Appendix– III

Uncertainty Analysis

Uncertainty analysis involves systematic procedures for calculating error estimates for experimental data. When estimating errors in heat engine experiments, it is usually assumed that data is gathered under fixed (known) conditions and detailed knowledge of all system components is available. Measurement errors arise from various sources, but they can be broadly classified as bias errors and precision (or random) errors. Bias errors remain constant during a set of measurements. They are often estimated from calibration procedures or past experience. Alternatively, different methods of estimating the same variable can be used, so that comparisons between those results would indicate the bias error. Elemental bias errors arise from calibration procedures or curve-fitting of calibrated data.

To quantify errors in experimental work, some calculations and estimation have to be applied on sensors, devices and machines that have been used to measure the experimental parameters. As the experiments performed as it need to express measurement uncertainty as

$$x' = x \pm u_x \quad (P\%) \qquad \text{AIII.1}$$

where, x', x, u^x and P% are true value, tested value, uncertainty of the measurement and confidence respectively. To do this with assurance, total uncertainty of each component or portion of the experiment or procedure is determined. Total uncertainty is determined by finding error due to equipment (bias) and due to environment (precision). Wherever possible, uncertainty of each component or portion of experiment is found to determine where uncertainty may be minimized. Uncertainties are propagated in post-processing phase, to quantities that are non-linear functions of a measurement or functions of multiple measurements with uncertainties based upon the functional relationship.

Both bias and precision errors are present in an experiment. The precision is measured whereas the bias error is usually determined from equipment vendor specs. The total error is the vector sum of these errors and it is to be noted that errors in estimating each error affect the value of the total error.

$$u_X = (B_X^2 + P_X^2)^{1/2} \qquad \text{AIII.2}$$

where, B_x and P_x are bias and precision error respectively.

In case of several measurements of the same quantity like engine load, the uncertainty is estimated using statistical measures of spread. Several measurements of the same quantity are: $X_1, X_2, X_3, X_4, X_5, ---, X_n$. Average load of the dynamometer is calculated as

$$\text{average} = (X1, X2, X3, X4, X5, ---, Xn)/n \qquad \text{AIII.3}$$

Now, there are two ways to describe the scatter in these measurements. The mean deviation from the mean is the sum of the absolute values of the differences between each measurement and the average, divided by the number of measurements:

$$\text{mean deviation from mean} = \frac{\sum_{i=1}^{n} |X_i - \text{average}|}{n} \qquad \text{AIII.4}$$

The standard deviation from the mean is the square root of the sum of the squares of the differences between each measurement and the average, divided by one less than the number of measurements:

$$\text{standard deviation from mean} = \frac{\sqrt{\sum_{i=1}^{n} (X_i - \text{average})^2}}{1-n} \qquad \text{AIII.5}$$

Either the mean deviation from the mean, or the standard deviation from the mean, gives a reasonable description of the scatter of data around its mean value.

Tested load - mean deviation < true load < tested load + mean deviation AIII.6

Tested load - mean deviation < true load < tested load + standard deviation AIII.7

For parameter that have been evaluated depending on two or more independent parameters, propagation of uncertainty is carried out using

$$\frac{U_y}{y} = \sqrt{\left(\frac{u_{x1}}{x1}\right)^2 + \left(\frac{u_{x2}}{x2}\right)^2 + \ldots + \left(\frac{u_{xn}}{xn}\right)^2} \qquad \text{AIII.8}$$

Where, U_y and y are uncertainty and the testing value of the evaluated parameter x1, x2,…., xn respectively.

The uncertainty analysis carried out in this Appendix is based on the lines suggested by Kline and McClintock [90]. It should be noted that the uncertainty analysis presented here considers only the errors that relate to the measurements made during testing. Δ is used here to symbolize the error in the quantity.

III.1 Uncertainty in Thermal Performance Parameters

III.1.1Brake thermal Efficiency:

The brake thermal efficiency for the compression ignition engine fuelled by hydrogen – diesel blends is calculated using the equation

$$\eta_{B.Th} = \frac{BP}{IP} \times 100$$

$$\frac{\Delta \eta_{B.Th}}{\eta_{B.Th}} = \sqrt{\left(\frac{\Delta BP}{BP}\right)^2 + \left(\frac{\Delta IP}{IP}\right)^2} \qquad \text{AIII.1}$$

- Uncertainty in BP

$$BP = \frac{N \times W}{C}$$

Where C=4234

$$\frac{\Delta BP}{BP} = \sqrt{\left(\frac{\Delta N}{N}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta C}{C}\right)^2} \qquad \text{AIII.2}$$

