Chapter 2

Finite Element Method & Transformer Design

2.1 Finite Element Method

The finite element method is a numerical technique for solving a wide variety of engineering problems. Finite Element Analysis (FEA) using Finite Element Method (FEM) was developed initially to carry out the structural analysis in civil and aeronautical engineering in early 1940s. However it was not until 1960 that Clough made the term finite element popular. Later on application of FEA is being expanded to simulation in mechanical and electrical engineering also, to solve complex design problems such as fluid flow, heat transfer, electric field plotting etc.

There are variety of practical engineering problems for which one cannot identify the exact solution either due to the complex nature of the governing differential equations or due to the difficulties in dealing with their initial and boundary conditions. Numerical techniques are the best alternate to deal with such problems. There are several numerical techniques such as finite difference method, boundary element method, finite element method, etc. Among all these finite element method (FEM) is practically well suited for the problems involving complex geometries. It provides a standard process for converting the governing energy principles or governing differential equations in to a system of matrix equations to be solved for an approximate solution.

The working of finite element method and its advantages over other methods can be described as under [106].

"In a continuum (a body of matter – solid, liquid or gas or simply a region of space in which a particular phenomenon is occurring), problem of any dimensions, the field variable – whether it is pressure, temperature, displacement, stress or some other quantity – possesses infinitely many values because it is a function of each generic point in the body or solution region. Consequently, the problem is one with an infinite number of unknowns. The finite element discretization procedure reduces the problem to one of a finite number of unknowns by dividing the solution region into elements and by expressing the unknown field variables in terms of assumed approximating functions also called interpolation functions. The interpolation functions are defined in terms of the values of the field variables at specified points called nodes or nodal points. Nodes usually lay on the element boundaries where adjacent elements are connected. In addition to the boundary nodes, an element may also have a few interior nodes. The nodal values of the field variable and the interpolation functions for the elements completely define the behavior of the field variable within the elements. For the finite element representation of a problem the nodal values of the field variable become the unknowns. Once these unknowns are found, the interpolation functions define the field variable throughout the assemblage of elements.

Clearly, the nature of the solution and the degree of approximation depend not only on the size and number of the elements used but also on the interpolation functions selected. While choosing the function, certain compatibility conditions should be satisfied. Often the functions are chosen such that the field variable or its derivatives are continuous across adjoining element boundaries.

An important feature of the finite element method that sets it apart from other numerical methods is its ability to formulate the solutions for individual elements before putting them together to represent the entire problem e.g. while solving a problem of stress analysis, the force-displacement or stiffness characteristics of each individual element is found first and then all elements are assembled to find out the stiffness of the whole structure. This reduces a complex problem into a series of greatly simplified problems.

Another advantage of finite element method is the variety of ways in which the properties of the individual elements can be formulated. There are basically three different approaches – Direct Approach which can be used only for relatively simple problems, Variational Approach and Weighted Residuals Approach.

The variational approach relies on the calculus of variations and involves extremizing a functional. For the problems of solid mechanics the functional turns out to be the potential energy, the complementary energy or some variant of these, such as the Reissner variational principle. Whereas the direct approach can be used to formulate the element properties for only the simplest element shapes, the variational approach can be employed for both simple and sophisticated element shapes.

The weighted residuals approach begins with the governing equations of the problem and proceeds without relying on a variational statement. This is advantageous because it thereby becomes possible to extend the finite element method to problems where no functional is available. The method of weighted residuals (Galerkin Approach)is widely used to derive element properties for nonstructural applications such as heat transfer, fluid mechanics, electromagnetism etc.

The above discussion reveals that the finite element analysis of any problem involves a step-by-step procedure as stated below [113], [117], [106].

Step 1 – Discretization:

The first step in finite element method is to divide the continuum (the structure or the solution region) into subdivisions or elements. Hence the structure is to be modeled with suitable finite elements. A variety of element shapes may be used, and different element shapes may be employed in the same solution region. The numbers of elements, their size and their arrangements are to be decided.

Step 2 – Selection of Interpolation Function:

In this step nodes have been assigned to each element and then interpolation function has been chosen to represent the variation of the field variable over the element. The field variable may be a scalar, a vector or a higher order tensor. Normally polynomials are selected as interpolation functions for the field variable as they are easy to integrate or differentiate.

Step 3 – Determination of Element Properties:

Once the elements and their interpolation functions have been selected i.e. finite element model has been established, determine the matrix equations describing the properties of the individual elements. For this any one of the three approaches – direct approach, variational approach or the weighted residual approach may be adopted.

Step 4 – To Obtain System Equations:

Assemble all the elements i.e. combine the matrix equations of individual elements to form the matrix equations that represent the entire system problem.

