CHAPTER 6

ON THE CAUSALITY BETWEEN GOVERNMENT EXPENDITURE AND NATIONAL INCOME IN INDIA*

6.1. INTRODUCTION

The proposition put forward by the German economist Adolf Wagner regarding the relation between economic development and the size of the government activity has held the interest of economists for over a century since he formulated the law in 1863. The Wagner's Law, as it is often referred to, raises some uncertainty regarding its precise interpretation. There has also been a wide-ranging debate regarding its empirical validity. However, Wagner's Law of "increasing state activity" (considered in great detail in Chapter 3) still remains the most frequently tested theoretical proposition. Several studies have been carried out for different countries, many lending support to Wagner's hypothesis and some rejecting it depending upon the formulation tested. As discussed earlier in Chapter 3, Wagner's law has been substantiated for India.

The assumption inherent in the Wagner's hypothesis is that the causality (or the relation between cause and effect) runs from the level of economic development to the size of government expenditure. However, in most econometric models and in a Keynesian mode of thought, it is stressed that changes in government expenditure lead to changes in national income.

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During the great depression of 1930's, Keynes advocated increased investment by the government to revive the economy from widespread unemployment and bring about economic stability. In other words, Keynes believed that an increase in the volume of expenditure would bring about economic growth.

In either case, the decision regarding the causative process on the basis of empirical testing is lacking. Hence, the causal connection between economic development and government expenditure can not be established unless the interaction of cause and effect is determined either empirically or logically.

6.2. STUDIES RELATING TO CAUSALITY BETWEEN GOVERNMENT EXPENDITURE AND NATIONAL INCOME

The studies concerning the causality between government expenditure and national income were inducted into the public expenditure analysis almost a decade ago. The pioneering effort was made by Singh and Sahni [1984a, 1984b] for studies relating to India and Canada. The analysis was carried out by them using the Granger Causality Test framework between GNP and government expenditure at aggregate level and at disaggregate level by functions like health, education, transport & communication, law & order, debt charges etc. Their results revealed a bidirectional causality between government expenditure and national income at the aggregate level and a mix of unidirectional and bidirectional causality at the disaggregate level. In another of their studies for U.S.A. [1986] also, a bidirectional causality was observed between GNP and government expenditure at the aggregate level.

Sarma and Rao's study [1991] for India for the period from 1960 to 1990 employed Geweke's Canonical Causality formulation in addition to the standard Granger test of causality. This study concluded that the causal flow runs from national income to government expenditure, which is in contrast to the Singh and Sahni study for India [1984a]. It may be mentioned here that the period covered and the test procedure followed are different in both the studies.

While the above studies have used single-country samples, namely those for India, Canada and U.S.A., a couple of studies have also been carried out for multi-country samples. The pattern of causality between national income and government expenditure shows substantial variations in the study conducted by Ram [1986] for 63 countries. In this study, the Guilkey and Salemi version of the Granger-Sargent procedure for causality has been used. However, the Ahsan, Kwan & Sahni study on 24 OECD (Organization for Economic Cooperation and Development) countries finds a bidirectional causality between national income and government expenditure for a majority of the countries. This finding is supportive of the earlier results obtained for single-country samples.

The above studies carried out by Ram [1986] and Singh & Sahni [1989] have not been without criticism. The first point of criticism is that, with the use of a bivariate system, some significant variables may be omitted because of which the causality results may be obscured. This point was made by Lutkepohl [1982]. Secondly, "no causality" indicates that the past and present values of an economic variable do not help in predicting another economic variable. This is incorrect if there exists a functional relationship between these economic variables which can be expressed within a theoretical framework. This point was raised by Nagarajan and Spears [1989]. For example, the cause and effect relationship between economic growth and expenditure has a theoretical backing in terms of Wagner's Law in that, as an economy progresses, an increase occurs in the government expenditure, and economic growth leads to growth in government expenditure with the rate of expenditure growth exceeding that of economic growth.