- Uncertainty in IP

$$IP = \dot{m}_f \times CV_{diesel} + \dot{m}_{H_2} \times CV_{H_2}$$

$$\frac{\Delta IP}{IP} = \sqrt{\left(\frac{\Delta \dot{m}_f}{\dot{m}_f}\right)^2 + \left(\frac{\Delta CV_{diesel}}{CV_{diesel}}\right)^2 + \left(\frac{\Delta \dot{m}_{H_2}}{\dot{m}_{H_2}}\right)^2 + \left(\frac{\Delta CV_{H_2}}{CV_{H_2}}\right)^2} \qquad \text{AIII.3}$$

By substitute III.2 and III.3 in III.1 yield the uncertainty in break thermal efficiency in term of basic parameters :

$$\frac{\Delta \eta_{B.Th}}{\eta_{B.Th}} = \sqrt{\left(\frac{\Delta N}{N}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta C}{C}\right)^2 + \left(\frac{\Delta \dot{m}_f}{\dot{m}_f}\right)^2 + \left(\frac{\Delta CV_{diesel}}{CV_{diesel}}\right)^2 + \left(\frac{\Delta \dot{m}_{H_2}}{\dot{m}_{H_2}}\right)^2 + \left(\frac{\Delta CV_{H_2}}{CV_{H_2}}\right)^2} \qquad \text{AIII.4}$$

III.1.2 Uncertainty in Volumetric Efficiency:

The volumetric efficiency for the compression ignition engine fuelled by hydrogen – diesel blends is calculated using the equation:

$$\eta_{Vol.} = \frac{V_{actual}}{V_{swept}}$$

$$\frac{\Delta \eta_{Vol.}}{\eta_{Vol.}} = \sqrt{\left(\frac{\Delta V_{actual}}{V_{actual}}\right)^2 + \left(\frac{\Delta V_{swept}}{V_{swept}}\right)^2} \qquad \text{AIII.5}$$

- Uncertainty in V_{actual}

  $V_{actual} = C_d A\sqrt{2gh_a}$

  $A = \pi d^2/4$

  $h_a = \frac{h_w \rho_w}{\rho_a} \Rightarrow h_a = \frac{h_w \rho_w}{\frac{P_a}{RT_a}}$

$$\frac{\Delta h_a}{h_a} = \sqrt{\left(\frac{\Delta h_w}{h_w}\right)^2 + \left(\frac{\Delta \rho_w}{\rho_w}\right)^2 + \left(\frac{\Delta P_a}{P_a}\right)^2 + \left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta T_a}{T_a}\right)^2}$$

$$\frac{\Delta Vactual}{Vactual} = \sqrt{\left(\frac{\Delta C_d}{C_d}\right)^2 + \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta g}{g}\right)^2 + \left(\frac{\Delta h_w}{h_w}\right)^2 + \left(\frac{\Delta \rho_w}{\rho_w}\right)^2 + \left(\frac{\Delta P_a}{P_a}\right)^2 + \left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta T_a}{T_a}\right)^2} \qquad \text{AIII.6}$$

- Uncertainty in V_{swept}

$$V_s = 4 \times \frac{\left(\frac{\pi D^2}{4} \times L \times N\right)}{120}$$

$$\frac{\Delta\,Vswept}{Vswept} = \sqrt{\left(\frac{\Delta D}{D}\right)^2 + \left(\frac{\Delta\,L}{L}\right)^2 + \left(\frac{\Delta\,N}{N}\right)^2} \qquad \text{AIII.7}$$

By substitute III.4 and III.5 in III.6 yield the uncertainty in volumetric efficiency in term of basic parameters:

$$\frac{\Delta\,\eta_{Vol.}}{\eta_{Vol.}} = \sqrt{\left(\frac{\Delta C_d}{C_d}\right)^2 + \left(\frac{\Delta\,d}{d}\right)^2 + \left(\frac{\Delta\,g}{g}\right)^2 + \left(\frac{\Delta h_w}{h_w}\right)^2 + \left(\frac{\Delta\,\rho_w}{\rho_w}\right)^2 + \left(\frac{\Delta\,P_a}{P_a}\right)^2 + \left(\frac{\Delta\,R}{R}\right)^2 + \left(\frac{\Delta\,T_a}{T_a}\right)^2 + \left(\frac{\Delta D}{D}\right)^2 + \left(\frac{\Delta\,L}{L}\right)^2 + \left(\frac{\Delta\,N}{N}\right)^2}$$

AIII.8

III.1.3 Uncertainty in Equivalence Ratio

The equivalence ratio equation is:

$$\emptyset = \left[\frac{\dfrac{m_f}{m_a - \dfrac{m_{H_2}}{\left(\frac{H_2}{m_a}\right)_{stoch.}}}}{\left(\frac{m_f}{m_a}\right)_{stoch.}}\right]$$

