5

REGRESSION MODELLING AND EXPERIMENTS

5.1 Introduction

In many problems two or more variables are related, and it is of interest to model and explore this relationship. For example, in a manufacturing process the yield of product is related to the quality of job. The mechanical engineer may want to build a model related to surface roughness and then use the model of prediction, process optimization, or process control.

In general, suppose that there is a single dependent variable or response y that depends on k independent or regressor variables, for example, $x_1, x_2, \dots x_k$ .The relationship between these variables is characterized by a mathematical model called a regression model. The regression model is fit to a set of sample data. In some instances, the experimenter knows the exact form of the true functional relationship between y and $x_1, x_2, \dots, x_k$, say $y = \emptyset(x_1, x_2, \dots, x_{k)}$. However, in most cases, the true functional relationship is unknown, and the experimenter chooses an appropriate function to approximate $\emptyset$. Low order polynomial models are widely used as approximating functions.

Regression methods are frequently used to analyze data from unplanned experiments, such as might arise from observation of uncontrolled phenomena or historical records. Regression methods are also very useful in designed experiments where something has “gone wrong”. We will demonstrate some of these situations for turning operations of different materials in this chapter.

5.2 Linear Regression Models

We will focus on fitting linear regression models. To illustrate, suppose that we wish to develop an empirical model relating the viscosity of polymer to the temperature and the catalyst feed rate. A model that might describe this relationship is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon \tag{5.1}$$

where *y* represents the viscosity, x_1 represents the temperature, and x_2 represents the catalyst feed rate. This is a multiple linear regression model with two independent variables. We often call the independent variables predictor variables or regressor. The term linear is used because Equation 5.1 is a linear function of the unknown parameters β_0, β_1 and β_2. The model describes a plane in the two – dimensional x_1, x_2 space. The parameters β_0 defines the intercept of the plane. We sometimes call β_1 and β_2 partial regression coefficients because β_1 measures the expected change in *y* per unit change in x_1 when x_2 is held constant and β_2 measures the expected change in *y* in per unit change in x_2 and x_1 is held constant.

In general, the response variable y may be related to *k* regressor variables. The model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon \tag{5.2}$$

is called a *multiple linear regression model* with *k* regressor variables. The parameters β_j, *j* = 0,1,….., *k*, are called regression coefficients. This model describes a hyper plane in the *k* – dimensional space of the regressor variables $\{ x_j \}$. The parameter β_j represents the expected change in response *y* per unit change in x_j when all the remaining independent variables x_i $(i \neq j)$are held constant.

Models those are more complex in appearance than Equation 5.2 may often still be analyzed by multiple linear regression techniques. For example, consider adding an interaction term to the first – order model in two variables, say

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \epsilon \tag{5.3}$$

If we let $x_3 = x_1 x_2$ and $\beta_3 = \beta_{12}$, then Equation 6.3 can be written as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon \tag{5.4}$$

which is a standard multiple linear regression model with three regressor. As another example, consider the second – order response surface model in two variables:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 + \epsilon \tag{5.5}$$

If we let $x_3 = x_1^2$, $x_4 = x_2^2$, $x_5 = x_1 x_2$, $\beta_3 = \beta_{11}$, $\beta_4 = \beta_{22}$ and $\beta_5 = \beta_{12}$ then this becomes

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \epsilon \tag{5.6}$$

which is a linear regression model. We have also been this model in examples earlier in the text. In general, any regression model that is linear in the parameters (the β's) is a linear regression model, regardless of the shape of the response surface that it generates.

5.3 Estimation of the Parameters in Linear Regression Models

The method of least squares is typically used to estimate the regression coefficients in a multiple linear regression model. Suppose that n>k observations on the response variable are available, say $y_1, y_2, \ldots, y_n$. Along with each observed response y_i, we will have an observation on each regressor variable and let x_{ij} denote the ith observation or level of variablex_j. The data will appear as in Table 5.1.We assume that the error term ϵ in the model has $E(\epsilon) = 0$ and $V(\epsilon) = 0^2$ and that the $\{\epsilon_i\}$ are uncorrelated random variables.

We may write the model equation (Equation 6.2) in terms of the observations in Table 5.1 as

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik} + \epsilon_i \\ &= \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \epsilon_i \qquad i = 1,2 \ldots \ldots, n \end{aligned} \tag{5.7}$$

The method of least squares chooses the β's in Equation 5.7 so that the sum of the squares of the errors, ϵ_i, is minimized. The least squares function is

$$L = \sum_{i=1}^{n} \epsilon_i{}^2 = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{k} \beta_j \, x_{ij} \right)^2 \qquad (5.8)$$

The function L is to be minimized with respect to $\beta_0, \beta_1, \ldots \ldots, \beta_k$. The least squares estimators, say $\hat{\beta}_0, \hat{\beta}_1, \ldots \ldots, \hat{\beta}_k$,must satisfy

$$\left.\frac{\partial L}{\partial \beta_0}\right|_{\hat{\beta}_0, \hat{\beta}_1, \ldots\ldots, \hat{\beta}_k} = -2 \sum_{i=1}^{n} \left(y_i - \hat{\beta}_0 - \sum_{j=1}^{k} \hat{\beta}_j \, x_{ij} \right) = 0 \qquad (5.9a)$$

and

$$\left.\frac{\partial L}{\partial \beta_j}\right|_{\hat{\beta}_0, \hat{\beta}_1, \ldots\ldots, \hat{\beta}_k} = -2 \sum_{i=1}^{n} \left(y_i - \hat{\beta}_0 - \sum_{j=1}^{k} \hat{\beta}_j \, x_{ij} \right) x_{ij} = 0 \qquad j = 1,2, \ldots \ldots .., k \quad (5.9b)$$

Table 5.1 Data for Multiple Linear Regressions

Y	X_1	X_2	…	X_k
y_1	X_{11}	X_{12}	…	X_{1k}
y_2	X_{21}	X_{22}	…	X_{2k}
. . .	. . .	. . .		. . .
y_n	X_{n1}	X_{n2}	…	X_{nk}

Simplifying Equation 5.9, we obtain

$$n\hat{\beta}_0 + \hat{\beta}_1 \sum_{i=1}^{n} x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{i2} + \ldots + \hat{\beta}_k \sum_{i=1}^{n} x_{ik} = \sum_{i=1}^{n} y_i$$

$$\hat{\beta}_0 \sum_{i=1}^{n} x_{i1} + \hat{\beta}_1 \sum_{i=1}^{n} x^2{}_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{i1} x_{i2} + \ldots + \hat{\beta}_k \sum_{i=1}^{n} x_{i1} x_{ik} = \sum_{i=1}^{n} x_{i1} y_i$$

$$\hat{\beta}_0 \sum_{i=1}^{n} x_{ik} + \hat{\beta}_1 \sum_{i=1}^{n} x_{ik} x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{ik} x_{i2} + \ldots + \hat{\beta}_k \sum_{i=1}^{n} x^2{}_{ik} = \sum_{i=1}^{n} x_{ik} y_i \quad (5.10)$$

These equations are called the least squares normal equations. Note that there are $p = k + 1$ normal equations, one for each of the unknown regression coefficients. The solution to the normal equations will be the least squares estimators of the regression coefficients $\widehat{\beta}_0, \hat{\beta}_1, \ldots \ldots, \hat{\beta}_k$.

$$y = \mathrm{X}\beta + \epsilon$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad X = \begin{bmatrix} 1 & x_{11} & x_{12} & \ldots & x_{1k} \\ 1 & x_{21} & x_{22} & \ldots & x_{2k} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{n1} & x_{n2} & \ldots & x_{nk} \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}, \text{and } \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

In general, y is an (*n* x 1) vector of the observations, **X** is an (*n* x *p*) matrix of the levels of the independent variables, $\boldsymbol{\beta}$ is a (*p* x 1) vector of the regression coefficients, and $\boldsymbol{\epsilon}$ is an (*n* x 1) vector of random errors.

