Chapter 3

Low energy unification

3.1 Higgs Effect in SU(15) GUT

Recently a new paradigm of GUT models have evolved[1, 2, 6, 7] following an observation that at least one symmetry breaking chain of a GUT based on the group SU(15) can be unified at a very low energy $M_u \sim O(10^7)$ GeV[1]. Because baryon number B is a gauge symmetry in this model, proton decay can be suppressed, and one possible Higgs structure has been proposed to this end[2]. Low energy unification makes these models free from problems of grand unified monopoles[6] and the gauge hierarchy problem is also much less severe [1].

All the present activity on SU(15) GUT relys on two important claims, namely, (i) there exists at least one symmetry breaking pattern of SU(15) grand unification, where the gauge coupling constants evolve very fast and can be unified at an energy scale $M_u \sim O(10^7)$ GeV and (ii) there exists at least one choice of Higgs fields which can (a) allow the above symmetry breaking chain, (b) forbid any gauge boson mediated proton decay, (c) suppress Higgs mediated proton decay and (d) make this low energy unification consistent with the nonobservation of proton decay.

Here we analyze these two claims. We discuss in a general way proton decay and the choice of Higgs fields required for any symmetry breaking in these GUTs along with their effect on the evolution of the gauge coupling constants. We find this cannot be neglected: for SU(15), unification below $M_u \sim O(10^{14})$ GeV is impossible for the breaking pattern proposed by Frampton and Kephart [FK][2]. However other interesting patterns exist which yield unification at $\sim 10^9 GeV$ and violate baryon number symmetry $U(1)_B$ at about the electroweak breaking scale, although there is no proton decay. The low energy ($\sim 250GeV$) symmetry includes phenomenologically interesting chiral color symmetry[8] and quark-lepton un-unified electroweak symmetry[9].

Our notation is the following. When we write the semisimple group $SU(n)^q_L \times SU(m)^l_R \times U(1)_X$ the subscript implies either the charge of the U(1) group or that right (left) handed particles are non-singlets under SU(n) (SU(m)) and the superscript q (l) means that only quarks (leptons) transform under this group. The gauge coupling constants of the groups $SU(n)^q_L$ and $U(1)_X$ will be written as $\alpha_{nqL} = \frac{g^2_{nqL}}{4\pi}$ and $\alpha_{1X} = \frac{g^2_{1X}}{4\pi}$ respectively. For the breaking $G_i \longrightarrow G_{i-1}$, the G_{i-1} singlet component of the Higgs ϕ_i acquires a vev at a scale M_i. In

[1]This section is based on Ref [3]

denotes a totally antisymmetric n^{th} rank tensor; hence 1^m1^n denotes a Young tableaux of m and n in the first and second columns respectively.

$$
\begin{aligned}
G_1[SU(15)] &\overset{\langle\phi_1\rangle}{\longrightarrow} G_2[SU(12)^q \times SU(3)^l] \\
&\overset{\langle\phi_2\rangle}{\longrightarrow} G_3[SU(6)_L \times SU(6)_R \times U(1)_B \times SU(3)^l] \\
&\overset{\langle\phi_3\rangle}{\longrightarrow} G_4[SU(3)_{cL} \times SU(2)^q_L \times SU(6)_R \times U(1)_B \times SU(3)^l] \\
&\overset{\langle\phi_4\rangle}{\longrightarrow} G_5[SU(3)_{cL} \times SU(2)^q_L \times SU(3)_R \times U(1)_R \times U(1)_B \times SU(2)^l \times U(1)^l_Y] \\
&\overset{\langle\phi_5\rangle}{\longrightarrow} G_6[SU(3)_c \times SU(2)_L \times U(1)_B \times U(1)^{\prime}_Y] \\
&\overset{\langle\phi_6\rangle}{\longrightarrow} G_7[SU(3)_c \times SU(2)_L \times U(1)_Y] \\
&\overset{\langle\phi_7\rangle}{\longrightarrow} G_8[SU(3)_c \times \times U(1)_Q] \qquad (3.1)
\end{aligned}
$$

$\langle\phi_i\rangle = M_i$. We shall denote this pattern by {1234567}; the pattern of ref. [1] is {1267}, for which $M_3 = M_4 = M_5 = M_6$.

We turn next to the Higgs fields required to ensure this pattern, taking minimal representations whenever possible. Our Higgs structure is very similar to that of [FK][2]. We choose ϕ_1 to be a 1^3, *i.e.*, a **455**-plet. The G_2 singlet component of ϕ_1 can then acquire a vev to break the group $G_1 \longrightarrow G_2$. The vev of the G_3 singlet component of $1^{14}1$ (**224**-plet) can break G_2, leaving $U(1)_B$ unbroken. Breaking $SU(6)_L$ to its special maximal subalgebra $SU(3)_{cL} \times SU(2)^q_L$ requires a somewhat large Higgs representation. Although self-conjugate representations can break any group to its maximal subalgebra, in this case the adjoint representation does not work and the next higher dimensional self-conjugate representation is required. These are contained in the self-conjugate representations of the higher groups, and the particular $SU(6)_L \longrightarrow SU(3)_{cL} \times SU(2)^q_L$ symmetry breaking can be accomplished with a **10800** dimensional $(1^{13}1^2)$ Higgs of $SU(15)$ which is contained in $\mathbf{105} \otimes \overline{\mathbf{105}}$. This is the lowest dimensional Higgs to break $G_3 \longrightarrow G_4$; [FK] considered a **14175**-plet $(1^{14}1^{14}11)$ $\subset \mathbf{120} \otimes \overline{\mathbf{120}}$ *i.e.* the next-highest one. Appropriate components of the adjoint (**224**-plet) can break $G_4 \longrightarrow G_5$. For the next stage a 1^3 ($\phi_5 \equiv$ **455** – *plet*) can be used; this breaks global lepton number in addition to the local groups.

The surviving group is now G_6 $[SU(3)_c \times SU(2)_L \times U(1)_{Y'} - 1_B]$. Note that $U(1)_{Y'}$ is orthogonal to $U(1)_B$, while the hypercharge Y in the standard model does not commute with B. In fact Y is a linear combination of B and Y'. [FK] break G_6 with a 1^5 (**3003**-plet) by giving a vev to the $Y = 0$ component labeled (10,11,12,13,14). To find out whether there exists any lower dimensional Higgs representation one can check that it is not possible to write any B-violating operator only with the fermions invariant under G_6. However with a 1^3 (**455**-dimensional) or a 1^4 (**1365**-dimensional) Higgs field there exists a G_6-invariant B-violating dimension-7 operator. But under G_7 one can write down B-violating dimension-6 operators only with fermions. Hence one can have $\phi_6 =$ **455** or **1365**. Both have B and Y' nonzero; the $Y = 0$ component can acquire a vev. Either of $\phi_7 =$ **105** or a **120** can be used to break the standard electroweak symmetry; [FK] had considered both for this purpose, but this is not necessary.

Considering next proton decay, since quark-lepton unification is broken at a scale M_1, the lepto-quark gauge bosons (X_μ) acquire a mass $\approx M_1$, while the di-quark bosons (Y_μ) acquire mass at a scale where the quark-antiquark unification is broken $(\approx M_2)$. Since $U(1)_B$ is a local gauge symmetry X_μ and Y_μ do not mix at this level. These transform under G_3 as $X_\mu \equiv [(6,1,\frac{1}{3},\overline{3})+ (1,\overline{6},-\frac{1}{3},\overline{3}) + (1,6,\frac{1}{3},3) + (\overline{6},1,-\frac{1}{3},3)]$ and $Y_\mu \equiv [(6,6,\frac{2}{3},1) + (\overline{6},\overline{6},-\frac{2}{3},1)]$, with $m_X^2 \sim \langle\phi_1\rangle$ and $m_Y^2 \sim \langle\phi_2\rangle$. The mixing between X_μ and Y_μ takes place when the Higgs fields ϕ_a and ϕ_b acquire vevs in the term $X_\mu\phi_a Y^\mu\phi_b \subset D_\mu\phi_a D^\mu\phi_b$. Since X_μ and Y_μ carry different B, the mixing can occur only at M_6, suppressing the amplitude for gauge boson mediated proton decay $\sim O(\frac{M_5 M_6}{M_1^2 M_2^2})$. Thus if $M_1 \approx M_2 \approx M_u$ and $M_5 \approx M_6 \approx 10^2$ GeV, then $M_u \geq 10^9$ GeV from the present limit on the proton lifetime.

