

CHAPTER 3

MATHEMATICAL MODELLING OF CONTROLLERS

Describe the mathematical modelling of single machine infinite bus system along with Heffron Philips model, Modelling of Intelligent control based Fuzzy PSS, Modelling of Smart control based ANN PSS and simulation of power circuit for multimachine system.

3.1 Introduction

Numbers of methods have been reported in literature for control performance both for SMIB & Multimachine systems in the last decades. The control performance assessment is the key operation for power system stabilization aiming to meet the technological challenges to cope with the changing requirements.

3.2 Performance evaluation: Intelligent control (Fuzzy Logic Control)

3.2.1 Mathematical modeling of conventional power system stabilizer

The basic function of a PSS is to add damping to the generator rotor oscillations by controlling its excitation using auxiliary stabilizing signal. To provide damping, the stabilizer must produce a component of electrical torque in phase with the rotor speed deviation. The CPSS design consists of Stabilizer gain, signal Washout and phase compensation [4].

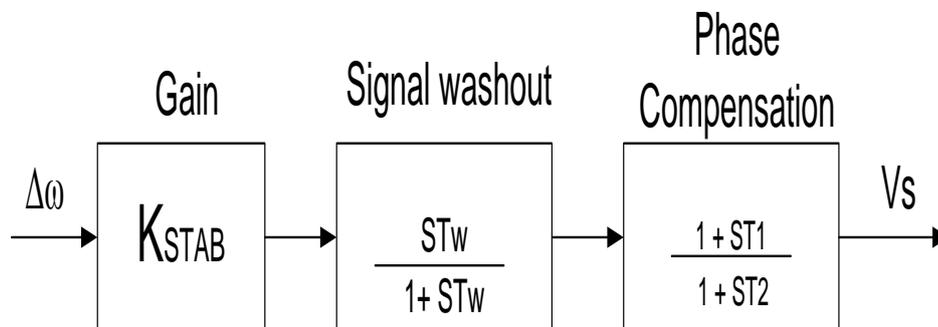


Fig. 3. 1 The basic block diagram of CPSS

✚ The **phase compensation block** provides the appropriate phase lead characteristics to compensate for the phase lag between exciter input and generator electrical torque.

✚ The **signal washout block** serves as high pass filter, with time constant T_w high enough to allow signals associated with oscillations in ω_r to pass unchanged, which removes D.C. signals. Without it, steady changes in speed would modify the terminal voltage. It allows PSS to respond only to changes in speed [5].

✚ The **stabilizer gain** K_{STAB} determines the amount of damping introduced by PSS. Ideally, the gain should be set at a value corresponding to maximum damping [6]. The Generator is connected to the infinite bus through transmission line as shown in Fig. 3.2. The main function of each block is discussed below:

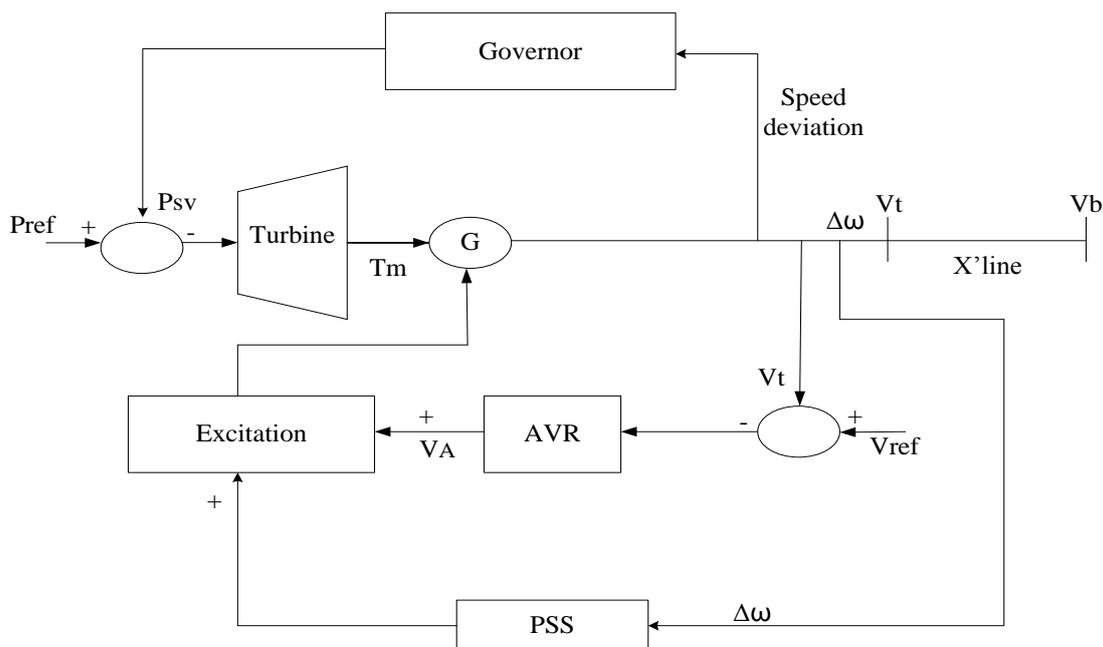


Fig. 3. 2Basic block diagram of Power system configuration for SMIB system

Synchronous generator: The Synchronous Generator is most common source of Electric power because it is simple to control, robust and maintenance free in a brushless system.

Turbine: Turbine converts Kinetic energy of a steam, fuel, water into rotational energy. It works as prime mover and thus provides mechanical input required to the Generator.

