow Disturbance Response In (%)	Reduction Temperature Disturbance Response In (%)
NSGA-II (Solution No-177) under IAE $[1.363, 0.052, 6.855, 0.601, 0.439]$	4.8	33.4	78
NSGA-II (Solution No-82) under ISE $[1.677, 0.0454, 4.886, 0.619, 0.215]$	12.59	31	77
MOPSO (Solution No-115) under ITAE $[1.090, 0.035, 5.876, 0.433, 0.40]$	11.35	28.5	76

The settling time for set point tracking response, flow disturbance rejection response, and temperature disturbance rejection response is derived from the above figures (Figure 7.1 to Figure 7.3), shown in following Table 7.9.

Table 7.9: Settling time of the system from the best rank solution under IAE, ISE, and ITAE.

Optimization of 2DOF controller parameters $[K_p, K_i, K_d, \alpha, \beta]$	Set point response in (second)	Flow disturbance response in (second)	Temperature disturbance response in (second)
NSGA-II (Solution No-177) under IAE $[1.363, 0.052, 6.855, 0.601, 0.439]$	52	120	89
NSGA-II (Solution No-82) under ISE $[1.677, 0.0454, 4.886, 0.619, 0.215]$	150	186	187
MOPSO (Solution No-115) under ITAE $[1.090, 0.035, 5.876, 0.433, 0.40]$	62	149	100

It is derived from Table 7.9, that settling time of the system is minimum under IAE criterion of NSGA-II (Solution No-177) [1.363,0.052, 6.855,0.601,0.439] algorithm.

7.4 Conclusion

Evolutionary (NSGA-II and NSGA-III) and swarm intelligence (MOPSO) based algorithms enhanced with Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is employed to optimize five parameters of Two Degree Of Freedom (2DOF) controller for the problem of shell and tube heat exchanger system. The Pareto set of solutions are obtained after optimizing all the five parameters of 2DOF controller using evolutionary (NSGA-II and NSGA-III) and swarm intelligence (MOPSO) algorithms. TOPSIS a multiple criteria decision making method is used to rank the set of Pareto optimal solutions for reducing number of Pareto optimal solutions to a single solution. The best rank solution obtain for 2DOF controller parameters after applying TOPSIS on set of Pareto optimal solutions using evolutionary (NSGA-II and NSGA-III) algorithms are compared with swarm intelligence (MOPSO) algorithm. In order to obtain the comparative analysis of optimization algorithms (NSGA-II, NSGA-III, and MOPSO) all the Pareto optimal solutions are combined under three separate evaluation criteria IAE, ISE, and ITAE. The combined nondominated set of solutions are 202, 133, and 116 under evaluation criteria IAE, ISE, and ITAE respectively. TOPSIS is used to obtain top 10 high rank individual solution from combined set of solution. To evaluate the performance optimization of 2DOF controller tuning, we compared the values of peak overshoot of step response, set point tracking error, disturbance rejection (both flow and temperature), settling time, and the percentage of solutions obtained from optimization algorithms under all three evaluation criteria.

From the results shown in Table 7.5, 7.6, and 7.7 , it is concluded that after merging the solutions of algorithms, the percentage of solutions from NSGA-II is greater than NSGA-III and MOPSO under all three evaluation criteria IAE, ISE, and ITAE. Also, the best rank solution is obtained from NSGA-II (under IAE and ISE) and MOPSO (under ITAE). From the Figure 7.1 to Figure 7.3, it is concluded that IAE criteria of NSGA-II (Solution No-177) algorithm has minimum peak overshoot of step response(4.8%), maximum rejection of flow (33.4%), and temperature (78%) disturbances for non-dominated set of solution [1.363,0.052, 6.855,0.601,0.439]. The minimum peak overshoot of step response, maximum rejection of flow, and temperature disturbances are achieved under ISE and ITAE criteria using NSGA-II (Solution No-82) and MOPSO(Solution No-115) algorithms respectively. It is derived from Table 7.9, that settling time of the system is minimum under IAE criterion of NSGA-II (Solution No-177) [1.363,0.052, 6.855,0.601,0.439]. From this, it is concluded that, NSGA-II algorithm outperforms NSGA-III and MOPSO algorithms.