Chapter 2

DESIGN AND IMPLEMENTATION OF A 3 PHASE PWM FOR 3 PHASE IDH

2.1 Introduction

The overall performance and the cost of the heating system will be one of the important issues to be considered during the design process for the next generation of Induction Dielectric Heating (IDH) applications. The power conversion circuit (Three phase Pulse Width Modulation (PWM) inverter) of IDH applications must achieve high efficiency, low harmonic distortion, high reliability and low electromagnetic interference (EMI) noise. Three phase PWM inverters are becoming more and more popular in present day induction heating system [92], [107], [109].

Sinusoidal Pulse Width Modulation (SPWM) has been used to control the three phase inverter output voltage [8]. To maintain a good performance of the drive the operation has been restricted between 0 to 78 % of the value that would be reached by square wave operation [30], [64], [92].

Since the concept of PWM inverter was introduced, the various modulation strategies have been developed [30], [47], [51], [75], [76], [79], [80], [83], [92], [104], [107], [113], and analyzed. The space vector modulation (SVM) [25], [96] offers significant flexibility to optimize switching waveforms. It has been well suited for digital implementation.

For the IDH application, full utilization of the DC bus voltage is extremely important

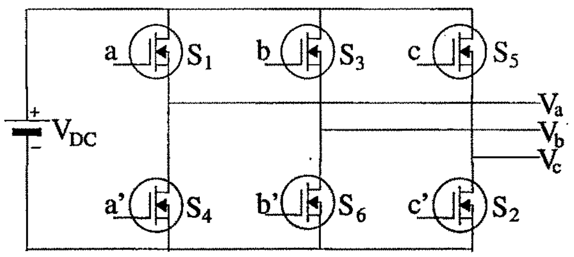

Figure 2.1: 3 Phase PWM Inverter Circuit for IDH

to achieve the maximum temperature under all conditions. The current ripple in three phase pulse width modulation inverter under steady state operation can be minimized using SVM compared to any other PWM methods for voltage control mode.

A symmetrical space vector modulation pattern has been proposed in this chapter, to reduce Total Harmonic Distortion (THD) without increasing the switching losses [42]. The design and implementation of a 3 phase PWM inverter for 3 phase IDH to control temperature using space vector modulation (SVM) has been described.

2.2 Principle of Space Vector Modulation

The circuit model of a typical three-phase voltage source PWM inverter is shown in Figure 2.1. S_1 to S_6 are the six power switches that shape the output voltage, which are controlled by the switching variables a, a', b, b' and c, c'. When an upper MOSFET is switched on, i.e., when a, b, or c is 1, the corresponding lower MOSFET is switched off, i.e., the corresponding a', b', or c' is 0. Therefore, the on and off states of the upper MOSFET S_1, S_3 and S_5 can be used to determine the output voltage [85].

The relationship between the switching variable vector $[a, b, c]^t$ and the line-to-line voltage vector $[V_{ab}, V_{bc}, V_{ca}]^t$ are given by equation 2.1 in the following:

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = V_{DC} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{2.1}$$

Also, the relationship between the switching variable vector $[a, b, c]^t$ and the phase voltage

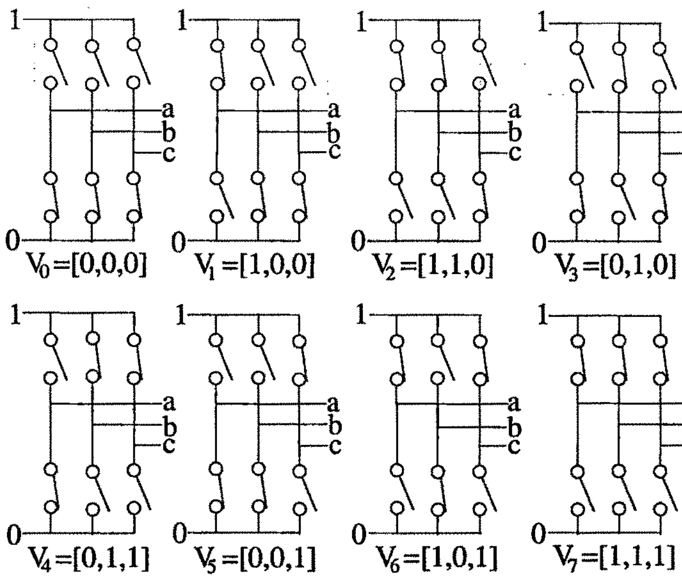

Figure 2.2: Eight Inverter Voltage Vectors

vector $[V_{an}, V_{bn}, V_{cn}]^t$ can be expressed as below.

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \frac{V_{DC}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{2.2}$$

As illustrated in Figure 2.1, there are eight possible combinations of on and off patterns for the three upper power switches. The on and off states of the lower power devices are opposite to the upper one and so are easily determined once the states of the upper power MOSFET's are determined. According to equation 2.1 and equation 2.2, the eight switching vectors, output line to neutral voltage (phase voltage) and output line-to-line voltages in terms of DC-link V_{DC}, are given in Table 2.1 and Figure 2.2 show the eight inverter voltage vectors [75] (V_0 to V_7).

Space Vector Modulation (SVM) refers to a special switching sequence of the upper three power MOSFETs of a three-phase inverter. The source voltage has been utilized most efficiently by the space vector modulation (SVM) compared to sinusoidal pulse width modulation [75] as shown in Figure 2.3.

Table 2.1: Switching Vectors, Phase Voltages and Output Line to Line Voltage

Voltage Vectors	Switching Vector						Line to line voltage			Vector
	a	b	c	a'	b'	c'	V_{ab}	V_{bc}	V_{ca}	
$V_0(000)$	OFF	OFF	OFF	ON	ON	ON	0	0	0	Zero
$V_1(100)$	ON	OFF	OFF	OFF	ON	ON	V_{DC}	0	$-V_{DC}$	Active
$V_2(110)$	ON	ON	OFF	OFF	OFF	ON	0	V_{DC}	$-V_{DC}$	Active
$V_3(010)$	OFF	ON	OFF	ON	OFF	ON	$-V_{DC}$	V_{DC}	0	Active
$V_4(011)$	OFF	ON	ON	ON	OFF	OFF	$-V_{DC}$	0	V_{DC}	Active
$V_5(001)$	OFF	OFF	ON	ON	ON	OFF	0	$-V_{DC}$	V_{DC}	Active
$V_6(101)$	ON	OFF	ON	OFF	ON	OFF	V_{DC}	$-V_{DC}$	0	Active
$V_7(111)$	ON	ON	ON	OFF	OFF	OFF	0	0	0	Zero

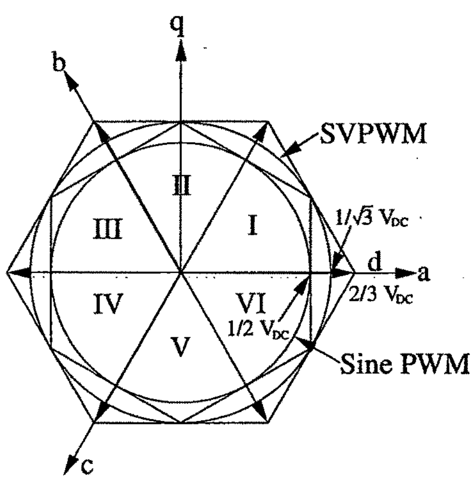

Figure 2.3: Locus of Maximum Linear Control Voltage in Sine PWM and SVPWM

In the vector space, according to the equivalence principle, the following operating rules are obtained:

$$\begin{aligned} V_1 &= -V_4 \\ V_2 &= -V_5 \\ V_3 &= -V_6 \\ V_0 &= V_7 = 0 \\ V_1 + V_3 + V_5 &= 0 \end{aligned} \tag{2.3}$$

In one sampling interval, the output voltage vector V_t can be written as

$$V_t = \frac{t_0}{T_s}V_0 + \frac{t_1}{T_s}V_1 + ... + \frac{t_7}{T_s}V_7 \tag{2.4}$$

Where

$t_0, t_1, ...t_7$ are the turn-on time of the vectors $V_0, V_1, ...V_7$;

$t_0, t_1, ...t_7 > 0$,

$$\sum_{i=0}^{7} t_i = T_s$$

Where

T_s = Sampling time.