Doubts have also been raised regarding the inconsistent results obtained in the above mentioned studies. It remains to be further investigated if the non-homogeneity in the results obtained can be attributed to deficiencies in the underlying model, the estimation procedure or some inadequacy in the data set or the sample. It is in this light that the causality testing for public expenditure and national income is undertaken for the period 1950-51 to 1989-90 in this study. This makes it the longest period covered by any causality study pertaining to India. The results can help in understanding whether the differences in the estimation procedure and the period covered affect the causality pattern for India.

6.3. THE GRANGER TEST OF CAUSALITY

Granger's conception of causality depends on the flow of time and, in view of this, Granger [1969] distinguished four patterns of causality. Let U_t be all the information in the universe accumulated since time (t-1) and let $(U_t - Y_t)$ denote all this information apart from the specified series Y_t.

The definitions of the patterns of causality can be written as follows :-

(1) Simple Causality : If $\sigma^2(X|U) < \sigma^2(X|U-Y)$, we say that Y is causing X denoted by $Y_t \longrightarrow X_t$. We say that Y_t is causing X_t if we are better able to predict X_t using all available information than if only the information apart from Y_t had been used.

(2) Instantaneous Causality : If $\sigma^2(X|U,Y) < \sigma^2(X|U)$ we say that instantaneous causality $Y_t \longrightarrow X_t$ is occurring. In other words, the current value of X_t is better 'predicted' if the present value of Y_t is included in the 'prediction' than if it is not.

(3) Causality Lag : If $Y_t \longrightarrow X_t$, we define the (integer) causality lag m to be the least value of K such that $\sigma^2(X|U-Y(K)) < \sigma^2(X|U-Y(K+1))$. Thus, knowing the values Y_{t-j}, $j=0,1,\ldots,m-1$ will be no help in improving the prediction of X_t.

(4) Feedback : If $\sigma^2(X|U) < \sigma^2(X|U-Y)$ and $\sigma^2(Y|U) < \sigma^2(Y|U-X)$, we say that feedback is occurring, which is denoted by $Y \longleftrightarrow X$. In other words, feedback is said to occur when X_t is causing Y_t and also Y_t is causing X_t.

The patterns (1), (2) and (3) refer to unidirectional causality $(X \longrightarrow Y)$ and (4) suggests bidirectional causality. The above definitions have assumed that X_t and Y_t are stationary stochastic variables[1,*]. In light of the above definitions, the simple causal model is :

$$
\begin{aligned}
X_t &= \sum_{j=1}^{m} a_j X_{t-j} + \sum_{j=1}^{m} b_j Y_{t-j} + U_t \\
Y_t &= \sum_{j=1}^{m} c_j X_{t-j} + \sum_{j=1}^{m} d_j Y_{t-j} + V_t
\end{aligned}
\qquad \ldots(6.1)
$$

where U_t and V_t are taken to be white-noise[2] series, i.e. $E(U_t U_s)=0=E(V_t V_s)$, $s \neq t$ and $E(U_t U_s)=0$, all t, s, and X_t and Y_t are two stationary time series[3] with zero means.

The definition of causality given above implies that Y_t causes X_t provided some $j_j \neq 0$. Similarly, X_t causes Y_t if some $c_j \neq 0$. If both of these events occur, there is a feedback relationship between X_t and Y_t.

The more general model with instantaneous causality is :

$$
\begin{aligned}
X_t + b_o Y_t &= \sum_{j=1}^{m} a_j X_{t-j} + \sum_{j=1}^{m} b_j Y_{t-j} + U_t \\
Y_t + c_o X_t &= \sum_{j=1}^{m} c_j X_{t-j} + \sum_{j=1}^{m} d_j Y_{t-j} + V_t
\end{aligned}
\qquad \ldots(6.2)
$$

If the variables are such that this kind of representation is required, then instantaneous causality occurs and the current value of X_t is better 'predicted' by including Y_t and this will improve the goodness of fit of the first equation for X_t.