We wish to find the vector of least squares estimators, $\widehat{\boldsymbol{\beta}}$, that minimizes

$$L = \sum_{i=1}^{n} \epsilon_i^2 = \epsilon' \epsilon = (y - \mathrm{X}\beta)' (y - \mathrm{X}\beta)$$

Note that *L* may be expressed as

$$L = y'y - \beta'\mathrm{X}'\mathrm{y} - \mathrm{y}'\mathrm{X}\beta + \beta'\mathrm{X}'\mathrm{X}\beta \quad (5.11)$$

$$= y'y - 2\beta'\mathrm{X}'\mathrm{y} + \beta'\mathrm{X}'\mathrm{X}\beta$$

because $\boldsymbol{\beta}'\mathbf{X}'\mathbf{y}$ is a (1 x 1) matrix, or a scalar, and its transpose $(\boldsymbol{\beta}'\mathbf{X}'\mathbf{y})' = \boldsymbol{y}'\mathbf{X}\boldsymbol{\beta}$ is the same scalar. The least squares estimators must satisfy

$$\left.\frac{\partial L}{\partial \beta}\right|_{\hat{\beta}} = -2\mathrm{X}'\mathrm{y} + 2\mathrm{X}'\mathrm{X}\hat{\beta} = 0$$

which simplifies to

$$X'X\hat{\beta} = X'y \tag{5.12}$$

Equation 5.12 is the matrix form of the least squares normal equations. It is identical to Equation 5.10. To solve the normal equations, multiply both sides of Equation 6.12 by the inverse of $\mathbf{X'X}$. Thus, the least squares estimator of $\boldsymbol{\beta}$ is

$$\hat{\beta} = (X'X)^{-1}X'y \tag{5.13}$$

It is easy to see that the matrix form of the normal equations is identical to the scalar form. Writing out Equation 6.12 in detail, we obtain

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i2} & \cdots & \sum_{i=1}^{n} x_{ik} \\ \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}^2 & \sum_{i=1}^{n} x_{i1}\, x_{i2} & \cdots & \sum_{i=1}^{n} x_{i1}\, x_{ik} \\ \vdots & \vdots & \vdots & & \vdots \\ \sum_{i=1}^{n} x_{ik} & \sum_{i=1}^{n} x_{ik}\, x_{i1} & \sum_{i=1}^{n} x_{ik}\, x_{i2} & \cdots & \sum_{i=1}^{n} x_{ik}^2 \end{bmatrix} \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{i1} y_i \\ \vdots \\ \sum_{i=1}^{n} x_{ik} y_i \end{bmatrix}$$

If the indicated matrix multiplication is performed, the scalar form of the normal equations (i.e., Equation 5.10) will result. In this form it is easy to see that $\mathbf{X'X}$ is a (p x p) symmetric matrix and $\mathbf{X'y}$ is a (p x 1) column vector. Note the special structure of the $\mathbf{X'X}$ matrix. The diagonal elements of $\mathbf{X'X}$ are the sums of squares of the elements in the columns of $\mathbf{X}$, and the off – diagonal elements are the sums of cross products of the elements in the columns of $\mathbf{X}$. Furthermore, note that the elements of $\mathbf{X'y}$ are the sums of cross products of the columns of $\mathbf{X}$ and the observations $\{y_i\}$.

The fitted regression model is

$$\hat{y} = X\hat{\beta} \tag{5.14}$$

In scalar notation, the fitted model is

$$\hat{y}_i = \hat{\beta}_0 + \sum_{j=1}^{k} \hat{\beta}_j\, x_{ij} \qquad i = 1,2, \ldots\ldots, n$$

The difference between the actual observations y_i and the corresponding fitted value $\hat{y}_i$ is the residual, say $e_i = y_i - \hat{y}_i$. The (n x 1) vector of residuals is denoted by

$$e = y - \hat{y} \tag{5.15}$$

Estimating σ^2. It is also usually necessary to estimate σ^2. To develop an estimator of this parameter, consider the sum of squares of the residuals, say

$$SS_E = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} e_i^2 = e'e$$

Substituting $\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}$, we have

$$\begin{aligned} SS_E &= (y - X\hat{\beta})'(y - X\hat{\beta}) \\ &= y'y - \hat{\beta}'X'y - y'X\hat{\beta} + \hat{\beta}'X'X\hat{\beta} \\ &= y'y - 2\hat{\beta}'X'y + \hat{\beta}'X'X\hat{\beta} \end{aligned}$$

Because $\mathbf{X}'\mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}'\mathbf{y}$, this last equation becomes

$$SS_E = y'y - \hat{\beta}'X'y \tag{5.16}$$

Equation 5.16 is called the error or residual sum of squares, and it has $n - p$ degrees of freedom associated with it. It can be shown that

$$E(SS_E) = \sigma^2 (n - p)$$

So an unbiased estimator of σ^2 is given by

$$\hat{\sigma}^2 = \frac{SS_E}{n - p} \tag{5.17}$$

Properties of the Estimators. The methods of least squares produce an unbiased estimator of the parameter $\boldsymbol{\beta}$ in the linear regression model. This may be easily demonstrated by taking the expected value of $\hat{\boldsymbol{\beta}}$ as follows:

$$E(\hat{\beta}) = E[(X'X)^{-1} X'y] = E[(X'X)^{-1}X'(X\beta + \epsilon)]$$

$$= E[(X'X)^{-1} X'X\beta + (X'X)^{-1}X'\epsilon] = \beta$$

because $E\ (\epsilon) = 0$ and $(X'X)^{-1}X'X = I$. Thus, $\hat{\beta}$ is an unbiased estimator of β.

The variance property of $\hat{\beta}$ is expressed in the covariance matrix:

$$\text{Cov}\left(\hat{\beta}\right) = E\left\{[\hat{\beta} - E\ (\hat{\beta})][\hat{\beta} - E\ (\hat{\beta})]'\right\} \tag{5.18}$$

which is just a symmetric matrix whose *i*th main diagonal element is the variance of the individual regression coefficient $\hat{\beta}_i$ and whose (*ij*)th element is the covariance between $\hat{\beta}_i$ and $\hat{\beta}_j$. The covariance matrix of $\hat{\boldsymbol{\beta}}$ is

$$\text{Cov}\left(\hat{\beta}\right) = \sigma^2\ (X'X)^{-1} \tag{5.19}$$

If σ^2 in Equation 5.19 is replaced with the estimate $\hat{\sigma}^2$ from Equation 5.12, we obtain an estimate of the covariance matrix of$\hat{\beta}$. The square roots of the main diagonal elements of this matrix are the standard errors of the model parameters.

Using the computer, Regression model fitting is almost always done using a statistical software packages, such as Minitab or JMP or Design Expert.

5.4 Hypothesis Testing in Multiple Regression

In multiple linear regression problems, certain tests of hypotheses about the model parameters are helpful in measuring the usefulness of the model. In this section, we describe several important hypothesis – testing procedures. These procedures require that the errors ϵ_i in the model be normally and independently distributed with mean zero and variance σ^2, abbreviated$\epsilon \sim$, NID (0, σ^2). As a result of this assumption, the observations $y_i$are normally and independently distributed with mean $\beta_0 + \sum_{j=1}^{k} \beta_j\, x_{ij}$ and variance σ^2.

The test for significance of regression is a test to determine whether a linear relationship exists between the response variable y and a subset of the regressor variables$x_1, x_2, \ldots, x_k$. The appropriate hypotheses are

$$H_0 : \beta_1 = \beta_2 = \ldots = \beta_k = 0 \tag{5.20}$$

$$H_1 : \beta_j \neq 0 \text{ for at least one } j$$

Rejection of H_0 in Equation 5.20 implies that at least one of the regressor variables $x_1, x_2, \ldots, x_k$ contributes significantly to the model. The test procedure involves an analysis of variance partitioning of the total sum of squares SS_T into a sum of squares due to the model (or to regression) and a sum of squares due to residual (or error), say

$$SS_T = SS_R + SS_E \quad (5.21)$$

Now if the null hypothesis $H_0 : \beta_1 = \beta_2 = \ldots = \beta_k = 0$ is true, then SS_R / σ^2 is distributed as X_k^2,where the number of degrees of freedom for X^2 is equal to the number of regressor variables in the model. Also, we can show that SS_E / σ^2 is distributed as X_{n-k-1}^2 and that SS_E and SS_R are independent. The test procedure for $H_0 : \beta_1 = \beta_2 = \ldots = \beta_k = 0$ is to compute

$$F_0 = \frac{SS_R \,|k}{SS_E \,|(n-k-1)} = \frac{MS_R}{MS_E} \quad (5.22)$$

and to reject H_0 if F_0 exceeds $F_{\alpha,k,n-k-1}$. Alternatively, we could use the P – value approach to hypothesis testing and, thus, reject H_0 if the P – value for the statistic $F_0$is less than α. The test is usually summarized in an analysis of variance table such as Table 5.2.