According to equation 2.3 and equation 2.4, there are infinite ways of decomposition of V into V_1, V_2, ... V_6. However, in order to reduce the number of switching actions and make full use of active turn-on time for space vectors, the vector V is commonly split into the two nearest adjacent voltage vectors and zero vectors V_0 and V_7 in an arbitrary sector. For example, in sector I, in one sampling interval, vector V can be expressed as

$$V = \frac{T_1}{T_s}V_1 + \frac{T_2}{T_s}V_2 + \frac{T_0}{T_s}V_0 + \frac{T_7}{T_s}V_7 \tag{2.5}$$

Where

$T_s - T_1 - T_2 = T_0 + T_7 \geq 0$, $T_0 \geq 0$ and $T_7 \geq 0$

Let the length of V be mV_{DC}, then

$$\frac{m}{sin\frac{2\pi}{3}} = \frac{T_1}{T_s}\frac{1}{sin(\frac{\pi}{3} - \alpha)} = \frac{T_2}{T_s}\frac{1}{sin\alpha} \tag{2.6}$$

Where

m = Modulation index

Table 2.2: Space Vector Modulation Algorithm

Sector I ($0 \le \omega t \le \pi/3$)	Sector II ($\pi/3 \le \omega t \le 2\pi/3$)
$T_1 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + \pi/6)$ $T_2 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + 3\pi/2)$ $T_0 + T_7 = T_c - T_1 - T_2$	$T_2 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + 11\pi/6)$ $T_3 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + 7\pi/6)$ $T_0 + T_7 = T_c - T_2 - T_3$
Sector III ($2\pi/3 \le \omega t \le \pi$)	Sector IV ($\pi \le \omega t \le 4\pi/3$)
$T_3 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + 3\pi/2)$ $T_4 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + 5\pi/6)$ $T_0 + T_7 = T_c - T_3 - T_4$	$T_4 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + 7\pi/6)$ $T_5 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + \pi/2)$ $T_0 + T_7 = T_c - T_4 - T_5$
Sector V ($4\pi/3 \le \omega t \le 5\pi/3$)	Sector VI ($5\pi/3 \le \omega t \le 2\pi$)
$T_5 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + 5\pi/6)$ $T_6 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + \pi/6)$ $T_0 + T_7 = T_c - T_5 - T_6$	$T_6 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + \pi/2)$ $T_1 = \frac{\sqrt{3}}{2} T_c m \cos(\omega t + 11\pi/6)$ $T_0 + T_7 = T_c - T_6 - T_1$

Thus,

$$\begin{aligned}
\frac{T_1}{T_s} &= \frac{2}{\sqrt{3}} m sin(\frac{\pi}{3} - \omega t) = \frac{2}{\sqrt{3}} m cos(\frac{\pi}{6} + \omega t) \\
\frac{T_2}{T_s} &= \frac{2}{\sqrt{3}} m sin \omega t = \frac{2}{\sqrt{3}} m cos(\frac{3\pi}{2} + \omega t) \\
T_0 + T_7 &= T_s - T_1 - T_2
\end{aligned} \tag{2.7}$$

Where

$$2n\pi \le \omega t = \alpha \le 2n\pi + \pi/3.$$

The length and angle of V determined by vectors $V_1, V_2, ...V_6$ are called as active vectors and V_0, V_7 are called zero (space) vectors. The decomposition of voltage V in different sectors has been presented in Table 2.2. Equation 2.5 and equation 2.6 have been commonly used for formulation of the space vector modulation. It has been shown that the turn on times $T_i (i = 1, ...6)$ for active vectors are identical in different space vector modulation [92], [104], [107], [113]. The different distribution of T_0 and T_7 for zero vectors yields different space vector modulation.

There are not separate modulation signals in each of the three space vector modulation technique [54]. Instead, a voltage vector is processed as a whole [25]. For space vector

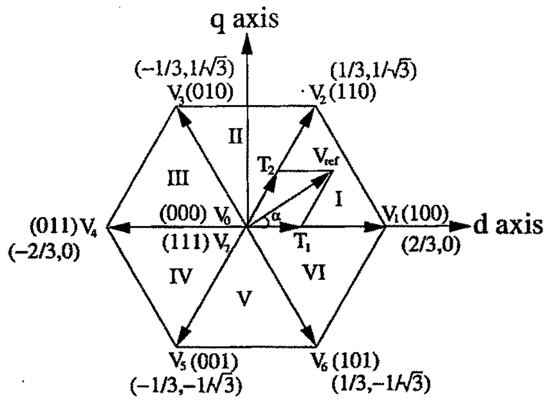

Figure 2.4: The Relationship of abc and Stationary dq Reference Frame

modulation, the boundary condition for sector I is:

$$\begin{aligned} T_s &= T_1 + T_2 \\ T_0 &= T_7 = 0 \end{aligned} \tag{2.8}$$

From equation 2.6 to equation 2.8;

$$\frac{m}{1} = \frac{sin\frac{\pi}{3}}{sin(\frac{2\pi}{3} - \alpha)} \tag{2.9}$$

The boundary of the linear modulation range is the hexagon [54], [67] as shown in Figure 2.3. The linear modulation range is located within the hexagon. If the voltage vector V exceeds the hexagon, as calculated from equation 2.7, then $T_1 + T_2 > T_s$ and it is unrealizable. Thus, for the over modulation region space vector modulation is outside the hexagon. In six step mode, the switching sequence is $V_1 - V_2 - V_3 - V_4 - V_5 - V_6$ [54]. Furthermore, it should be pointed out that the trajectory of voltage vector V should be circular while maintaining sinusoidal output line-to-line voltages. From Figure 2.4, it has been seen that for linear modulation range, the length of vector mV_{DC} should be $V = (\sqrt{3}/2)V_{DC}$, the trajectory of V becomes the inscribed circle of the hexagon and the maximum amplitude of sinusoidal line-to-line voltages is the source voltage V_{DC}.

Moreover, for space vector modulation, there is a degree of freedom in the choice of zero vectors in one switching cycle, i.e., whether V_0 and V_7 or both.

For continuous space vector schemes, in the linear modulation range, both V_0 and V_7 are used in one cycle, that is, $T_7 \geq 0$ and $T_0 \geq 0$.

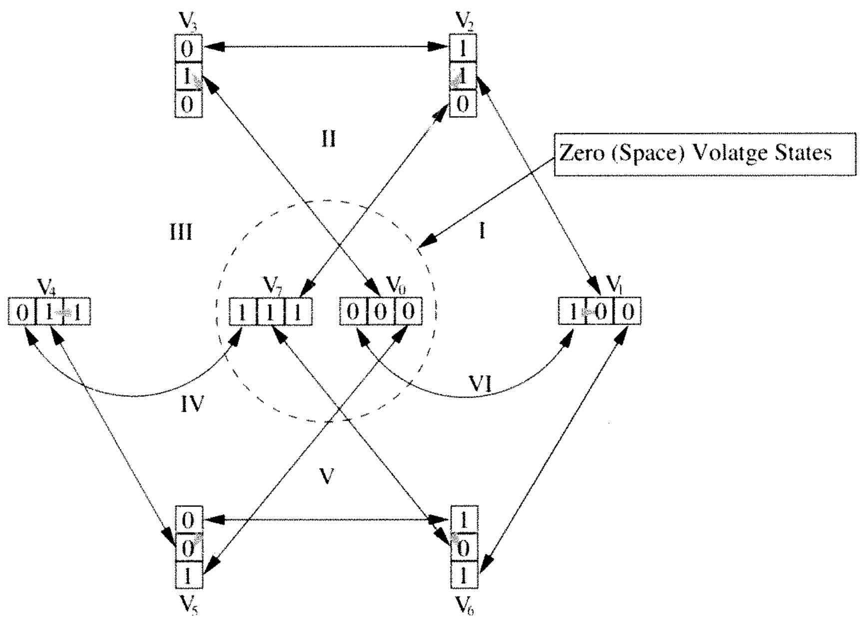

Figure 2.5: Transitions Between Different Switching States

For discontinuous space vector schemes, in the linear modulation range, only V_0 or only V_7 is used in one cycle, that is $T_7 = 0$ and $T_0 = 0$.

