SUMMARY

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The main aim of the thesis is to investigate the basic properties of controllability analysis such as controllability, trajectory controllability and exact controllability of nonlinear and singular controlled systems using the tools of nonlinear functional analysis.

We begin with the exact controllability of third order nonlinear dispersion system,

$$\frac{\partial w(x,t)}{\partial t} + \frac{\partial^3 w(x,t)}{\partial x^3} = (Gu)(x,t) + f(t, w(x,t)) \tag{1}$$

on the domain $t \geq 0$, $0 \leq x \leq 2\pi$; with periodic boundary condition

$$\frac{\partial^k w(0,t)}{\partial x^k} = \frac{\partial^3 w(2\pi,t)}{\partial x^k}; k = 0, 1, 2.$$

and initial condition

$$w(x,0) = 0$$

where

$$(Gu)(x,t) = g(x)\{u(t,x) - \int_0^{2\pi} g(s)u(s,t)ds\}$$

where $g(x)$ is a piecewise continuous non negative function on $[0, 2\pi]$ such that

$$[g] \stackrel{\text{def}}{=} \int_0^{2\pi} g(x)dx = 1$$

and $f : [0, \infty] \times R \longrightarrow R$

Later on we extended the nonlinear system given in (1) in to nonlinear integro-differential system.

For trajectory controllability we investigate a integro-differential system governed by the operator equation

$$\left.\begin{aligned} \tfrac{dx}{dt} &= Ax(t) + Bu(t) + f\big(t, x(t), \textstyle\int_0^t g(t,s,x(s))ds\big); \\ x(t_0) &= x_0; \quad t \in J = [0,T] \end{aligned}\right\} \tag{2}$$

where the state $x(t)$ and the control $u(t)$ lie in some Hilbert space H with the norm $\|.\|$, for each $t \in J$, $A : H \to H$ is a linear operator not necessarily bounded, $B : J \times H \to H$, $g : \Delta \times H \to H$ and $f : J \times H \times H \to H$ are nonlinear operators; here $\Delta = \{(t,s) \in J^2 : 0 \le s \le t \le T\}$.

In the continuation of exact controllability, we have generalised the Volterra integral equation by Hammerstein type integral equation and studied its exact controllability. Sufficient number of examples were given to illustrate the theory. The system was of the form

$$x(t) = \int_0^t h(t,s)u(s)ds + \int_0^t k(t,s,x)f(s,x(s))ds; \quad 0 \le t \le T \le \infty \qquad (3)$$

where the state of the system $x(.)$ lies in a Hilbert space X for each time $t \in J = [0,T]$. The control function $u(.)$ is from $U = L^2([0,T] : V)$,V is assumed to be a Hilbert space, for each $t \in J$. The kernal function $h(t,s)$ is defined from V to X, while the kernal function $k(t,s,x)$ is defined from X to X; both are bounded linear operators. $f : J \times X \to X$ is a nonlinear operator for each $t, s \in J$. From system (3) it is clear that the initial state of (3) is zero, that is, $x(t_0) = 0 \in X$.

We studied controllability analysis of n-dimensional abstract singular system which is of the form

$$\left.\begin{array}{l} t\frac{d^2x}{dt^2} - (\alpha - 1 + tA)\frac{dx}{dt} + (\alpha-1)Ax(t) = 0 \\ x(0) = 0; \qquad lim_{t\to 0}\Gamma(\alpha)t^{1-\alpha}\frac{d}{dt}x(t) = u \end{array}\right\} \qquad (4)$$

where $0 < \alpha < 1, t > 0, x(t) \in R^n$ for each t and A is a constant $n \times n$ matrix; u is considered as a control applied to the system at initial time. u is considered as a controlled applied to the system at initial time. $u \in L^2([0,T], R^n)$
We have also dealt with the singular integro-differential system whose controllability can be studied by α-times integrated Cosine function. The form of this system is

$$\left.\begin{array}{rl} t\frac{d^2x}{dt^2} + (1-\alpha)\frac{dx}{dt} - tAx(t) - (1-\alpha)A\int_0^t x(\tau)d\tau & = 0 \\ x(0) = 0; \; lim_{t\to 0}\Gamma(\alpha)t^{1-\alpha}\frac{d}{dt}x(t) & = u \end{array}\right\} \qquad (5)$$

where $0 < \alpha \le 1, u \in R^n, x(t) \in R^n$ for all t, A is a constant $n \times n$ matrix. Such type of systems are very relevant in the applied fields of mathematics.