Governor: The job of the Governor is to maintain the speed that changes with variation in load. It provides correct amount of mechanical power input to the turbine to match the electrical output of the corresponding Generator.

- 1) The composition of the electrical torque influenced by $\Delta\delta$ over constant K1 and the internal incremental voltage $\Delta e'q$ over constant K2.
- 2) The effect of the field winding determined by the field winding constant K3 and influenced by $\Delta\delta$ over constant K4 and
- 3) The effect of the excitation system influenced by $\Delta\delta$ over constant K5 and $\Delta e'q$ over constant K6. The excitation system itself is modeled by a first-order transfer function including the amplification factor KA and the time constant TA [7].

Here, consider a single machine infinite bus system. Model 1.0 is used for Synchronous generator representation.

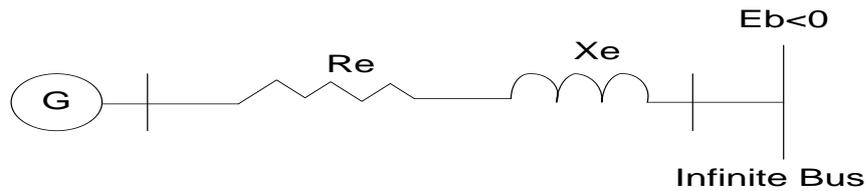


Fig. 3. 4 A single machine infinite bus system [8]

The algebraic equation of stator and rotor are,

$$E'_q + x'_d i_d = V_q \quad (3.1)$$

$$-x_q i_q = V_d \quad (3.2)$$

The complex terminal voltage can be expressed as,

$$V_Q + V_D = (i_q + j i_d)(R_e + j X_e) e^{j\delta} + E_b < 0 \quad (3.3)$$

Separating real and imaginary parts of above equation,

$$V_q = R_e i_q - X_e i_d + E_b \cos \delta \quad (3.4)$$

$$V_d = i_q X_e + R_e i_d - E_b \sin \delta \quad (3.5)$$

Put value of V_q and V_d in equation of stator and rotor, Thus we get the expression for i_d and i_q .

$$i_q = \frac{(X'_d + X_e) E_b \sin \delta - R_e E_b \cos \delta + R_e E'_q}{(X_q + X_e)(X'_d + X_e) + R_e^2} \quad (3.6)$$

$$i_d = \frac{1}{A} [R_e E_b \sin \delta + (X_q + X_e)(E_b \cos \delta - E'_q)] \quad (3.7)$$

$$\text{Where, } A = (X_q + X_e)(X'_d + X_e) + R_e^2 \quad (3.8)$$

The expression for i_q and i_d are,

$$\Delta i_d = C1 \Delta \delta + C2 \Delta E'_q \quad (3.9)$$

$$\Delta i_q = C3 \Delta \delta + C4 \Delta E'_q \quad (3.10)$$

$$\text{Where, } C1 = \frac{1}{A} [R_e E_b \cos \delta - (X_q + X_e)(E_b \sin \delta)] \quad (3.11)$$

$$C2 = -\frac{1}{A} (X_q + X_e) \quad (3.12)$$

$$C3 = \frac{1}{A} [(X'_d + X_e) E_b \cos \delta + R_e E_b \sin \delta] \quad (3.13)$$

$$C4 = \frac{1}{A} R_e \quad (3.14)$$

✚ Rotor mechanical equation and torque angle loop

The rotor mechanical torque equations are,

$$\frac{d\delta}{dt} = \omega_B (s_m - s_{m0}) \quad (3.15)$$

$$2H \frac{ds_m}{dt} = -D s_m + T_m - T_e \quad (3.16)$$

Linearize the equation of torque,

$$\Delta T_e = [E_{q0} C3 - (X_q - X'_d) i_{q0} C1] \Delta \delta + [E_{q0} C4 + i_{q0} - (X_q - X'_d) i_{q0} C2] \Delta E'_q \quad (3.17)$$

$$\text{Put, } K1 = E_{q0} C3 - (X_q - X'_d) i_{q0} C1 \quad (3.18)$$

$$K2 = E_{q0} C4 + i_{q0} - (X_q - X'_d) i_{q0} C2 \quad (3.19)$$

$$\text{Thus, } \Delta T_e = K1 \Delta \delta + K2 \Delta E'_q \quad (3.20)$$

$$\Delta s_m = \frac{1}{2Hs} [-D s_m + \Delta T_m - \Delta T_e] \quad (3.21)$$

From these two equations (3.20) and (3.21), Torque angle loop can be represented by the block diagram shown below [8]:

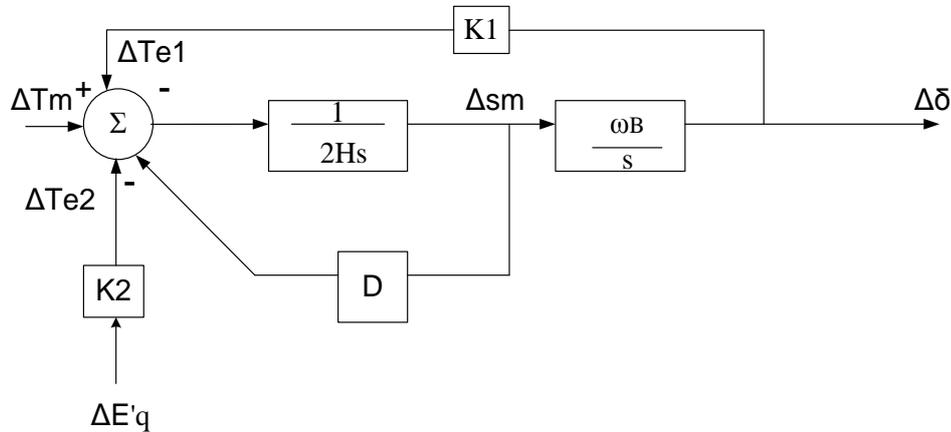


Fig. 3. 5 Torque angle loop

✚ Representation of flux decay

The equation for the field winding can be expressed as,

$$T'_{d0} \cdot \frac{dE'_q}{dt} = E_{fd} - E'_q + (X_d - X'_d)i_d \quad (3.22)$$