2.3 SVM Technique for Three Phase Inverter

Figure 2.1 shows a typical three phase inverter. There are eight possible states: six active vectors and two zero vectors, as shown in Table 2.1 and Figure 2.2. Each inverter switching vectors specifics as the space vector for the output voltage of inverter. The six active switching space vector are evenly distributed 60^o intervals with length of $\sqrt{3}V_{DC}/2$ and from a hexagon. Also two zero space vectors are located at a center of hexagon in the complex plane, as shown in Figure 2.5.

Eight space vectors depending on the switching condition can be represented as complex vector can be given as,

$$\begin{aligned} V_k &= \frac{2}{3}V_{DC}e^{j(k-1)\frac{\pi}{3}}, \qquad k = 1, 2, - - - -, 6 \\ &= 0, \qquad k = 0, 7 \end{aligned} \tag{2.10}$$

When the location of V_{ref} has been fixed, for example, in sector I of Figure 2.4 the integral

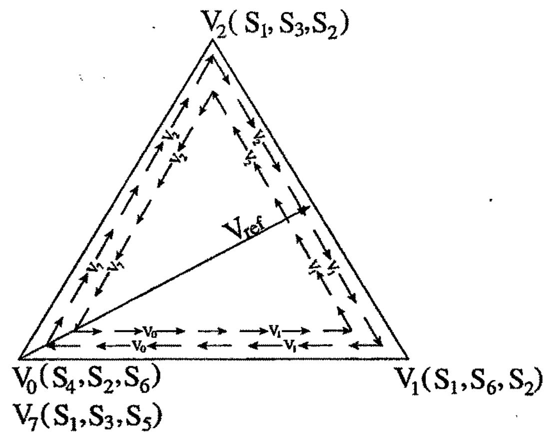

Figure 2.6: Optimal Vector Commutation for Sector I

equation for V_{ref} over a single space vector modulation cycle give.

$$\int_0^{T_s} V_{ref}dt = \int_0^{T_1} V_1 dt + \int_{T_1}^{T_1+T_2} V_2 dt + \int_{T_1+T_2}^{T_s} V_0 dt \quad (2.11)$$

T_1 and T_2 are the switching time spent on the output voltage vectors V_1 and V_2 respectively. The representation of V_{ref} by V_1 and V_2 and switching sequences has been shown in Figure 2.6

2.3.1 Six space vectors of three phase inverter

Assuming the three phase induction dielectric heating by using the proposed technique operates ideally. Balanced set of output voltage v_A, v_B, v_C and a set of output currents i_A, i_B, i_C for the three phase PWM inverter has been expressed by

$$\begin{aligned} v_A &= \frac{V_{DC}}{2} msin\omega t = V_o sin\omega t \\ v_B &= V_o sin(\omega t - 120) \\ v_C &= V_o sin(\omega t - 240) \end{aligned} \quad (2.12)$$

Where

$-1 < m < 1$

$$\begin{aligned} i_A &= I_o sin(\omega t - \phi) \\ i_B &= I_o sin(\omega t - \phi - 120) \\ i_C &= I_o sin(\omega t - \phi - 240) \end{aligned} \quad (2.13)$$

Where
V_o = Maximum amplitude of the output voltage in *Volt*
I_o = Maximum amplitude of the output current in *Amp*
Determined by modulation index m.

There are six switching vectors in three phase inverter when the switches S_1, S_2, S_3, S_4, S_5 and S_6 are turn on and off, as shown in Figure 2.2. Based on the six possible combination of the six individual switches signified by its switching states labelled as $[S_1, S_2, S_3]$, the six space voltage vectors can be produced in the complex plane, as shown in Figure 2.4 and Figure 2.5. Six space vectors are evenly distributed 60^o interval with length of $\sqrt{3}V_{DC}/2$. However, two zero vectors as (1,1,1) and (0,0,0) are available in three phase inverter. Accordingly, the maximum trajectory locus has been the form of an exact square wave with corner points defined by the realizable space voltage vector $(V_1, V_2, V_3, V_4, V_5, V_6)$. In the Figure 2.4, each '1' represents an output line attached to the positive DC link, whereas '0' denotes connection to the negative DC link of source.

If the balanced three phase sinusoidal waveforms are required, the reference voltage vector should be controlled in a circular manner.

2.3.2 Determination of switching times in the proposed SVM

The realization method for SVM technique of three phase inverter has been proposed with zero space vectors. To determine the switching times for the reference vector V_{ref} by adjusting six voltage space vectors.

The V_{ref} in sector I has four voltage space vectors V_1, V_2, V_0 and V_7 which are adjacent to the V_{ref}. The T_1 and T_2 are the switching time spent on the V_1 and V_2, respectively. The T_1 and T_2 do not satisfy the constant switching interval T_s except for the reference vector reaching to the maximum voltage. The remainder of the switching interval should be utilized in the main sector and diagonal sector because of zero space vectors in three phase inverter. Accordingly, the new reference vectors replacing the role of zero space vectors should be set again. For example, sector I sets as the main sector and sector IV sets as the diagonal sector in Figure 2.5.

The amount of the switching times T_1, T_2, T_0 and T_7 should be equal to switching interval T_s.

$$T_s = T_1 + T_2 + T_0 + T_7 \tag{2.14}$$

Where

$$T_0 = T_7 = T_o/2$$

The T_1 and T_2 is the remainder time spent on the V_1 and V_2 in the sector I and also the same time spent on the V_4 and V_5 in the sector IV, respectively. ΔV is defined as the difference between desired voltage vectors for V from V_{ref} in the Figure 2.4. The ΔV is divided into halves on the main sector I, $\Delta V/2$. Then, it is added to V_{ref} and the new reference vector $(V_{ref} + \Delta V/2)$ has been formed again in main sector. Let $(V_{ref} + \Delta V/2)$ called "the re-formed reference vector".

On the contrary, the vector $(-\Delta V/2)$ to restrain the reference vector in main sector I should be inserted in the diagonal sector IV. Let $(-\Delta V/2)$ called "the restraint reference vector". Its direction is opposite to V_{ref} and its $(\Delta V/2)$ in the re-formed reference vector $(V_{ref} + \Delta V/2)$ of the main sector I.

The T_1, T_2, T_0 and T_7, can be calculated as the following process. Absolute value of maximum space vector to the square locus at $\omega t = \alpha$ is given by

$$|V_{max}| = |V_{ref} + \Delta V| = \frac{\sqrt{3}V_{DC}}{2}\frac{1}{sin\alpha + cos\alpha} \tag{2.15}$$

Where

$$\alpha\ (0^o < \alpha < 60^o) = \text{Phase angle in } Deg$$

From equation 2.15, the absolute value of a reference vector difference ΔV is given by

$$|\Delta V| = \frac{\sqrt{3}V_{DC}}{2}\frac{1}{sin\alpha + cos\alpha} - |V_{ref}| \tag{2.16}$$

Absolute value of the re-formed reference vector $(V_{ref} + \Delta V/2)$ in a main sector is given by

$$|V_{ref} + \frac{\Delta V}{2}| = \frac{\sqrt{3}V_{DC}}{4}\frac{1}{sin\alpha + cos\alpha} + |\frac{V_{ref}}{2}| \tag{2.17}$$

Absolute value of a restraint reference vector $-\Delta V/2$ in the diagonal sector is given by

$$|-\frac{\Delta V}{2}| = \frac{\sqrt{3}V_{DC}}{4}\frac{1}{sin\alpha + cos\alpha} - |\frac{V_{ref}}{2}| \tag{2.18}$$

When the re-formed reference vector $(V_{ref} + \Delta V/2)$, for instance, is located in the main sector I, the integral for $(V_{ref} + \Delta V/2)$ can be divided into the integral for the V_1 and V_2.

$$\int_0^{T_1+T_2} \left(V_{ref} + \frac{\Delta V}{2}\right) dt = \int_0^{T_1} V_1 dt + \int_{T_1}^{T_1+T_2} V_2 dt \tag{2.19}$$

Similarly, the integral equation for $-\Delta V/2$ in the diagonal sector IV can be divided into the integral for the V_4 and V_5 .