Now both X_μ and Y_μ are contained in the SU(15) gauge boson G_μ, which transforms as a self-adjoint **224**-plet of SU(15). As a result, the SU(15) multiplets $\phi_a(\supset \Phi_a)$ and $\phi_b(\supset \Phi_b)$ can allow the coupling $X^\mu\Phi_a Y_\mu \Phi_b$ iff $\phi_a = \phi_b^\dagger(\equiv \phi)$. If only one component of ϕ acquires a vev, *i.e.* the Higgs multiplet which breaks $U(1)_B$ takes part in no other symmetry breaking, then $\langle\Phi_a\rangle$ and $\langle\Phi_b\rangle = \langle\Phi_a^\dagger\rangle$ will carry equal and opposite B, forbidding mixing between X_μ and Y_μ. Gauge boson mediated proton decay is then absent, at least to this order. Couplings of $\phi_a^\dagger$ with other Higgs fields will determine the higher order terms. Since ϕ_a is the Higgs field which breaks $U(1)_B$, in our case $\phi_a = \phi_6 = \mathbf{1365}$. The couplings of $\mathbf{1365}^\dagger$ of the form $\langle\mathbf{1365}\rangle\langle\mathbf{1365}^\dagger\rangle$ with other Higgs fields cannot have any B-violating effect. If we also consider $\phi_7 = \mathbf{120}$ then the only $U(1)_B$-breaking term is of the form $\langle\mathbf{1365}\rangle\langle\mathbf{1365}\rangle\langle\mathbf{1365}\rangle\langle\mathbf{455}\rangle$, for which $B = 3$. Thus this also cannot contribute to proton decay. Since there is no linear coupling of **1365** with other Higgses, in this scenario there is absolutely no gauge boson mediated proton decay with $\phi_6 = \mathbf{1365}$ and $\phi_7 = \mathbf{120}$. If different components of the same Higgs field (which break $U(1)_B$) acquire vevs, then there can be gauge boson mediated proton decay· for example if $\phi_6 = \mathbf{455}$, then since $\phi_5 = \mathbf{455}$, mixing between X_μ and Y_μ will occur. The amplitude will be proportional to $\sim \frac{\langle\phi_5\rangle\langle\phi_6\rangle}{M_1^2 M_2^2}$, which is not suppressed by Yukawa couplings.

There is no straightforward way to understand the Higgs mediated proton decay; such processes will depend on the choice of all the Higgs fields in the theory. For ϕ_6, the types of operators which can lead to proton decay are of the form $\psi\psi\psi\psi\langle\phi_6\rangle$. But the Higgs fields necessary to couple the fermions with $\phi_6 = \mathbf{1365}$ are **105** dimensional, and ϕ_6 does not have any linear couplings with combinations of other Higgs fields; hence this operator cannot give rise to proton decay. Considering higher dimensional operators, with one ϕ_6 there does not exist any other higher dimensional operator, and as a result there is also no Higgs mediated proton decay for this choice. Hence to avoid proton decay we choose $\phi_6 = \mathbf{1365}$ and $\phi_7 = \mathbf{120}$.

We next compute the effect of the Higgs fields considered in the evolution of the coupling constants[10]. We use the one-loop renormalization group equations which have the form,

$$\mu\frac{d\alpha_i(\mu)}{d\mu} = 2\beta_i\alpha_i^2(\mu) \tag{3.2}$$

where $\alpha_i = \frac{g_i^2}{4\pi}$, the β-functions are defined as, $\beta_i = -\frac{b_i}{(4\pi)}$, and $b_i = T_g[i] - \frac{4}{3}T_f[i] - \frac{1}{6}T_s[i]$, corresponding to the contributions from gauge bosons, fermions and Higgs scalars, respectively. The fermionic contributions to the various subgroups are the same and are given by $T_f = n_f$, where n_f is the number of generations; these cancel out in the equation of $\sin^2\theta_w$ and $(1 - \frac{8}{3}\frac{\alpha}{\alpha_s})$. The gauge contributions are

$$T_g[12] = 176; \quad 2T_g[3_{cL}] = 2T_g[3_R] = T_g[3^l] = 4T_g[3_c] = 44;$$

$$T_g[6_L] = T_g[6_R] = 88; \quad 3T_g[2^q_L] = T_g[2^l_L] = 4T_g[2_L] = \frac{88}{3}; \tag{3.3}$$

with $T_g = 0$ for all $U(1)$ groups. For our choice of Higgs the T_s are given in Table 3.1.

To include the Higgs contributions we assumed the extended survival hypothesis[11] and the Applequist-Carrazone decoupling theorem[12] (standard assumptions made in calculating Higgs effects in evolution of coupling constants).

$M_1 \to M_2$	$M_2 \to M_3$	$M_3 \to M_4$	$M_4 \to M_5$	$M_5 \to M_6$	$M_6 \to M_7$
$[12] = 3052$	$[6_L] = 264$	$[3_{cL}] = [2^q_L] = 48$	$[3_{cL}] = [2^q_L] = [3_R] = 18$	$[3_c] = 0$	$[3_c] = 0$
$[3_l] = 608$	$[6_R] = 114$	$[6_R] = 114$	$[1_R] = 18.33$	$[2_L] = 0.5$	$[2_L] = 0.5$
	$[1_B] = 93$	$[1_B] = 93$	$[1_B] = 1.5$	$[1_B] = 1.5$	$[1_Y] = .3$
	$[3_l] = 136$	$[3_l] = 136$	$[1_l] = 13.33$	$[1_{Y'}] = .5$	
			$[2^l_L] = 36$		

Table 3.1: Contributions to $T_s[n]$ at various scales

Denoting $\alpha_G^{-1}(M_J)$ by $\mathcal{A}_G(J)$, we employ the appropriate boundary conditions: (i) $\mathcal{A}_{12}(1) = \mathcal{A}_{3l}(1) = \mathcal{A}_{15}(1)$, (ii) $\mathcal{A}_{6L}(2) = \mathcal{A}_{6R}(2) = \mathcal{A}_{1B}(2) = \mathcal{A}_{12}(2)$, (iii) $\mathcal{A}_{3cL}(3) = \mathcal{A}_{2qL}(3) = \mathcal{A}_{6L}(3)$, (iv) $\mathcal{A}_{1R}(4) = \mathcal{A}_{3R}(4) = \mathcal{A}_{6R}(4)$ and $\mathcal{A}_{2l}(4) = \mathcal{A}_{1l}(4) = \mathcal{A}_{3l}(4)$, (v) $\mathcal{A}_{3c}(5) = \frac{1}{2}\mathcal{A}_{3cL}(5) + \frac{1}{2}\mathcal{A}_{3R}(5)$; $\mathcal{A}_{2L}(5) = \frac{3}{4}\mathcal{A}_{2qL}(5) + \frac{1}{4}\mathcal{A}_{2l}(5)$ and $\mathcal{A}_{1Y'}(5) = \frac{1}{2}\mathcal{A}_{1R}(5) + \frac{1}{2}\mathcal{A}_{1l}(5)$, (vi) $\mathcal{A}_{1Y}(6) = \frac{9}{10}\mathcal{A}_{1Y'}(6) + \frac{1}{10}\mathcal{A}_{1B}(6)$. With this information we can relate the SU(15) coupling constants (at energy $M_u \approx M_1$) to the low energy ($M_7 \approx M_w \approx 10^2 GeV$) $SU(3)_c \times SU(2)_L \times U(1)_Y$ coupling constants:

