Chapter 4

Evolution of the Yukawa Couplings of MSSM

Minimal Supersymmetric Standard Model(MSSM) is arguably the most promising extension of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ Standard Model. The model introduces one superpartner to all the fermionic scalar and gauge fields and demands invariance of the Lagrangian under the supersymmetric transformations of the fields. The most interesting feature of the model is that due to the cancellation of infinities between scalar and fermion loops the model does not suffer from the gauge hierarchy problem. Presently much interest is generated in this model as it leads to successful gauge coupling unification when the supersymmetry breaking scale is around 1 TeV[1]. On the other hand the model introduces more free parameters like the masses of the superpertners and the quantity $tan\beta$ defined as the ratio of the vacuum expectation values of the the two Higgs scalars present to give masses to the up and down type quarks respectively.

Recently a lot of effort has gone into trying to constrain these free parameters of MSSM by embedding into a grand unified framework. Particularly in one approach [10] one assumes a GUT group of SO(10), E_6 or and at the same time assumes Georgi-Jarlskog[9] form of mass matrices at the unification scale. In a second approach [7] it is assumed that at the unification scale $y_b = y_\tau$ but y_t does not have to satisfy such an unification condition. In a complete two loop analysis it is shown that in this case $tan\beta$ can have two solutions one of which is substantially larger than the other. In yet another approach [8] it is assumed that in a SO(10) GUT framework the third family fermions get mass from the operator $\mathbf{16} \times \mathbf{16} \times \mathbf{10}$ and hence at the unification scale $y_b = y_\tau = y_t$. In this approach one gets a large value of $tan\beta$.

Our aim is to find a lower bound on the quantity $tan\beta = \frac{v_2}{v_1}$ without imposing any specific boundary condition on the Yukawa couplings at the GUT scale. To do that we first note that the Yukawa coupling $y_i(m_t)$ s are related to the corresponding fermion masses $m_i(m_t)$ and $tan\beta$ (See the exact relation below). Where we have called the top quark mass as m_t. Now let us consider the specific case of the coupling y_t. By solving the Renormalization Group (RG) equations of the Yukawa couplings one can find out the maximum value of the top quark Yukawa coupling at the scale m_t for which the top Yukawa coupling will remain perturbative in the entire range upto M_U which is the unification scale of the gauge couplings (From the proton decay experiments we know that the lifetime of proton is more than 10^{32} years which puts lower bound on the scale M_U. We take $M_U = 2 \times 10^{16}$ GeV[2]). This will give an upper bound on y_t. Now for fixed value of m_t this will give a lower bound on $tan\beta$. Hence now by varying m_t in the entire parameter space of interest one gets an absolute lower bound of the quantity $tan\beta$.

Our starting point is the following Lagrangian

$$\mathcal{L}_{int} \quad = \quad \sqrt{4\pi y_\tau}\,(L_3 H_1 \bar{E}_3)_F + \sqrt{4\pi y_b}\,(Q_3 H_1 \bar{D}_3)_F + \sqrt{4\pi y_t}\,(U_3 \bar{H}_2 Q_3)_F \qquad (4.1)$$

Where y_τ,y_t and y_b are tau lepton top quark and bottom quark Yukawa couplings respectively. Whereas Q_3, $\bar{D}_3$, $\bar{U}_3$, $\bar{E}_3$, L_3 are the chiral fields and the subscript signifies the generation to which they belong. H_1 and H_2 signifies the scalar fields which couple to down and up type quarks respectively. We describe the transformation properties and the anomalous dimensions of the fields in the Table 4.1. α_i is defined as $\frac{g_i^2}{4\pi}$, where g_i s are the gauge couplings.