Step 5 – Boundary Conditions & Initial Conditions:

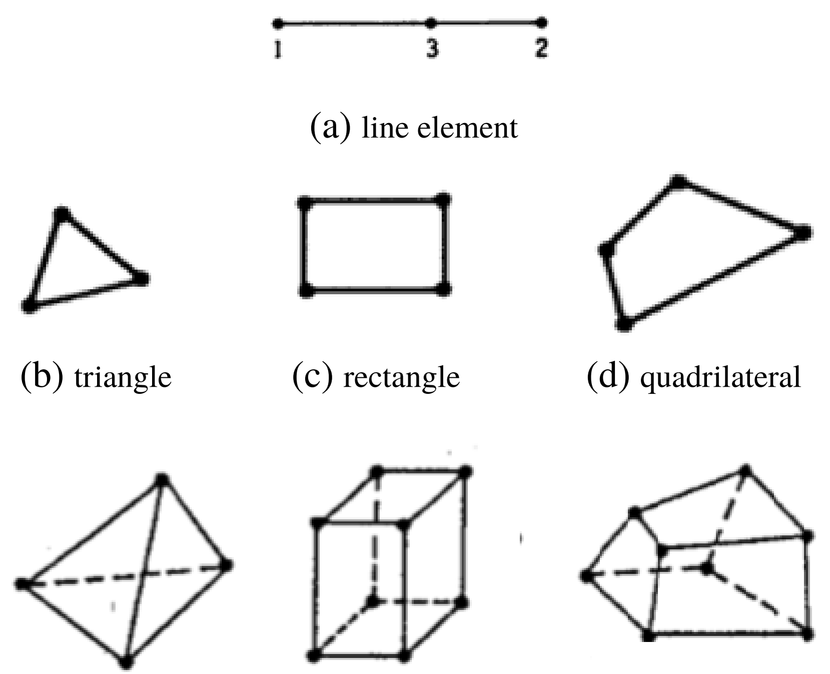
Once the system equations are ready for the solution, they must be modified to account for the boundary conditions and initial conditions of the problem. Known nodal values of the dependent variables or nodal loads are imposed at this stage.

Step 6 – Solution Phase:

In this step, a set of linear or non linear simultaneously equations are solved to obtain the unknown nodal values of the problem if the problem describes steady or equilibrium behavior. If the problem is unsteady, the nodal unknowns will be a function of time and then a set of linear or nonlinear ordinary differential equations will have to be solved. Make additional computations if required to obtain the other unknowns based on the solutions achieved for the system equations e.g. in an electric heat flow analysis problem, if the nodal unknowns are temperature values then overall temperature rise of the system can be calculated.”

2.1.1 Finite Elements

"A very important aspect of finite element method is the selection of particular type / shape of finite element and defining an appropriate interpolation function. Various element shapes such as – one dimensional line element, two dimensional triangle, rectangle and general quadrilateral elements and three dimensional tetrahedron, right prism (also known as brick) and general hexahedron elements have been shown in figure 2.1 (a) to (g). To approximate the curved boundaries with only few elements, isoparametric elements such as triangle, quadrilateral, tetrahedron or hexahedron elements as shown in figure 2.2 (a) to (d) are normally used. This helps in solving the three dimensional problems with less complexity and computation time.



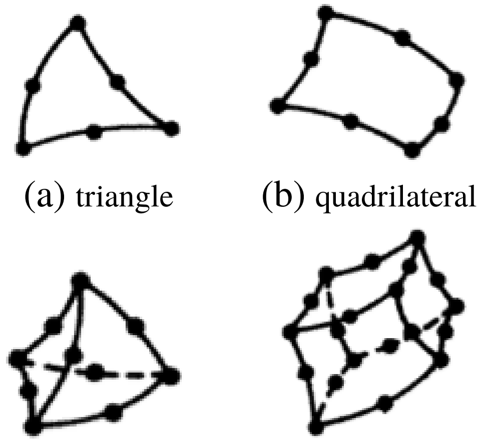
(a) line element

(b) triangle (c) rectangle (d) quadrilateral

(e) tetrahedron (f) right prism (g) hexahedron

Figure 2.1: Basic Element Shapes

The nodes assigned to all these elements have been shown by a black dot. Nodes may be classified as either exterior or interior depending on their



(a) triangle (b) quadrilateral

(c) tetrahedron (d) hexahedron

Figure 2.2: Common Quadratic Isoparametric Elements

location relative to the geometry of an element. Exterior nodes lie on the boundary of an element and they represent the points of connection between bordering elements. Nodes positioned at the corners of an element, along the edges or on the surfaces are all exterior nodes. Nodes that do not connect with neighboring elements are called interior nodes.

Apart from element shape, two other features that characterize a particular element are – (1) the number of nodes assigned to the element and (2) the number and type of nodal variables chosen for the element. The numbers of nodal variables or the parameters assigned to an element are called the degrees of freedom of the element. They can be exterior or interior in relation to the element boundaries depending upon whether they are assigned to exterior or interior nodes.”

