Chapter 4

Design Optimization

4.1 General

In engineering applications, optimization is an essential process. In today's highly competitive market, any design need to be efficient, reliable and cost effective. This demands optimization design stage as well as at design implementation and production stage. In present resaearch work, a dry type low and medium voltage transformer designing using finite element method has been discussed so far. The main emphasis was on determination of short circuit forces (impedance value), prediction of temperature rise and hot spot evaluation, defining magnetizing characteristics (inrush current) and identifying the minimum value of air clearances to avoid failures due to high voltage surges.

The parameters being obtained during the design stage need to be optimized to have efficient, reliable and cost effective design. The parameters – magnetizing characteristic and air clearance are mainly governed by the inherent characteristics of the raw material. Also the magnetizing properties are being constrained by the requirement of the customer for no load losses whereas air clearances are being restricted due to the properties of insulating material available in the market. Hence in present work, the design optimization has been worked out for impedance value and temperature rise parameters using interior point method as discussed in section 4.2.

4.2 Optimization Problem and Optimization Techniques

The conventional design procedure leads to a design that mostly satisfies the functional and other requirements of the design problem. However there may be more than one acceptable design and the best among them can be identified using the optimization techniques based on the selected criteria. The statement of an optimization problem can be stated as in equation 4.1 [116] and [114].

$$Find \quad X = \begin{Bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{Bmatrix} which\ minimizes\ f(X) \tag{4.1}$$

subject to the consstraints

$g_j(X) \leq 0 \qquad j = 1, 2, ..., m$

$l_j(X) = 0 \qquad j = 1, 2, ..., p$

Where, X is an n-dimensional vector called the design vector; $f(X)$ is called the objective function, $g_j(X)$ and $l_j(X)$ are known as inequality and equality constraints respectively. The number of variables n and the number of constraints m and / or p need not be related to each other. Since this equation contains the constraints, it is called constrained optimization problem.

An optimization problem can be classified in different ways as stated below.

- Constrained or unconstrained problem depending on the existence of the constraints.
- Parametric (static) or trajectory (dynamic) problem based on the nature of design variable.
- Optimal control or non-optimal control problem depending on the physical structure of the problem.
- Linear or nonlinear programming problem based on the nature of equations involved.
- Integer or real valued programming problems in accordance to the permitted values for the design variables.
- Deterministic and non-deterministic (stochastic) programming problems based on the deterministic nature of the design variables.
- Separable or non separable problem based on the separability of the objective and constraint functions.
- Single objective or multi-objective programming problems based on the number of objective functions.

As shown in table 4.1, there are various optimization techniques and selection of any one among them depends on the type and nature of

Table 4.1: Optimization Techniques

Optimization Techniques	Stochastic Process Techniques	Statastical methods
Calculas method	Statistcal desicion theory	Regression analysis
Calculas of variations	Markovv processes	Cluster Analysis,
Nonlinear Programming	Queuing theory	Pattern Recognition
Geometric Programming	Renewal theory	Design of experiments
Quadratic Programming	Simulation methods	Discriminate analysis
Linear Programming	Reliability theory	factir analysis
Dynamic programming		
Integer programming		
Stochastic programmming		
Seperable programming		
Multiobjective programming		
Particle swarm optimization		
Neural Networks		
Fuzzy Optimization		

optimization problem as described above. The present research problem is a non linear constrained problem involving both equality and inequality constraints. For solving such problems, indirect methods are normally found suitable because in direct methods, the constraints are handled in an explicit manner whereas in indirect methods, the constrained problem is solved as a sequence of unconstrained minimization problems. Interior Penalty Function Method also known as barrier method with mixed equality and inequality constraints has been used in conjunction with Kuhn-Tucker conditions to solve the present design problem.

4.2.1 Interior Penalty Function Method with Equality and Inequality Constraints

The optimization problem represented in equation 4.1 can be converted into an unconstrained minimization problem by formulating a function as in equation 4.2.

$$\phi_k = \phi(X, r_k) = f(X) + r_k \sum_{j=1}^{m} G_j \left[g_j(X) \right] + H\left(r_k\right) \sum_{j=1}^{p} l_j^2(X) \tag{4.2}$$

Where, G_j is a function of g_j tending to infinity as the constraint boundary is approached and $H(rk)$ is a function of parameter r_k tending to infinity as r_k tends to zero.