Field	*Quantum number*	*Anomalous Dimension*
L_3	$(1,2,-\frac{1}{2})$	$\frac{1}{4\pi}[y_\tau - \frac{3}{2}\alpha_2 - \frac{3}{10}\alpha_y]$
$\bar{E}_3$	$(1,1,1)$	$\frac{1}{4\pi}[2y_\tau - \frac{6}{5}\alpha_y]$
$\bar{D}_3$	$(\bar{3},1,\frac{1}{3})$	$\frac{1}{4\pi}[2y_b - \frac{8}{3}\alpha_3 - \frac{4}{30}\alpha_y]$
$\bar{U}_3$	$(\bar{3},1,-\frac{2}{3})$	$\frac{1}{4\pi}[2y_t - \frac{8}{3}\alpha_3 - \frac{8}{15}\alpha_y]$
Q_3	$(3,2,\frac{1}{6})$	$\frac{1}{4\pi}[y_t + y_b - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{1}{30}\alpha_y]$
H_1	$(1,2,-\frac{1}{2})$	$\frac{1}{4\pi}[y_\tau + 3y_b - \frac{3}{2}\alpha_2 - \frac{3}{10}\alpha_y]$
H_2	$(1,2,\frac{1}{2})$	$\frac{1}{4\pi}[y_t - \frac{3}{2}\alpha_2 - \frac{3}{10}\alpha_y]$

Table 4.1: Transformation properties and anomalous dimensions

From the anomalous dimensions one can immediately write down the evolution equations of the Yukawa couplings. The variable t is defined as $t = \frac{1}{2\pi}\ ln\ \mu$ (GeV) [5].

$$\frac{\partial \alpha_y}{\partial t} = \alpha_y^2[2n_f + \frac{3}{5}] \qquad (4.2)$$

$$\frac{\partial \alpha_2}{\partial t} = \alpha_2^2[2n_f - 6 + 1] \qquad (4.3)$$

$$\frac{\partial \alpha_3}{\partial t} = \alpha_3^2[2n_f - 9] \qquad (4.4)$$

$$\frac{\partial y_\tau}{\partial t} = y_\tau[\gamma_{L_3} + \gamma_{H_1} + \gamma_{\bar{E}_3}] = y_\tau[4y_\tau + 3y_b - 3\alpha_2 - \frac{9}{5}\alpha_y] \qquad (4.5)$$

$$\frac{\partial y_b}{\partial t} = y_b[\gamma_{Q_3} + \gamma_{H_1} + \gamma_{\bar{D}_3}] = y_b[y_\tau + y_t + 6y_b - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{17}{15}\alpha_y] \qquad (4.6)$$

$$\frac{\partial y_t}{\partial t} = y_t[\gamma_{U_3} + \gamma_{H_2} + \gamma_{\bar{Q}_3}] = y_t[6y_t + y_b - \frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{13}{15}\alpha_y] \qquad (4.7)$$

While solving this set of differential equations we have made the following inputs of experimental numbers.

$$\sqrt{4\pi y_t(m_t)} = \frac{m_t(m_t)\sqrt{1+tan^2\beta}}{174\tan\beta} \tag{4.8}$$

$$\sqrt{4\pi y_b(m_t)} = \frac{m_b(m_b)\sqrt{1+tan^2\beta}}{174\ \eta_b} \tag{4.9}$$

$$\sqrt{4\pi y_\tau(m_t)} = \frac{m_\tau(m_\tau)\sqrt{1+tan^2\beta}}{174} \tag{4.10}$$

η_b takes into account the 3-loop Q.C.D. plus 1-loop Q.E.D. evolution of y_b from the bottom mass energy scale to the top mass energy scale [4]. The value of η_b depends on the value of α_s. we have used

$$\begin{aligned} \alpha_s(M_z) &= 0.123 \pm .004 \\ sin^2\theta(M_z) &= 0.2334 \pm .0008 \\ \frac{1}{\alpha_{em}(M_z)} &= 127 \pm 0.2 \end{aligned}$$