2.1.2 Interpolation Functions with Generalized Coordinates:

“The functions used to represent the behavior of a field variable within an element (element equations) are called the interpolation functions or shape functions or approximating functions. These functions can be of many types; however polynomials have been used widely to express these functions due to ease of their integration or differentiation in one, two or three dimensions. The interpolation function cannot be chosen arbitrarily. Certain continuity requirements must be there to satisfy the convergence criteria. The degree of continuity of a field variable is said to be C^0 if the field variable is continuous at element interfaces. It is said to be C^1 if the first derivatives are continuous and C^2 if the second derivatives of these equations are continuous. If the function appearing under the integrals in the element equations contain derivatives up to $(r + 1)^{th}$ order, then for rigorous assurance of the convergences – compatibility requirement and completeness requirement must be satisfied as element size decreases i.e. at element interfaces one must have C^r continuity to satisfy compatibility requirement and C^{r+1} continuity within an element to satisfy completeness requirement. In one, two and three dimension, a general complete nth order polynomial can be expressed as in equation 2.1 to 2.3.

$$P_n(x) = \sum_{k=1}^{T_n(1)} \alpha_K x^i, \qquad i \leq n \tag{2.1}$$

where,

$T_n^{(1)} = n + 1$
for $n = 1, T_1^{(1)} = 2, \quad and \quad P_1(x) = \alpha_1 + \alpha_2 x$
for $n = 2, T_2^{(1)} = 3 \quad and \quad P_2(x) = \alpha_1 + \alpha_2 x + \alpha_3 x^2$ and so on

$$P_n(x, y) = \sum_{k=1}^{T_n(2)} \alpha_K x^i y^j, \qquad i + j \leq n \tag{2.2}$$

where,

$T_n^{(2)} = (n+1)(n+2)/2$
for $n = 1, T_1^{(2)} = 3, \quad and \quad P_1(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y$
for
$n = 2, T_2^{(2)} = 6, P_2(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy + \alpha_5 x^2 + \alpha_6 y^2$
and so on

$$P_n(x,y,z) = \sum_{l=1}^{T_n(3)} \alpha_l x^i y^j Z^k, \qquad i+j+k \leq n \tag{2.3}$$

where the number of terms in polynomial is

$T_n^{(3)} = (n+1)(n+2)(n+3)/6$

for $n = 1, T_1^{(3)} = 4, \quad and \quad P_1(x,y,z) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z;$

for $n = 2, T_2^{(3)} = 10, P_2(x,y,z) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z + \alpha_5 xy + \alpha_6 xz + \alpha_7 yz + \alpha_8 x^2 + \alpha_9 y^2 + \alpha_{10} Z^2$ and so on ;

For two and three dimensional polynomials, if the terms are placed in triangular array, a triangle similar to Pascal triangle as shown in figure 2.3 for two dimensional polynomial and Pascal tetrahedron as in figure 2.4 for three dimensional polynomial.

The coefficients α_k in all above polynomials which are representing the field variables, are called the generalized coordinates of the element. They are the independent parameters and specify the magnitude of the prescribed distribution for ϕ- a polynomial function. The shape of the prescribed distribution is decided by the order of the polynomial which depends on the