$$\begin{aligned}
\alpha_{3c}^{-1}(M_w) &= \alpha_{15}^{-1}(M_1) + 2\beta_{12} ln(\frac{M_1}{M_2}) + (\beta_{6L} + \beta_{6R}) ln(\frac{M_2}{M_3}) \\
&+ (\beta_{3cL} + \beta_{6R}) ln(\frac{M_3}{M_4}) + (\beta_{3cL} + \beta_{3R}) ln(\frac{M_4}{M_5}) \\
&+ 2\beta_{3c} ln(\frac{M_5}{M_6}) + 2\beta_{3c} ln(\frac{M_6}{M_w})
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
\alpha_{2L}^{-1}(M_w) &= \alpha_{15}^{-1}(M_1) + (\frac{3}{2}\beta_{12} + \frac{1}{2}\beta_{3l}) ln(\frac{M_1}{M_2}) + (\frac{3}{2}\beta_{6L} + \frac{1}{2}\beta_{3l}) ln(\frac{M_2}{M_3}) \\
&+ (\frac{3}{2}\beta_{2qL} + \frac{1}{2}\beta_{3l}) ln(\frac{M_3}{M_4}) + (\frac{3}{2}\beta_{2qL} + \frac{1}{2}\beta_{2lL}) ln(\frac{M_4}{M_5}) \\
&+ 2\beta_{2L} ln(\frac{M_5}{M_6}) + 2\beta_{2L} ln(\frac{M_6}{M_w})
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\alpha_{1Y}^{-1}(M_w) &= \alpha_{15}^{-1}(M_1) + (\frac{11}{10}\beta_{12} + \frac{9}{10}\beta_{3l}) ln(\frac{M_1}{M_2}) \\
&+ (\frac{9}{10}\beta_{6R} + \frac{1}{5}\beta_{1B} + \frac{9}{10}\beta_{3l}) ln(\frac{M_2}{M_3}) \\
&+ (\frac{9}{10}\beta_{6R} + \frac{1}{5}\beta_{1B} + \frac{9}{10}\beta_{3l}) ln(\frac{M_3}{M_4}) \\
&+ (\frac{9}{10}\beta_{1R} + \frac{1}{5}\beta_{1B} + \frac{9}{10}\beta_{1l}) ln(\frac{M_4}{M_5})
\end{aligned}$$

$$+(\frac{9}{5}\beta_{1Y'} + \frac{1}{5}\beta_{1B})ln(\frac{M_5}{M_6}) + 2\beta_{1Y}ln(\frac{M_6}{M_w}) \quad (3.6)$$

The relevant linear combinations are those which yield

$$\sin^2(\theta_w) = \frac{3}{8} - \frac{5}{8}\alpha(\alpha_{1Y}^{-1} - \alpha_{2L}^{-1}) \;\; and \;\; (1 - \frac{8}{3}\frac{\alpha}{\alpha_s}) = \alpha(\alpha_{2L}^{-1} + \frac{5}{3}\alpha_{1Y}^{-1} - \frac{8}{3}\alpha_{3c}^{-1}) \quad (3.7)$$

namely,

$$\begin{aligned} 2.4(16\pi^2) &= (52.8 - 162.9h)ln(M_{12}) + (35.2 - 36.7h)ln(M_{23}) + (17.2h - 82.1)ln(M_{34}) \\ &\quad +(29.3 - 2.7h)ln(M_{45}) + (14.7 + .1h)ln(M_{56}) + (14.7 - .1h)ln(M_{6w}) \quad (3.8) \\ 8.3(16\pi^2) &= (264 - 814.7h)ln(M_{12}) + (117.3 - 23h)ln(M_{23}) + (58.7 - 19h)ln(M_{34}) \\ &\quad +(88 - .5h)ln(M_{45}) + (44 + .5h)ln(M_{56}) + (44 + .3h)ln(M_{6w}) \quad (3.9) \end{aligned}$$

where $h = 0$ denotes the pure gauge case and $h = 1$ includes Higgs effects. Here $M_{ij} \equiv M_i/M_j$ and the current experimental values[13] of $\sin^2\theta_w(= .233)$ and $\alpha_s(= .11)$ have been used.

For the pattern {1267} the unification scale $M_1 \approx M_u \approx 10^7 GeV$ in the pure gauge case, which is the [FL] result[1]. Large gauge contributions to the evolution equations enhance the coefficients of the first two terms; as a result unification is reached faster than in the usual GUTs like SU(5) (for which $M_1 = M_2 = M_A = M_u \sim O(10^{14})GeV$). However when Higgs effects are included ($h = 1$) we find no solution to (3.8,3.9) for the {1267} scenario other than $M_1 = M_u \geq O(10^{14})GeV$, forbidding the low energy unification of [FL].

For $h = 1$ we find three other interesting three-stage patterns: (A) {2467} with $M_1 = M_2 = M_u$; $M_3 = M_4 = M_x$, $M_5 = M_6 = M_y$; (B) {3467} with $M_1 = M_2 = M_3 = M_u$; $M_4 = M_x$; $M_5 = M_6 = M_y$; and (C) {2567} with $M_1 = M_2 = M_u$; $M_3 = M_4 = M_5 = M_x$; $M_6 = M_y$ each having a 1-parameter family of solutions for M_y. (Although (C) does not have full unification at low energy, it does have interesting TeV physics.) Sample values are given in Table 3.2

	{2467}		{3467}		{2567}	
M_y	M_u	M_x	M_u	M_x	M_u	M_x
250	7.91×10^8	2.96×10^8	8.87×10^8	3.50×10^2	1.97×10^{14}	1.77×10^3
500	1.11×10^9	4.06×10^8	1.25×10^9	7.05×10^2	1.98×10^{14}	3.53×10^3
1000	1.56×10^9	5.56×10^8	1.76×10^9	1.42×10^3	1.98×10^{14}	7.05×10^3
1500	1.91×10^9	6.68×10^8	2.15×10^9	2.15×10^3	1.99×10^{14}	1.06×10^4

Table 3.2: Mass scales (in GeV) for patterns (A)-(C)

The most interesting pattern is {3467}, which has both low energy unification at $\sim 10^9$ GeV and interesting TeV physics. We can decouple the electroweak breaking scale with the other symmetry breakings and have TeV scale chiral color symmetry and the quark-lepton un-unified electroweak symmetry breaking, which will raise the unification scale a little. The existence of chiral color symmetry at the TeV scale or lower will imply the presence of axigluons, whose phenomenological consequences have been studied[14]. The presence of the un-unified electroweak symmetry at low energy will imply the existence of extra charged and neutral gauge bosons, whose mixing with the Z-boson will affect various asymmetry parameters in the e^+e^- deep-inelastic scattering[15].

To summarize, we have shown that Higgs fields play a significant role in the evolution of gauge coupling constants in GUTs where baryon number is a symmetry. The consistency of the symmetry breaking scenario presented here with present-day proton decay data along with its interesting TeV scale physics make it a model worthy of further investigation.

3.2 Implications of SUSY SU(15) GUT

Supersymmetry offers a very interesting theoretical possibility which places fermions and bosons at equal footing via its transformation laws. Though supersymmetry itself can solve the problem of gauge hierarchy it is nevertheless an interesting proposition to endow the SU(15) GUT model with supersymmetric transformation laws and see the consequences. This is simply because supersymmetry is a rich symmetry by itself and nature seems to use all the symmetries available to her. Particularly, if one wants to unify these theories with gravity without causing naturalness problem, then supersymmetric version seems more promising. Another important aspect of checking the consistency of the supersymmetric $SU(15)$ GUT is to find out whether experimental findings of supersymmetry will still allow the possibility of low energy unification.

In the suspersymmetric version of the theory [2]every particle will imply the presence of its supersymmetric partner. When all this new particles run in the loops they will alter the renormalization procedure of the conventional theory and hence the beta functions. The supersymmetric beta functions (to one loop order) which will control the evolution of the coupling constants given by the following expression [16]. Let us also note here that due to unequal normalizations of the generators at different stages of the symmetry breaking chain in the calculations of the mass scales one has to multiply the beta functions with proper normalization factors

$$\beta(N) = -\frac{1}{(4\pi)}\left[3N - T - n_f\right] \tag{3.10}$$

Here, N stands for the SU(N) group of which the coupling constant is under consideration, T denotes the contribution of the scalar loops and n_f stands for the number of fermion generations which is constrained to be three by LEP data.

