

Summary

Controllability problems and stability problems of some linear and nonlinear discrete-time systems are investigated. To obtain controllability results, some tools from analysis such as fixed point theorem, inverse function theorem, implicit function theorems etc are used. Along with controllability results, we made an attempt to obtain a computational algorithm for the actual computation of steering control.

- Steering control for the semilinear discrete-time system

$$x(t+1) = A(t)x(t) + B(t)u(t) + f(t, x(t)), \quad t \in N_0 \triangleq \{0, 1, 2, \dots\} \quad (1)$$

is developed and using Banach's fixed point theorem its well definedness is proved under the conditions that the its linear counterpart is controllable and nonlinear function is Lipschitz. Under the same assumptions it is proved that the controllability and reachability are equivalent for the system (1). An algorithm for the actual computation of steering control for the semilinear system (1) is provided.

- Global controllability of discrete-time Volterra system

$$\Sigma_L : x(t+1) = \sum_{i=0}^t A(i)x(t-i) + B(t)u(t), \quad t \in N_0 \quad (2)$$

is established via controllability Grammian and Kalman type rank condition. Using inverse function theorem and implicit function theorem, sufficient condition for the local controllability of semi-linear Volterra system of the form

$$\Sigma_N : x(t+1) = \sum_{i=0}^t A(i)x(t-i) + B(t)u(t) + f(x(t), u(t)), \quad t \in N_0 \quad (3)$$

is obtained.

- Also asymptotic stability of nonlinear discrete-time system

$$x(t+1) = Ax(t) + f(t, x(t)), \quad t \in N_0 \quad (4)$$

is investigated using the concept of (sp) matrix. Using the concept of generalized subradius the asymptotic stability of system

$$x(t+1) = A(t)x(t) + f(t, x(t)), \quad t \in N_0 \quad (5)$$

is analyzed. To prove these results, discrete Gronwall inequality is used. Also accurate estimate for the bound of solution of system (5) is derived.

- The problem of the asymptotic relationship between the solutions of a linear Volterra difference equation of convolution type

$$x(t+1) = A(t)x(t) + \sum_{r=t_0}^t B(t-r)x(r), \quad x(t_0) = x_0, \quad t \in N_{t_0} \triangleq \{t_0, t_0+1, \dots\} \quad (6)$$

and its nonlinear perturbation

$$y(t+1) = A(t)y(t) + \sum_{r=t_0}^t B(t-r)y(r) + f(t, y(t)), \quad y(t_0) = x_0, \quad t \in N_{t_0} \quad (7)$$

is studied by using the dichotomic behavior of the linear Volterra system.

- The problem of optimal control for discrete Volterra system

$$x(t+1) = \sum_{i=0}^t A(i)x(t-i) + Bu(t), \quad t \in N_0 \quad (8)$$

is studied by using the classical minimization technique of Lagrangian multipliers. Here quadratic performance index for the finite time process ($0 \leq t \leq N$)

$$J = \frac{1}{2}x^*(N)Sx(N) + \frac{1}{2} \sum_{t=0}^{N-1} [x^*(t)Qx(t) + u^*(t)Ru(t)] \quad (9)$$

is chosen. A controller is obtained which minimize J as given by equation (9), when it is subjected to the constraint equation specified by (8) and when initial condition on state vector is specified as

$$x(0) = c. \quad (10)$$

Numerical examples are included to give more clarity and understanding in each case.