$$\Delta E'_q [ST'_{d0} + 1 - (X_d - X'_d)C2] = E_{fd} + (X_d - X'_d)C1\Delta\delta \quad (3.23)$$

$$\text{Consider, } K4 = -(X_d - X'_d)C1 \text{ and } K3 = \frac{1}{1 - (X_d - X'_d)C2} \quad (3.24)$$

$$\Delta E'_q [K3 ST'_{d0} + 1] = K3 E_{fd} - K3 K4 \Delta\delta \quad (3.25)$$

The above equation (3.25) represents the flux decay loop which is shown below:

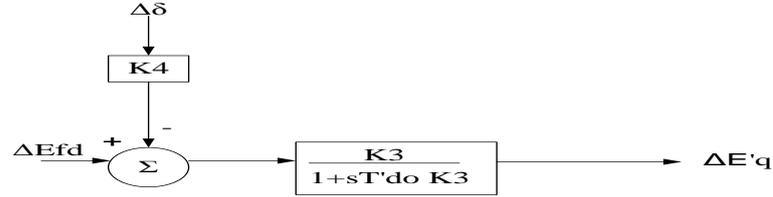


Fig. 3. 6 Flux decay loop [8]

Representation of Excitation system

For the present analysis, we ignore auxiliary signal Vs. The perturbation in the terminal voltage Vt can be expressed in terms of direct and quadrature axis.

$$\Delta V_t = \frac{V_{d0}}{V_{t0}} \Delta V_d + \frac{V_{q0}}{V_{t0}} \Delta V_q \quad (3.26)$$

$$\Delta V_t = \frac{V_{d0}}{V_{t0}} (-X_q C3 \Delta\delta - X_q C4 \Delta E'_q) + \frac{V_{q0}}{V_{t0}} (X'_d C1 \Delta\delta + (1 + X'_d C2) \Delta E'_q) \quad (3.27)$$

$$\Delta V_t = K5 \Delta\delta + K6 \Delta E'_q \quad (3.28)$$

$$\text{Where, } K5 = -\frac{V_{d0}}{V_{t0}} (X_q C3) + \frac{V_{q0}}{V_{t0}} (X'_d C1) \quad (3.29)$$

$$K6 = -\frac{V_{d0}}{V_{t0}} (X_q C4) + \frac{V_{q0}}{V_{t0}} (1 + X'_d C2) \quad (3.30)$$

The above equation (3.28) represents the block diagram of excitation system [8].

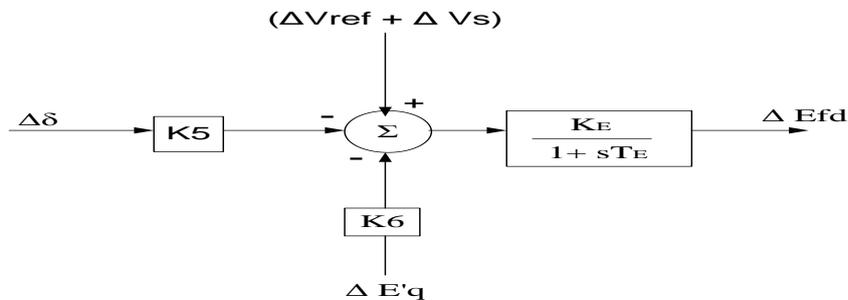


Fig. 3. 7 Excitation system loop

Computation of Heffron Phillips constants for lossless network

Here, $R_e=0$. We are neglecting the armature resistance. So, the loss less network on stator side.

$$\text{From, } K1 = E_{q0} C3 - (X_q - X'_d) i_{q0} C1 \quad (3.31)$$

$$C1 = \frac{1}{A} [R_e E_b \cos \delta o - (X_q + X_e)(E_b \sin \delta o)] \quad (3.32)$$

$$\text{Where, } A = (X_q + X_e)(X'_d + X_e) + R_e^2 \quad (3.33)$$

$$\text{So that, } C1 = \frac{-E_b \sin \delta o}{X'_d + X_e} \quad (3.34)$$

$$C3 = \frac{1}{A} [(X'_d + X_e) E_b \cos \delta o + R_e E_b \sin \delta o] \quad (3.35)$$

$$C3 = \frac{E_b \cos \delta o}{X_q + X_e} \quad (3.36)$$

$$\text{Thus, } K1 = \frac{E_b E_{q0} \cos \delta o}{X_e + X_q} + \frac{(X_q - X'_d)}{(X_e + X'_d)} E_b i_{q0} \sin \delta o \quad (3.37)$$