A computational formula for SS_R may be found easily. We have derived a computational formula for SS_E in Equation 5.16- that is,

$$SS_E = \mathrm{y'y} - \hat{\beta}'\mathrm{X'y}$$

Now, because $SS_T = \sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} y_i)^2/n = \mathbf{y}'\mathbf{y} - (\sum_{i=1}^{n} y_i)^2/n$, we may rewrite the foregoing equation as

$$SS_E = \mathrm{y'y} - \frac{(\sum_{i=1}^{n} y_i)^2}{n} - \left[\hat{\beta}'\mathrm{X'y} - \frac{(\sum_{i=1}^{n} y_i)^2}{n}\right]$$

or

$$SS_E = SS_T - SS_R$$

Therefore, the regression sum of square is

$$SS_R = \hat{\beta}'X'y - \frac{(\sum_{i=1}^{n} y_i)^2}{n} \tag{5.23}$$

and the error sum of squares is

$$SS_E = y'y - \hat{\beta}'X'y \tag{5.24}$$

and the total sum of squares is

$$SS_T = y'y - \frac{(\sum_{i=1}^{n} y_i)^2}{n} \tag{5.25}$$

Table 5.2 Analysis of variance for Significance of Regression in Multiple Regression

Source of variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Regression	SS_R	k	MS_R	MS_R/MS_E
Error or	SS_E	$n-k-1$	MS_E	
Total	SS_T	$n-1$		

These computations are almost always performed with regression software. The test of significance of regression in this example involves the hypotheses

$$H_0 : \beta_1 = \beta_2 = 0$$

$$H_1 : \beta_j \neq 0 \text{ for at least one } j$$

The P – value for the F statistic (Equation 5.22) is very small, so we would conclude that at least one of the two variables – temperature (x_1) and feed rate (x_2) - has a nonzero regression coefficient.

The coefficient of multiple determination R^2, where

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T} \tag{5.26}$$

Just as in designed experiments, $R^2$is a measure of the amount of reduction in the variability of y obtained by using the regressor variables$x_1, x_2, \ldots, x_k$ in the model.

However, as we have noted previously, a large value of R^2 does not necessarily imply that the regression model is a good one. Adding a variable to the model will always increaseR^2, regardless of whether the additional variable is statistically significant or not. Thus, it is possible for models that have large values of R^2 to yield poor predictions of new observations or estimates of the mean response.

Because R^2 always increase as we add terms to the model, some regression model builders prefer to use an adjusted R^2 statistic defined as

$$R^{\,2}_{adj} = 1 - \frac{SS_E/(n-p)}{SS_T/(n-1)} = 1 - \left(\frac{n-1}{n-p}\right)(1 - R^2) \qquad (5.27)$$

In general, the adjusted R^2 statistic will not always increase as variables are added to the model. In fact, if unnecessary terms are added, the value of $R^{\,2}_{adj}$ will often decrease.

$$R^{\,2}_{adj} = 1 - \left(\frac{n-1}{n-p}\right)(1 - R^2)$$

which is very close to the ordinary R^2. When R^2 and $R^{\,2}_{adj}$ differ dramatically, there is a good chance that nonsignificant terms have been included in the model.

5.5 Introduction to Surface Roughness

Surface roughness has received serious attention for many years. It has formulated an important design feature in many situations such as parts subject to fatigue loads, precision fits, fastener holes and aesthetic requirements. In addition to tolerances, surface roughness imposes one of the most critical constraints for the selection of machines and cutting parameters in process planning. A considerable number of studies have investigated the general effects of the speed, feed, depth of cut, nose radius and others on the surface roughness.

A popular model [38] to estimate the surface roughness with a tool having none zero radius is

$$\text{Ra} = \frac{0.032\, f^2}{r} \qquad (5.28)$$

Ra is the surface roughness (μm), f is the feed rate (mm/rev), r is the tool nose radius (mm). Although a qualitative analysis of the machining variables of speed, feed, and depth of cut on the surface roughness has been widely available in the literature, very few comprehensive predictive models have been developed. In this work, an empirical model will be developed based on metal cutting results from fractional factorial experiments, and it will include the workpiece hardness, feed, tool nose radius , depth of cut, Cutting velocity and cutting time

5.5.1 Ideal Surface Roughness

The ideal surface roughness represents the best possible finish that may be obtained for a given tool shape and feed and can be approached only if built up edge, chatter, inaccuracies in machine tool movement, and so on, are eliminated. The ideal surface finish for a turning operation in which a sharp-cornered tool is used is shown in Fig. 5.1 (a).

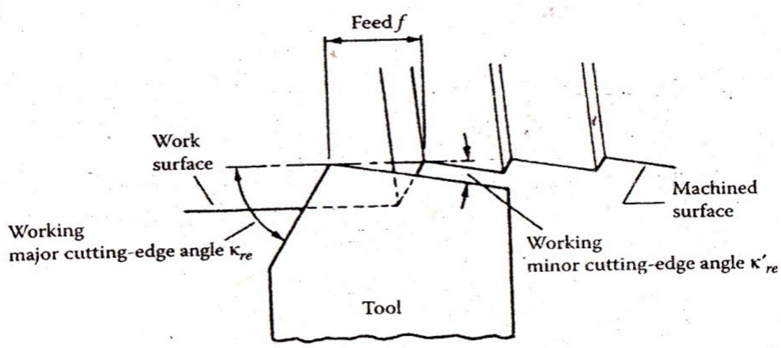

Fig. 5.1(a) Idealized model of surface roughness with sharp corner cutting tool [38]

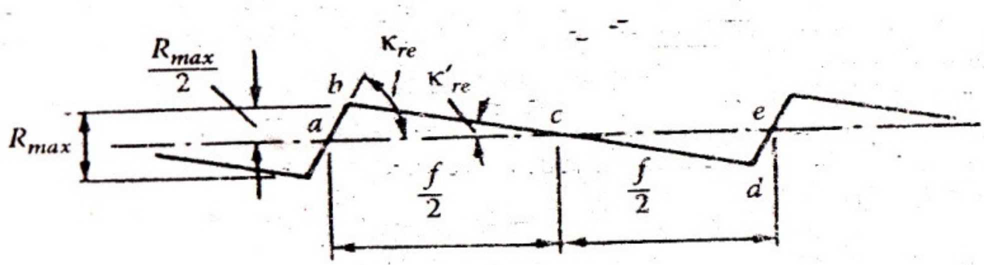

Fig. 5.1(b) Cross section through surface irregularities [38]

For the purpose of quantitative comparisons and analysis, it is useful to be able to express the roughness of machined surface in terms of a single factor or index. The index most commonly used is known as the arithmetical mean value R_a and may be found as follows. In the Fig. 5.1(b), which shows a cross section through the surface under consideration, a mean line is first found that is parallel to the general surface direction and divides the surface in such a way that the sum of the areas formed above the line is equal to the sum of the areas formed below the line. The surface roughness R_a is now given by the sum of the absolute values of all the areas above and below the mean line divided by the sampling length.

Thus, the surface roughness value is given by

$$R_a = \frac{|area - abc| + |area - cde|}{f}$$

where f is the feed.