$$\int_{T_4+T_5}^{T_s} \frac{-\Delta v}{2} dt = \int_{T_4+T_5}^{T_4+T_5+T_0} V_4 dt + \int_{T_4+T_5+T_0}^{T_s} V_5 dt \tag{2.20}$$

Assuming the V_{ref} is constant and the switching frequency is high during a switching period T_s, equation 2.19 is arranged by

$$T_1 + T_2 \left(V_{ref} + \frac{\Delta V}{2} \right) = T_1 V_1 + T_2 V_2 \tag{2.21}$$

And equation 2.20 is arranged by

$$T_4 + T_5 \frac{-\Delta v}{2} dt = T_4 V_4 + T_5 V_5 \tag{2.22}$$

From equation 2.17 to equation 2.22, the re-formed reference vector in a main sector is represented by

$$T_s \left| V_{ref} + \frac{\Delta V}{2} \right| \begin{bmatrix} cos\alpha \\ sin\alpha \end{bmatrix} = T_1 \frac{\sqrt{3}V_{DC}}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + T_2 \frac{\sqrt{3}V_{DC}}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2.23}$$

And the restraint reference vector in a diagonal sector is represented by

$$T_s \left| -\frac{\Delta V}{2} \right| \begin{bmatrix} cos\alpha \\ sin\alpha \end{bmatrix} = T_1 \frac{\sqrt{3}V_{DC}}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + T_2 \frac{\sqrt{3}V_{DC}}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2.24}$$

Six switching times can be solved by equation 2.23 and equation 2.24 and given by the followings;

$$\begin{aligned} T_1 &= \frac{T_s}{V_{DC}} \left[\frac{\sqrt{3}V_{DC}}{2} \frac{1}{sin\alpha + cos\alpha} + |V_{ref}| \right] cos\alpha \\ T_2 &= \frac{T_s}{V_{DC}} \left[\frac{\sqrt{3}V_{DC}}{2} \frac{1}{sin\alpha + cos\alpha} + |V_{ref}| \right] sin\alpha \\ T_0 + T_7 &= T_s - T_1 - T_2 \\ T_4 &= \frac{T_s}{V_{DC}} \left[\frac{\sqrt{3}V_{DC}}{2} \frac{1}{sin\alpha + cos\alpha} + |V_{ref}| \right] cos\alpha \\ T_5 &= \frac{T_s}{V_{DC}} \left[\frac{\sqrt{3}V_{DC}}{2} \frac{1}{sin\alpha + cos\alpha} + |V_{ref}| \right] sin\alpha \\ T_0 + T_7 &= T_s - T_4 - T_5 \end{aligned} \tag{2.25}$$

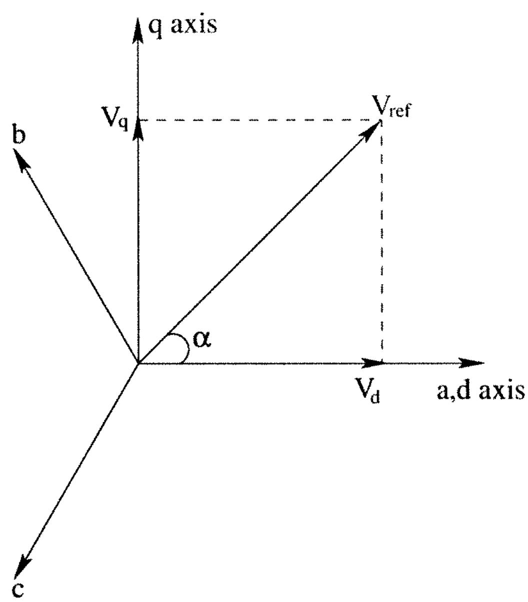

Figure 2.7: Voltage Space Vector and its Components in (d,q)

2.4 Steps of Design for SVM Generation

To implement the space vector modulation, the voltage equations in the abc reference frame can be transformed into the stationary dq reference frame [75] as shown in Figure 2.7.

From Figure 2.7, the relation between these two reference frames is as below

$$f_{dq0} = K_s f_{abc} \tag{2.26}$$

Where

$$K_s = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

$$f_{dq0} = [f_d, f_q, f_0]^T$$

$$f_{abc} = [f_a, f_b, f_c]^T$$

and f denotes either a voltage or a current variable.

As described in Figure 2.7, this transformation is equivalent to an orthogonal projection of $[a, b, c]^t$ onto the two-dimensional perpendicular to the vector $[1, 1, 1]^t$ (the equivalent $d - q$ plane) in a three-dimensional coordinate system. As a result, six non-zero vectors and two zero vectors are possible. Six non-zero vectors (V_1 - V_6) shape the axis of a hexagonal as shown in Figure 2.4 and feed electric power to the load. The angle between any adjacent two non-zero vectors is 60 degrees. Meanwhile, two zero vectors (V_0 and V_7) are at the origin and apply zero voltage to the load. The eight vectors are called the basic space vectors and are denoted by V_0, V_1, V_2, V_3, V_4, V_5, V_6 and V_7. The same transformation can be applied to the desired output voltage to get the desired reference voltage vector V_{ref} in the $d-q$ plane.

The objective of space vector modulation technique is to approximate the reference voltage vector V_{ref} using the eight switching patterns.

The space vector modulation can be implemented by the following steps:

Step 1. Determine V_d, V_q, V_{ref} and angle (α).

Step 2. Determine time duration T_1, T_2, T_0.

Step 3. Determine the switching time of each MOSFET (S_1 to S_6).

2.4.1 Determination V_d, V_q, V_{ref} and angle (α)

From Figure 2.7, the V_d, V_q, V_{ref} and angle (α) can be determine as follows:

$$\begin{aligned}
V_d &= V_{an} - V_{bn}\cdot cos60 - V_{cn}\cdot cos60 \\
&= V_{an} - \frac{1}{2}V_{bn} - \frac{1}{2}V_{cn} \\
V_q &= 0 + V_{bn}\cdot cos30 - V_{cn}\cdot cos30 \\
&= \frac{\sqrt{3}}{2}V_{bn} - \frac{\sqrt{3}}{2}V_{cn} \\
\begin{bmatrix} V_d \\ V_q \end{bmatrix} &= \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \\
|V_{ref}| &= \sqrt{V_d^2 + V_q^2} \\
\alpha &= tan^{-1}\left(\frac{V_d}{V_q}\right) = \omega t = 2\pi f t,
\end{aligned} \tag{2.27}$$

Where

f = Fundamental frequency

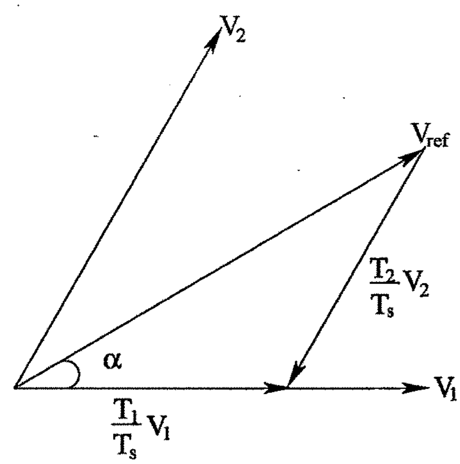

Figure 2.8: Reference Vector as a Combination of Adjacent Vectors at Sector I

2.4.2 Determination time duration T_1, T_2, T_0

From Figure 2.8, the switching time duration can be calculated as follows:

- Switching time duration at sector I

$$\begin{aligned} \int_0^{T_s} V_{ref}dt &= \int_0^{T_1} V_1 dt + \int_{T_1}^{T_1+T_2} V_2 dt + \int_{T_1+T_2}^{T_s} V_0 dt \\ T_s \cdot V_{ref} &= (T_1 \cdot V_1 + T_2 \cdot V_2) \\ T_s \cdot |V_{ref} \cdot \begin{bmatrix} cos(\alpha) \\ sin(\alpha) \end{bmatrix} &= T_1 \cdot \frac{2}{3} \cdot V_{DC} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + T_2 \cdot \frac{2}{3} \cdot V_{DC} \begin{bmatrix} cos(\pi/3) \\ sin(\pi/3) \end{bmatrix} \end{aligned} \tag{2.28}$$