After estimating the two linear equations 6.1 or 6.2 as the case may be, the null hypothesis $b_j=c_j=0$ for all j $(j=0,1,\ldots,m)$

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* All terms or words in this chapter with superscripted numbers are explained at the end of this chapter in APPENDIX VIA.

is tested against the alternative hypothesis that b_j=0 and c_j=0 for at least some j's, using the F-test. It is, however, important to note that, since the equations involve lagged independent and dependent variables, the error terms might be serially correlated and the test statistic is invalid in the presence of serial correlation. Therefore, all the variables used in the regressions are passed through filters[4] such as those of Sims[5] [1972], Nerlove[6] [1964] or Box and Jenkins[7] [1976], so that the regression residuals would be white-noise with this prefiltering.

6.4. STATIONARITY TESTS

Subjecting the Granger causality test to actual or raw data series (unadjusted series) poses problems as the data series may not be stationary. However, without identifying the cause of non-stationarity, first differencing or employing a filter may result in over-differencing of the series. Also, it has been found that most economic variables exhibit some regular patterns of movement and it is common practice to remove the trend component prior to the causality analysis, which is called 'detrending' of the series. Mostly, this is achieved by including time as an explanatory variable. In addition, the decision regarding the lag length to be used in the regression equations is also to be taken so as to make the Granger test applicable to the economic variables being considered.

To view whether a series is stationary or not, the correlogram[8] or auto correlation function of the series can be generated. A non-damping autocorrelation function would indicate that the series is not stationary. However, the pattern shown in the correlogram cannot suggest the alternatives of converting the series into stationarity. There are basically two different classes of non-stationary processes, the trend stationary (TS)[9] process and difference stationary process (DS)[10]. Here, we are concerned with the roots of the autoregressive (AR) polynomial[11] of the two processes mentioned above. The stationarity condition for an autoregressive process is that the roots of the characteristic equation[12] should be less than one in absolute value. Several testing procedures have been devised to test not only the existence of unit root (i.e. whether one of roots of the AR polynomial of the statistical process is 1), but also to test the existence of a regular trend. Existence of a unit root in the AR polynomial means that the series is a DS process and, hence, it can be converted to stationarity by first differencing[13] the series. If the time trend coefficient turns out to be significant then the series is a TS process and this implies that the series should be converted to logs to make it stationary.

Dickey and Fuller [1979] have devised a method for testing the presence of unit root in the AR polynomial of the statistical process. In the present study, the Nelson and Plosser [1982] version of the Dickey-Fuller (DF) test has been used which gives an indication about the existence of a unit root and also the presence of time trend. After the series are tested for the existence of unit root as well as time trend effect, depending upon the result, the series are converted to stationarity using suitable methods of transformations (to logs or first differencing). The Granger test is then applied to the stationarised series.

6.5. FINDINGS OF THE STATIONARITY TESTING

The stationarity testing for Central Government expenditure and its economic categories (commodities & services, wages & salaries, transfer payments, and gross capital formation) is done for the period 1950-51 to 1989-90. Similar testing for the functional categories (defence, education, medical & public health, agriculture, industry and transport and communication) is done for the period 1966-67 to 1989-90.

Using the RATS (Version 2.12) software developed by VAR Econometrics and Doan Associates, the plotting of correlograms and unit-root testing is done. The lag length to be used in the model is determined on the basis of both Akaike [1974] and Schwarz [1978] criteria (also with the help of the above mentioned programme). From the first two autocorrelations for the GNP series it has been found that the autocorrelation function does not damp out quickly, which is an indication that the series is not stationary. As mentioned earlier, the correlogram cannot suggest the method to be used for converting a non-stationary series into stationarity. Hence, the Dickey-Fuller test is applied to look for the existence or absence of unit root in the AR polynomial. This is done with the help of τ statistic. The cumulative distribution of τ is given in Fuller [1976].