$$\text{Now, } K2 = E_{q0} C4 + i_{q0} - (X_q - X'_d) i_{q0} C2 \quad (3.38)$$

$$\text{And } C2 = -\frac{1}{A} (X_q + X_e) \quad (3.39)$$

$$C4 = \frac{1}{A} R_e = 0 \quad (3.40)$$

$$\text{Thus, } K2 = i_{q0} \left[1 + \frac{(X_q - X'_d)}{(X_e + X'_d)} \right] = \frac{(X_e + X_q)}{(X_e + X'_d)} i_{q0} \quad (3.41)$$

$$\text{Now, } K3 = \frac{1}{1 - (X_d - X'_d) C2} = \frac{1}{1 - (X_d - X'_d) \left(\frac{-1}{X_e + X'_d} \right)} \quad (3.42)$$

$$\text{Thus, } K3 = \frac{X_e + X'_d}{X_d + X_e} \quad (3.43)$$

$$\text{Now, } K4 = -(X_d - X'_d) C1 = -(X_d - X'_d) \left(\frac{-E_b \sin \delta o}{X'_d + X_e} \right) \quad (3.44)$$

$$\text{Thus, } K4 = \frac{(X_d - X'_d)}{(X'_d + X_e)} E_b \sin \delta o \quad (3.45)$$

$$\text{Now, } K5 = -\frac{V_{d0}}{V_{t0}} (X_q C3) + \frac{V_{q0}}{V_{t0}} (X'_d C1) \quad (3.46)$$

$$= -\frac{V_{d0}}{V_{t0}} \cdot X_q \cdot \left(\frac{E_b \cos \delta o}{X_q + X_e} \right) + \frac{V_{q0}}{V_{t0}} \cdot X'_d \cdot \left(\frac{-E_b \sin \delta o}{X'_d + X_e} \right) \quad (3.47)$$

$$\text{Thus, } K5 = \frac{-X_q V_{d0} E_b \cos \delta o}{(X_e + X_q) V_{t0}} - \frac{X'_d V_{q0} E_b \sin \delta o}{(X_e + X'_d) V_{t0}} \quad (3.48)$$

$$\text{Now, } K6 = -\frac{V_{d0}}{V_{t0}} (X_q C4) + \frac{V_{q0}}{V_{t0}} (1 + X'_d C2) \quad (3.49)$$

$$K6 = \left(1 - \frac{X'_d}{X'_d + X_e} \right) \frac{V_{q0}}{V_{t0}} \quad (3.50)$$

$$\text{Thus, } K6 = \frac{X_e}{X_e + X'_d} \left(\frac{V_{q0}}{V_{t0}} \right)$$

State space modelling of SMIB system

Consider the single machine system and assuming a synchronous Generator is represented by model 1.0 with neglecting damper winding. The mechanical loop of synchronous machine can be represented torque angle loop. Considering torque angle loop of synchronous machine which is shown Fig 3.5 [8].

From This loop,

$$x1 = \Delta\delta \tag{3.51}$$

$$\dot{x}1 = x2 = \Delta Sm \cdot \omega B \tag{3.52}$$

$$\dot{x}2 = -\frac{K1}{2H} \Delta\delta - \frac{D}{2H} \Delta Sm - \frac{K2}{2H} \Delta E'q \tag{3.53}$$

Now, the field winding equation can be represented by Flux decay loop of the synchronous machine. This is shown in Fig 3.6.

From this loop,

$$\dot{x}3 = -\frac{K4}{T'do} \Delta\delta - \frac{1}{T'doK3} \Delta E'q + \frac{1}{T'do} \Delta Efd \tag{3.54}$$

Now, the Excitation of the machine is represented by Excitation loop of the synchronous machine is shown in Fig 3.7.

From this loop,

$$\dot{x}4 = -\frac{KE K5}{TE} \Delta\delta + \frac{-KE K6}{TE} \Delta E'q - \frac{1}{TE} \Delta Efd + \frac{KE}{TE} (\Delta Vref + \Delta Vs) \tag{3.55}$$

Now, overall system block diagram consist of torque angle loop (rotor swing equation), flux decay and excitation system. We have to combine this three loop to obtain the overall system block diagram is shown below.

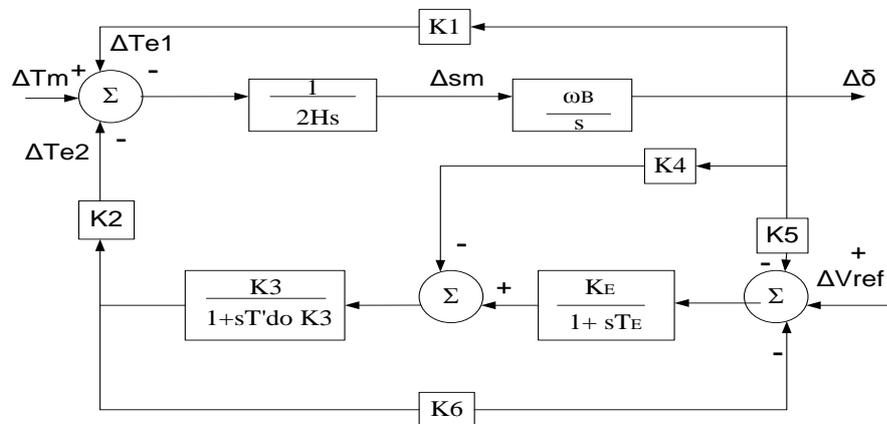


Fig. 3. 8 Overall block diagram of the system

Here, the damping term D in the swing equation is neglected because the value of d is generally small. We can also represent the system equations in the state space form. From the above block diagram, the following equations can be obtained:

$$\dot{x} = [A]x + [B](\Delta V_{ref} + \Delta V_s) \quad (3.56)$$

$$\dot{x} = \begin{bmatrix} 0 & \omega B & 0 & 0 \\ \frac{-K1}{2H} & \frac{-D}{2H} & \frac{-K2}{2H} & 0 \\ -K4 & 0 & -1 & 1 \\ \frac{T'do}{TE} & 0 & \frac{T'doK3}{TE} & \frac{T'do}{TE} \\ \frac{-KE K5}{TE} & 0 & \frac{-KE K6}{TE} & \frac{-1}{TE} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta S_m \\ \Delta E'q \\ \Delta Efd \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{KE}{TE} \end{bmatrix} (\Delta V_{ref} + \Delta V_s) \quad (3.57)$$

Here, $K1$ to $K6$ are Heffron Phillips constants. They are mainly depends on machine operating condition and the parameters of the machines. Generally $K1$, $K2$, $K3$ and $K6$ are positive. $K4$ is almost positive except for the higher value of Re . $K5$ is positive for low to medium external impedances and low to medium loadings. $K5$ is usually negative for higher value of impedances and heavy loadings [8].

➤ Formulation of Multimachine systems

A typical modern power system consists of a few thousands of nodes with heavy interconnections. Computation simplification and memory reduction have been two major issues in the development of mathematical models and algorithms for digital computation of transient stability [9].

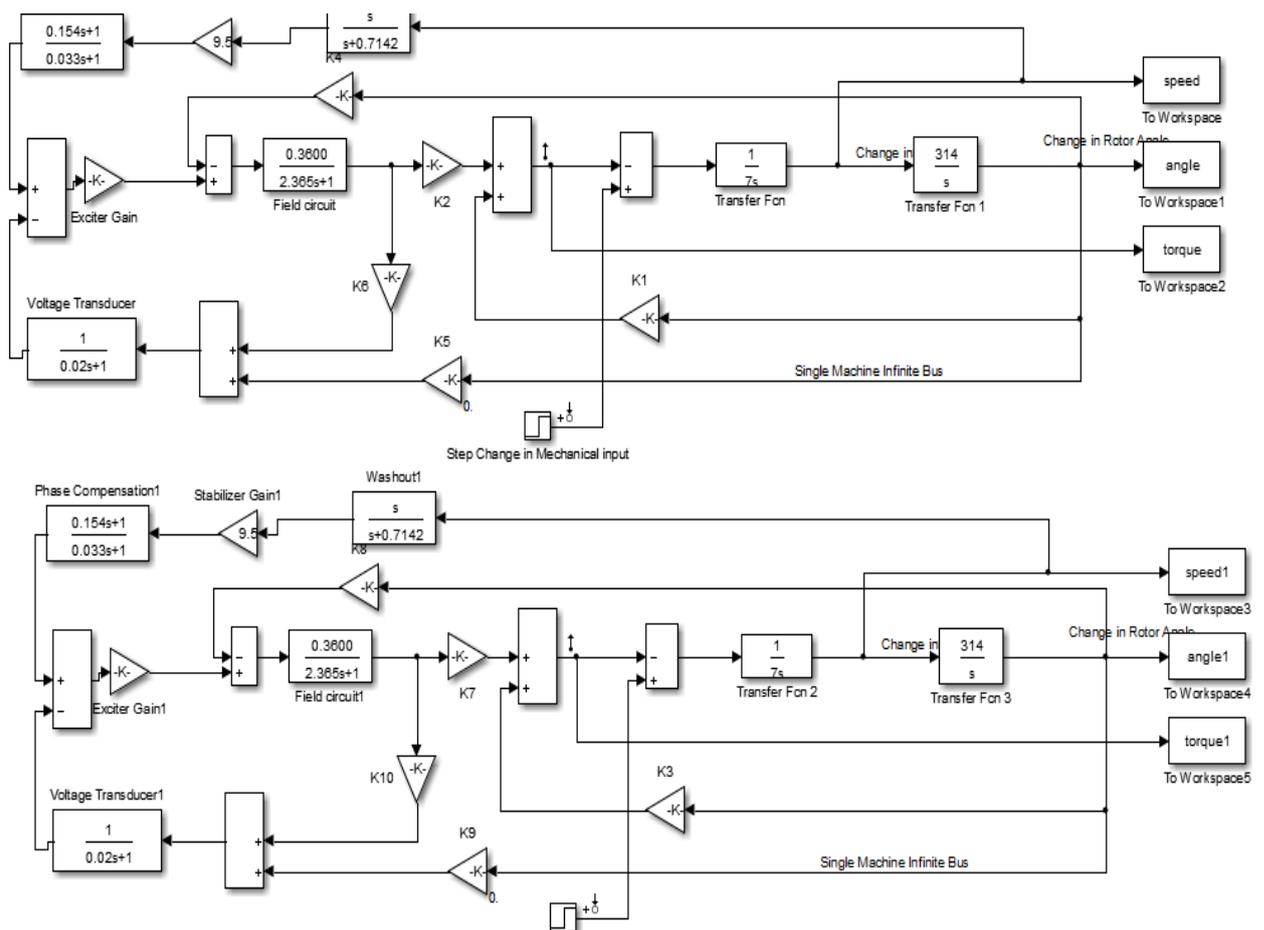


Fig. 3. 9Modelling of multimachine Infinite bus system

The system consists of synchronous generator, turbine, governor, and tie-line mode. The power flow over the transmission line will appear as a positive load to one area and equal but negative load to the other, or vice versa, depending on the direction of power flow. A 1000 MW hydraulic generation plant (machine M1) is connected to a load center through a long 500 kV, 700 km transmission line. The load center is modeled by a 5000 MW resistive load. The load is fed by the remote 1000 MW plant and a local generation of 5000 MW (machine M2). The system has been initialized so that the line carries 950 MW which is close to its surge impedance loading (SIL = 977 MW) (5). In order to maintain system stability after faults, the transmission line is shunt compensated at its center by a 200-Mvar Static Var Compensator (SVC) [9].