Equation 4.2 can be represented in the form as in equation 4.3. Here, if ϕ_k is minimized for a decreasing sequence of values r_k then it can be proved that the unconstrained minima Xk will converge to the solution X of the original problem as expressed in equation 4.2.

$$\phi_k = \phi(X, r_k) = f(X) - r_k \sum_{j=1}^{m} \frac{1}{g_j(X)} + \frac{1}{\sqrt{r_k}} \sum_{j=1}^{p} l_j^2(X) \tag{4.3}$$

The iteration procedure for solving the above equation can be summarized as follows:

- Start with an initial feasible point X_1 such that it satisfies all the constraints. Take initial value of $r_1 > 0$ and set k = 1.
- Minimize $\phi(X, rk)$ and obtain the solution for X_k.
- Check whether X_k is the optimum solution for the original problem or not. If yes then terminate the process else go to the next step.
- Find the value of next penalty parameter, r_{k+1} as r_{k+1}= cr_k, where c <1
- Set the new value of k = k+1 and take the new starting point as $X_1 = X_k$ and go to step 2 till the convergence is achieved.

4.3 Optimization Problem Formulation

As discussed above, the optimization of a transformer design is to be carried out for committed impedance value and temperature rise (hot spot temperature) value. The impedance of a transformer referred to the primary side can be represented as in equation 4.4. The hot spot temperature of a respective windings – primary and secondary can be expressed as in equations 4.5 and 4.6 respectively.

$$\begin{aligned} Z_1 &= \sqrt{R_1^2 + X_1^2} \\ &= \sqrt{K_1^2(L_{mt1}\sigma_1 + L_{mt2}\sigma_2)^2 + K_2^2(3a + b_1 + b_2)^2(L_{mt1} + L_{mt2})^2} \end{aligned} \tag{4.4}$$

where
$K_1 = \rho N_1 / I_1$
$K_2 = \pi \mu_0 f N_1^2 / (3L_c)$

$$\theta_{cp} = q_p \rho_i (1 - s_{fp}^{0.5}) b_1^2 / 2 = K_p b_1^2 \tag{4.5}$$

where
$K_p = q_p \rho_i (1 - s_{fp}^{0.5}) / 2$

$$\theta_{cs} = q_s \rho_i (1 - s_{fs}^{0.5}) b_1^2 / 2 = K_s b_1^2 \tag{4.6}$$

where

$K_s = q_s \rho_i (1 - s_{fs}^{0.5})/2$

ρ=Resistivity of Winding material in Ω–m

N_1=Number of turns of Primary Winding

I_1 = Primary Current in A

L_{mt1}=Mean Length of Primary turn in m

L_{mt2}=Mean length of Secondary turn in m

σ_1=Current Density of Primary Winding in A/ sq m

σ_2=Current Density of Secondary Winding in A/ sq m

a = Thickness of Duct between Primary and Secondary Winding in m

b_1 = Thickness of Primary Winding in m

b_2 = Thickness of Secondary Winding in m

ρ_i = Thermal Resistivity of insulating material in $^{\circ}Cm/watt$

S_f = Space factor = Copper Area / Winding Area

While designing a transformer, parameters like no load losses, load losses (winding losses or I^2R losses), impedance value, temperature rise of the winding as well as overall temperature rise etc. are having restrictions as per customer's specifications as well as relevant standards. However tolerances do exist in accordance to the respective reference standards [67], [108]. A preliminary design is required to be carried out based on the prior experience. FEM can be used for obtaining preliminary design as discussed in chapter 3 as well as in [60], [61], [62], [64], [65], [69], [70], [76], [84] etc. Based on this, the optimization for the selected parameters can be done as represented in [71], [79], [85] and [96].