We use the one-loop renormalization group equations which have the form,

$$\mu\frac{d\alpha(\mu)}{d\mu} = 2\beta\alpha^2(\mu) \tag{3.11}$$

Where $\alpha(\mu)$ stands for the value of the coupling constant at the energy scale μ

Solving the renormalization group equations (3.6) using the combinations of the couplings given in the equation (3.7) and the Higgs contributions given in Table(3.3) we can find out the unification scale. Afterwards using the value of the unification scale as an input we can find the value of α_{15} at the unification scale using the expressions of α_3. We have calculated

[2]This section is based on Ref [5]

$M_1 \to M_2$	$M_2 \to M_3$	$M_3 \to M_4$	$M_4 \to M_5$	$M_5 \to M_6$	$M_6 \to M_7$
$[12] = -2908$ $[3_l] = -572$	$[6_L] = -192$ $[6_R] = -42$ $[1_B] = -93$ $[3_l] = -100$	$[3_{cL}] = -30$ $[6_R] = -42$ $[1_B] = -93$ $[3_l] = -100$ $[2^q_L] = -40$	$[3_{cL}] = 0$ $[1_R] = -18.3$ $[1_B] = -1.5$ $[1_l] = -13.3$ $[2^l_L] = -15.6$ $[2^q_L] = -10$ $[3_R] = 0$	$[3_c] = 9$ $[2_L] = 5$ $[1_B] = -1.5$ $[1_{Y'}] = -0.5$	$[3_c] = 11$ $[2_L] = 7.2$ $[1_Y] = -.02$

Table 3.3: Value of [3N-T] at various scales with proper normalizations

these quantities for all possible chains coming from SU(15) GUT. None of the chains can give a consistent low energy unification scheme. For a few breaking chain where unification is apparently achieved at a scale around 10^{12} GeV the value of α_{15} at the unification scale becomes undefined hence forbidding a consistent perturbative unification scheme. What it means is that the coupling constants evolve so fast that they become more than unity much before the unification scale. Then the α^{-1} evolves to zero and the coupling constants becomes undefined.

To outline the procedure of solutions in brief let us set the notation that $m_{ij}=ln\frac{m_i}{m_j}$. Now solving in the case when $m_{12} = 0, m_{34} = 0, m_{56} = 0$ we get by solving for m_{23} and m_{45}:

$$
\begin{aligned}
m_{23} &= -1.35 + 0.005 m_{67} \\
m_{45} &= 17.78 + 0.60 m_{67}.
\end{aligned}
$$

Now for $m_i > m_j$, m_{ij} has to be positive definite which immediately sets the bound $m_{67} = 27$ when $m_{23} = 0$ hence for the breaking chain 2467 m_6 has the minimum value of the order of 10^{14} GeV. Furthermore by using the minimum value of m_{67} in the second equation we can see that the minimum value of m_{45} is 34. Hence forbidding any unification of coupling at all (within the plank scale).

Similarly let us consider the case when $m_{12} = 0, m_{23} = 0, m_{56} = 0$ we get by solving for m_{34} and m_{45}:

$$
\begin{aligned}
m_{34} &= 11.2 - 0.41 m_{67} \\
m_{45} &= -7.03 + 0.31 m_{67}.
\end{aligned}
$$

Now in this case to make m_{45} at least positive m_{67} has to be atleast 22.67 and hence m_{34} has to be atleast 1.9 which leads to an apparent unification scale of approximately 10^{12} GeV. But if one checks the value of the inverse of the coupling constant at the unification scale using the α_3 equation, for example, one sees that it has crossed the value zero and has become negative. Hence for the chain 3467 there is no consistent unification framework. In this simple way all possible symmetry breaking chains can be analyzed.

To see the result for the chain 1367 let us solve the equations for m_{12} and m_{23} in terms of m_{67}, making all other m_{ij}s vanish. The solutions are:

$$
\begin{aligned}
m_{12} &= -0.27 + 0.009 m_{67} \\
m_{23} &= -0.558 + 0.023 m_{67}
\end{aligned}
$$

this means that to make m_{12} positive, m_{67} has a minimum value of 30. Which immediately means that the scale m_6 is at least 10^{15} GeV, and the unification scale is even higher. For the chain 2567 we will solve for m_{23} and m_{56} and making all the other m_{ij}s vanish. The solutions are:

$$\begin{aligned} m_{12} &= 0.26 - 0.004 m_{67} \\ m_{23} &= 34.03 - 1.16 m_{67}. \end{aligned}$$

Here though m_{67} can be low yet the unification of couplings still occur at a very high value. This is simply because as m_{67} becomes smaller the value of m_{23} increases. Similarly, for most of the chains, we find the unification scale becomes larger than 10^{14} GeV, and the possibility of low energy unification is lost. For these chains we have first calculated by taking the supersymmetry breaking scale to be same as M_6. Taking M_{susy} to be lower, or around the TeV scale, we find the situation worsens.

In the Table 3.4 we state a sample of these values for those chains which was considered earlier in the nonsupersymmetric model, and some more sample chains.

Breaking Chain	Unification Scale	$\alpha_{15}^{-1}(M_U)$
2467	No Unification	–
3467	4.63 10^{12}	Undefined
2567	Greater than 10^{14}	–
1367	Greater than 10^{14}	–
4567	Greater than 10^{14}	–

Table 3.4: Mass scales in GeV

In this section we have attempted to ask the question that if supersymmetry is discovered in near future how is it going to affect the new paradigm of the low energy unification of the SU(15) GUT model. These conclusions will also be true for the $SU(16)$ GUT, with similar symmetry breaking chains. We find that the low energy unification with SU(15) in the supersymmetric framework is not allowed. Most of the symmetry breaking chains do not allow for low energy unifications, and a few symmetry breaking chains which allow low energy unification fails to satisfy the perturbative unification constraint (coupling constants to be less than one). Hence the signals of the existence of supersymmetry in future colliders will rule out the possibility of low energy unification.

The scenario of symmetry breaking in nonsupersymmetric SU(15) GUT, which allows low energy unification, has some interesting features. It is essential for the low energy unification to have chiral color $SU(3)_{cL} \times SU(3)_{cR}$ group and the quark-lepton ununified group $SU(2)_L^q \times SU(2)_L^l$ survive till very low energy, for the gauge coupling constants to evolve very fast and get united at an energy scale around 10^8 GeV. Thus the existence of these groups and the leptoquarks are some of the essential critereons of the low energy unification, which can be tested in the laboratory in near future. Thus any signatures of these groups may seriously question the existence of supersymmetry and if the signatures of the low energy unification and also that of supersymmetry are found, then it will cast a serious question on our understanding of the grand unification scenario.

3.3 Low energy unification with SU(16)

We have already discussed a new paradigm of low energy unification in which we have considered $SU(15)$ as the unification group[1]. Here we extend the idea to the left-right symmetric version of such a theory. We show that retaining all the good features of $SU(15)$ we can also incorporate left-right symmetry in intermediate stages. Unlike the $SU(15)$ GUT here lepton number is also a local gauge symmetry which may survive to a low energy scale. Right handed neutrino can be accommodated naturally as all the fermions transform in the fundamental representation of $SU(16)$.

At the level of highest symmetry the theory is invariant under the gauge group $SU(16)$ [3]. At and above this level the coupling constant is that of the group $SU(16)$. With the decrease in energy, the group goes through a number of symmetry breaking phases, and the theory becomes least symmetric at the present energies with the residual symmetry of $SU(3)$ color and the symmetry of electromagnetic interactions. It is noteworthy that the baryon number symmetry remains exact upto a very low energy scale of a few TeV. This makes the proton stable in the sense that the gauge boson mediated proton decay is absent. Interestingly the completely un-unified symmetry group of the quarks and leptons also appears at a low energy scale together with the chiral color symmetry. The appearance of this group at a comparatively low scale makes this model worthy of phenomenological studies[8],[9],[14].