Where

$0 \leq \alpha \leq 60^o$

$$\begin{aligned} T_1 &= T_s \cdot a \cdot \frac{sin(\pi/3 - \alpha)}{sin(\pi/3)} \\ T_2 &= T_s \cdot a \cdot \frac{sin(\alpha)}{sin(\pi/3)} \\ T_0 &= T_s - (T_1 + T_2) \end{aligned} \tag{2.29}$$

Where

$$\begin{aligned} T_s &= \frac{1}{f_s} \\ a &= \frac{|V_{ref}|}{\frac{2}{3}V_{DC}} \end{aligned}$$

- Switching time duration at any sector

$$\begin{aligned}
T_1 &= \frac{\sqrt{3}\cdot T_s\cdot|V_{ref}|}{V_{DC}}\left(sin\left(\frac{\pi}{3}-\alpha+\frac{n-1}{3}\pi\right)\right) \\
&= \frac{\sqrt{3}\cdot T_s\cdot|V_{ref}|}{V_{DC}}\left(sin\frac{n}{3}\pi-\alpha\right) \\
&= \frac{\sqrt{3}\cdot T_s\cdot|V_{ref}|}{V_{DC}}\left(sin\frac{n}{3}\pi cos\alpha - cos\frac{n}{3}\pi sin\alpha\right) \\
T_2 &= \frac{\sqrt{3}\cdot T_s\cdot|V_{ref}|}{V_{DC}}\left(sin\left(\alpha-\frac{n-1}{3}\pi\right)\right) \\
&= \frac{\sqrt{3}\cdot T_s\cdot|V_{ref}|}{V_{DC}}\left(-cos\alpha\cdot sin\frac{n-1}{3}\pi + sin\alpha\cdot cos30\frac{n-1}{3}\pi\right) \\
T_0 &= T_s - T_1 - T_2
\end{aligned} \tag{2.30}$$

Where

n = 1 through 6 (that is, Sector I to VI)

$(n-1)\frac{\pi}{3} \leq \alpha \leq \frac{n\pi}{3}$

2.4.3 Determination of the switching time for MOSFET (S_1 to S_6)

Based on Figure 2.9, Figure 2.10 and Figure 2.11, the switching time at each sector has been summarized in Table 2.3 and it will be built in Simulink model to implement SVM.

2.5 Simulation Results

Simulation results were performed using simulink block as shown in Figure 2.12. The DC bus V_{DC} is equal to $325V$, is connected to the input of the inverter. For the linear operating range the V_{ref} must not exceeds the boundary of the hexagon. Therefore the maximum amplitude of the desired V_{ref} is calculated as

$$|V_{ref}|_{max} = \sqrt{\left(\frac{2}{3}V_{DC}\right)^2 - \left(\frac{2}{6}V_{DC}\right)^2} \tag{2.31}$$

Sample circuit parameters are given in Table 2.4. Simulation space vector generator has been shown in Figure 2.12. Three phase PWM inverter output line to line voltage, output current, 3 phase to 2 phase dq transformation voltages and 3 phase to 2 phase dq transformation currents are shown in Figure 2.13, Figure 2.14, Figure 2.15, Figure