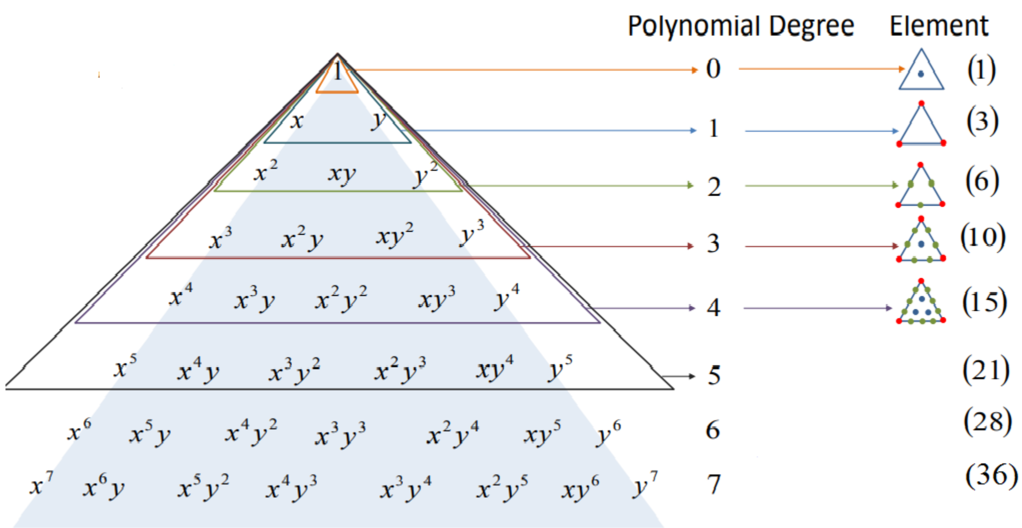


Figure 2.3: Pascal Triangle in Two Dimension

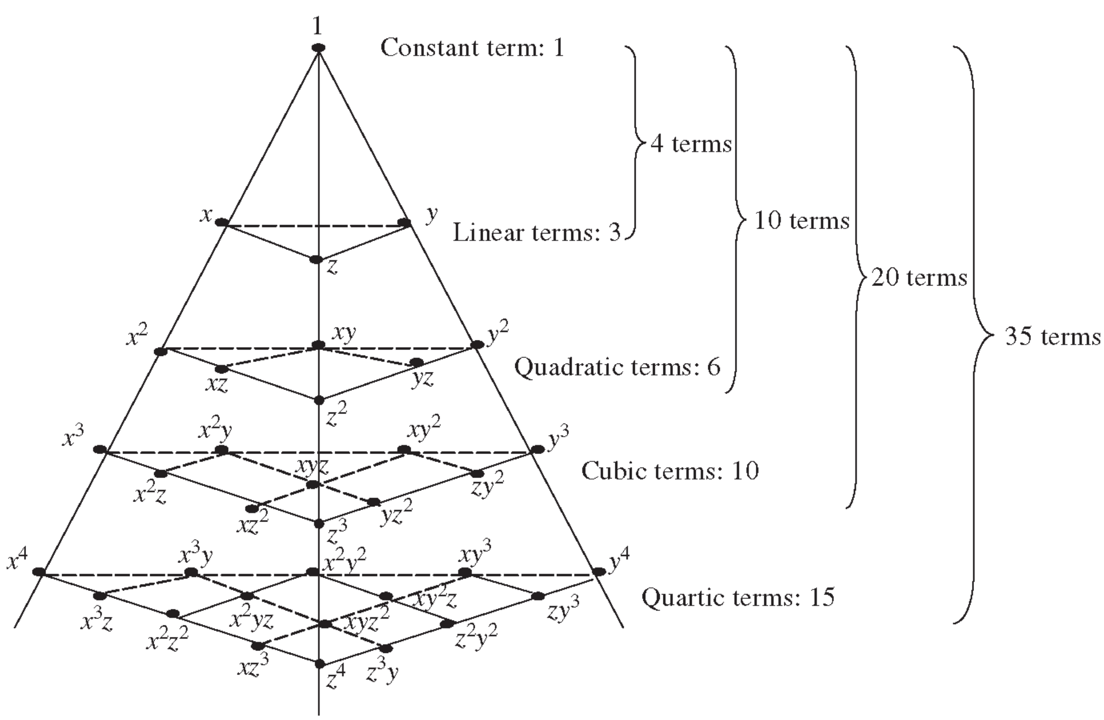


Figure 2.4: Pascal Tetrahedron

number of degree of freedom assigned to the element. Accordingly the number of coefficients in a polynomial should be equal to the number of nodal variables.

While choosing a polynomial expression as an interpolation function for an element, compatibility and completeness requirements are to be satisfied to ensure continuity of the field variable and convergence to the correct solution as the element mesh size is made smaller and smaller. Apart from these two requirements, a polynomial needs to possess geometric isotropy that is the polynomial expansion for the element must remain unchanged under a linear transformation from one Cartesian coordinate system to another. Two axioms that allow constructing polynomial series with this desired property are:

1. Polynomials of order n those are complete – contain all their terms – are said to have geometric isotropy.

2. Polynomials of order n those are incomplete – yet to contain the appropriate terms to preserve symmetry are said to have geometric isotropy.

According to the first axiom, the polynomials expressed by equations 2.1 to 2.3, when used as an interpolation function, remain invariant under linear coordinate transformation. The second axiom talks about symmetry during truncation of the polynomial. The polynomial series must be truncated by dropping the terms those occur in symmetric pairs to preserve the geometric isotropy. Equation 2.4 represents once such truncated cubic polynomial showing eight terms that actually contain ten terms.

$$P(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_7 x^3 + \alpha_{10} y^3 \quad (2.4)$$

OR

$$P(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 xy + \alpha_6 y^2 + \alpha_8 x^2 y + \alpha_9 xy^2$$

An interpolation function represented by a polynomial series – for a rectangular element as shown in figure 2.5, with its nodes positioned at the corners – is shown in equation 2.5.

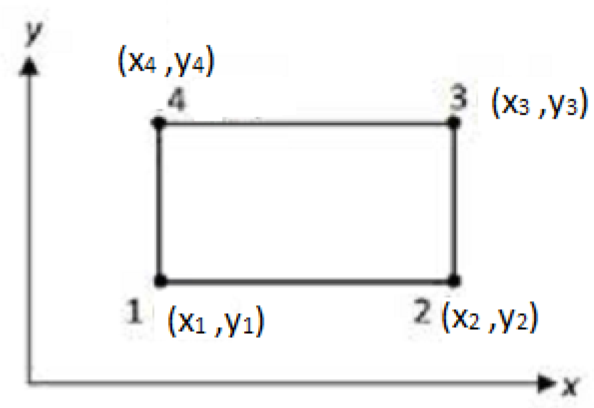


Figure 2.5: A Rectangular Element with Global Coordinate

$$\phi(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy \tag{2.5}$$

The coefficients of this series are generalized coordinates. Since one value of ϕ is assigned to each node, the element has four degrees of freedom and a set of simultaneous equations may be represented as in equation 2.6

$$\phi_1 = \alpha_1 + \alpha_2 x_1 + \alpha_3 y_1 + \alpha_4 x_1 y_1$$
$$\phi_2 = \alpha_1 + \alpha_2 x_2 + \alpha_3 y_2 + \alpha_4 x_2 y_2$$
$$\phi_3 = \alpha_1 + \alpha_2 x_3 + \alpha_3 y_3 + \alpha_4 x_3 y_3$$
$$\phi_4 = \alpha_1 + \alpha_2 x_4 + \alpha_3 y_4 + \alpha_4 x_4 y_4$$