Table 6.1. gives the τ values and the significance level of the coefficient of time trend, for various economic and functional categories. The tabulated τ value at 5% level of significance is -2.97 and that at 1% is -3.65. The criterion used for unit root testing is that, if the calculated τ value is less than the tabulated τ value, then it indicates the significance of the τ value and the absence of the unit root. In such a case, the series is said to be stationary. As for the significance level of the coefficient of time-trend, if the significance level is less than 1%, then the time-trend coefficient is significant.

Using the above criterion, it can be seen from Table 6.1. that the actual data series is non-stationary since the τ values for all the heads of expenditure and GNP are insignificant at 1% level. The log-transformed series on commodities & services, medical & public health and agriculture are stationary with the τ values significant at 1% level, while the series on transport & communication is stationary with the τ value significant at 5% level. The series for the rest of the categories are all non-stationary because of the insignificance of the τ values. Finally, it can be observed that the log-first differenced series for all the categories are stationary, with τ values significant at 1% level.

It may be noted that, in spite of the series being stationary with first differences, the time-trend coefficient happened to be statistically significant in almost all the cases, except for education and total expenditure. This is indicative

TABLE 6.1

ESTIMATED τ-VALUE AND SIGNIFICANCE LEVEL OF THE COEFFICIENT OF TIME TREND

Series	Actual data series		Log transformed		Log first differenced	
	τ-value	Significance level of time-trend	τ-value	Significance Level of time-trend	τ-value	Significance Level of time-trend
	(1)	(2)	(3)	(4)	(5)	(6)
Gross National Product	6.2992	55.3	-2.9553	0.1	-5.1318	0.4
Central Govt. Expenditure	5.1667	64.9	-1.8502	6.3	-5.7205	63.8
Commodities & Services	3.2569	49.3	-4.0148	0.02	-5.4086	36.4
Wages & Salaries	0.7669	20.1	-2.9347	0.4	-3.9423	29.8
Transfer Payments	4.5983	97.6	-2.1711	2.3	-6.5424	5.7
Gross Capital Formation	4.7011	43.8	-2.0733	4.0	-5.4782	93.9
Defence Services	-0.1738	9.5	-1.9847	4.7	-4.3464	10.9
Education	-1.2191	5.4	-1.2843	11.1	-3.9625	8.7
Medical & Public Health	-0.8473	9.0	-4.4426	0.03	-6.5886	73.9
Agriculture	-0.6655	8.8	-5.6513	0.0	-6.3495	74.6
Industry	-1.0795	9.4	-2.7371	1.3	-4.1963	97.0
Transport & Communication	-1.6034	4.4	-3.1742	0.3	-4.6270	51.2

Source : Based on Tables 3.1, 4.2 and 4.6.

Note : The significance level of the coefficient of time trend is given in per cent.

of the fact that the time-trend effect can not be neutralized. To sort out this problem, the log transformation of the series was done and the Dickey-Fuller test was repeated. The test results indicated that the first differences in logs were fairly stationary and the Dickey-Fuller test statistics (τ values) were not only statistically significant but the trend effect was also insignificant or neutralized. This fact is well illustrated in Table 6.2., which gives the results of the stationarity testing for GNP and Central Government expenditure series for India at current prices.

After the series are converted to stationarity the next step is to decide on the optimal lag length to be used in the equations. As already mentioned the software used also gives an indication about the optimal lag length using both the Akaike and Schwarz criteria. The lag length was taken to be 2. After having done so, the causality analysis is carried out using the F-test on first differences of logarithmic series keeping the lag length at 2.