Here to begin with we give the breaking chain that can give rise to the standard model. We note here that there can be in general a number of chains of descent to the standard model.

$$\begin{aligned}
SU(16) &\xrightarrow{M_U} G[SU(12)\times SU(4)^l]\\
&\xrightarrow{M_1} G_1[SU(6)_L\times SU(6)_R\times U(1)_B\times SU(4)^l]\\
&\xrightarrow{M_2} G_2[SU(3)_L\times SU(2)^q_L\times SU(6)_R\times U(1)_B\times SU(4)^l]\\
&\xrightarrow{M_3} G_3[SU(3)_L\times SU(2)^q_L\times SU(3)_R\times U(1)^q_R\times U(1)_B\times SU(2)^l_L\times SU(2)^l_R\times U(1)^{lep}]\\
&\xrightarrow{M_4} G_4[SU(3)_L\times SU(2)^q_L\times SU(2)^l_L\times SU(3)_R\times U(1)_R\times U(1)_B\times U(1)^l]\\
&\xrightarrow{M_5} G_5[SU(3)_c\times SU(2)_L\times U(1)_B\times U(1)_h]\\
&\xrightarrow{M_6} G_6[SU(3)_c\times SU(2)_L\times U(1)_Y]\\
&\xrightarrow{M_z} G_7[SU(3)_c\times U(1)_{em}]
\end{aligned}$$

Here the superscript q or l denotes that quarks or leptons have nontrivial transformation law under these groups and the subscripts L and R mean so for the left and right handed fermions. The subscript c stands for the color gauge group of QCD.

In a previous section we have shown that in $SU(15)$ GUT the effect of Higgs bosons play a significant role in the evolution of the coupling constants with increasing energy and hence on the values of the mass scales. This is due to the presence of high dimensional Higgs fields required to obtain the desired symmetry breaking pattern. The influence of the Higgs fields on the evolution of coupling constants can be so serious that they can alter the symmetry breaking pattern altogether. In $SU(16)$ GUT The symmetry breaking pattern is very similar to that of its $SU(15)$ counterpart. Hence in $SU(16)$ or in $SU(15)$ GUT the Higgs effects must

[3] This section is based on Ref. [4]

be taken seriously. Here we shall consider the Higgs fields required to obtain the breaking chain and their contribution in the renormalization group equations in detail.

The Higgs structure is similar to that we proposed for $SU(15)$ GUT. We denote 1^n as the totally antisymmetric nth rank tensor and $1^n 1^m$ as the representation which has m and n vertical boxes in the first and second columns of its Young's tableau. For the transition from the group G_1 to group G_2 the G_2 singlet component of the Higgs field should acquire vacuum expectation value. Turning to the specific case of $SU(16)$ we note that at the scale M_U the breaking can be achieved by giving the vacuum expectation value to the $SU(12)\times SU(4)$ singlet component of 1^4. Using the exactly same procedure we see that the breaking at the scale M_1 can be done by $1^{14}1$ which leaves $U(1)_B$ unbroken. At the scale M_2 the breaking of $SU(6)_L$ to its special maximal subalgebra requires a somewhat large dimensional Higgs field representation. We use the **14144** dimensional Higgs field $1^{14}1^2$ to break this group. As a passing comment we note here that this Higgs field will contribute significantly to the beta functions of the renormalization group equations and make its presence strongly felt in the determination of the mass scales. The group $SU(4)^l$ can be broken by a Higgs field which transforms as a 15-plet under $SU(4)^l$ and which is contained in **255** under $SU(16)$. At the stage M_3 the breaking of $SU(6)_R$ to $SU(3)_R\times$ $U(1)_R$ is a bit complicated. **255** breaks $SU(6)_R$ to $SU(3)\times$ $SU(3)\times U(1)_R$ and subsequently the two $SU(2)_L$ groups of the quark and leptonic sectors respectively are glued by $1^{14}1^2$. The breaking of the lepton number local gauge symmetry $U(1)^{lep}$ can be achieved by either **16** or the two index symmetric Higgs field of dimension **136**. In the first case it carries a lepton number one unit and in the second case it carries that of two units. We shall see that the choice of specific Higgs field shall give interesting difference of physics in the context of neutrino oscillations. At the scale M_5 the breaking is done by the 1^4 Higgs field which is **1820** dimensional. The baryon number is broken by either 1^5 or 1^6. In both the cases we get interesting physics. As an example in the first case we get processes where baryon number changes by 3 units and in the second case it changes by 2 units. It is welknown that to give masses to the fermions vacuum expectation value has to be given to the component $(1,2,-\frac{1}{2})$ which is contained in either 1^2 or 11. These Higgs Field representations are summarized in Table A

Let us now turn our attention to the group theoretic transformation properties of the fermions under the different symmetry groups in the symmetry breaking scheme. A minimal left-right symmetric theory should have at least one right handed neutrino (ν_R) on top of the standard quarks which includes three left handed doublets and six right handed singlets under the weak interaction gauge group $SU(2)_L$ and three leptons namely one left handed doublet and one right handed singlet. At grand unification energies and above this sixteen fermions should transform under some representation of the unification group. This requirement makes $SU(16)$ a very natural choice of the unification gauge group which has a 16 dimensional fundamental representation. In the model the fermions transform under the fundamental representation of $SU(16)$. Now as the energy becomes lower the symmetry breakings occur and the transformation properties of the fermions change in each symmetry breaking scale. In the following we summarize these transformation properties. We use the notation that (m,n) is a representation which transforms under the semisimple group $SU(M)\times SU(N)$ as a m-plate under the former group and as a n-plate under the the later group.

$$\begin{aligned} SU(16) &\longrightarrow 16 \\ G &\longrightarrow (12,1)+(1,4) \\ G_1 &\longrightarrow (1,\bar{6},n,1)+(6,1,-n,1)+(1,1,0,4) \end{aligned}$$

$SU(16)$	G	G_1	G_2	G_3	G_4	G_5	G_6
1820							
255	(143,1)						
14144	(4212,1)	(189,1,0,1)					
255	(1,15)	(1,1,0,15)	(1,1,0,1,15)				
255	(143,1)	(1,35,0,1)	(1,1,35,0,1)				
14144	(4212,1)	(1,189,0,1)	(1,1,189,0,1)				
136	(1,10)	(1,1,0,10)	(1,1,1,0,10)	$(1,1,1,0,0,1,3,\frac{2}{\sqrt{8}})$			
16	(1,4)	(1,1,0,4)	(1,1,1,0,4)	$(1,1,1,0,0,1,2,\frac{1}{\sqrt{8}})$			
1820	(66,6)	$(6,\bar{6},0,6)$	$(3,2,\bar{6},0,6)$	$(3,2,\bar{3},-\frac{1}{\sqrt{12}},0,2,2,0)$	$(3,2,\bar{3},-\frac{1}{\sqrt{12}},0,2,\frac{1}{\sqrt{12}})$		
$4368(1^5)$	(220,6)	$(1,\bar{20},\frac{3}{2\sqrt{6}},6)$	$(1,1,\bar{20},\frac{3}{2\sqrt{6}},6)$	$(1,1,1,\frac{1}{\sqrt{12}},\frac{3}{2\sqrt{6}},1,1,-\frac{2}{\sqrt{6}})$	$(1,1,1,\frac{1}{\sqrt{12}},\frac{3}{2\sqrt{6}},1,-\frac{2}{\sqrt{12}})$	$(1,1,\frac{3}{2\sqrt{6}},-\frac{1}{\sqrt{24}})$	
136	(78,1)	$(6,\bar{6},0,1)$	$(3,2,\bar{6},0,1)$	$(3,2,\bar{3},\frac{1}{\sqrt{12}},0,1,1,0)$	$(3,2,\bar{3},\frac{1}{12},0,1,0)$	$(1,2,0,\frac{1}{\sqrt{24}})$	$(1,2,-\frac{1}{2}\sqrt{\frac{3}{20}})$