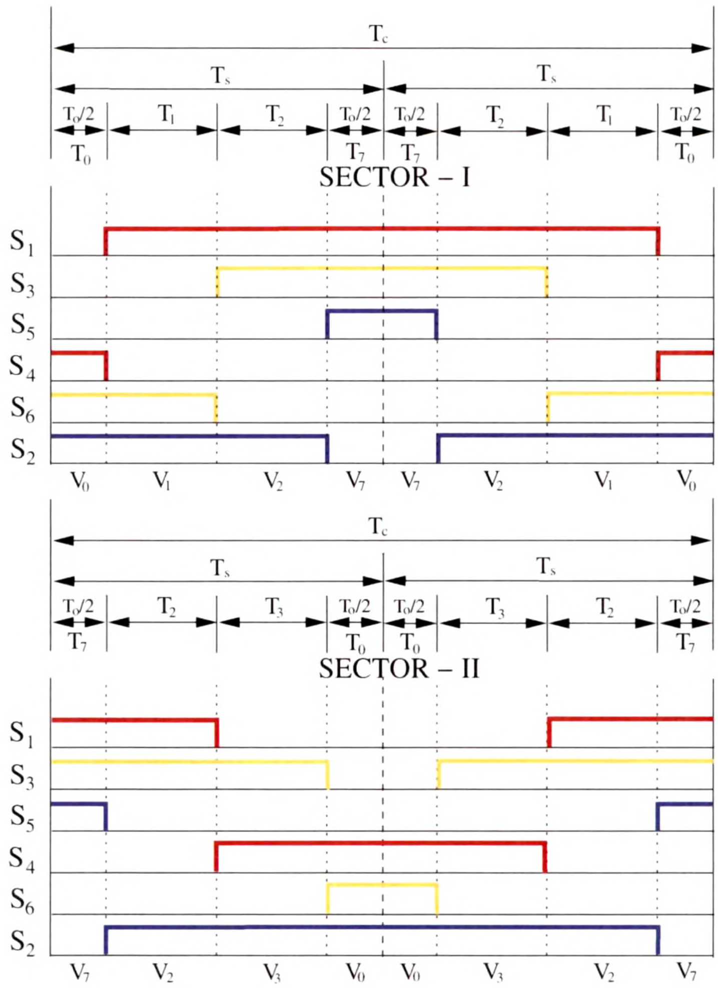

Figure 2.9: Optimal Switching Sequences for Sector I & II

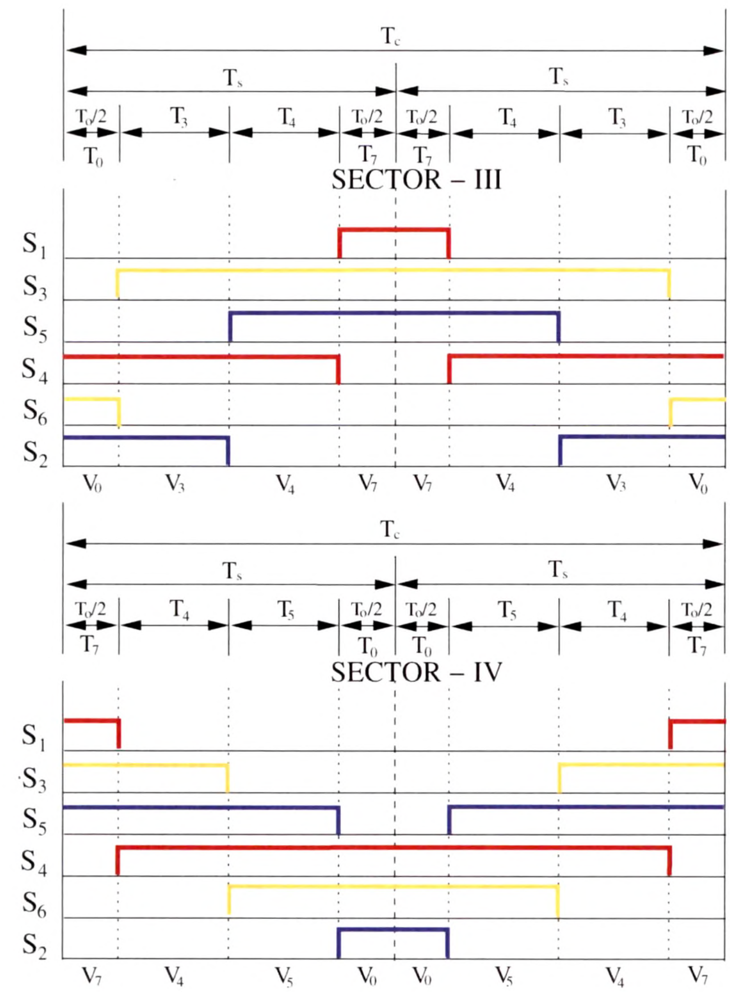

Figure 2.10: Optimal Switching Sequences for Sector III & IV

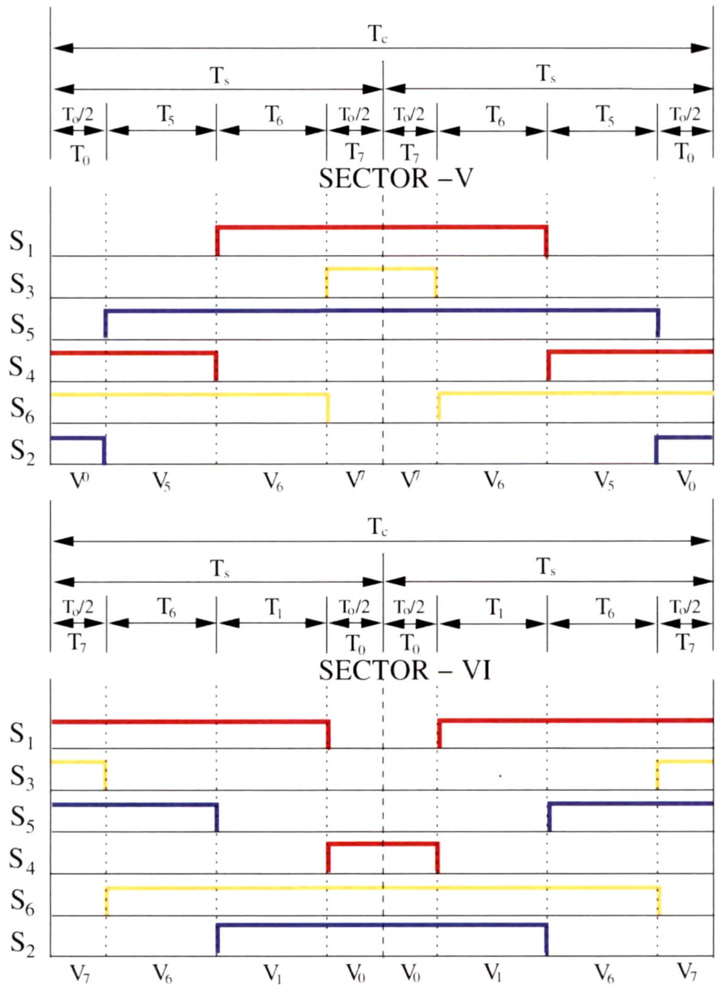

Figure 2.11: Optimal Switching Sequences for Sector V & VI

Table 2.3: Switching Time Calculation at Each Sector

Sector	Upper Switches (S_1, S_3, S_5)	Lower Switches (S_4, S_6, S_2)
1	$S_1 = T_1 + T_2 + T_7$ $S_3 = T_2 + T_7$ $S_5 = T_7$	$S_4 = T_0$ $S_6 = T_1 + T_0$ $S_2 = T_1 + T_2 + T_0$
2	$S_1 = T_2 + T_7$ $S_3 = T_2 + T_3 + T_7$ $S_5 = T_7$	$S_4 = T_3 + T_0$ $S_6 = T_0$ $S_2 = T_2 + T_3 + T_0$
3	$S_1 = T_7$ $S_3 = T_3 + T_4 + T_7$ $S_5 = T_4 + T_7$	$S_4 = T_3 + T_4 + T_0$ $S_6 = T_0$ $S_2 = T_3 + T_0$
4	$S_1 = T_7$ $S_3 = T_4 + T_7$ $S_5 = T_4 + T_5 + T_7$	$S_4 = T_4 + T_5 + T_0$ $S_6 = T_5 + T_0$ $S_2 = T_0$
5	$S_1 = T_6 + T_7$ $S_3 = T_7$ $S_5 = T_5 + T_6 + T_7$	$S_4 = T_5 + T_0$ $S_6 = T_5 + T_6 + T_0$ $S_2 = T_0$
6	$S_1 = T_1 + T_6 + T_7$ $S_3 = T_7$ $S_5 = T_6 + T_7$	$S_4 = T_0$ $S_6 = T_1 + T_6 + T_0$ $S_2 = T_1 + T_0$
	$T_7 = T_o/2$	$T_0 = T_o/2$

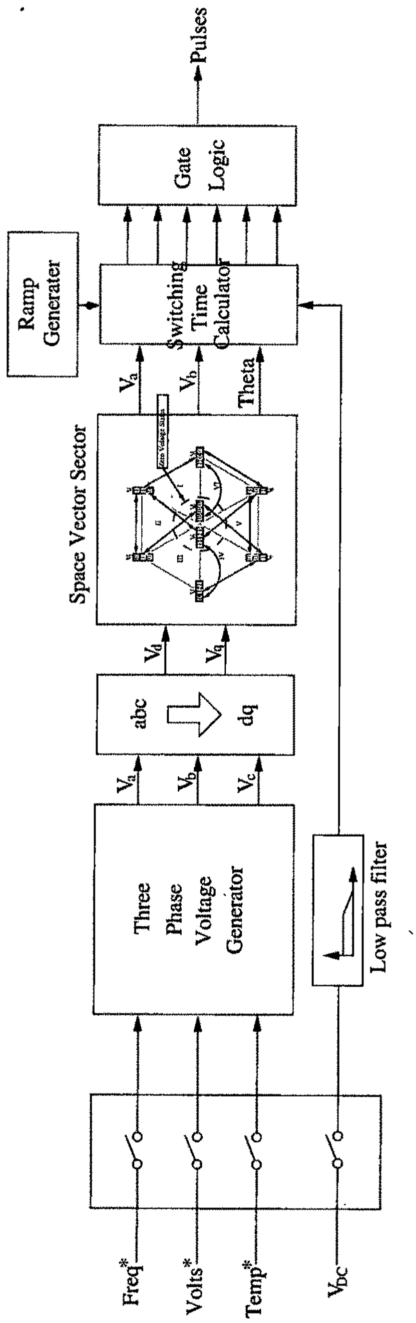

Figure 2.12: SVM Generator

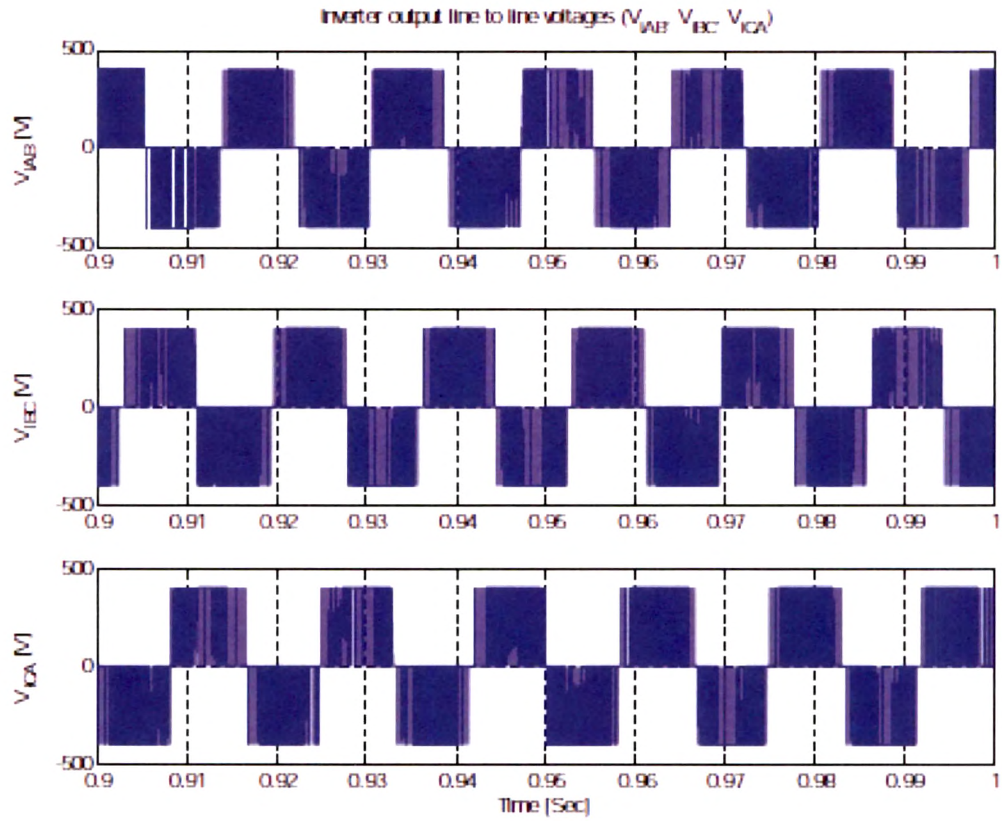

Figure 2.13: Simulation of Inverter Output Line to Line Voltages ($V_{lAB}, V_{lBC}, V_{lCA}$)

2.16 respectively. Simulation summaries and results are given in Table 2.5, Table 2.6 respectively.