6.6. RESULTS OF THE GRANGER CAUSALITY TESTS

Table 6.3 summarizes the results of the Granger Causality tests on various series. It can be seen that there exists a bidirectional causality between GNP and Central Government expenditure. This is in line with the earlier result obtained by Singh and Sahni [1984a] for India. The bidirectional causality can be justified on the grounds that, with an increase in GNP, the ability of the government to spend rises. This, in turn, leads to increased expenditure, thereby supporting Wagner's

TABLE 6.2

RESULTS OF STATIONARITY TESTING FOR GNP AND CENTRAL GOVERNMENT EXPENDITURE SERIES FOR INDIA AT CURRENT PRICES

Series	Actual data series		Log transformed		Log first differenced	
	Unit Root	Time-trend	Unit Root	Time-trend	Unit Root	Time-trend
	(1)	(2)	(3)	(4)	(5)	(6)
Gross National Product	*		*	*		*
Central Govt. Expenditure	*		*			
Commodities & Services	*			*		
Wages and Salaries	*			*		
Transfer Payments	*		*	*		
Gross Capital Formation	*		*	*		
Defence Services	*		*	*		
Education	*		*			
Medical & Public Health	*			*		
Agriculture	*			*		
Industry	*		*	*		
Transport & Communication	*	*	*	*		

Source : Based on Table 6.1.

Note : 1) * indicates existence of unit root or time trend, as the case may be.

2) The tabulated τ-value for sample size n=40 is -2.97 at 5% level of significance and -3.65 at 1% level. The stationarity testing for total Central Government expenditure, commodities & services, wages & salaries, transfer payments and gross capital formation is done for the period 1950-51 to 1989-90, and that for defence, education, medical & public health, agriculture, industry, and transport & communication is done for the period 1966-67 to 1989-90.

TABLE 6.3

RESULTS OF GRANGER CAUSALITY BETWEEN GNP AND GOVERNMENT EXPENDITURE AND ITS COMPONENTS FOR INDIA AT CURRENT PRICES

Dependent Variables	Lag length	Durbin's Test II	F-value of regression	Degrees of freedom	F-value of Granger test	Degrees of freedom	Inference
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Central Government Expenditure	2	-0.2550	21.086	(4,33)*	7.7283	(2,33)*	Gross National Product causes Central Govt. Expenditure
Gross National Product	2	-0.2505	41.788	(4,33)*	9.7977	(2,33)*	Central Govt. Expenditure causes Gross National Product
Wages and Salaries	2	-0.4679	22.579	(4,33)*	3.4975	(2,33)**	Gross National Product causes Wages & Salaries
Gross National Product	2	-0.7398	26.168	(4,33)*	1.5885	(2,33)	....
Commodities and Services	2	-0.4604	20.377	(4,33)*	4.8375	(2,33)*	Gross National Product causes Commodities & Services
Gross National Product	2	-0.8311	28.268	(4,33)*	2.6921	(2,33)	....
Transfer Payments	2	-0.2986	20.377	(4,33)*	9.8375	(2,33)*	Gross National Product causes Transfer Payments
Gross National Product	2	-0.2764	48.439	(4,33)*	13.294	(2,33)*	Transfer Payments causes Gross National Product
Gross Capital Formation	2	-0.6002	2.2795	(4,33)*	4.2540	(2,33)**	Gross National Product causes Gross Capital Formation
Gross National Product	2	-0.0967	36.703	(4,33)*	7.1253	(2,33)*	Gross Capital Formation causes Gross National Product

......(Table 6.3. continued)

......(Table 6.3 continued)