In present problem, it is aimed to optimize the design for impedance and temperature rise values so as to minimize the cost. The cost function can be represented as in equation 4.7as a function of impedance and hot spot temperature. While carrying out the optimization process, design ambient temperature has not been included as it remains unchanged during the entire process. Hence the hot spot temperature will be considered as the temperature rise of the winding and objective function value has been calculated.

$$f_c(Z,\theta) = f\left\{Z_1\right\} + f\left\{\theta_{cp}\right\} + f\left\{\theta_{cs}\right\} \tag{4.7}$$

where
Z_1=Impedance of the transformer referred to Primary Side
θ_{cp}=Hot Spot Temperature of Primary Winding in $^\circ$ C
θ_{cs}=Hot Spot Temperature of Secondary Winding in $^\circ$ C

All the parameters or variables narrated in equation 4.7 are interdependent variables. To achieve convergence simultaneously for all these variables is very time consuming and may lead to the problem of non convergence. This necessitates further simplification of the objective function to an extent possible. The difficulties encountered with objective function and the procedures adopted for its simplification are as described below.

- The impedance value has normally a tolerance band of +/- 10%. This imposes an inequality constraint for the objective function. $0.9Z \leq Z \leq 1.1Z$
- The hot spot temperatures for primary and secondary windings are already imposing inequality constraints and cannot be transformed to have unconstrained problem.

- The one way of reducing the interdependence of the variable is to fix some of the variables so that the process accuracy is not lost. This can be done by assuming the current densities for primary and secondary windings, respectively σ_1 and σ_2 as constants by taking their values same as considered during the initial design run.

- The other way of reducing the interdependence of the variables is to simplify the impedance function by eliminating the resistance part. Normally the resistance contributes around 10% and inductive reactance contributes around 90% in building up the impedance. If we eliminate the resistance part from impedance function and set the objective function value accordingly, the inductance value can be optimized without much compromise in the objective function value and entire optimization process converges fast. However this approach will affect the design finalization during reverse design approach and hence not advisable to be adopted unless and until convergence is not achievable.

- After evaluating all the variables, a reverse design will have to be carried out with practical considerations for all the variables as per availability of the raw material and other design limitations from specification point of view and / or manufacturing point of view.

4.4 Design Optimization in an Isolation Transformer

The optimization process has been carried out for an isolation transformer having a rating of 100 kVA, 415 V / 415 V. The specification for the same are narrated in table 4.2.

As evident from the specifications, although the insulation class is H, the permissible temperature rise is $70^{\circ}C$ only in accordance to the customer's requirement. This increases the complexity in the design and ultimately leads to the increase in costing if design is not being optimized. An initial design was carried out for this transformer and the values obtained for different design parameters and variables are given in table 4.3.

The results of initial design indicate that, the temperature rise is critical and any minor variation in the losses or change in the quality of the material will lead to the violation of temperature rise limits. Also the impedance value obtained is 3.73% for a single winding. This value of impedance is due to the resistance of the winding and self inductance only. The effect of mutual inductance is difficult to account for while calculating the impedance. When an actual transformer is being manufactured, due to the effect of mutual inductance an actual impedance value increases by around 3 - 5%. Thus there is a scope in the improvement in impedance value since the tolerance band on impedance value is of 10%. For this we can change the wire gauge that will change the mean length of turns and there by the effective resistance of the winding will change and ultimately improving the impedance value and the temperature rise, both.

Using equation 4.7, an optimization has been carried out by using Interior Point Method with Penalty Function having inequality constraints as defined hereunder as in equations 4.8, 4.9 and 4.10.

$$3.6\% \leq Z_1 \leq 4.4\% \tag{4.8}$$

$$50^{\circ}C \leq \theta_{cp} \leq 70^{\circ}C \tag{4.9}$$

Table 4.2: Specifications of 100 kVA Isolation Transformer

Specifications	**Parameter Value**
Rating	100 kVA, 3 Phase, 50 Hz
Primary / Secondary Voltage	415 V (Delta) / 415 V (Star)
Type of Connection	Dyn1
% Impedance at 75$°C$	4% (+/-10%)
No Load Losses (Maximum)	650 W
Copper Losses (Maximum) at 75 $°C$	1550 W
Insulation Class	H
Permissible Temperature Rise	70$°C$

