Chapter 5

Conclusions

To conclude, the works represented in this thesis may be divided into three parts. In the first part we have studied the constraints imposed by the LEP data on various GUT models. In the second part we have considered the new paradigm of low energy unification and found various interesting results. In the third part we have worked out some constraints on the parameter space of MSSM using the evolution of Yukawa couplings. In the following we summarize this three parts.

5.1 LEP Constraints on Unified Models

Our analysis shows that by including the effects of higher dimensional operators arising due to quantum gravity or spontaneous compactification of extra spatial dimensions in Kaluza-Klein theories (or due to a group G' which breaks to $SU(5)$ above the unification scale), it is possible to show that the predictions of a minimal $SU(5)$ GUT is in conformity with the latest LEP values of $\sin^2\theta_W$ and α_s, and also with the experimental constraints on proton lifetime.

The most recent experimental data provide very strong constraints on left-right symmetric models. We have shown that if a left-right symmetric group coming from either a grand unified or partially unified group breaks at an intermediate mass scale, M_R, then the tightly constrained values of $\sin^2\theta_W$ and α_s can be used to put a lower bound on the value of M_R. This lower bound is $\approx 10^9$ GeV, irrespective of the unification group. Grand unified theories and partially unified theories, therefore, completely rule out the possibility of seeing the right handed partners of $W^\pm$ at the energies available in current experiments or those planned in the near future. Conversely, the discovery of these particles at such energies can be used to refute unification models. It is of importance to note, however, that our analysis puts no constraints whatsoever on the existence of extra Z at low energies, as an extra $U(1)_R$ can survive down to electroweak breaking scales. The inclusion of the Higgs or supersymmetry increases the lower bound on M_R.

We have studied the non-perturbative unification scenario first proposed by Maiani, Parisi and Petronzio. We point out that the non-supersymmetric version of this scenario is ruled out by LEP data. However, the supersymmetric extension of this scenario remains a viable alternative to conventional grand unified theories and is capable of predicting the precision values of couplings determined from LEP. Our numerical results show that the non-perturbative scale, Λ, at which all couplings are large, is around 0.7–0.8$\times 10^{17}$ GeV, with the supersymmetric threshold M_s around 1.0–1.4 TeV. If the scale M_s gets either larger or smaller it is then not possible to reproduce the values of the couplings at M_Z. We should note that the agreement with the data is obtained only for a constrained range of parameters of this scenario. In principle, the effect of higher-order corrections could be large and this may ruin the agreement. It is also likely that more accurate measurements of the strong coupling α_3 at low energies may be sufficient to either put strong constraints or completely rule out this scenario. It is nevertheless interesting that this scenario, at the two-loop level, is a possible alternative to conventional grand unification.

5.2 Studies on Low Energy Unification

We have shown that Higgs fields play a significant role in the evolution of gauge coupling constants in GUTs where baryon number is a symmetry. The consistency of the symmetry breaking scenario presented here with present-day proton decay data along with its interesting TeV scale physics make SU(15) GUT a model worthy of further investigation. The most interesting pattern is {3467} (see section 3.1), which has both low energy unification at $\sim 10^9$ GeV and interesting TeV physics. We can decouple the electroweak breaking scale with the other symmetry breakings and have TeV scale chiral color symmetry and the quark-lepton un-unified electroweak symmetry breaking, which will raise the unification scale a little. The existence of chiral color symmetry at the TeV scale or lower will imply the presence of axigluons, whose phenomenological consequences have been studied in the literature. The presence of the un-unified electroweak symmetry at low energy will imply the existence of extra charged and neutral gauge bosons, whose mixing with the Z-boson will affect various asymmetry parameters in the e^+e^- deep-inelastic scattering.

The scenario of symmetry breaking in nonsupersymmetric SU(15) GUT, which allows low energy unification, has some interesting features. It is essential for the low energy unification to have chiral color $SU(3)_{cL} \otimes SU(3)_{cR}$ group and the quark-lepton ununified group $SU(2)^q_L \otimes SU(2)^l_L$ survive till very low energy, for the gauge coupling constants to evolve very fast and get united at an energy scale around 10^8 GeV. Thus the existence of these groups and the leptoquarks are some of the essential critereons of the low energy unification, which can be tested in the laboratory in near future. Thus any signatures of these groups may seriously question the existence of supersymmetry and if the signatures of the low energy unification and also that of supersymmetry are found, then it will cast a serious question on our understanding of the grand unification scenario.

We have seen that there exists one possible breaking chain in a Grand Unified Theory based on the group $SU(16)$ where a unification scale of the order of 10^{11} GeV is possible. There exists a very low energy scale (M_B) which may be almost anywhere between the unification scale and the electroweak scale where completely ununified symmetry of quarks and leptons may exist together with chiral color symmetry. The scale M_B comes lower when the separation between the scale M_A and the scale M_B is increased. Qualitatively we understand it in the following way. The beta function coefficients can be looked into as the slope of the lines if one plots the inverse coupling constants with respect to energy. It can be easily checked that as at the $SU(16)$ level all the fermions transform under the fundamental representation of the group and in the other levels they transform in a more complicated way under the various groups in the intermediate stages all the groups cannot be normalized in the same way. To compensate for the mismatch in the normalizations the beta function coefficients has to be multiplied by appropriate factors. Due to that the slope of the curves representing the inverse couplings also gets multiplied by the appropriate factors and the couplings get united earlier giving rise to low energy unification. We have also seen that this model satisfies the experimental constraints coming from proton decay experiments in the sense that proton decay is suppressed. We have shown that there exists atleast one choice of the Higgs sector where there is no Higgs mediated proton decay either. For some specific choice of the Higgs fields there may exist interesting phisical consequences like the $N - \bar{N}$ oscillation. There is also the possibility of having the sea-saw mechanism to give Majorana mass to the neutrinos and this also may have observable consequences. Last but not the least we emphasize again that there exists very rich low energy physics coming from this model hence keeping in mind the forthcoming high-energy experiments at SSC,LHC and other places this model is worthy of further investigation.

5.3 Evolution of Yukawa Couplings

We have asked the question that "what is the minimum value of $tan\beta$ that can be achieved without assuming any specific boundary conditions on the Yukawa couplings at the GUT scale?". We have assumed that there is a perturbative supersymmetric theory upto the scale of 2×10^{16} GeV though we have not assumed any specific model of grand unification. We have seen that the requirement that all the Yukawa couplings should be in the perturbative domain upto M_U forces $tan\beta$ to be atleast 0.70 for m_t = 108 GeV. This lower limit rises with higher values of m_t. We have checked that if we have $M_{susy} = m_t$ the lower bound does not vary much. It stays at 0.71 for m_t = 108 GeV. In Figure 6 the upper curve is for the case when $M_{susy} = m_t$. We have also checked that the lower bound remains insensitive to the variation the bottom quark mass in the range 4.10 to 4.40 GeV. It is interesting to note that evenif we increase M_{susy} upto 10 TeV the lower bound on $tan\beta$ still remains just above 0.71 when m_t is 108 GeV and for other values of m_t it remains just above the lower bound for $M_{susy} = m_t$ case. As $tan\beta$ is a free parameter in the MSSM we consider such a bound as important. As a practical example it will have important implications in the search of supersymmetric Higgs bosons in colliders.