Since the area abc and cde are equal,

$$R_a = \frac{2}{f}(area - abc) = \frac{R_{max}}{4}$$

Where $\frac{R_{max}}{2}$ is the height of the triangle abc.

It is interesting to note at this stage that the arithmetical mean value of surface roughness for a surface having uniform irregularities is equal to one quarter the maximum height of the irregularities. Now by the geometry,

$$R_{max} = \frac{f}{4\left(cotK_{r_\varepsilon} + cotK'_{r_\varepsilon}\right)}$$

where $K_{r_\varepsilon}$and $K'_{r_\varepsilon}$are the working major and minor cutting edge respectively.

Putting the value of R_{max}in the above equation of R_a, gives

$$R_a = \frac{f}{4\left(cotK_{r_\varepsilon} + cotK'_{r_\varepsilon}\right)} \tag{5.29}$$

Equation 5.29 shows that that arithmetical mean value for such a surface is directly proportional to the feed and the curve in Figure 5.2 shows how the arithmetical mean value is affected by the working minor –cutting edge angle.

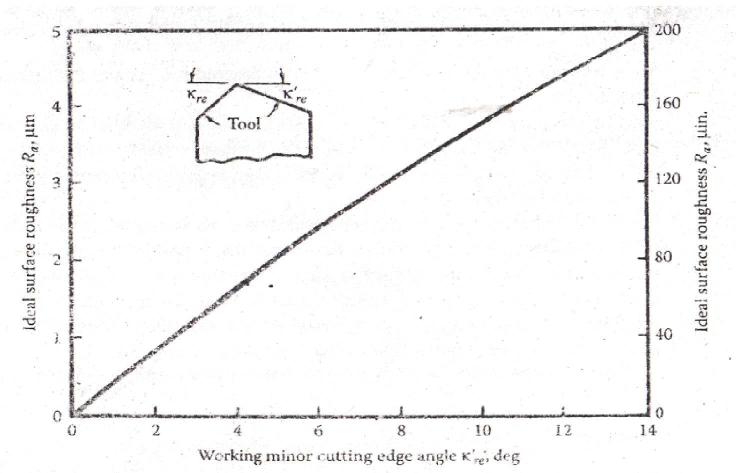

Fig. 5.2 Effect of minor cutting edge angle on surface roughness (K_{r_ε}=45^0, f=0.1mm)

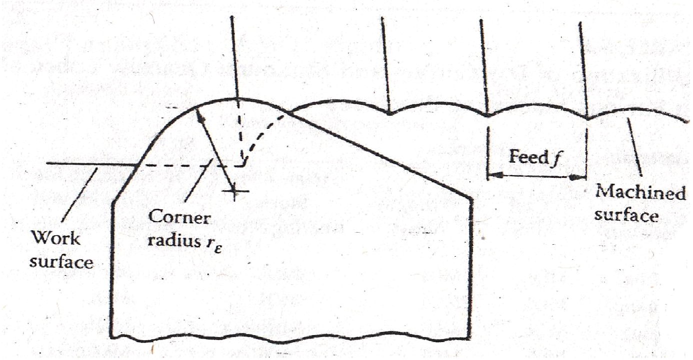

Fig. 5.3 Idealized model of surface roughness for round corner tool [38]

Cutting tools are usually provided with a rounded corner, and Figure 5.3 shows the surface produced by such a tool under ideal conditions. Deriving a theoretical equation giving the arithmetical mean value for such a surface is rather more difficult than in the preceding example, but it can be shown that this roughness value is closely related to the feed and corner radius by the following expression.

$$R_a = \frac{0.0321\, f^2}{r_\varepsilon} \qquad (5.30)$$

where r_ε is the corner radius.

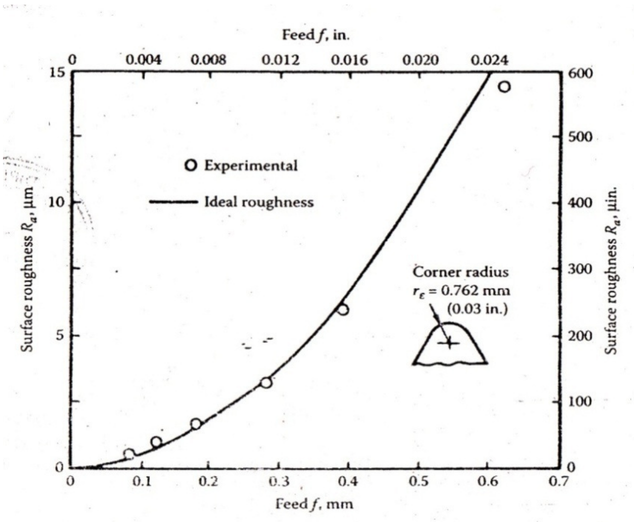

Fig. 5.4 Comp. of experimental results with an idealized model of surface roughness

In Figure 5.4 the theoretical relationship between the surface roughness value and the feed given by Equation 5.30 is compared with experimental results. In these experiments, work material (copper) and cutting conditions were carefully chosen such that the natural surface roughness was extremely low and no imperfections from the cutting action (chatter, built-up edge, etc.) were visible on the specimens. The operation was one of turning, and before each test the tools were carefully ground with the correct corner radius. Figure 5.4 shows that in these experimental results, the actual roughness of the specimens was close to the "ideal" for each feed used.

5.5.2 Natural Surface Roughness

In practice it is not usually possible to achieve conditions such as those described above, and normally the natural surface roughness forms a large proportion of the actual surface roughness. One of the main factors contributing to natural surface is occurrence of a built-up edge. The built up edge may be continually building up and breaking down, the fractured particles being carried away on the under surface of the chip and on the new work piece surface. Thus, it would be expected that the larger built-up edge, the rougher would be the surface produced, and factors tending to reduce the built up edge would give improved surface finish. Such factors would therefore be an increase in cutting speed, a change form say, high speed steel tool material to cemented carbide, the introduction of free machining materials such as leaded or resulfurezed steel, the application of the correct cutting lubricant at low cutting speeds, and so on. An example of the effect of cutting speed is shown in Fig. 5.5, where the actual surface roughness for a turned component is large at low cutting speeds and becomes reduced as the cutting speed is increased, until it approaches the ideal surface roughness at high cutting speeds.

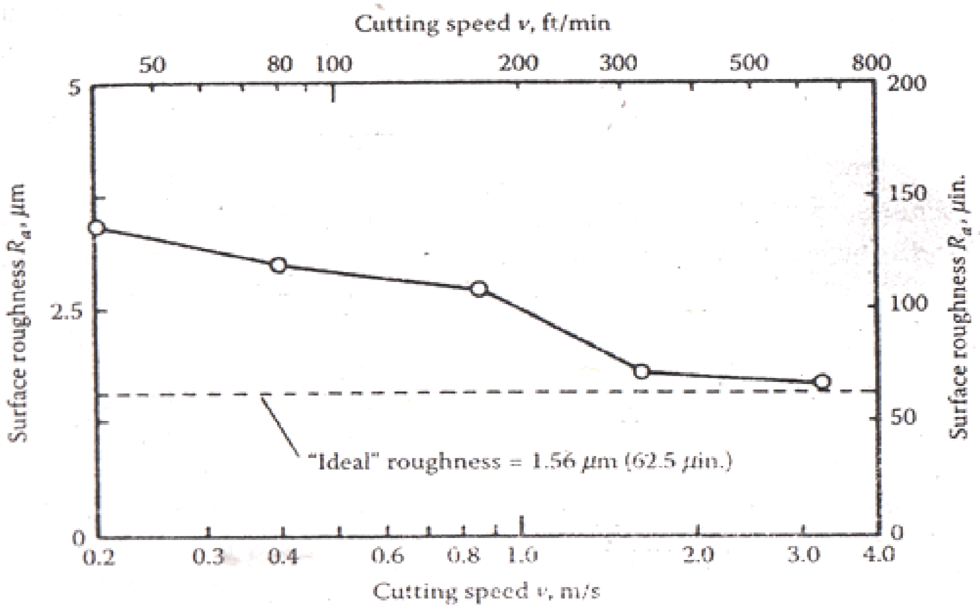

Fig. 5.5 Effect of cutting speed on the surface roughness for turning M.S.