500 kV Transmission System

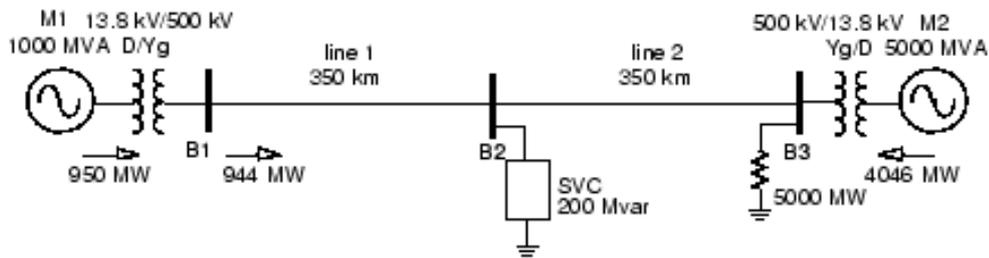


Fig. 3. 10 Multimachine system

Notice that this SVC model is a pharos model valid only for transient stability solution. The SVC does not have a Power Oscillation Damping (POD) unit. The two machines are equipped with a Hydraulic Turbine and Governor (HTG), Excitation system and Power System Stabilizer (PSS). These blocks are located in the two 'Turbine and Regulator' subsystems. Three types of stabilizers can be selected: a conventional model, Intelligent model & Smart model stabilizer [10] [11]. This is the simplest model used in stability analysis and requires minimum data. The following assumptions are made:

- Mechanical power input to each synchronous machine is assumed to be constant.
- Damping is neglected.
- Synchronous machines are modeled as constant voltage sources behind transient reactance.
- Loads are represented as constant impedances.

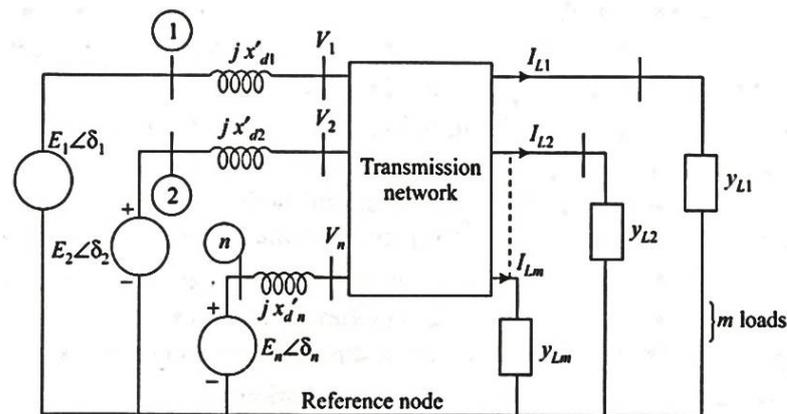


Fig. 3. 11 Transmission network for multimachine system

(5) Paper entitled, "Development of Intelligent controller for Power System stabilization for Single Machine Infinite Bus system", *Discovery International Daily journal* ISSN 2278 – 5469.

Nodes 1, 2 n are introduced in the model and are called internal nodes (the terminal node is the external node connected to the transmission network). The swing equations are formed for the various generators using the following steps [12]:

Step 1: All system data is converted to a common base.

Step 2: A Prefault load flow is performed, to determine the prefault steady state voltages, At all the external buses. Using the prefault voltages, the loads are converted into equivalent shunt admittance, connected between the respective bus and the reference node.

Step 3: The internal voltages are calculated from the terminal voltages.

Step 4: The bus admittance matrix Y_{bus} formed to run the load flow is modified to include the following.

(i) The equivalent shunt load admittance given by connected between the respective load bus and the reference node.

(ii) Additional nodes are introduced to represent the generator internal nodes. Appropriate values of admittances corresponding to x' , connected between the internal nodes and terminal nodes are used to update the Y_{bus} .

(iii) Y_{bus} corresponding to the faulted network is formed. Generally transient stability analysis is performed, considering three phase faults, since they are the most severe. The Y_{bus} during the fault is obtained by setting the elements of the row and column corresponding to the faulted bus to zero.

(iv) Y_{bus} corresponding to the post-fault network is obtained, taking into account line outages if any. If the structure of the network does not change, the Y_{bus} of the post-fault network is same as the prefault network.

Step 5: The admittance form of the network equations is $I = Y_{bus}V$

Since loads are all converted into passive admittances, current injections are present only at the n generator internal nodes. The injections at all other nodes are zero.

Step 8: The n second order differential equations can be decomposed into $2n$ first order differential equations which can be solved by any numerical method. Though reduced order models, also called classical models, require less computation and memory, their results are not reliable. Further, the interconnection of the physical network of the system is lost.