Table 4.3: Design Variables / Parameters after Initial Design

Design Parameter / Variable	Obtained Value	Design Parameter / Variable	Obtained Value	Objective Function Value & Copper Weight
Lmt1	0.80 m	a	18 mm	128.4598 48 Kg
Lmt2	0.57m	b_1	13.8 mm	
σ_1	2.00 A / sq.mm	b_2	13.4 mm	
σ_2	1.93 A / sq.mm	% Z	3.73%	
θ_{cp}	67.48 $°C$	θ_{cs}	60.78 $°C$	

Table 4.4: Lower and Upper Bound for Design Variables

Design Variable	**Lower Bound**	**Upper Bound**
L_{mt1}	0.7 m	0.9 m
L_{mt2}	0.5 m	0.7 m
a	12 mm	24 mm
b_1	12 mm	16 mm
b_2	12 mm	16 mm

$$50°C \leq \theta_{cs} \leq 70°C \tag{4.10}$$

The lower and upper bound for the design variables are as shown in table 4.4.

After running an optimization program using Matlab R 2014 with an Interior Point Method, the obtained values of design variables are as shown in table 4.4 and design parameters and objective function value have been shown in table 4.5.

As evident from the values of design parameters shown above in table 4.5, all the parameters are within the required limits. Using reverse design approach [14], the winding losses have been also calculated and are found in limits as per customer's requirement. However, above solution cannot be worked out exactly in actual implementation due to the restrictions in the availability of wire sizes and spacers to be kept between windings. The winding thickness b1 and b2 for primary and secondary winding respectively will have to be corrected according to the available wire sizes. This will change the values of Lmt1 and Lmt2 also. Apart from this, the duct size "a" is also required to be rounded off to a next higher or lower integer as per availability of the duct. Accordingly the values of "a", "b1" and "b2" are selected as per raw material availability, as shown in table 4.7. Fortunately this has not forced to change

Table 4.5: Obtained Values of Design Variables

Design Variable	Obtained Value
L_{mt1}	0.8048 m
L_{mt2}	0.6947 m
a	14.9 mm
b_1	13.0 mm
b_2	13.3 mm

Table 4.6: Obtained Values of Design Parameters

Design Variable	Obtained Value	Objective Function
$\%Z_1$	3.612%	110.5287
θ_{cp}	59.58 $°C$	
θ_{cs}	50.76 $°C$	

the current densities and hence they have been kept unchanged. Further to this as mean length of secondary turns is also required to be increased, an interlayer duct of 10 mm thickness has been considered between the two layers of secondary winding. With reverse designing, the values obtained for design variables, design parameters and objective function are as shown in table 4.7.

Table 4.7: Design Variables / Parameters after Optimized Design

Design Parameter / Variable	TObtained Value	Design Parameter / Variable	Obtained Value	Objective Function Value & Copper Weight
L_{mt1}	0.805 m	a	14 mm	103.782 47 Kg
L_{mt2}	0.72 m	b_1	12.3 mm	
σ_1	2.00 A / sq.mm	b_2	13.4 mm	
σ_2	1.93 A / sq.mm	% Z	3.521%	
θ_{cp}	53.54 $°C$	θ_{cs}	50.26 $°C$	

Table 4.8: Obtained Values of Design Parameters

Design Variable	Obtained Value
$\%Z_1$	3.66%
θ_{cp}	64.5 $^{\circ}C$
θ_{cs}	63.8 $^{\circ}C$
No load loses	630 W
Copper Losses at 75 $^{\circ}C$	1475 W

4.5 Results

After running this optimization program, an actual transformer has been built and checked for the design parameters. The values obtained after actual testing have been summarized in table 4.8. The values indicated in the table for temperature rises are obtained after the complete heat run test for no load and simulated load condition as per IS 11171, 1985. They indicate the final temperature rise of the windings considering the effect of core losses.

4.6 Conclusion

The results of this chapter prove that the optimization of an objective function leads to a more realistic assumption during the design stage. Based on the obtained values of design variables and their feasibility of selection during the practical realization of an actual transformer, the need for prototype development can be avoided. Any parametric criticality observed during the initial stage – as in present case , the primary temperature value was out of bound during the initial design that can be handled during the design stage itself. This saves time, efforts and cost during the development of any new design. Apart from parametric achievements, it can also be seen that the copper content has been reduced by around 1 kg per coil. This makes the design cost effective and economical in present competitive environment.