TABLE A

$$
\begin{aligned}
G_2 &\longrightarrow (1,1,\bar{6},n,1)+(3,2,1,-n,1)+(1,1,1,0,4)\\
G_3 &\longrightarrow (1,1,\bar{3},p,n,1,1,0)+(1,1,\bar{3},-p,n,1,1,0)+(3,2,1,0,-n,1,1,0)\\
&\quad +(1,1,1,0,0,1,2,m)+(1,1,1,0,0,2,1,-m)\\
G_4 &\longrightarrow (1,1,\bar{3},p,n,0)+(1,1,\bar{3},-p,n,0)+(3,2,1,0,-n,0)+(1,1,1,0,0,-2\sqrt{\frac{2}{3}}m)\\
&\quad +(1,2,1,0,0,\sqrt{\frac{2}{3}}m)+(1,1,1,0,0,0)\\
G_5 &\longrightarrow (\bar{3},1,n,n)+(\bar{3},1,n,-n)+(3,2,-n,0)+(1,2,0,n)+(1,1,0,-2n)+(1,1,0,0)\\
G_6 &\longrightarrow (\bar{3},1,-\frac{2}{3}K)+(\bar{3},1,\frac{1}{3}K)+(3,2,\frac{1}{6}K)+(1,1,K)+(1,1,-\frac{1}{2}K)+(1,1,0)
\end{aligned}
$$

Here the $U(1)$ normalization are defined in terms of

$$
n=\frac{1}{2\sqrt{6}};\quad m=\frac{1}{2\sqrt{2}};\quad p=\frac{1}{2\sqrt{3}};\quad K=\sqrt{\frac{3}{20}}.
$$

We know that in the electroweak breaking scale M_Z the generators of electromagnetic symmetry group $U(1)_{em}$ arises out as a linear combination of the generator of the $U(1)$ part of the weak isospin group $SU(2)_L$ and that of the weak hypercharge $U(1)_Y$ by the following equation,

$$
Q=T_L^3+Y. \tag{3.12}
$$

Let us call this equation as the $U(1)$ *matching condition* at the scale M_Z. Similarly at the various symmetry breaking scales in the above breaking chain we have used different matching conditions for the groups. These matching conditions are stated below.

At the scale M_4 the lepton number symmetry breaks as the generator of $U(1)^{lep}$ and the diagonal generator of $SU(2)_R^l$ mixes with each other in the following way to generate the group $U(1)^l$,

$$
Y^l=\sqrt{\frac{1}{3}}T^3_{2^l_R}+\sqrt{\frac{2}{3}}Y^{lep}. \tag{3.13}
$$

At the scale M_5, $U(1)_R$ and $U(1)^l$ breaks to make $U(1)_h$.

$$
Y_h=\sqrt{\frac{1}{2}}Y_R+\sqrt{\frac{1}{2}}Y^l. \tag{3.14}
$$

At the scale M_6, baryon number cease to be a local gauge symmetry and conventional hypercharge appears from the linear combination of $U(1)_B$ and $U(1)_h$.

$$
Y=-\sqrt{\frac{1}{10}}Y_B-\sqrt{\frac{9}{10}}Y_h. \tag{3.15}
$$

Now we briefly touch two more mathematically involved topics . To begin with we note that the generators of $SU(16)$ and that of the standard model groups cannot be normalized in the same way. We proceed further by giving a short discussion of the process of calculating the contribution of the Higgs fields to the beta functions. Let us fix that all the generators of $SU(16)$ are normalized to 2. In that case at the standard model energies the generators of $SU(3)_C$ and $SU(2)_L$ automatically becomes the generators of $SU(16)$. In contrast the generators of $U(1)_Y$ are normalized to $\frac{1}{2}$. So in the renormalization group equations we have to multiply the beta function corresponding to $U(1)_Y$ group by the appropriate factor of 4. Similarly it is easy to see that all other $U(1)$ groups in the symmetry breaking chain has to be multiplied by 4. Turning to the non-abelian groups it can be checked that the group $SU(2)^q_L$ in all stages is normalized to $\frac{3}{2}$ hence to treat it at par with all other groups one has to multiply the beta function corresponding to this by a factor of $\frac{4}{3}$. $SU(3)_L$ and $SU(3)_R$ in all the stages are normalized to 1 hence one finds the aforesaid factor to be 2. To complete the discussion on the normalization factors we note that all other groups are normalized to $\frac{1}{2}$ hence the relevant factor is 4

At this point let us turn our attention to the expression of the beta function for the group $SU(N)$

$$b(N) = -\frac{1}{(4\pi)^2}\left[\frac{11}{3}N - \frac{1}{6}T - \frac{4}{3}n_f\right] \tag{3.16}$$

For $U(1)$ groups N vanishes. Here n_f denotes the number of families of fermions and $T(R)$ denotes the contribution of the Higgs fields which transform nontrivialy under the group under consideration. To calculate T we have followed the following sum rule[18]:

Suppose R_i and r_i ($i = 1, 2, ..$) are different representations of a group $SU(N)$, which when vectorically multiplied satisfies the following relation.

$$R_1 \times R_2 = \sum_{i=1} r_i \tag{3.17}$$

Also let for the representation of dimension r, the contribution to the renormalization group equation is $T(R)$. Then,

$$T(R_1 \times R_2) = R_2 T(R_1) + R_1 T(R_2) = \sum_{i=1} T(r_i) \tag{3.18}$$

To use these equations one uses the following information to start with

$$\begin{aligned} T(N) &= \frac{1}{2,} \\ T(N^2-1) &= N, \\ T\left[\frac{N(N-1)}{2}\right] &= \frac{N-2}{2,} \\ T\left[\frac{N(N+1)}{2}\right] &= \frac{N+2}{2,} \\ T(1) &= 0. \end{aligned}$$

As an example consider 3 and $\bar{3}$ representations of $SU(3)$. When vectorically multiplied they give

$$3 \times \bar{3} = 1 + 8,$$

so using the sumrule

$$T(8) = 3T(3) + 3T(\bar{3}) - T(1) = 3.$$

To evaluate the mass scales we use the standard procedure of evolving the couplings with energy The energy dependence of the couplings with energy[10]. The energy dependence of the couplings are completely determined by the particle content of the theory and their couplings inside the loop diagrams of the guage boson's. This is expressed by the renormalization group equation The one-loop RG equation is given by the following equation.

$$\mu \frac{d}{d\mu}\alpha(\mu) = 2\ b\ \alpha^2(\mu), \tag{3.19}$$

where

$$\alpha = \frac{g^2}{4\pi}. \tag{3.20}$$

Using the above information and the matching conditions given with each symmetry breaking chain one can relate the $SU(16)$ coupling constant $\alpha_{SU(16)}$ with the standard model couplings α_{3c}, α_{2L} and α_{1Y} at the scale M_Z. At this point let us remember that there are three quark doublets and one leptonic doublet under the group $SU(2)_L$ in the standard model hence in the evolution of coupling α_{2L} the quark and leptonic groups $SU(2)_L^q$ and $SU(2)_L^l$ do not contribute equally to the standard model group $SU(2)_L$ instead they contribute with a relative factor 3.