5.6 Factors affecting on surface roughness

1. Vibration and chatter: During the machining vibrations may occur either as forced type or self induced type. Vibrations and chatter badly affect tool life, surface finish and accuracy of machined surface. Forced vibrations occur under the action of rhythmically varying force due to mechanical causes on the cutting tool. The tool or workpiece is pushed bodily by varying force at a frequency of mechanical sources. Forced vibrations may cause by continuous chips with built up edge and by fractures occurring ahead of the tool in case of segmental chips. These also occur if the finish of work pieces is very poor due to previous chatter cut. Self induced vibrations (chatter), more serve type, occur because of unstable equilibrium of the potential vibrating member and once initiated are of self perpetuating type and occur at a frequency close to natural frequency of the vibrating member. These may occur due to variation in cutting force with cutting speed.

These are least likely to occur at low cutting speeds, high tool frequencies or with a freshly sharpened tool.

2. Accuracy of Machined Surface: There are several factors which affect accuracy of machined surface. In general accuracy is affected by static alignment and steady state effects due to the elastic forces set up in the machine structure and workpiece during cutting and dynamic machining considerations (forced vibrations). Proper leveling of machine tool and alignment of various parts needs to be tested properly. The rigidity of machine tool and work piece combination should be increased by proper selection and setting of tool to minimize deflection due to overhang and by proper clamping and support of the work piece. Forced vibrations are caused by unbalanced rotating masses or by periodic vibrations caused by the teeth of milling cutter while engaging workpiece. Rigidity of the machine tool structure also improves the chatter.

3. Effect of lubrication and coolant: At present coolants and lubricants are increasingly recognized as harmful factors for environment and machine operators health. Industry and research institutions are looking for new means of reducing or eliminating the use of cutting fluids, both for economical and ecological reasons. This can be done if quality properties of machined surfaces and process parameters in dry and wet machining are comparable. Here investigation into the influence of cutting zone cooling and lubrication on surface roughness is done for turning of C45 steel. Dry cutting and minimum quantity lubrication (MQL) results are compared with conventional emulsion cooling. Cutting forces and their components were put under examination as well. The experimental outcomes indicate that the cooling and lubrication conditions affect significantly the investigated process and surface properties. However, the impact of the cooling and lubricating technique depends to a large extent on the applied cutting parameters, namely the cutting speed and feed rate. Turning dry or with MQL with properly selected cutting parameters makes it possible to produce better surface topography characteristics than turning with conventional emulsion cooling. Apart from improving the surface properties the MQL mode of cooling and lubrication also provides environmental friendliness (Fig.5.6)[66]

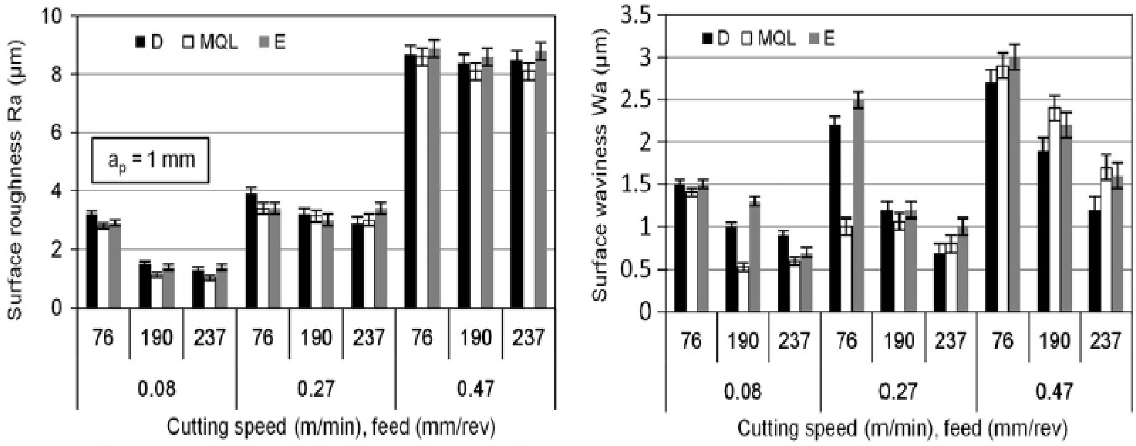

Fig. 5.6 Surface roughness and waviness depending on cooling and lubrication conditions and cutting parameters [66]

4. **Effect of the work piece material**: High hardness, strength and low ductility result in good surface roughness. Also defect in the structure of the work piece material affects the surface roughness. Addition of sulpher, lead to steel or lead to brass results in improves the surface roughness.

5. **Effect of tool material**: A material which permits high cutting speed will produce better surface finish. Thus the order would be carbon steel. HSS, cast alloy, sintered carbide tools. Diamond produces best finish due to smaller built up edge as a result of low friction of the metal on the face of the diamond.

6. **Geometry of chip formation:** The discontinuous chips are found to occur while machining brittle materials. In this case the roughness of machined surface is believed to depend on the size of chips.

5.7 Measurement of Surface Roughness

Measurement of surface roughness is important to realize that other kinds of deviation from perfectly smooth surface can occur. These deviations are called surface flows and waviness. Surface flows are widely separated irregularities that occur at random over the surface. They may be scratches, cracks, or similar flows.

The standard instruments for the determination of surface roughness operate by amplifying the vertical motion of a stylus as it is drawn slowly across the surface; they can produce, in addition, a continuous recording of the profile. In modern instruments the surface profile is digitally sampled and the arithmetical mean surface roughness value is calculated continuously over a selected cutoff distance. The cutoff length is selected to eliminate waviness from the measurements. Roughness tester SJ 400 used for this work is given in Fig. 5.7 and Table 5.3 with detailed specification.

Fig. 5.7 Surface roughness tester SJ-400

Table 5.3 Specification of surface roughness tester

Order No.*	SJ-401 SJ-402	178-947-3A (inch/mm) 178-945-3A (inch/mm)	178-957-3A (inch/mm) 178-959-3A (inch/mm)
Measuring method		Skidless/Skidded measurement	
Measuring range	Z-axis	32000µin, 3200µin, 320µin (800µm, 80µm, 8µm) (Up to 2,400µm with an option stylus)	
	X-axis	SJ-401: 1” (25mm) SJ-402: 2” (50mm)	
Drive method	Straightness	SJ-401: 12µin/1” (0.3µm/25mm) SJ-402: 20µin/2” (0.5µm/50mm)	
	Measuring speed	.002”, .004”, .02”, .04”/s (0.05, 0.1, 0.5, 1.0mm/s)	
	Return speed	.02”, .04”, .08”/s (0.5, 1.0, 2.0 mm/s)	
Height-Tilt adjustment unit	Tilt adjustment range	±1.5°	
Assessed profile		Primary profile (P), Roughness profile (R), Filtered waviness profile (W), DIN4776, MOTIF (R, W)	
Evaluation parameters		Ra, Ry, Rz, Rq, Pc, R3z, mr, Rt, Rp, Rv, Sm, S, δc, Rk, Rpk, Rvk, Ppi, R, AR, Rx, Δa, Δq, Ku, HSC, mrd, Sk, W, AW, Wte, Wx, Vo	
Number of sampling length		X1, X3, X5, XL* (*=arbitrary length)	
Arbitrary length		SJ-401: .01” to 2” (.01” increments) [0.1 to 25mm (0.1mm increments)] SJ-402: .04” to 2” (.01” increments) [0.1 to 50mm (0.1mm increments)]	
Sampling length (L)		.003”, .01”, .03”, .1”, .3” (0.08, 0.25, 0.8, 2.5, 8mm)	
Printing width		1.89” (48mm)/paper width: 2.28” (58mm)	
Recording magnification	Vertical magnification	10 to 100K magnification, Auto	
	Horizontal magnification	1 to 1K magnification, Auto	
Detector	Detection method	Differential inductance method	
	Minimum resolution	.005µin (320µin range)/0.000125µm (8µm range)	
	Stylus tip	Corn 90°, Radius 5µm, Diamond	Corn 60°, Radius 2µm, Diamond
	Measuring force	4mN	0.75mN
	Radius of skid	1.57”(40mm)	
	Skid force	Less than 400mN	
Function	Customize	Display/Roughness parameter selectable	
	Data compensation	R-surface, Tilt compensation	
	Ruler function	Displays the coordinate difference of any two points	
	Displacement detection mode	Enables the stylus displacement to be input while the drive unit is stopped	
	Statistical processing	Maximum value, Minimum value, Mean value, Standard deviation (s)	
Cut-off length		.003”, .01”, .03”, .1”, .3” (0.08, 0.25, 0.8, 2.5, 8mm)	
Calibration		Ra, Step (Automatic calibration entering the value of roughness specimen)	
Battery	Charging time	15 hours	
	Number of measurements	600 maximum without printing	
Power consumption		43W (max.)	
Dimension	Control unit	12.09”x6.50”x3.7” (307x165x94mm)	
	Height-Tilt adjustment unit	5.16”x2.48”x3.90” (131x63x99mm)	
	Drive unit	SJ-401: (128x36x47mm) SJ-402: (155x36x47mm)	
Roughness standard		JIS (JIS B0601-1994-1982), DIN, ISO, ANSI	
LCD size		Touch panel	
Data output		RS-232C input/output, SPC output	
External control		Connection to data processing system (option)	
Mass	Control unit	2.64lbs. (1.2kg)	
	Height-Tilt adjustment unit	1.88lbs. (0.4kg)	