y_τ does not vary much in this interval as tau lepton does not carry color. Once the values of the couplings y_t, y_b and y_τ are specified at the top mass scale they can be evolved to the supersymmetry breaking scale M_{susy} using the non-supersymmetric renormalization group equations [6]. In our calculation we have considered two cases in one case M_{susy} is taken to be 1 TeV and M_{susy} is taken to be m_t in the other. From 1 TeV to M_U we have used the supersymmetric evolution equation which we have described above. We have used $m_b(m_b) = 4.25 \pm 0.15$ GeV and $m_\tau(m_\tau) = 1.777$ GeV. The top quark mass is taken in the range 108[3] to 175 GeV. We have not assumed any unification of the Yukawa couplings at the GUT scale. The minimum value of $tan\beta$ is 0.70 and it is achieved when m_t is minimum that is 108 GeV.In Figure 5 have plotted the evolution of y_t when m_t is 135 GeV. Curve A represents the case when $tan\beta$ is 1.01 which is lower than the lower bound 1.03 (see Table 4.2) for bounds. Hence we see that curve A reaches the nonperturbative region earlier than the scale M_U. On the other hand curve C which represents $tan\beta = 1.05$ becomes nonperturbative after the scale M_U. Curve B is obtained when $tan\beta = 1.03$. The variation of the lower bound with respect to m_t is plotted in Figure 6.

To conclude we have asked the question that "what is the minimum value of $tan\beta$ that can be achieved without assuming any specific boundary conditions on the Yukawa couplings at the GUT scale?". We have assumed that there is a perturbative supersymmetric theory upto the scale of 2×10^{16} GeV though we have not assumed any specific model of grand unification. We have seen that the requirement that all the Yukawa couplings should be in the perturbative domain upto M_U forces $tan\beta$ to be atleast 0.70 for $m_t = 108$ GeV. This lower limit rises with higher values of m_t. We have checked that if we have $M_{susy} = m_t$ the lower bound does not vary much rather it stays at 0.71 for $m_t = 108$ GeV. In Figure 6 the upper curve is for the case when $M_{susy} = m_t$. We have also checked that the lower bound remains insensitive to the variation the bottom quark mass in the range 4.10 to 4.40 GeV. It is interesting to note that evenif we increase M_{susy} upto 10 TeV the lower bound on $tan\beta$ still remains just above 0.71 when m_t is 108 GeV and for other values of m_t it remains just above the lower bound for $M_{susy} = m_t$ case. As $tan\beta$ is a free parameter in the MSSM we consider such a bound