$$\begin{aligned}
g_{3c}^{-2}(M_z) &= g_{SU(16)}^{-2}(M_U) + \\
&\quad 2b_{12}M_{U1} + (b_{6L} + b_{6R})M_{12} + (b_{3L} + b_{6R})M_{23} + \\
&\quad (b_{3L} + b_{3R})M_{34} + (b_{3L} + b_{3R})M_{45} + 2b_{3c}M_{56} + 2b_{3c}M_{6z} \\
g_{2L}^{-2}(M_z) &= g_{SU(16)}^{-2}(M_U) + \\
&\quad (\frac{3}{2}b_{12} + \frac{1}{2}b_4^l)M_{U1} + (\frac{3}{2}b_{6L} + \frac{1}{2}b_4^l)M_{12} + \\
&\quad (\frac{3}{2}b_{2L}^q + \frac{1}{2}b_4^l)M_{23} + (\frac{3}{2}b_{2L}^q + \frac{1}{2}b_{2L}^l)M_{34} + (\frac{3}{2}b_{2L}^q + \frac{1}{2}b_{2L}^l)M_{45} + \\
&\quad 2b_{2L}M_{56} + 2b_{2L}M_{6z} \\
g_{1Y}^{-2}(M_z) &= g_{SU(16)}^{-2}(M_U) + \\
&\quad (\frac{11}{10}b_{12} + \frac{9}{10}b_4^l)M_{U1} + (\frac{9}{10}b_{6R} + \frac{1}{5}b_{1B} + \frac{9}{10}b_4^l)M_{12} + \\
&\quad (\frac{9}{10}b_{6R} + \frac{1}{5}b_{1B} + \frac{9}{10}b_4^l)M_{23} + (\frac{9}{10}b_{1R} + \frac{1}{5}b_{1B} + \frac{6}{10}b_1^{lep} + \frac{3}{10}b_{2R}^l+)M_{34} + \\
&\quad (\frac{9}{10}b_{1R}^q + \frac{1}{5}b_{1B} + \frac{9}{10}b_1^l)M_{45} + \\
&\quad (\frac{9}{5}b_{1h} + \frac{1}{5}b_{1B})M_{56} + 2b_{1Y}M_{6z}
\end{aligned} \tag{3.21}$$

Here M_{ij} is defined as $ln(\frac{M_i}{M_j})$

To calculate the mass scales we also have to know the numerical values of the beta function coefficients To know them one has to know the contribution of the Higgs scalars to the beta functions (T). In the Table 3.5 we give these values.

With the quantities $g_{1Y}^{-2}(M_z)$ $g_{2L}^{-2}(M_z)$ and $g_{3c}^{-2}(M_z)$ at hand one can construct two different linear combinations with them to form the experimentally measured quantities at the energy

G	G_1	G_2	G_3	G_4	G_5	G_6
$[12] = 1492$	$[6_L] = 69$	$[3_L] = 42$	$[3_L] = 15$	$[3_L] = 9$	$[3_c] = 0$	$[3_c] = 0$
$[4^l] = 293$	$[6_R] = 93$	$[2_L^q] = 63$	$[2_L^q] = 22.5$	$[2_L^q] = 13.5$	$[2_L] = 0.5$	$[2_L] = 0.5$
	$[1_B] = 45$	$[6_R] = 93$	$[3_R] = 15$	$[3_R] = 9$	$[1_B] = .375$	$[1_Y] = .075$
	$[4^l] = 63$	$[1_B] = 45$	$[1_R^q] = 7.58$	$[1_R^q] = 3.16$	$[1_h] = .083$	
		$[4^l] = 63$	$[1_B] = .375$	$[1_B] = .375$		
			$[2_L^l] = 18$	$[2_L^l] = 9$		
			$[1^{lep}] = 2$	$[1_l] = 3.16$		

Table 3.5: Contributions of the Higgs scalar to the R-G equation at various energy scales

scale M_z.It easy to check that the following relations hold between them.

$$\begin{aligned} sin^2(\theta_w) &= \frac{3}{8} - \frac{5}{8}e^2(g_{1Y}^{-2} - g_{2L}^{-2}), \\ 1 - \frac{8}{3}\frac{\alpha}{\alpha_s} &= e^2(g_{2L}^{-2} + \frac{5}{3}g_{1Y}^{-2} - \frac{8}{3}g_{3c}^{-2}). \end{aligned} \tag{3.22}$$

From the present experimental measurements at LEP the value of $Sin^2(\theta_w)$ and α_s has been very accurately measured.We use for our purpose the following values[2] of them and the $U(1)$ coupling α at the scale M_z

$$\begin{aligned} sin^2(\theta_w) &= .233, \\ \alpha_s &= .11, \\ \alpha &= \frac{1}{127.9.} \end{aligned} \tag{3.23}$$

Having these informations at hand one can straightaway go to calculate the mass scales of symmetry breaking.

Let us discuss the calculation of the first chain in some detail. Let us now assume that $M_4 = M_3 = M_A$. This means that the groups $SU(6)_L$ $SU(6)_R$ and $SU(4)^l$ happens to break at the same scale. Similarly let us also assume that $M_4 = M_5 = M_B$. Solving for M_{U1} and M_{B6} in terms of the other variables one gets,

$$\begin{aligned} M_{U1} &= -.28 - .10M_{1A} - .10M_{6z} + .04M_{AB} \\ M_{B6} &= 19.80 - 4.81M_{1A} - 2.93M_{6z} - .21M_{AB} \end{aligned} \tag{3.24}$$

As the symmetry breaking at M_U precedes thet at M_1, M_{U1} is at least positive. So from the first equation one infers that for a specific set of values of the other parameters in the right-hand side there is a minimum value to M_{AB}.Varying the parameters of the equations one gets the following subset of the solution set allowed by the equations.Taking M_z to be around 91 GeV one can also calculate the unification scale and the scaleM_6 where the completely un-unified symmetry of the quarks and leptons and the chiral color symmetry is broken.We note that as the parameter M_{AB} increases i.e. as the separation between the scale M_A and the scale M_B increases the scale M_B comes down. Results are summarized in Table 3.6.

M_{AB}	M_{1A}	M_{6z}	M_{B6}	M_{U1}	M_B	M_U
7	0	0	18.4	0	10^9	10^{12}
9.5	1	0	12.9	0	10^8	10^{12}
10.75	1.5	0	10.3	0	10^7	10^{11}
12	2	0	8.7	0	10^6	10^{11}
14.5	3	0	2.3	0	10^4	10^{11}

Table 3.6: Mass scales of SU(16) GUT

3.3.1 Proton Decay

Having the mass scales and Higgs structure in hand we proceed in this paper to discuss the issue of proton decay now. In all the breaking chains that we have considered here, the quark lepton unification is broken at the scale M_U while the quark antiquark unification is broken at the scale M_1. As a result the leptoquark gauge bosons (X_μ) will acquire mass at the scale M_U while the diquark gauge bosons (Y_μ) acquire mass at the scale M_1. Under the group G_1 their transformation properties are

$$\begin{aligned} X_\mu &\Longrightarrow (6,1,-B,\bar{4})+(1,\bar{6},B,\bar{4})+ \\ &\quad\ (\bar{6},1,B,4)+(1,6,-B,4) \\ Y_\mu &\Longrightarrow (6,6,-2B,1)+(\bar{6},\bar{6},2B,1) \end{aligned} \tag{3.25}$$

where B is defined as,

$$B = \frac{1}{2\sqrt{6}} \tag{3.26}$$

Now $U(1)_B$ being an explicit local gauge symmetry of the model, X_μ and Y_μ contains different " Barion Numbers " and hence cannot mix directly to form an $SU(16)$ invariant operator.

The mixing can be induced indirectly through the term $D_\mu\phi_a D^\mu\phi_b$, where D_μ is the covariant derivative of the $SU(16)$ invariant theory.$D_\mu\phi_a D^\mu\phi_b$ will contain a term $X_\mu\phi_a X^\mu\phi_b$. When ϕ_a and ϕ_b acquires vacuum expectation value the mixing between X_μ and Y^μ occurs. But this can occur only at the scale M_6 hence the amplitude is suppressed by a factor of $O\frac{(M_5M_6)}{(M_1^2M_2^2)}$.

To see how the gauge bosons couple to the Higgs fields we note that all the gauge bosons at the $SU(16)$ level transform under the **224** dimensional adjoint representation. We also note the following tensor product at the $SU(16)$ level

$$224 \times 224 = 1 + 224 + 224 + 14175 + 10800 + 12376 + 12376 \tag{3.27}$$

Being the product of two selfconjugate representations all the terms in the right hand side are selfconjugate which couples to only self conjugate representations. From the Table A that the the Higgs field that carries Baryon Number is 1^5. So the only Higgs field which can induce a Baryon Number violating effect is 1^5 which is **4368** dimensional.

The only self conjugate combination made up with 1^5s is $< 4368 >< \bar{4368} >$ which again corries no baryon number hence not giving rise to any baryon number violating process[17].