There are many different roughness parameters in use, but Ra is by far the most common. Other common parameters include R_y , R_q and R_z . Some parameters are used only in certain industries or within certain countries. Each of the roughness parameters is calculated using a formula for describing the surface. The average roughness Ra is expressed in units of height. In the Imperial system, Ra is typically expressed in "millionths" of an inch. This is also referred to as "micro inches" or sometimes just as "micro".

5.8 Tool Geometries for Improved Surface Finish

As has been shown the geometric surface roughness developed in machining is dependent on the tool geometry, machine kinematics and feed rate. This has led to the development of specific type of edge preparation for cutting tool inserts aimed at improved surface finish or more often to allow increased feed rates for a specified roughness, with associated reduction in cycle time. These inserts are known as wiper inserts, and they have become widely used for both turning and milling tools.

Similar to conventional turning inserts, wiper inserts remove the chip with the leading cutting edge and this leaves the expected geometric surface roughness through the mechanism described in the previous section. However, wiper inserts have an additional radius or flat behind the tool that is kept in contact with the work piece after the initial cut (Fig. 5.8). This burnishes the peaks, leaving a smoother surface finish.

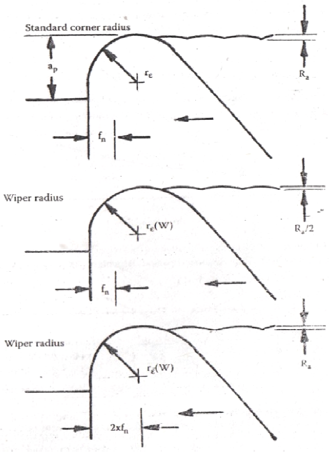

Fig. 5.8 Wiper inserts edge geometries and improvement in Roughness **[38, 45]**

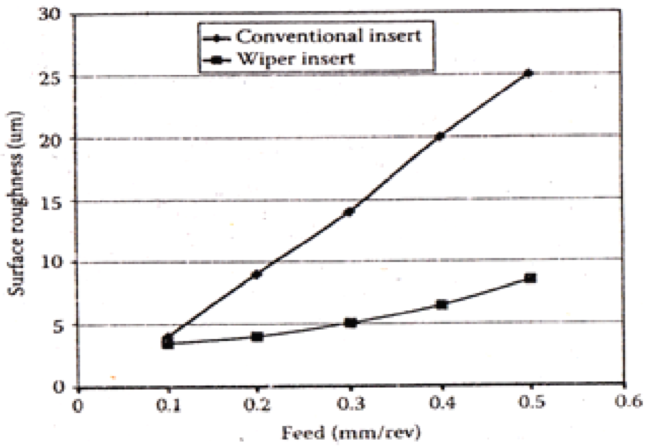

Fig. 5.9 Effect of feed on the effectiveness of a wiper inserts **[38]**

These wiper geometries are not effective for all applications. For example they are not suitable for light finishing cuts because they require slightly higher uncut chip thickness value to work well. Also higher feed rates are necessary to take full advantage of wiper geometry. Figure 5.9 shows the effect of the wiper edge on surface finish for different feed rates. In turning, these inserts are mainly effective on surface parallel and perpendicular to axis of rotation of the workpiece.

5.9 Experimental Design and Conditions

To develop a second-order surface roughness model, the experiments examined the impact of the following parameters on the surface roughness in finish turning:

1. Workpiece hardness.
2. Feed.
3. Tool nose radius
4. Depth of cut.
5. Cutting speed
6. Cutting time.

In this study, ceramic inserts (supplied by Ceratizit) were used. ISO code TNMG160404 EN-TMF, and TNMG 160408 EN-TM with different nose radius (60^0 triangular shaped inserts) are used. The inserts were mounted on a commercial tool. The factors and levels in the experiments are presented in Table 5.4 and Table 5.5. All the experiments were carried out on Jobber X_L model made by Ace designer CNC lathe machine with variable spindle speed 50 to 3500 rpm and 7.5 Kw motor drive was used for machining tests. Surface finish of the work piece material was measured by Surf test model No. SJ-400 (Mitutoyo make). The surface roughness was measured at three equally spaced locations around the circumference of the work pieces to obtain the statistically significant data for the test. We assume that the three, four and five-factor interactions are negligible, because these higher-order interactions are normally assumed to be almost impossible in practice. Therefore, a 2^{5-1} fractional design is selected. This resolution V design leads to 16 runs of the experiments. To consider system variations, such as tool wear and vibration in particular, the cutting time and a replicate number of three are selected, respectively and average is taken and presented in Table 5.6 and Table 5.7.

Two set of materials are used for this test with different hardness and chemical composition.

1. AISI 1040 steel and Aluminium
2. AISI 410 steel and Aluminium

Table 5.4 Factors and levels for AISI 1040 steel and Aluminium

Level	Hardness (A)	Feed (B)	Tool radius (C)	Cutting Velocity (D)	Depth of Cut (E)
-1	AISI 1040 steel (92HRB)	0.06mm/rev	0.4mm	220m/min	0.3mm
1	Aluminium (60 HRB)	0.14mm/rev	0.8mm	280m/min	0.9mm

Table 5.5 Factors and levels for AISI 410 steel and Aluminium

Level	Hardness (A)	Feed (B)	Tool radius (C)	Cutting Velocity (D)	Depth of Cut (E)
-1	AISI 410 steel (99HRB)	0.06mm/rev	0.4mm	220m/min	0.3mm
1	Aluminium (60 HRB)	0.14mm/rev	0.8mm	280m/min	0.9mm

5.10 Regression Analysis

To establish the prediction model, regression analysis is conducted with MINITAB using the above experimental data. Regression analysis is considered to be one of the most important and most popularly used data mining techniques. Feng and Wang [25] have shown that for a reasonably large set of data from structurally designed experiments, such as those presented in this work, regression analysis generates comparable results with its competing data mining method. Table 5.8 and 5.9 presents the regression result when considering cutting time. Six variables are involved in the prediction model: workpiece hardness; feed; tool nose radius; depth of cut; cutting speed; and cutting time.