or, in matrix form

$$\{\phi\} = [G]\{\alpha\} \tag{2.6}$$

Where

$$\{\phi\} = \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{Bmatrix}$$

$$[G] = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2 y_2 \\ 1 & x_3 & y_3 & x_3 y_3 \\ 1 & x_4 & y_4 & x_4 y_4 \end{bmatrix}$$

$$\{\alpha\} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{Bmatrix}$$

The generalized coordinates can be expressed as the solution of equation 2.6 for $\{\alpha\}$,

$$\{\alpha\} = [G]^{-1}\{\phi\} \tag{2.7}$$

Expressing the terms of equation 2.5 as a product of a row and a column vector,

$$\phi = \lfloor P \rfloor \{\alpha\} \tag{2.8}$$

where,

$\lfloor P \rfloor = \lfloor 1 \quad x \quad y \quad xy \rfloor$

Substituting equation 2.7 into equation 2.8

$$\phi = \lfloor P \rfloor [G]^{-1}\{\phi\} = \lfloor N \rfloor \{\phi\} \tag{2.9}$$

with

$$\lfloor N \rfloor = \lfloor P \rfloor [G]^{-1} \tag{2.10}$$

Equations 2.7 to 2.10, though represent a case of rectangular element, are generally applicable to all straight elements. The original interpolation polynomial $\lfloor P \rfloor \{\alpha\}$ should not be confused with the interpolation functions N_i associated with the nodal degree of freedom. $\lfloor P \rfloor \{\alpha\}$ is an interpolation function that applies to the whole element and expresses the field variable behavior in terms of the generalized coordinates, whereas the interpolation functions N_i refer to the individual nodes and individual nodal degrees of freedom; collectively representing the field variable behavior. It takes on unit value at node i and zero value at all the other nodes of the element.

Solution to above equations for a large system is very time consuming as it requires very large computational efforts in obtaining $[G]^{-1}$. The alternative to this method is the use of Natural Coordinate system as described in section 2.1.3 for a three dimensional tetrahedron element."

2.1.3 Interpolation Functions with Natural Coordinates

"A local coordinate system that relies on the element geometry for its definition and whose coordinates range between zero and unity within the element is known as a natural coordinate system. In this system, one particular coordinate has unit value at one node of the element and zero value at the other node(s) and its variation between nodes is linear. The main advantage of using Natural Coordinates is that it allows the use of closed-form integration formulae to evaluate the integrals in the element equations. Also the use of Natural coordinates makes it easier to develop curve sided elements. The natural coordinate system makes it easy to describe the location of a point inside an element in terms of the coordinates associated with the nodes of that element. Natural coordinates are denoted as $L_i(i = 1, 2, \ldots, n)$, where n represents the numbers of external nodes of an element. The natural coordinates are the functions of the global Cartesian coordinate system in which normally an element is defined. The natural coordinates for a four node tetrahedron element as shown in figure 2.6 is a set of volume coordinates. They are related with the global Cartesian coordinates as shown in equation 2.11.

$$\begin{aligned} x &= L_1x_1 + L_2x_2 + L_3x_3 + L_4x_4 \\ y &= L_1y_1 + L_2y_2 + L_3y_3 + L_4y_4 \\ z &= L_1z_1 + L_2z_2 + L_3z_3 + L_4z_4 \\ L_1 + L_2 &+ L_3 + L_4 = 1 \end{aligned} \tag{2.11}$$

Further solving equation 2.11, it gives equation 2.12

$$L_i = \frac{1}{6V}(a_i + b_ix + c_iy + d_iz), \qquad i = 1, 2, 3, 4 \tag{2.12}$$

where,

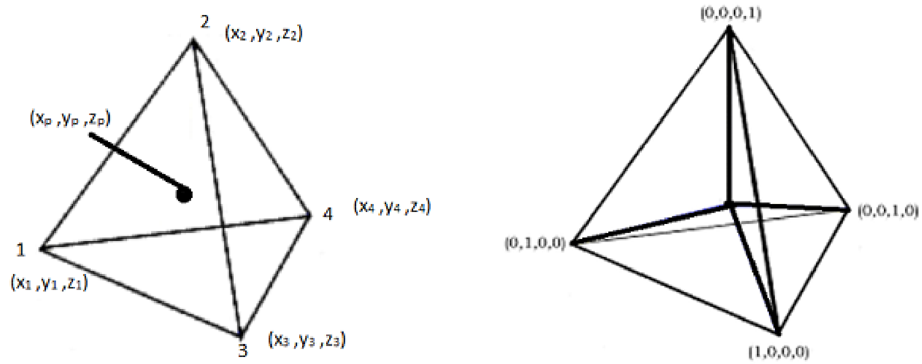
$$6V = \begin{vmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix} = 6(\text{volume of the tetrahedron defined by nodes 1,2,3,4 }) \tag{2.13}$$

The other constants can be derived using a cyclic permutation of the subscripts 1 to 4. As these constants are the cofactors of the determinant in equation 2.13, they must be represented with proper sign i.e. negative or positive.