At the end, we study the controllability on infinite time horizon for second-order semilinear neutral functional differential inclusions of the form:

$$\left.\begin{array}{rcl} \frac{d}{dt}[x'(t) - f(t,x_t)] & \in & Ax(t) + Bu(t) + F(t,x_t,x'(t)), \quad t \in [0,\infty) \\ x_0 & = & \phi, x'(0) = x_0 \end{array}\right\} \qquad (6)$$

where, the state $x(t)$ takes the values in the real Banach space X with norm $|.|$ and the control $u(.)$ given in $L^2(J, U)$, a Banach space of all admissible control function with U as a Banach space. B is a bounded linear operator from U to X, A is a linear infinitesimal generator of a strongly continuous cosine family $\{C(t) : t \in R\}$ in X. $F : J \times C \times X \to 2^X, f : J \times C \to X, \phi \in C$, where $C = C([-r, 0], X)$. To study the controllability of the system (6), we invoke the fixed point theorem due to Ma, set-valued analysis and cosine operator theory. In this thesis we give an integrated functional analytic approach for investigating

- Exact controllability of nonlinear third order dispersion system (1),
- Controllability of nonlinear integro-differential third order dispersion system of (1),
- Trajectory controllability of nonlinear integro-differential system (2),
- Exact controllability of generalized Hammerstein type integral equations (3) and its applications,
- Controllability of singular systems (4) and (5), using α-times integrated semigroups and cosine functions, respectively, and
- Controllability of system (6) described by second-order neutral functional differential inclusions on infinite time horizon.

Although controllability problem concerning the linear systems has been widely studied, there is relatively a modest attempt towards the analysis of nonlinear and integro-differential systems in infinite dimensional space. In Chapter 3, we have discussed the exact controllability of a nonlinear dispersion system. This work extends the work of Russell and Zhang, in which authors considered a linear dispersion system. We have obtained controllability result by using two types of nonlinearities, namely, Lipschitzian and monotone. In the last section, we use the weaker notion of Lipschitz continuity called integral contractor to study the exact controllability of system(1). In Chapter 4, we generalize the dispersion system(1) by considering the integro-differential dispersion system and study the controllability by using Schaefer fixed point theorem.

In Chapter 5, we have investigated the T-controllability of nonlinear integro-differential system (2). Here, first we studied the trajectory (T) controllability of nonlinear system in finite dimensional space and then the result has been extended to infinite dimensional set up. For this we use the tools of monotone operator theory and set valued analysis. We have also used Lipschitzian and monotone nonlinearities with coercive property.

We now come to the useful concept of exact controllability of Hammerstein type integral equation in Chapter 6, where we have used the monotone operator theory. Here the controllability problem is reduced to solvability problem in appropriate spaces and proved main result on exact controllability of system (3). The main theorem of this chapter allows us to study several applications arisen in connection with the exact controllability. Some nonlinear evolution systems and some systems of partial differential equations can be put in the frame-work of system (3).

Chapter 7 deals with controllability of the singular system (4) using α-times integrated semi-groups. Also, we have studied the controllability of the integro-differential singular system (5) using α-times integrated cosine functions.

Chapter 8 concludes with controllability of second-order neutral functional differential inclusion of system (6), in noncompact interval. We rely on the fixed point theorem due to Ma and mutivalued analysis to study the controllability of the system (6). Example is given to illustrate the theory.