A spectral analysis of all waveforms is performed and all harmonics are presented in Table 2.7. These results show that acceptable performances can be obtained at all testing frequencies since the total harmonic distortion (THD) did never reach 10%. At high switching frequency the PWM converter generate a voltage having amplitude close to the desired value.

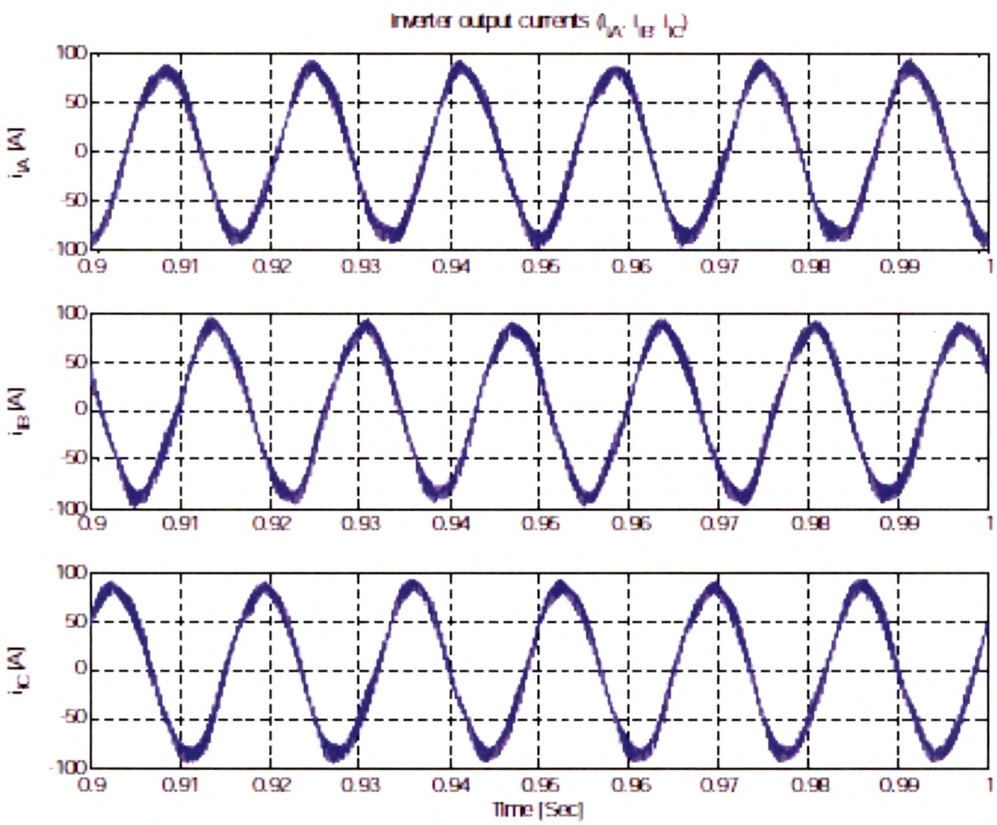

Figure 2.14: Simulation Results of Inverter Output Currents (i_{iA}, i_{iB}, i_{iC})

Table 2.4: Circuit Parameters

Parameter	Value
Utility	220V/50Hz
V_{DC}	325 volt
L_m	69.31mH
f_{sw}	2Khz

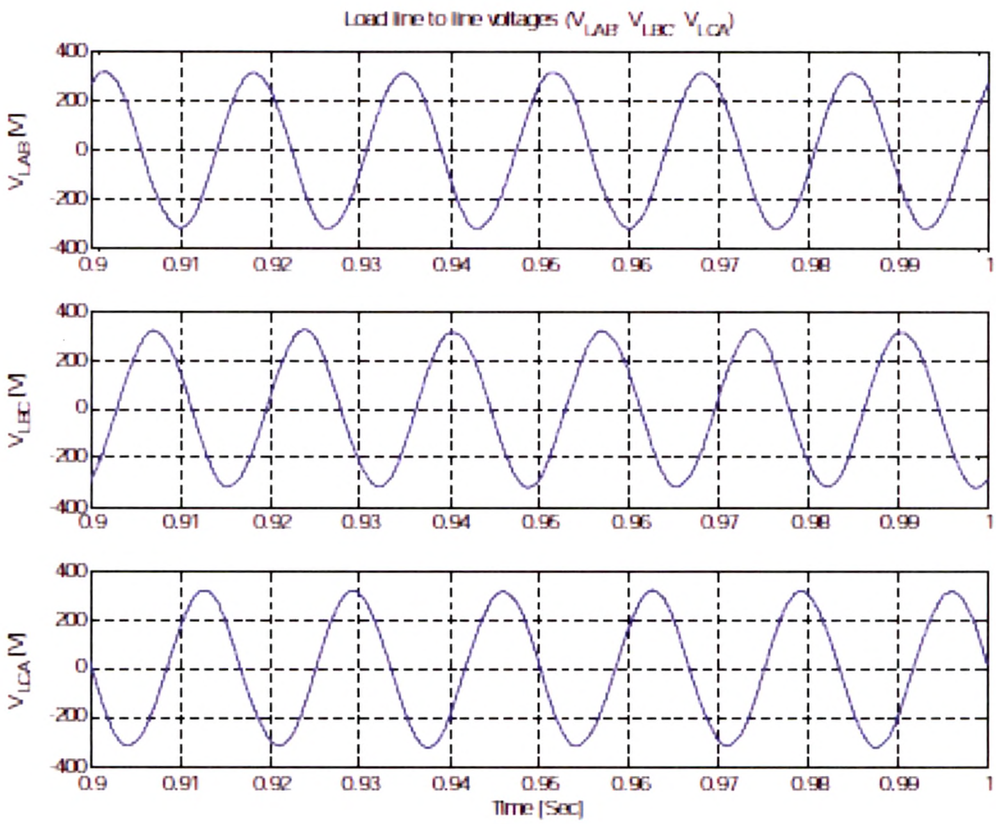

Figure 2.15: Simulation Results of Load Line to Line Voltages ($V_{LAB}, V_{LBC}, V_{LCA}$)

Table 2.5: Simulation Results

Switching Freq. in Hz	Set Temp. in ^0C	Final Temp. in ^0C	V_{ab} in Volt	Frequency in Hz	Load current in Amp.
200	1200	1167	139.21	41.01	9.036
2000	1200	1170	153.49	41.74	6.107
20000	1200	1168	151.33	41.74	5.186
200000	1200	1190	175.95	41.74	4.765