Table 5.6 Design of experiment and data for AISI 1040 steel and Aluminium

Sr. No.	Material (A)	Feed (B) (mm/rev)	Tool Radius(C) (mm)	Speed (D) (m/min)	Depth of Cut(E) (mm)	Time(t) (sec)	Roughness (Ra) (μm)
1	-1	-1	-1	-1	1	20.62	1.35
2	1	-1	-1	-1	-1	27.50	0.52
3	-1	1	-1	-1	-1	11.00	2.25
4	1	1	-1	-1	1	08.25	1.85
5	-1	-1	1	-1	-1	27.50	1.10
6	1	-1	1	-1	1	20.62	0.35
7	-1	1	1	-1	1	08.25	1.52
8	1	1	1	-1	-1	11.00	0.82
9	-1	-1	-1	1	-1	18.30	0.66
10	1	-1	-1	1	1	13.72	0.39
11	-1	1	-1	1	1	05.47	1.65
12	1	1	-1	1	-1	07.30	1.50
13	-1	-1	1	1	1	13.72	0.89
14	1	-1	1	1	-1	18.30	0.29
15	-1	1	1	1	-1	07.30	0.92
16	1	1	1	1	1	05.47	0.70

Table 5.7 Design of experiment and data for AISI 410 steel and Aluminium

Sr. No.	Material (A)	Feed (B) (mm/rev)	Tool Radius(C) (mm)	Speed (D) (m/min)	Depth of Cut(E) (mm)	Time(t) (sec)	Roughness (Ra) (μm)
1	-1	-1	-1	-1	1	22.10	0.78
2	1	-1	-1	-1	-1	27.10	0.52
3	-1	1	-1	-1	-1	13.01	2.08
4	1	1	-1	-1	1	08.52	1.85
5	-1	-1	1	-1	-1	22.98	0.41
6	1	-1	1	-1	1	20.31	0.35
7	-1	1	1	-1	1	08.55	0.91
8	1	1	1	-1	-1	11.89	0.82
9	-1	-1	-1	1	-1	20.12	0.79
10	1	-1	-1	1	1	12.88	0.45
11	-1	1	-1	1	1	04.25	1.69
12	1	1	-1	1	-1	05.10	1.50
13	-1	-1	1	1	1	12.99	0.34
14	1	-1	1	1	-1	16.55	0.29
15	-1	1	1	1	-1	06.80	0.73
16	1	1	1	1	1	04.75	0.70

Figs. 5.10 and 5.11 compare the fitted values and observed values. Figs. 5.12 and 5.13 show the relative percentage error between the fitted value and the observed values using Eq. (5.28). Table 5.8 and 5.9 indicates that material, feed, tool nose radius, speed and cutting time significantly affect the surface roughness independently. Depth of cut has not significant effect of the surface roughness. In addition, some interactions among these six variables also significantly affect the surface roughness. Recall that cutting time is dependent of feed and speed. Figures 5.14 and 5.16 show that the prediction model has a good precision and it appears to be a better model than Eq. (5.28).

Table 5.8 The prediction model with six variables (AISI 1040 steel and Aluminium)

Predictor	Coef	SE Coef	T	P	VIF
Constant	1.34801	0.03713	36.31	0.000	
Material (A)	-0.245	0.006577	-37.25	0.000	1.000
Feed (B)(mm/rev)	0.22482	0.017	13.22	0.000	6.682
Tool radius(C)(mm)	-0.22375	0.006577	-34.02	0.000	1.000
Speed (D)(m/min)	-0.232998	0.009868	-23.61	0.000	2.251
Time (t)(Sec)	-0.021434	0.002606	-8.22	0.001	7.938
AB	0.06125	0.006577	9.31	0.001	1.000
AC	-0.042447	0.006592	-6.44	0.003	1.005
AD	0.09	0.006577	13.68	0.000	1.000
AE	-0.02	0.006577	-3.04	0.038	1.000
BC	-0.1875	0.006577	-28.51	0.000	1.000
CD	0.04875	0.006577	7.41	0.002	1.000

$$\mathrm{Ra} = 1.35 - 0.245 * \mathrm{Material(A)} + 0.225 * \mathrm{Feed(B)} - 0.224 * \mathrm{Tool\ radius(C)} - 0.233 * \mathrm{Speed(D)} - 0.0214 * \mathrm{Time(t)} + 0.0612 * A * B - 0.0424 * A * C + 0.090 * A * D - 0.020 * A * E - 0.188 * B * C + 0.0488 * C * D \quad (31)$$

S = 0.0263076   R-Sq = 99.9%   R-Sq (adj) = 99.8%
PRESS = 0.0370968   R-Sq(pred) = 99.27%

Table 5.9 The prediction model with six variables (AISI 410 steel and Aluminium)

Predictor	Coef	SE Coef	T	P	VIF
Constant	0.81698	0.08260	9.89	0.000	
Material (A)	-0.07692	0.01207	-6.37	0.001	1.013
Feed (B))(mm/rev)	0.42696	0.03659	11.67	0.000	9.311
Tool radius(C) (mm)	-0.31668	0.01238	-25.57	0.000	1.067
Speed (D)(m/min)	-0.06022	0.02258	-2.67	0.037	3.547
Time (t)(sec)	0.005224	0.006001	0.87	0.417	12.187
AC	0.04674	0.01237	3.78	0.009	1.064
AE	0.03160	0.01199	2.63	0.039	1.001
BC	-0.17905	0.01262	-14.19	0.000	1.107
BD	-0.05602	0.01244	-4.50	0.004	1.077

$$Ra = 0.817 - 0.0769 * Material(A) + 0.427 * Feed(B) - 0.317 * Tool\ rdius(C) - 0.0602 * Speed(D) + 0.00522 * Time(t) + 0.0467 * A * C + 0.0316 * A * E - 0.179 * B * C - 0.0560 * B * D \quad (32)$$

S = 0.0479597 R-Sq = 99.7% R-Sq (adj) = 99.3%
PRESS = 0.106859 R-Sq (pred) = 97.84%

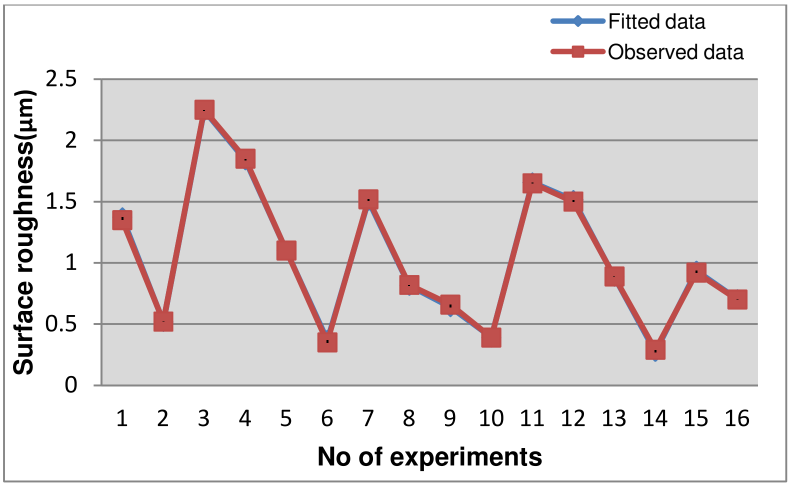

Fig. 5.10 Fitted value Vs observed value (AISI 1040 steel and Al)

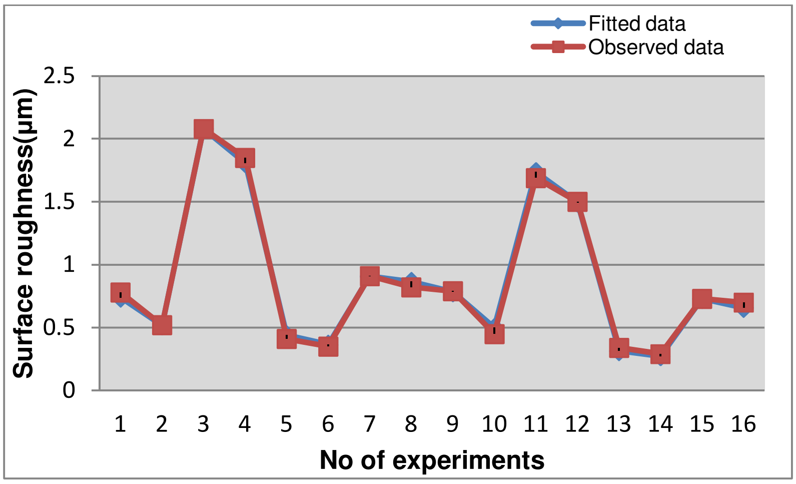

Fig. 5.11 Fitted value Vs observed value (AISI 410 steel and Al)