3.3 Design Steps for Fuzzy Logic (Intelligent) controller

The general design procedure for fuzzy control can be given as follow:

1. First, analyse whether the problem has sufficient elements to warrant a fuzzy logic application; otherwise, apply a conventional method.
2. Get all the information (design and operation characteristics of the plant) from the operator of the plant to be controlled.
3. If model is available, develop a simulation model and study the performance characteristics.
4. Identify the function elements where fuzzy logic can be applied.
5. Identify the input and output variables of each fuzzy system.
6. Formulate fuzzy sets and select the corresponding membership function shape of each.
7. Formulate the rule table.
8. Test the model, evaluate the results, tune the rules and membership functions, and retest until satisfactory results are obtained.

The power system stabilizer is used to improve the performance of synchronous generator. However, it results into poor performance under various loading conditions when implemented with conventional PSS. Therefore, the need for fuzzy logic PSS arises. The fuzzy controller used in power system stabilizer is normally a two-input and a single-output component [15]. It is usually a MIS0 system. The two inputs are change in angular speed and rate of change of angular speed whereas output of fuzzy logic controller is a voltage signal. A modification of feedback voltage to excitation system as a function of accelerating power on a unit is used to enhance the stability of the system. The stabilizing signals are computed using the standard fuzzy membership functions depending upon these variables [16]. For FL PSS as Intelligent controller design, generator speed deviation and acceleration can be observed and have been chosen as the input signal of the fuzzy PSS. The dynamic performance of the system could be evaluated by examining the response curve of these two variables. The voltage is taken as the output from the fuzzy logic controller which is shown below.

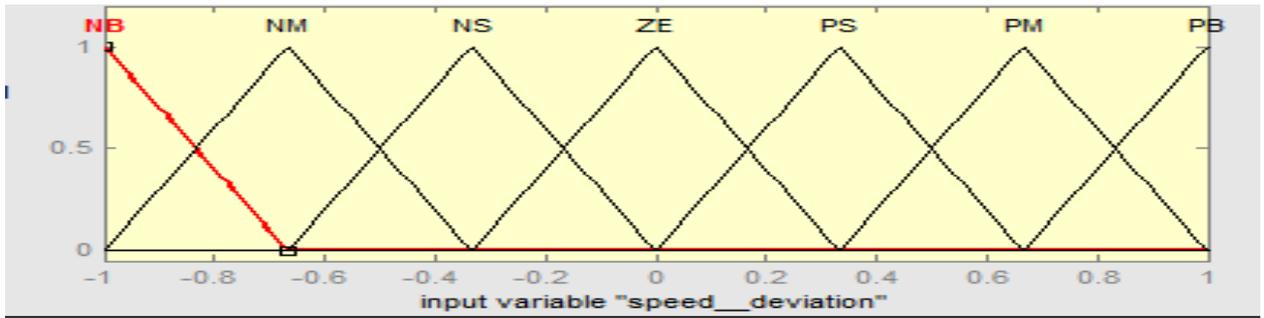


Fig. 3. 13 Membership function for speed deviation

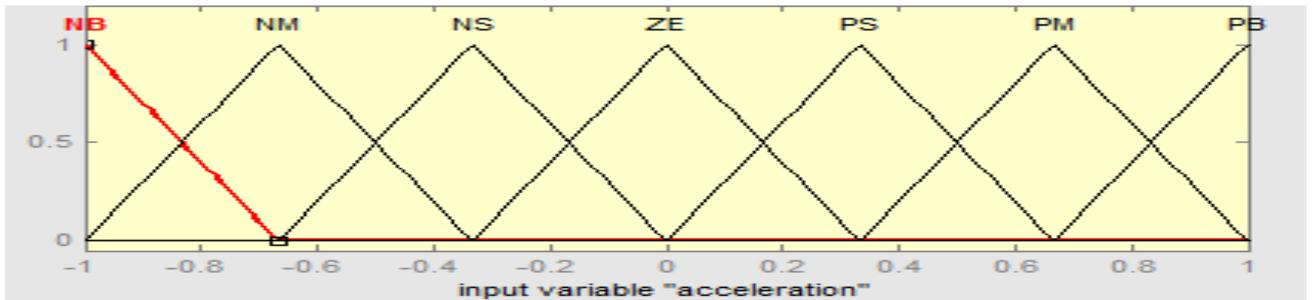


Fig. 3. 14 Membership function for acceleration

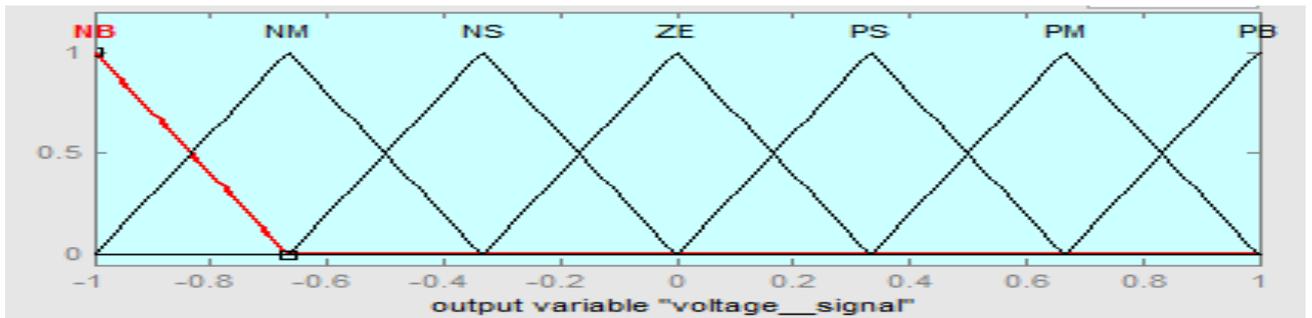


Fig. 3. 15 Membership function for voltage signal

🔧 Fuzzy rule base

A set of rules which define the relation between the input and output of fuzzy controller can be found using the available knowledge in the area of designing PSS. These rules are defined using the linguistic variables. The two inputs, speed and acceleration, result in 49 rules for each machine. The typical rules are having the following structure:

Table 3. 1Rule base table for fuzzy logic controller

SPEED DEVIATION	ACCELERATION						
	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NM	NM	NS
NM	NB	NM	NM	NM	NS	NS	ZE
NS	NM	NM	NS	NS	ZE	ZE	PS
ZE	NM	NS	NS	ZE	PS	PS	PM
PS	NS	ZE	ZE	PS	PS	PM	PM
PM	ZE	PS	PS	PM	PM	PM	PB
PB	PS	PM	PM	PB	PB	PB	PB

Total (7x7) 49 rules can be developed according to this rule base. But it is not convenient to take all 49 rules because it takes more simulation time. Thus from that 49 rules, selected 10 rules which gives better performance under wide range of operating condition.

All the 10 rules can be made based on this fuzzy rule base table listed below:

Rule 1: If speed is NM and Acceleration is NM then Voltage signal is NM

Rule 2: If speed is NS and Acceleration is NS then Voltage signal is NS

Rule 3: If speed is ZE and Acceleration is ZE then Voltage signal is ZE

Rule 4: If speed is PS and Acceleration is PS then Voltage signal is PS

Rule 5: If speed is PM and Acceleration is PM then Voltage signal is PM

Rule 6: If speed is NM and Acceleration is PM then Voltage signal is NS

Rule 7: If speed is ZE and Acceleration is PS then Voltage signal is PS

Rule 8: If speed is ZE and Acceleration is PM then Voltage signal is PB

Rule 9: If speed is PM and Acceleration is NS then Voltage signal is PS

Rule 10: If speed is PB and Acceleration is PS then Voltage signal is PB

3.4 Performance evaluation: Smart control

3.4.1 Design steps for ANN Implementation

1. Analyse the problem and find whether it has sufficient elements for a neural network [17].

2. If the ANN is to represent a static function then a three layer feed forward network should be sufficient. For a dynamic function, select either a recurrent neural network or a time delayed network.
3. Select an input and output signals. For a feed forward network, select the hidden layer neurons [18].
4. Select generally a sigmoid transfer network for unipolar output and a hyperbolic tan function for bipolar output.
5. Select a development system such as Neural Network toolbox in MATLAB.
6. Select appropriate learning coefficient (η) and momentum factor (μ).
7. Select an acceptable training error ξ and a no. of epochs.
8. After the training is complete with all patterns, test the network performance [19].

3.4.2 Artificial neural network based Power system stabilizer

The interconnection of artificial neurons results in an ANN, and its objective is to emulate the function of a human brain to solve scientific, engineering, and many other real-life problems. This network generally is classified as feed-forward and feedback types. In a feed-forward network, signals from neuron to neuron flow only in the forward direction, whereas in a recurrent network, the signals can flow in a forward as well as backward or lateral direction [20][21]. The ANN model is shown in Fig. 3.16.

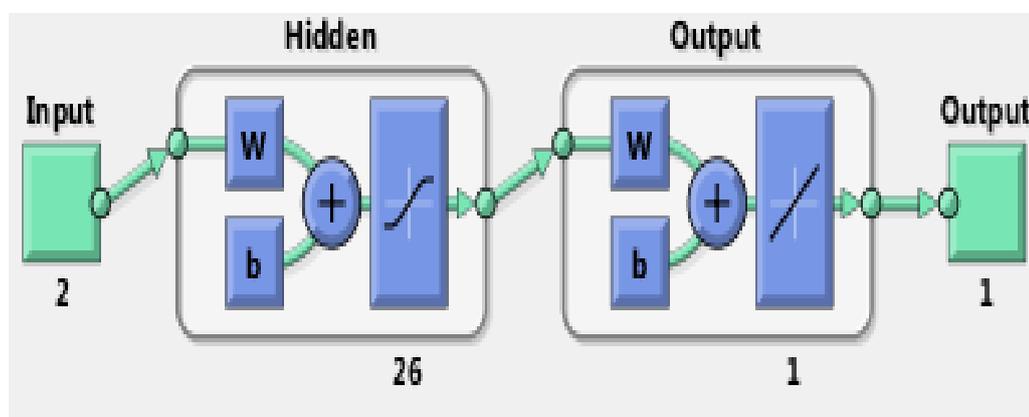


Fig. 3. 16 Artificial neural network models

Back propagation is the most popular training method for a multi-layer feed-forward network; therefore the standard network trained by this algorithm is often called the BP network. It is generalization of the delta learning rule developed by Widrow and Haft for Adaline training. Here aim is to train the ANN with the help of BKP algorithm to reduce oscillation in the SMIB system. The network structure is given in figure [22] [23].The design of Fuzzy based PSS is replaced by ANN based PSS for reducing the oscillation.

3.5 Concluding Remarks

In this chapter the mathematical modelling of the proposed controllers is developed for the power system stabilization considering single machine infinite bus system and multimachine system.