The uncertainty in equivalence ratio is:

$$\frac{\Delta\,\emptyset}{\emptyset} = \sqrt{\left(\frac{\Delta m_f}{m_f}\right)^2 + \left(\frac{\Delta\,m_a}{m_a}\right)^2 + \left(\frac{\Delta\,m_{H_2}}{m_{H_2}}\right)^2 + \left(\frac{\Delta\left(\frac{H_2}{m_a}\right)_{stoch.}}{\left(\frac{H_2}{m_a}\right)_{stoch.}}\right)^2 + \left(\frac{\Delta\left(\frac{m_f}{m_a}\right)_{stoch.}}{\left(\frac{m_f}{m_a}\right)_{stoch.}}\right)^2} \qquad \text{AIII.9}$$

III.1.4 Uncertainty in Brake Specific Energy Consumption

The brake specific energy consumption equation is:

BSEC= (diesel energy + hydrogen energy)/BP

BSEC= ($m_f \times CV_{diesel} + m_{H_2} \times CV_{H_2}$)/BP

$$\frac{\Delta BSEC}{BSEC} = \sqrt{\left(\frac{\Delta m_f}{m_f}\right)^2 + \left(\frac{\Delta CV_{diesel}}{CV_{diesel}}\right)^2 + \left(\frac{\Delta \dot{m}_{H_2}}{\dot{m}_{H_2}}\right)^2 + \left(\frac{\Delta CV_{H_2}}{CV_{H_2}}\right)^2 + \left(\frac{\Delta N}{N}\right)^2 + \left(\frac{\Delta\,W}{W}\right)^2 + \left(\frac{\Delta\,C}{C}\right)^2} \qquad \text{AIII.10}$$

III.2 Sample Calculations

In this section, a numerical estimation of the most probable error (uncertainty) is presented. The estimate is based on test run No. II.15.1 of Appendix II, the sample calculation of which is given in Appendix IV. Table III.1 gives the values of the quantities measured during this run and probable errors in each of them. The errors are based on the least counts and sensitivities of the measuring instruments used in the present experimental investigations.

Table III.1 Probable Errors in the Estimation of Thermal Performance of Hydrogen-Diesel Duel fuel Engine.

Quantity	Value	Probable Error, $\lvert\Delta\rvert$
Speed	1500 rpm	30 rpm
W	24 kg	0.02 kg
m_f	779.53×10^{-6} kg/sec	0.025×10^{-6} kg/sec
m_{H2}	0 kg/sec	0.02 kg/sec
m_a	18.72×10^{-3} kg/s	1×10^{-3} kg/s
h_w	177 mm	1 mm
d	25 mm	1 mm
L	73 mm	1 mm
D	89	1 mm
P_a	99.60 kN/m^2	0.996 kN/m^2
T_a	25.19 °C	0.5 °C

III.2.1 Brake thermal Efficiency:

From eq. III.4

$$\frac{\Delta \eta_{B.Th}}{\eta_{B.Th}} = \sqrt{\left(\frac{\Delta N}{N}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta C}{C}\right)^2 + \left(\frac{\Delta m_f}{m_f}\right)^2 + \left(\frac{\Delta CV_{diesel}}{CV_{diesel}}\right)^2 + \left(\frac{\Delta m_{H_2}}{m_{H_2}}\right)^2 + \left(\frac{\Delta CV_{H_2}}{CV_{H_2}}\right)^2}$$

$\left(\frac{\Delta C}{C}\right)$= 0, C is constant

$\left(\frac{\Delta CV_{diesel}}{CV_{diesel}}\right)$=0, CV_{diesel} is constant

$\left(\frac{\Delta CV_{H_2}}{CV_{H_2}}\right)$=0, CV_{H2} is constant

$$\frac{\Delta \eta_{B.Th}}{\eta_{B.Th}} = \sqrt{\left(\frac{30}{1500}\right)^2 + \left(\frac{0.02}{24}\right)^2 + \left(\frac{0.025 \times 10-6}{779.53 \times 10-6}\right)^2 + \left(\frac{0.02}{0}\right)^2}$$

$\frac{\Delta \eta_{B.Th}}{\eta_{B.Th}}$= 0.02 or 2%

III.2.2 Volumetric Efficiency

Equation III.8 shows the uncertainty in volumetric efficiency

$$\frac{\Delta \eta_{Vol.}}{\eta_{Vol.}} =$$

$$\sqrt{\left(\frac{\Delta C_d}{C_d}\right)^2 + \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta g}{g}\right)^2 + \left(\frac{\Delta h_w}{h_w}\right)^2 + \left(\frac{\Delta \rho_w}{\rho_w}\right)^2 + \left(\frac{\Delta P_a}{P_a}\right)^2 + \left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta T_a}{T_a}\right)^2 + \left(\frac{\Delta D}{D}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta N}{N}\right)^2}$$

