CHAPTER 9

SOME OPEN PROBLEMS AND APPLICATIONS

9.1 APPLICATIONS:

There are many applications of uniform and non-uniform local limit results to the entropy problems, occupation time problems, Abelian and Tauberian theorems, probabilistic number theory, probabilistic methods for obtaining mathematical results, etc.

We discuss two broad areas of application of our results.

(A): It is well known that stable laws are considered to be suitable replacements to normal law in various fields such as behaviour of stock market prices. The point of interest is to get results applicable in the cases where distributions involved have 'heavy-tail' behaviour.

Tail behaviour of convolutions of densities has been found to be of interest in some HIV-latency time problem (see: Berman (1992)). It is expected that rate of convergence type results will be required to get finer approximations of convolutions of densities in similar problems.

(B): As pointed out in the introduction, problems involving random number of random variables arise in a natural way in reliability studies. It is expected that results of Chapters 7 and 8 will be found useful when the components used in the reliability system could be coming from two or more populations the availability of the components being controlled by a random mechanism.

9.2 SUMMARY AND OPEN PROBLEMS:

We may summarize the various results obtained in the previous Chapters in a convenient tabulated form as follows.

Chapter No.	Assumptions on the original r.v.s	Type of result concerning local limit theorem
3	IID with d.f. $F \in \mathcal{D}_{NNA}(2)$	Uniform rate Non-uniform bound
4	IID with d.f. $F \in \mathcal{D}_{NNA}(\alpha)$ $\alpha \neq 1, \alpha \neq 2$	Uniform rate
5	Independent r.v.s with parent distributions $F_1 \in \mathcal{D}_{NA}(\alpha)$, $F_2 \in \mathcal{D}_{NA}(\alpha)$ $\alpha \neq 1, \alpha \neq 2$	Uniform rate Non-uniform rate
6	Independent r.v.s with parent distributions $F_1 \in \mathcal{D}_{NA}(\alpha_1)$, $F_2 \in \mathcal{D}_{NA}(\alpha_2)$ $0 < \alpha_1 < \alpha_2 < 2$	Uniform rate Non-uniform rate
7	Independent r.v.s coming randomly from F_1 and F_2, $F_1, F_2 \in \mathcal{D}_{NA}(\alpha)$, $\alpha < 2$	Central Limit Theorem
8	Independent r.v.s coming randomly from F_1 and F_2, $F_1, F_2 \in \mathcal{D}_{NA}(\alpha)$, $\alpha < 2$	Local Limit Theorem

(A): It is evident from the above table that results of Chapters 3 and 4 need extension to the level of results of Chapter 5 as a first step.

(B): The restriction of D_{NA} for F_1 and F_2 as made in Chapter 6 need extension to the D_{NNA} set up.

(C): In the random sampling scheme set up in Chapters 7 and 8, there appears to be hardly any work and that too with the assumptions of normal attraction. it will be worthwhile to consider non-normal attraction case as well as rate of convergence in the local limit theorem.

(D): Also non-uniform bound type results need to be strengthened to get non-uniform rate type results.

BIBLIOGRAPHY

[1] Banys, J.J. (1972): Estimate of convergence rate in integral limit theorem, Lithuanian Mathematical Journal, 12, No.1, 41-46.

[2] Banys, J.J. (1974): On non-uniform estimate of the remainder term in integral local limit theorem, Lithuanian Mathematical Journal, 14, No.3, 57-65.

[3] Banys, J.J. (1977): An estimate of the convergence rate of the limiting stable law in metric L_1, Lithuanian Mathematical Journal, 18, No.1, 13-18.

[4] O. Barndorff-Neilson(1964): On the limit distribution of maximum of a random number of independent random variables, Acta. Math. Acad. Sci. Hung., 15.

[5] Basu, S.K. (1974): On the rate of convergence in a local limit theorem, Proc. Cambridge Phil. Soc. , 76, 307-312.

[6] Basu, S.K. (1976): On a local limit theorem concerning variables in the domain of normal attraction of a stable law of index α, $1<\alpha<2$, Ann. Prob., 4, 486-489.

[7] Basu, S.K. (1978): A non-uniform rate of convergence in a local limit theorem, Math. Proc. Cambridge Phil. Soc. , 84, 351-359.

[8] Basu, S.K., Maejima, M., Patra N.(1979): A non-uniform rate of convergence in a local limit theorem concerning variables in the domain of normal attraction of a stable law, Yokohama Math. J., Vol. 27, 63-72.