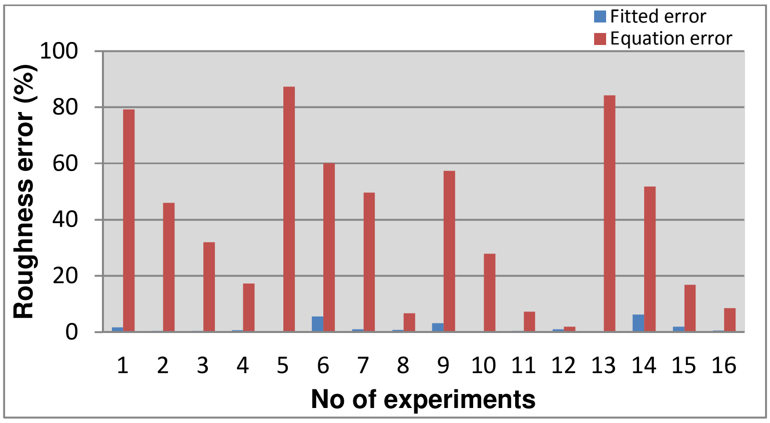

Fig. 5.12 Relative error of the fitted values vs. Computed values (AISI 1040 steel and Aluminium)

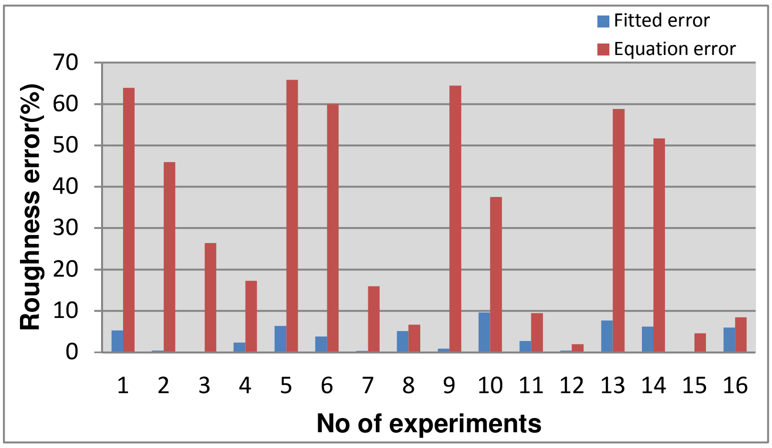

Fig. 5.13 Relative error of the fitted values vs. Computed values (AISI 410 steel and Aluminium)

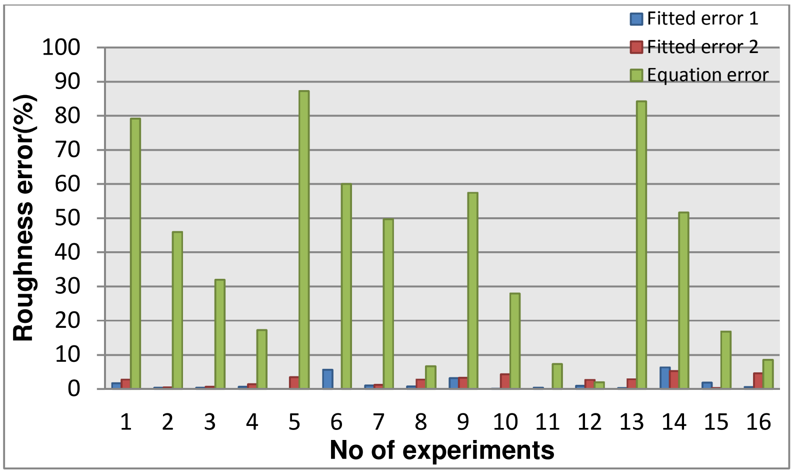

Fig. 5.14 Comparison of the fitted values and the estimated values (AISI 1040 steel and Aluminium)

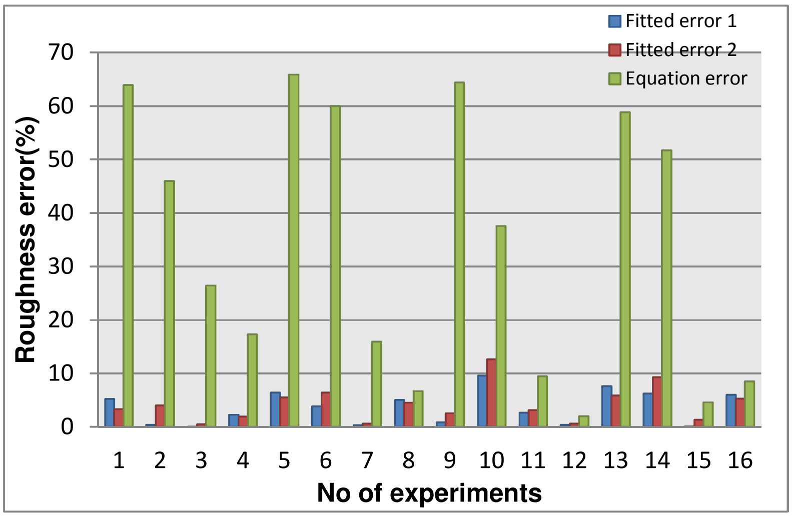

Fig. 5.15 Comparison of the fitted values and the estimated values (AISI 410 steel and Aluminium)

Figures 5.14 and 5.15 compares the relative percentage error among the fitted values considering the cutting time, the fitted values without considering the cutting time, and the computed values from Eq. 5.28. It shows that the fitted values have the highest precision, which shows that it represents the surface roughness better than Eq. 5.28.

5.11 Confirmation test

In order to verify the adequacy of the model developed, five confirmation run experiments have been performed (Table 5.10, 5.11) at different cutting conditions. The test condition for the first three validation run experiments are among the cutting conditions that are performed previously while the remaining two validation run experiments are the conditions that have not been used previously. The experimental results have been validated by asserting that the predicted values are very close to each other and hence, the developed models are suitable for predicting the surface roughness in machining with 4 % error.

Table 5.10 Confirmation test (1040 steel and Aluminium)

Sr. No.	Material (A)	Feed (B) (mm/rev)	Tool radius (C) (mm)	Speed (D) (m/min)	Depth of Cut(E) (mm)	Time (t) (sec)	Exp. (Ra) (μm)	Predicted (Ra) (μm)	Error (%)
1	1	1	1	1	1	5.50	0.72	0.7049	2.09
2	1	1	1	-1	-1	10.5	0.85	0.8263	2.78
3	1	-1	-1	-1	-1	25.25	0.55	0.5687	3.28
4	-1*	-1	-1	-1	-1	21.05	1.34	1.3173	1.69
5	1*	1	-1	-1	-1	8.54	1.85	1.8747	1.31

Table 5.11 Confirmation test (AISI 410 steel and Aluminium)

Sr. No.	Material (A)	Feed (B) (mm/rev)	Tool radius (C) (mm)	Speed (D) (m/min)	Depth of Cut(E) (mm)	Time (t) (sec)	Exp. (Ra) (μm)	Predicted (Ra) (μm)	Error (%)
1	1	1	1	1	1	5.10	0.68	0.6568	3.41
2	1	1	1	-1	-1	12.5	0.84	0.8646	2.84
3	1	-1	-1	-1	-1	26.2	0.53	0.5167	2.50
4	-1*	-1	-1	-1	-1	23.1	0.83	0.8109	2.30
5	1*	1	-1	-1	-1	9.2	1.81	1.746	3.53

In next chapter RSM and (3^4) full factorial design of experiment is used to develop the surface roughness prediction model for different materials like AISI 1040 steel, AISI 410 steel, Mild steel and Aluminium with 95% confidence level