To see the Higgsfield mediated proton decay at first we note that the fermions are in the **16** dimensional fundamental representation. To give mass to the fermions the coupling of the form $\bar{\psi_L}^c \psi_L \phi$ must exist. The minimum dimensional Higgs field which can do the job is **120**. This field can give rise to Higgs madiated proton decay if 1^6 breaks the Baryon Number due to the presence of the term $< 1^6 >< 1^6 >< 1^2 >< 1^2 >$ in the Lagrangian. In that case we can choose **136** to give mass to the fermions. In our choice 1^5 breaks the baryon number hence it does not couple to **120**. Hence there is no Higgs mediated proton decay.

3.3.2 $N - \overline{N}$ Oscillations

Let us consider the $SU(16)$ level operator $< 1^5 >< 1^5 >< 1^5 >< 16 >$. This forms a singlet under $SU(16)$ and hence allowed in the Lagrangian. This term give rise to $\Delta B = 3$ processes. If instead we choose **136** to break the lepton Number symmetry, then this process vanishes.

We have already noted that if 1^6 breaks the Baryon Number symmetry then one has to choose **136** to give mass to the fermions; here we note that then the term $< 1^{14}1^2 >< 136 >< 136 >< 1^6 >< 1^6 >$ will be be allowed in the Lagrangian which may give rise to $\Delta B = 3$ processes. As the term is of dimension five it will be suppressed by M_U. With 1^2 we can construct the $SU(16)$ level operator $< 1^5 >< 1^5 >< 1^4 >< 1^2 >$ which can break the Baryon Number by two units and hence give rise to gauge boson mediated $N - \bar{N}$ oscillations. To see the Higgs field mediated processes we note that if **120** dimensional Higgs field couples to the fermions and 1^6 breaks the Baryon number then the operator $< 1\bar{2}0 >< 1\bar{2}0 >< 1\bar{2}0 >< 1^6 >$ can give rise to Higgs field mediated N-$\bar{N}$ oscillations.

In this paper we have seen that there exists one possible breaking chain in a Grand Unified Theory based on the group $SU(16)$ where a unification scale of the order of 10^{11} GeV is possible. There exists a very low energy scale (M_B) which may be almost anywhere between the unification scale and the electroweak scale where completely ununified symmetry of quarks and leptons may exist together with chiral color symmetry. The scale M_B comes lower when the separation between the scale M_A and the scale M_B is increased. Qualitatively we understand it in the following way. The beta function coefficients can be looked into as the slope of the lines if one plots the inverse coupling constants with respect to energy. It can be easily checked that as at the $SU(16)$ level all the fermions transform under the fundamental representation of the group and in the other levels they transform in a more complicated way under the various groups in the intermediate stages, all the groups cannot be normalized in the same way. To compensate for the mismatch in the normalization the beta function coefficients has to be multiplied by appropriate factors. Because of this the slope of the curves representing the inverse couplings also gets multiplied by the appropriate factors and the couplings get united earlier giving rise to low energy unification.

We have also seen that this model satisfies the experimental constraints coming from proton decay experiments in the sense that proton decay is suppressed. We have shown that there exists atleast one choice of the Higgs sector where there is no Higgs mediated proton decay either.

For some specific choice of the Higgs fields there may exist interesting physical consequences like the $N - \bar{N}$ oscillation. There is also the possibility of having the sea-saw mechanism to give Majorana mass to the neutrinos and this also may have observable consequences.

Last but not the least we emphasize again that there exists very rich low energy physics coming from this model hence keeping in mind the forthcoming high-energy experiments at Superconducting Super Collider, CERN Large Hadron Collider and other places this model is worthy of further investigation.

3.4 Appendix

3.4.1 SU(16) Tensor Products

$$\begin{aligned}
16 \times 16 &= 120_a + 136_s \\
\bar{16} \times 16 &= 1 + 255 \\
16 \times 120 &= 560_a + 1360 \\
\bar{120} \times 120 &= 1 + 255 + 14144 \\
\bar{136} \times 136 &= 1 + 255 + 18240 \\
560_a \times 16 &= 1820_a + 7140 \\
1820_a \times 16 &= 4368_a + 24752
\end{aligned} \tag{3.28}$$

3.4.2 SU(16) Branching Rules

$$\begin{aligned}
SU(16) &\Longrightarrow SU(12) \times SU(4) \\
16 &= (12,1) + (1,4) \\
136 &= (78,1) + (12,4) + (1,10) \\
120 &= (66,1) + (12,4) + (1,6) \\
255 &= (143,1) + (12,\bar{4}) + (\bar{12},4) + \\
&\quad (1,15) + (1,1) \\
560 &= (220,1) + (66,4) + (12,6) + \\
&\quad (1,\bar{4}) \\
1820 &= (495,1) + (220,4) + (66,6) + \\
&\quad (12,\bar{4}) + (1,1) \\
14144 &= (1,1) + (1,35) + (12,\bar{4}) + \\
&\quad (12,\bar{20}) + (\bar{12},4) + (\bar{12},20) + \\
&\quad (\bar{66},6) + (66,\bar{6}) + (143,1) + \\
&\quad (143,15) + (70,\bar{4}) + (\bar{780},4) + \\
&\quad (4212,1)
\end{aligned} \tag{3.29}$$

3.4.3 SU(12) Tensor Products

$$12 \times 12 = 66_a + 78_s$$

$$
\begin{aligned}
\bar{12} \times 12 &= 1 + 143 \\
12 \times 66 &= 220_a + 572 \\
\bar{78} \times 78 &= 1 + 143 + 5940 \\
\bar{66} \times 66 &= 1 + 143 + 4212 \\
220_a \times 12 &= 495 + 2145 \\
495_a \times 12 &= 792 + 5148
\end{aligned}
\tag{3.30}
$$

3.4.4 SU(12) Branching Rules

$$
\begin{aligned}
SU(12) &\Longrightarrow SU(6)\times SU(6)\times U(1) \\
12 &= (6,1,-B) + (1,\bar{6},B) \\
66 &= (15,1,-2B) + (1,\bar{15},2B) + (6,\bar{6},0) \\
78 &= (21,1,-2B) + (1,\bar{21},2B) + (6,\bar{6},0) \\
143 &= (35,1,0) + (\bar{6},\bar{6},2B) + (6,6,-2B) + \\
&\quad (1,1,0) + (1,35,0) \\
220 &= (20,1,-3B) + (1,\bar{20},3B) + (6,\bar{15},B) + \\
&\quad (15,\bar{6},-B) \\
495 &= (15,1,-4B) + (20,\bar{6},-2B) + (15,\bar{15},0) + \\
&\quad (6,\bar{20},2B) + (1,\bar{15},4B) \\
792 &= (\bar{6},1,-5B) + (15,\bar{6},-3B) + (20,\bar{15},-B) + \\
&\quad (15,\bar{20},B) + (6,\bar{15},3B) + (1,6,5B) \\
572 &= (70,1,-3B) + (15,\bar{6},-B) + (6,\bar{15},B) + \\
&\quad (21,\bar{6},-B) + (6,\bar{21},B) + (1,\bar{70},3B) \\
4212 &= (189,1,0) + (15,15,-4B) + (6,6,-2B) + \\
&\quad (84,6,-2B) + (\bar{15},\bar{15},4B) + (1,35,0) + \\
&\quad (1,189,0) + (\bar{6},\bar{84},2B) + (\bar{84},\bar{6},2B) + \\
&\quad (6,84,-2B) + (1,1,0) + (35,1,0) + \\
&\quad (35,35,0) + (\bar{6},\bar{6},2B)
\end{aligned}
\tag{3.31}
$$

3.4.5 SU(6) Branching Rules

$$
\begin{aligned}
SU(6) &\Longrightarrow SU(3)\times SU(2) \\
6 &= (3,2) \\
15 &= (6,1) + (\bar{3},3) \\
20 &= (1,4) + (8,2) \\
21 &= (\bar{3},1) + (6,3)
\end{aligned}
$$

$$
\begin{aligned}
35 &= (1,3) + (8,1) + (8,3) \\
70 &= (1,2) + (8,4) + (8,2) \\
&\quad (10,2)
\end{aligned}
\tag{3.32}
$$

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