$\frac{\Delta C_d}{C_d}$=0, C_d= constant

$\left(\frac{\Delta g}{g}\right)$=0, g = constant

$\left(\frac{\Delta \rho_w}{\rho_w}\right) = 0, \rho_w = \text{constant}$

$\left(\frac{\Delta R}{R}\right)$ = 0, R= constant

Then,

$$\frac{\Delta \eta_{Vol.}}{\eta_{Vol.}} = \sqrt{\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta h_w}{h_w}\right)^2 + \left(\frac{\Delta P_a}{P_a}\right)^2 + \left(\frac{\Delta R}{R}\right)^2 + \left(\frac{\Delta T_a}{T_a}\right)^2 + \left(\frac{\Delta D}{D}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta N}{N}\right)^2}$$

$$\frac{\Delta \eta_{Vol.}}{\eta_{Vol.}} = \sqrt{\left(\frac{1}{25}\right)^2 + \left(\frac{1}{177}\right)^2 + \left(\frac{0.996}{99.60}\right)^2 + \left(\frac{0.5}{25.19}\right)^2 + \left(\frac{1}{89}\right)^2 + \left(\frac{1}{73}\right)^2 + \left(\frac{30}{1500}\right)^2}$$

$\frac{\Delta \eta_{Vol.}}{\eta_{Vol.}}$ = 0.0494 or 4.94%

III.2.3 Equivalence Ratio

Equation III.9 describes the uncertainty in the equivalence ratio:

$$\frac{\Delta \emptyset}{\emptyset} = \sqrt{\left(\frac{\Delta m_f}{m_f}\right)^2 + \left(\frac{\Delta m_a}{m_a}\right)^2 + \left(\frac{\Delta m_{H_2}}{m_{H_2}}\right)^2 + \left(\frac{\Delta \left(\frac{H_2}{m_a}\right)_{stoch.}}{\left(\frac{H_2}{m_a}\right)_{stoch.}}\right)^2 + \left(\frac{\Delta \left(\frac{m_f}{m_a}\right)_{stoch.}}{\left(\frac{m_f}{m_a}\right)_{stoch.}}\right)^2}$$

$\left(\frac{\Delta \left(\frac{H_2}{m_a}\right)_{stoch.}}{\left(\frac{H_2}{m_a}\right)_{stoch.}}\right)$=0, $\left(\frac{H_2}{m_a}\right)_{stoch.}$ is constant

$\left(\frac{\Delta \left(\frac{H_2}{m_a}\right)_{stoch.}}{\left(\frac{H_2}{m_a}\right)_{stoch.}}\right)$=0, $\left(\frac{H_2}{m_a}\right)_{stoch.}$ is constant

$$\frac{\Delta \emptyset}{\emptyset} = \sqrt{\left(\frac{\Delta m_f}{m_f}\right)^2 + \left(\frac{\Delta m_a}{m_a}\right)^2 + \left(\frac{\Delta m_{H_2}}{m_{H_2}}\right)^2}$$

$$\frac{\Delta \emptyset}{\emptyset} = \sqrt{\left(\frac{0.025}{779.53}\right)^2 + \left(\frac{1}{18.72}\right)^2 + \left(\frac{0.02}{0}\right)^2}$$

$\frac{\Delta \emptyset}{\emptyset}$ = 0.0534 or 5.34%

III.2.4 Brake Specific Energy Consumption

Eq. III.10 gives the uncertainty in brake specific energy consumption

$$\frac{\Delta BSEC}{\mathrm{BSEC}} = \sqrt{\left(\frac{\Delta \mathrm{m_f}}{\mathrm{m_f}}\right)^2 + \left(\frac{\Delta \mathrm{CV_{diesel}}}{\mathrm{CV_{diesel}}}\right)^2 + \left(\frac{\Delta \mathrm{m_{H_2}}}{\mathrm{m_{H_2}}}\right)^2 + \left(\frac{\Delta \mathrm{CV_{H_2}}}{\mathrm{CV_{H_2}}}\right)^2 + \left(\frac{\Delta N}{N}\right)^2 + \left(\frac{\Delta W}{W}\right)^2 + \left(\frac{\Delta C}{C}\right)^2}$$

$$\frac{\Delta BSEC}{\mathrm{BSEC}} = \sqrt{\left(\frac{0.025}{779.53}\right)^2 + \left(\frac{0.02}{0}\right)^2 + \left(\frac{30}{1500}\right)^2 + \left(\frac{0.02}{24}\right)^2}$$

$\frac{\Delta BSEC}{\mathrm{BSEC}}$= 0.02 or 2%