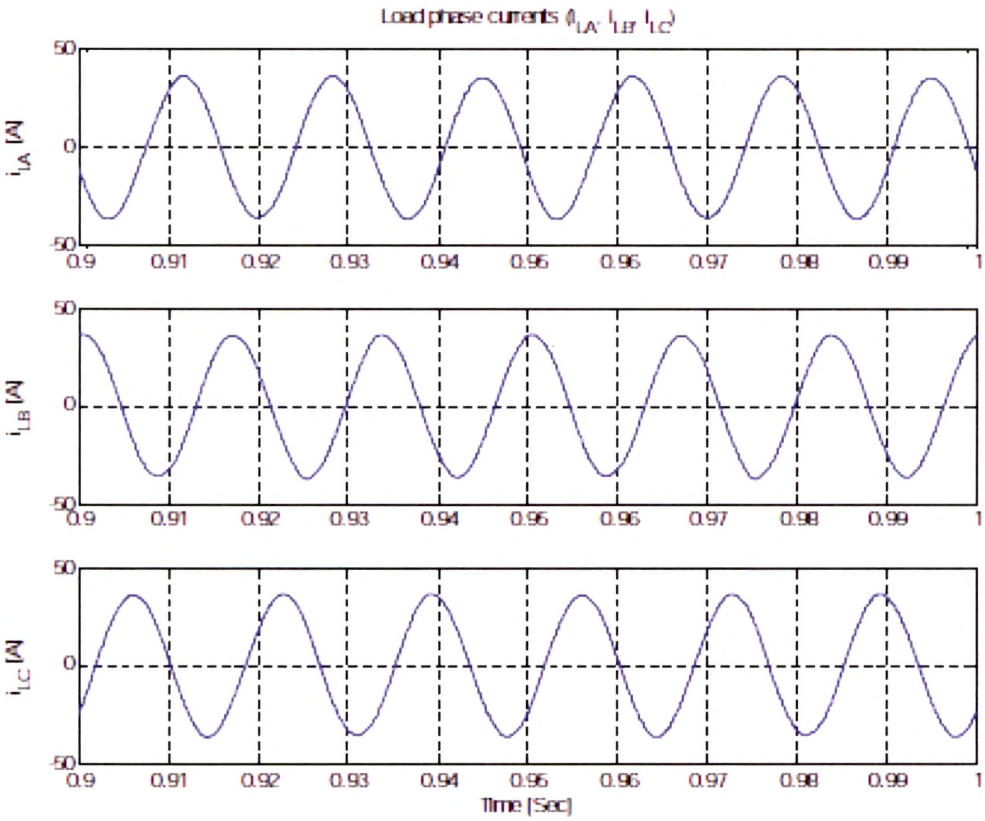

Figure 2.16: Simulation Results of Load Phase Currents (i_{LA}, i_{LB}, i_{LC})

Table 2.6: Simulation Summaries

Set Temp. in ^0C	Final Temp. in ^0C	V_{ab} in Volt	Frequency in Hz
150	145.8	15.20	16.42
530	499	67.22	18.44
1300	1275	165.84	45.21
1500	1484	191.75	52.17
1200	1170	153.49	41.74

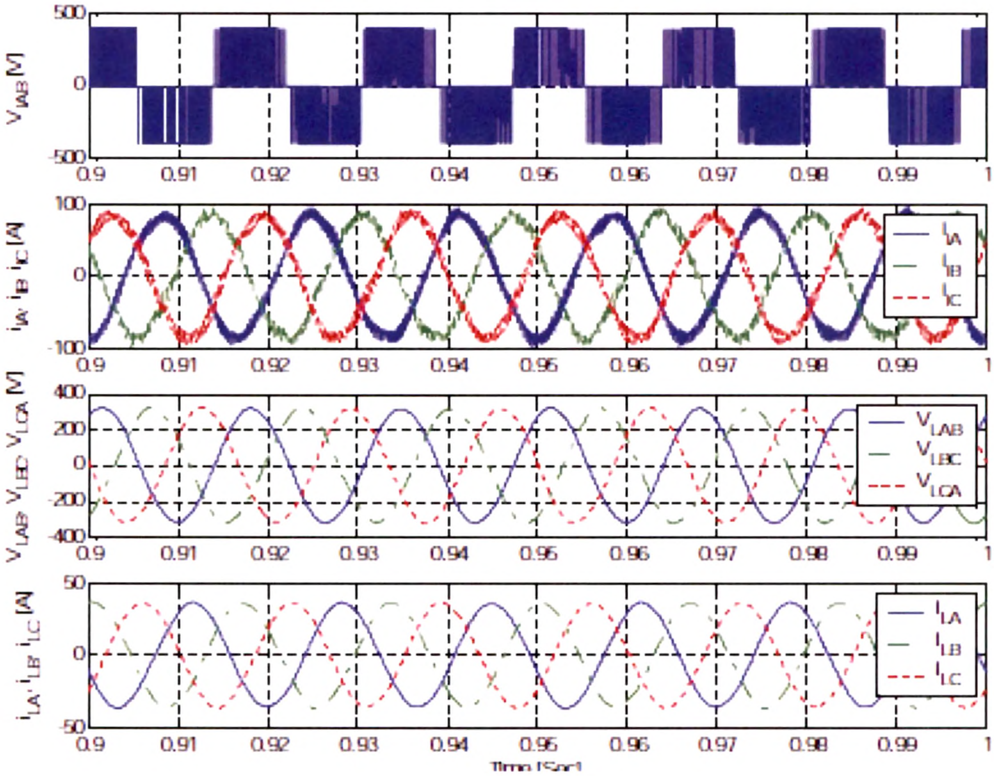

Figure 2.17: Simulation Waveforms. (a) Inverter Output Line to Line Voltage (V_{lAB}) (b) Inverter Output Current (i_{iA}) (c) Load Line to Line Voltage (V_{LAB}) (d) Load Phase Current (i_{LA})

Table 2.7: Spectral Analysis

h	Harmonic for different Switching frequencies			
	1 kHz	3 kHz	5 kHz	10 kHz
0	-4.58	-4.58	-4.16	-4.16
1	81.22	80.83	73.33	73.24
2	3.22	2.99	2.64	2.60
3	4.70	4.53	4.06	4.03
4	0.42	0.20	0.11	0.06
5	4.45	5.04	4.80	4.94
6	1.98	1.79	1.58	1.55
7	1.03	1.07	0.99	1.00

Table 2.8: Switching Vectors Part I

Sector	Switching Vector						Voltage	Time	Consecutive Time	Sample Time
	S_q	S_2	S_3	S_4	S_5	S_6				
SECTOR I	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$	T_{C11}	T_{S-I}
	1	0	0	0	1	1	V_1	T_1		
	1	1	0	0	0	1	V_2	T_2		
	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$		
	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$	T_{C12}	
	1	1	0	0	0	1	V_2	T_2		
	1	0	0	0	1	1	V_1	T_1		
	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$		
SECTOR II	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$	T_{C21}	T_{S-II}
	1	1	0	0	0	1	V_2	T_2		
	0	1	0	1	0	1	V_3	T_3		
	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$		
	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$	T_{C22}	
	0	1	0	1	0	1	V_3	T_3		
	1	1	0	0	1	1	V_2	T_2		
	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$		
SECTOR III	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$	T_{C31}	T_{S-III}
	0	1	0	1	0	1	V_3	T_3		
	0	1	1	1	0	0	V_4	T_4		
	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$		
	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$	T_{C32}	
	0	1	1	1	0	0	V_4	T_4		
	0	1	0	1	0	1	V_3	T_3		
	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$		

Table 2.9: Switching Vectors Part II

Sector	Switching Vector						Voltage	Time	Consecutive Time	Sample Time
	S_q	S_2	S_3	S_4	S_5	S_6				
SECTOR IV	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$	T_{C41}	T_{S-IV}
	0	1	1	1	0	0	V_4	T_4		
	0	0	1	1	1	0	V_5	T_5		
	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$		
	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$	T_{C42}	
	0	0	1	1	1	0	V_5	T_5		
	0	1	1	1	0	0	V_4	T_4		
	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$		
SECTOR V	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$	T_{C51}	T_{S-V}
	0	0	1	1	1	0	V_5	T_5		
	1	0	1	0	1	0	V_6	T_6		
	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$		
	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$	T_{C52}	
	1	0	1	0	1	0	V_6	T_6		
	0	0	1	1	1	0	V_5	T_5		
	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$		
SECTOR VI	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$	T_{C61}	T_{S-VI}
	1	0	1	0	1	0	V_6	T_6		
	1	0	0	0	1	1	V_1	T_1		
	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$		
	0	0	0	1	1	1	V_0	$\frac{T_0}{2}$	T_{C62}	
	1	0	0	0	1	1	V_1	T_1		
	1	0	1	0	1	0	V_6	T_6		
	1	1	1	0	0	0	V_7	$\frac{T_0}{2}$		

2.6 Conclusions

The main finding of this chapter reveals following:

1. Space vector modulation requires only a reference space vector to generate three phase sine waves.

2. The amplitude and frequency of load voltage can be varied by controlling the reference space vector.

3. This algorithm is flexible and suitable for advanced vector control.

4. The strategy of the switching minimizes the distortion of load current as well as loss due to optimum number of commutations in the inverter.

5. The effectiveness of the SVM to reduced the switching power losses is proved.

6. SVM is one of the best solutions to achieve good voltage transfer and reduce harmonic distortion in the output of three phase inverter for IDH.

7. It also provides excellent output performance optimized efficiency and high reliability compared to similar three phase inverter with conventional pulse width modulations.