If the field variable ϕ is represented as a function of L_i instead of x, y and z, then

$$\phi(x, y, z) = \phi_1L_1 + \phi_2L_2 + \phi_3L_3 + \phi_4L_4$$

The appropriate differentiation and integration formulae can be expressed as in equations 2.14 and 2.15."



(a) (b)

Figure 2.6: Tetrahedron Element (a) in Global Coordinates (b) in Natural (Volume) Coordinates

$$
\begin{aligned}
\frac{\partial \phi}{\partial x} &= \sum_{i=1}^{4} \frac{\partial \phi}{\partial L_i}\frac{\partial L_i}{\partial x} \\
\frac{\partial \phi}{\partial y} &= \sum_{i=1}^{4} \frac{\partial \phi}{\partial L_i}\frac{\partial L_i}{\partial y} \\
\frac{\partial \phi}{\partial z} &= \sum_{i=1}^{4} \frac{\partial \phi}{\partial L_i}\frac{\partial L_i}{\partial z}
\end{aligned}
\tag{2.14}
$$

$$
\int_{V^{(e)}} L_1^{\alpha} L_2^{\beta} L_3^{\gamma} L_4^{\delta} \mathrm{d}V^{(e)} = \frac{\alpha!\beta!\gamma!\delta!}{(\alpha+\beta+\gamma+\delta+3)!}6V \tag{2.15}
$$

2.2 FEM – Application in Transformer Designing - A Retrospective

As discussed in chapter 1, the circuit theory models for designing transformers are not much accurate in determining the transformer parameters such as winding impedance, leakage inductance, hot spot temperature etc. The physical realization of these parameters is needed on a prototype unit. The finite element method can play a vital role in deriving these parameters without any physical verification. Various research approaches and papers show the effectiveness of finite element method in determining the above mentioned parameters while designing the transformers - both oil cooled as well as dry type - for power and distribution sectors as well as to analyze and detect the internal faults in the transformer.

The main objective of applying finite element method while designing a transformer is, to estimate the temperature rise and hot spots under non linear load conditions, to determine the impedance value and short circuit forces to identify the electrical stresses and weakest insulation points and to estimate the correct insulation level to avoid the HV failures.

2.2.1 Transformer Design Parameters - Realization using FEM

The use and effectiveness of finite element method in designing the power transformer as compared to magnetic circuit theory [120] have been discussed in various papers like, a three dimensional finite element analysis of electric fields at winding ends of dry type transformer [23], hot spot and life evaluation of power transformer [26], [25] & [49], predicting hottest spot temperatures in ventilated dry type transformer windings [6], leakage inductance calculations [48], thermal modeling of disc type winding, foil type winding and transformer assembly for ventilated dry type transformers [39], [45] & [53], a generalized finite element analysis of three-dimensional heat transfer problems exhibiting sharp thermal gradients [41], thermal analysis of power transformer using an advanced coupled 3D heat transfer and fluid flow FEM model [59], leakage flux and force calculation on power transformer windings [4], experimental verification of short circuit electromagnetic forces in a dry type transformer using FEA [57], transformer over heating under non linear and unbalanced load conditions citepie95, [22], [24] and [54], internal winding fault detection and analysis [19], [27] & [37], analysis of short circuit performance of split winding transformer using coupled field circuit approach [31], etc.

In a paper presented by S C Bell and P S Bodger [29] and [14], a comparison has been made between magnetic circuit theory and finite element method for designing the power transformer. It summarizes the reverse method of transformer design.

In their paper, first the models for the resistive and inductive reactance components of the Steinmetz 'exact' transformer equivalent circuit have been developed from fundamental theory as presented in [25]. Then two and three dimensional linear and non linear magneto static finite element models were introduced as an alternative model for the inductive reactance components.

To demonstrate the reverse design method, the author have designed, built and tested two single phase, 50 Hz, high voltage transformers, the results of which are summarized in table 2.1

R_c =Transformer core resistance
R_w = Transformer winding resistance
X_m = Magnetizing reactance of the transformer
X_l = Leakage reactance of the transformer
DM = Direct Measurement
CTM = Circuit theory model
LFEM = Linear finite element model
NLFEM = Non-linear finite element model

Table 2.1: Calculated and Measured Equivalent Circuit Parameters for Sample Transformers

Method	Equivalent Circuit Parameters in Ω			
	R_c	R_w	X_m	X_l
Transformer 1 : Primary 240 V, Secondary 6240 V, 200 VA				
DM	3388	10.0	1987	2.8
CTM	1342	11.5	1383	1.9
LFEM	-	-	1905	1.6
NLFEM	-	-	1883	-
Transformer 2 : Primary 14 V, Secondary 4560 V, 617 VA				
DM	18	0.043	41	0.012
CTM	9.9	0.055	20	0.016
LFEM	-	-	25	0.015
NLFEM	-	-	54	-

The results show that the non linear finite element model most accurately calculated the magnetizing reactance value of the two sample transformers. For transformer 1 the non linear model seems to be less accurate however this may be due to the approximations made in the geometry of the finite element model.