[9] Basu, S.K., Maejima, M. (1979): An L_p-version of of the local limit theorem for densities, Keio Math. Sem. Rep., No.4, 1-6.

[10] Basu, S.K., Maejima, M. (1980): A local limit theorem for attractions under a stable law, Math. Proc. Cambridge Phil. Soc. , 87, 179-187.

[11] Basu, S.K. (1984): A local limit theorem for attraction to the standard normal law, Metrica, Vol.31, 245-252.

[12] Berry, A.C. (1941): The accuracy of the Gaussian approximation to the sum of independent variates, Trans. Amer. Math. Soc., 49, 122-136

[13] Bingham N.H., Goldie, C.M., Teugels, J.L.(1987): *Regular Variation*, Cambridge university Press, Cambrige.

[14] Esseen, C.G.(1945): Fourier Analysis of distribution functions: A mathematical study of the Laplace-Gaussian law, Acta Math., 77, 1-125.

[15] Feller, W. (1971): *An introduction to probability theory and its applications*, Vol II, John Wiley & sons, New York.

[16] Gnedenko, B.V. (1954): On local limit theory for densities, Dokl. Akad. Nauk. SSR, 95, 5-7.

[17] Gnedenko, B.V. (1962): On certain problems in probability theory, Selected Translations in Math. Stat. and Probability, Vol. 3, 167-180.

[18] Gnedenko, B.V., Kolmogorov, A.N. (1968): *Limit Theorems for Sums of Independent Random Variables*, Addison-Wesley.

[19] Hall, P. (1981): Comedy of errors: the canonical form for a stable characteristic function, Bull. London Math. Soc., 13, 23-27.

[20] Hoglund, T. (1970): On the convergence of convolutions of the distributions with regularly varying tails, Z. Wahr. verw. Geb., 15, 263-272.

[21] Ibragimov, I.A., Linnik, Yu.V. (1971): *Independent ans stationary sequences of random variables*, Wotters-Noordhof Publishing Co., Groningen.

[22] Kirkire, P.D.(1991): A CLT when summands come randomly from *r* populations, Cal. Stat. Association Bulletin, Vol.41, 161-164.

[23] Kirkire, P.D.(1994): A non-uniform rate of convergence in the local limit theorem for independent random variables, Sankhya, series A, Vol. 56, Part 3, 399-415

[24] Korolev Yu.V. (1992): On the accuracy of normal approximmation for the distributions of the sums of random num,ber of independent r.v.s, Th. Prob. Appl. Vol.31, 540-544.

[25] Kruglov, V.M. (1968): A local limit theorem for non-identically distributed random variabls, Th. Prob. Appl., 332-334.

[26] Maejima, M. (1978): A non-uniform estimate in local limiit theorem for densities-I and II, Yokohama Math. J., 26, 83-89, 119-135.

[27] Maejima, M. (1980): The reminder term in local limit theorem for independent r.v.s, Tokyo J. Math., 3 , 311-329.

[28] Mason, D.A.(1970): Convolutions of stable laws as limit distibutions of partial sums, Ann. Math. Stat., 41, 101-114.

[29] Petrov, V.V. (1956): Local limit theorem for densities, Th. Prob. Appl.,349-357.

[30] Petrov, V.V. (1975): Sums of independent r.v.s, Akademie-Verlag, Berlin.

[31] Smith, W.L. (1953): A frequency function form of the central limit theorem, Math. Proc. Cambridge Phil. Soc., 49, No.3, 462-472.

[32] Smith, W.L., Basu, S.K. (1974): General moment functions and a density version of the central limit theorem, Proc. Cambridge Phil. Soc., 75, 365-381.

[33] Sreehari, M. (1970): On a class of limit distributions for normalized sums of independent r.v.s, Th. Prob. Appl., Vol.15, 258-281.

[34] Survilla, P. (1964): A local limit theorem for densities, Lithuanian Mathematical Journal, 4, No.4, 535-540.

[35] Titchmarsh (1937): Theory of Functions, Oxford.

[36] Zinger, A.A. (1965:a): On a problem of B.V.Gnedenko, Dokl. Acad. Nauk. SSSR, Vol.162, 1238-1240.

[37] Zinger, A.A. (1965:b): On a class of limit distributions for normalized sums of independent r.v.s, Th. Prob. Appl., Vol.10, 607-626.

[38] Zolotarev, V.M., Korolyk, V.S.(1961): On a hypothesis proposed by B.V.Gnedenko, Th. Prob. Appl., 6, 431-435.