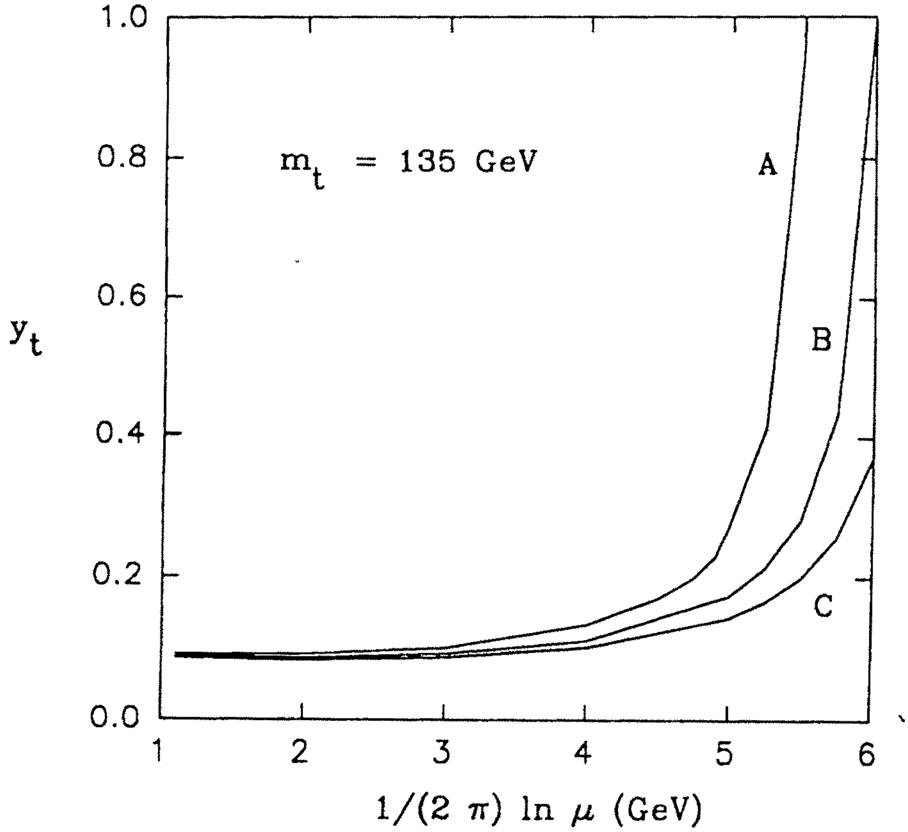

FIGURE 5

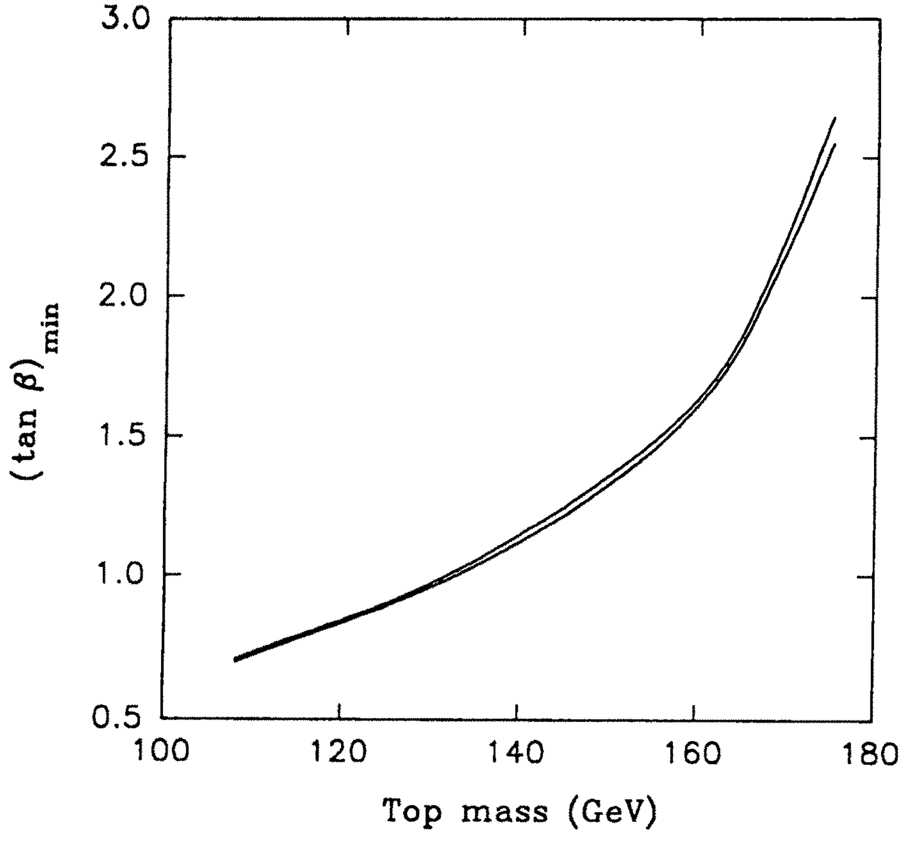

FIGURE 6

m_t	$M_{susy} = 1TeV$	$M_{susy} = m_t$
108	0.70	0.71
115	0.78	0.79
125	0.89	0.90
135	1.03	1.05
145	1.21	1.24
155	1.45	1.48
165	1.84	1.87
175	2.25	2.65

Table 4.2: Lower Bounds on $tan\beta$ for $M_{susy} = 1$ TeV and $M_{susy} = m_t$

as important. As a practical example it will have important implications in the search of supersymmetric Higgs bosons in colliders[7].

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