D. Azizian, M. Vakilian, and J. Faiz [48] have introduced analytical and FEM based models for electromagnetic modeling and inductance calculation in dry type multi-winding traction transformers. The accuracy of these models is verified against the experimental data for a traction transformer having specifications as given in table 2.2.

The test results are as shown in table 2.3 from which it is evident that the FEM has better and most accurate results. The accuracy of axi-symmetric 2D FEM model goes on increasing as we increase the complexity of modeling from simplest core (2D-S) to full core (2D-F) to improved core (2D-I). The accuracy of 3D model is much higher than the 2D models.

The experimental results of this paper also yield that the proper modeling of the transformer core further helps in determining the value of leakage inductance due to change in the cross section area of the core. This is especially very helpful in designing the converter duty transformers where leakage inductance between the transformer windings is of utmost importance to decide the capacitance value for the LCL filter. The leakage inductance of the transformer itself forms a part of the filter circuit.

Table 2.2: Specifications of Typical Traction Transformer

Power Rating 4000 KVA	Vector Group Dd0/y11
HV Line Voltage $(H1\|\|H2) = 20\ KV$	HV Line Current $(H1\|\|H2) = 115.5\ A$
LV Line Voltage $(L_1\|\|L2) = 750\ V$	LV Line Current $(L1\|\|L2) = 1540\ A$

Table 2.3: Leakage Inductances of Traction Transformer

Test / Measurement Method	Leakage Inductances %					
	L1H1	L1H2	L1L2	L2H1	L2H2	H1H2
Analytical Results	5.9	72.5	55.2	71.7	88.1	6.0
2D-Simplest core	5.679	44	45	44	42	5.686
2D-Full Core	5.808	111	112	112	110	5.84
2D–Improved Core	5.794	98	100	99	97	5.834
3D Model	5.892	88	88	88	88	5.914
Experimental	5.797	74	73	72	72	5.943

N. L. Allen and et.al [13] have discussed the behavior of air and its breakdown at elevated temperatures in non uniform electric fields. V. E. Gonzalez and et.al [50] have proposed the designing of insulating supports in medium voltage dry type transformers. The effect of the high operation temperatures of the transformer on the voltage breakdown in the insulating supports was investigated and the electric field distribution on different shapes/profiles of the supports was identified using finite element method.

The impact of inrush currents on the mechanical stresses developed in a high voltage power transformer coils during the switching of a transformer has been discussed in [21]. The behavior of superconducting transformer against inrush current has been studied by S. Nishimiya and et.al [32]. A pre-fluxing technique has been discussed for a single phase transformer to reduce the magnetizing inrush current at the instant of switching of a transformer in [58].

M. Enokizono and N. Soda [12] did the analysis for finding out the core losses using improved FEM. In [15], measurement of eddy current loss coefficient have been done and de-rating of single phase transformer has been suggested in comparison with k-factor approach. Effect of saturation and eddy currents in commercial variable transformer has been modeled using Finite element method by S. H. Khan and et.al [18]. Calculation of eddy current field in the flange for the bushing and tank wall of a large power transformer has been presented in [34]. M. A. Venegas Vega and et.al [36] have estimated the stray losses in a three phase transformers using 3D finite element method. Investigation of no load and load losses in a amorphous core dry type transformers has been done in [47].

2.2.2 Life evaluation of Transformer using FEM

One of the most important parameters governing the life expectancy of a transformer is the hot-spot temperature value. Stray losses and non linear loads in a transformer are one of the main contributing factors in creating such hot spots which all together decide the life of a transformer.

The stray losses in a transformer are caused by the time variable leakage flux which induces emf and circulates eddy currents in the winding as well as conducting parts of the transformers such as clamps, core, tank wall etc. Evaluation of stray losses can be done quite accurately by FEM as discussed by A. S. Reddy and M. Vijaykumar [26].

Dejan Susa [25] has developed the models to determine the hottest spot in a transformer based on heat transfer theory, application of the lumped capacitance method, the thermal-electrical analogy and a new definition of nonlinear thermal resistances at different locations within a power transformer. The changes in oil viscosity, loss variation with temperature and changes in transformer time constants due to changes in oil viscosity were also accounted for in the thermal models. The results showed that the top oil temperature time constant is shorter than the time constant suggested by the present IEC loading guide, especially in cases where the oil is guided through the windings in a zigzag pattern for the ONAN and ONAF cooling modes. The models are validated using experimental results, which have been obtained from a series of thermal tests performed on a range of power transformers.

With development of the electronic equipments such as Computers, UPS, and High Frequency Drives for motor loads, arc furnaces, Electronic Ballasts, Compact Fluorescent Lamps, etc. the harmonic content in the power distribution network has increased tremendously. These highly non linear loads are present not only in the industrial sectors but also in the commercial sector and the first victim to any such load is always a transformer feeding them. These non linear loads result into the overheating of the transformers. The non linearity of the loads is best judged by the K factor [51] as described by equation 2.16 and accordingly the transformer required will be designated as K-rated transformer.

The K-rated transformer does not mitigate the harmonic but is capable enough to sustain the overheating due to such non linear loads. The design and application considerations along with testing approach based on UL, NEMA and IEEE standard C57.110-1986 are best described by L. W. Pierce, member IEEE [8].

$$K = \sum_{1}^{h} \left(\frac{{I_h}^2}{{I_{RMS}}^2} \right) h^2 \tag{2.16}$$

where

h = order of harmonic
I_h= RMS value of the current for h order harmonic
I_{RMS} = RMS value of the total load current

A three dimensional finite element method using a magnetic scalar potential formulation is applied [24] to compute the magnetic field in free and iron spaces. The calculation is then combined with a mixed analytical and numerical form of the electrical circuit equation to take into account the skin and proximity effects in the rectangular windings in dry type transformers under non linear load conditions. A two step FEM using the both reduced and total magnetic scalar potentials throughout a penalty term is proposed in this paper.

Accordingly the magnetic field into the air gap (Ω_a) and in the core (Ω_c) based on the reduced and total magnetic scalar potential ϕ follows the governing equations:

$$\begin{aligned} ln\Omega_a : H = H_j - grad(\phi) \\ ln\Omega_c : H = -grad(\phi) \end{aligned} \tag{2.17}$$

Where, the source magnetic field H_j can be computed using either the Biot-Savart law or a fictitious field for the known current distribution [4]. The relationship given in equation 2.17 has been solved from a two step formulation using a difference magnetic field $h = H - H0$, considered as being a disturbance of $H0$ due to the core saturation. In first step the magnetic permeability μ was assumed to be infinite and a reduced magnetic scalar potential ϕ_1 was used to calculate the magnetic field $H0$. In the second step, a total magnetic scalar potential ϕ_2 formulation was applied with $H0$ as the new magnetic source field in order to calculate the difference field h.

Based on the magnetic field described above and the empirical formula of the ac resistance component as in [110], the additional losses in the winding due to the harmonic currents have been modeled in [24] on a sample 10 KVA dry type distribution transformer. The nominal current of the transformer is applied to the windings for all frequencies in the above paper. However with non linear loads the harmonics do not own the same amplitude. The power losses are then need to be weighted and modeled in accordance with the current to estimate the exact value of temperature rise.

The other major factor that decides the life of a transformer is the designing of insulation system. The determination of insulation level can best be identified by determining the electric fields at the winding ends. Using three dimensional FEM and ANSYS software, J. Hong, L Heyun and X. Zihong [23] have estimated the electric fields at the winding end in 10KV SG10 dry type transformer. The similar approach can be extended to other higher rating transformer too at the design stage itself to estimate the correct insulation level and consequently increase the life of a transformer.

2.2.3 Fault detection in Transformer using FEM

Majority times the faults in a transformer are turn to turn, disk to disk or turn/disk to earth short circuit. This occurs mainly because of the aging of the insulation or displacement of the insulation during transportation or maintenance. This leads to the overheating of the transformer and finally results into the transformer failure. Study shows that around 70-80% of the transformer failures are due to the short circuit between turns. Considering this damage occurring in the transformer and time and cost involved in rectifying the same, it seems that simulation involving modeling of the transformer for detecting the fault is the most economical and convenient way.

One of such methods of modeling distribution transformer with internal short circuit faults using Finite Element Analysis (FEA) is presented in [19]. Based on the physical information of the transformer, the finite element model for a normal transformer or a transformer with an internal fault was implemented by commercially available software. The resulting circuit model was exported and used in a circuit analysis package to study the terminal behavior of the transformer. The observations of the simulation are compared with the experimental results and it shows that the FEA model can provide an accurate estimation of the internal winding faults.

The electromagnetic quasi-static finite element approach [37] describes the detection of winding short circuit faults with the help of frequency response analysis. The principle of coupled circuit approach and electromagnetic quasi-static analysis approach were applied along with FEA for determining the short circuit current under different fault conditions.

A case study on 30 MVA, 63 KV/20 KV, Ynd1, and 50 Hz transformer has been discussed in this paper [37]. Parameters of transformer are estimated by means of finite element analysis and utilized in circuit based model and the input impedance is calculated in wide band frequency. In addition to classifying and analyzing main types of short circuit according to IEEE standard C57.140, frequency response of disk-disk short circuit state is also investigated in several points in the HV winding to identify the location of the short circuit fault along the winding. The results have been summarized in terms of deviation of resonance component which showed that the first resonance of the input impedance due to short circuit moves orderly giving a better idea about the location of the short in a winding.

2.3 Conclusion

The discussion above reveals that the finite element method is an efficient tool for designing the transformer. Whereas the determination of impedance, winding hot spots temperature, insulation class and insulation level etc. are the major challenges while designing a transformer, the FEA can serve a vital role and may provide a cost effective and efficient solution to the problem. The preliminary results obtained from finite element model may be useful to develop cost effective and efficient design. However from all above discussions, it is evident that a three dimensional finite element model is predominant over a two dimensional model. While initiating the analysis a two dimensional model may be adopted but for the betterment of results the analysis must be completed by developing a three dimensional model only.