Dependent Variables	Lag length	Durbin's Test II	F-value of regression	Degrees of freedom	F-value of Granger test	Degrees of freedom	Inference
Medical & Public Health	2	0.1514	7.9685	(4,17)*	11.376	(2,17)*	Gross National Product causes Medical & Public Health
Gross National Product	2	0.7047	44.246	(4,17)*	4.6905	(2,17)**	Medical & Public Health causes Gross National Product
Defence Services	2	0.4314	19.446	(4,17)*	7.8574	(2,17)*	Gross National Product causes Defence Services
Gross National Product	2	0.2582	57.802	(4,17)*	8.3776	(2,17)**	Defence Services causes Gross National Product
Education	2	-1.0105	6.4339	(4,17)*	4.3285	(2,17)**	Gross National Product causes Education
Gross National Product	2	-0.1120	32.370	(4,17)*	1.4604	(2,17)	....
Agriculture	2	0.9372	4.0172	(4,17)**	7.8694	(2,17)*	Gross National Product causes Agriculture
Gross National Product	2	-0.6235	30.502	(4,17)*	0.9521	(2,17)	....
Industry	2	0.5398	3.3238	(4,17)**	6.4542	(2,17)*	Gross National Product causes Industry
Gross National Product	2	-0.5484	30.485	(4,17)*	0.9474	(2,17)	....
Transport & Communication	2	-0.1064	4.7895	(4,17)*	8.5998	(2,17)*	Gross National Product causes Transport & Communication
Gross National Product	2	-0.5154	27.962	(4,17)*	0.2612	(2,17)	....

<u>Source</u> : Based on Tables 3.1 and 4.2

<u>Note</u> : * Significant at 1% level
** Significant at 5% level

views. The causality from expenditure to national income is supportive of Keynesian thought of pumping of investment into the economy by the government to promote economic growth and stability.

There also exists a bidirectional causality between GNP and transfer payments and gross capital formation. The transfers to individuals and regions as well as redistributive transfers (such as pensions and subsidies) increase as national income rises. The resource transfers to States and Union Territories for developmental purposes and subsidies (especially those for export promotion and industrial development of backward regions) lead to increase in GNP. The same can be said of gross capital formation, which increases relatively as national income rises and the increase in gross capital formation causes national income to attain a higher level. The direction of causality for expenditure on commodities and services and wages and salaries is unidirectional from GNP to these heads of expenditure. This is also in conformity with Wagner's thinking that expenditure on goods and services and wages and salaries will grow as national income rises. This, and many other hypothesis have been summarized under 'some potentially testable hypothesis on government expenditure' in the book by Bird [1970].

As far as the functional categories are concerned, the medical & public health and defence services show bidirectional causality, whereas education, agriculture, industry and transport & communication show unidirectional causality from GNP to these components of expenditure.

6.7. CONCLUDING REMARKS

The result of the present study, indicating a bidirectional causality between government expenditure, is in conformity with the earlier results of Singh and Sahni [1984a] for India. This seems to suggest that the difference in the length of the time-period chosen and the methodology adopted does not alter the outcome. One fundamental advantage of the Granger causality test has been to make an effort to go beyond mere 'correlation' and 'association' and to assess 'causation' across economic variables. The Wagner's hypothesis and many other economic relationships can be investigated through Granger test of causality.

APPENDIX VIA

GLOSSARY OF TERMS PERTAINING TO CAUSALITY TESTING

1. Stationary Stochastic Processes : A statistical phenomenon that evolves in time according to probabilistic laws is called a stochastic process. A very special class of stochastic process, called the stationary process, is based on the assumption that the process is in a particular state of statistical equilibrium. A stochastic process is said to be stationary if its properties are unaffected by a change of time origin; that is, if the joint probability distribution associated with m observations, Z_{t1}, $Z_{t2}, \ldots,$ Z_{tm}, made at any set of times, $t_1, t_2, \ldots t_m$, is the same as that associated with m observations Z_{t1+k}, $Z_{t2+k}, \ldots, Z_{tm+k}$, made at times $t_1+k, t_2+k, \ldots, t_m+k$. Thus, for a process to be strictly stationary, the joint probability distribution of any set of observations must be unaffected by shifting all the times of observation forward or backward by any integer amount k.
2. A 'white noise' is a serially uncorrelated process.
3. A time series is a set of observations generated sequentially in time. The two principal components of the time series are secular or growth component and a cyclic component. The secular component is in the domain of growth theory while the cyclic component is assumed to be transitory (stationary) in nature.
4. A filter may be defined as any series of arithmatical operations used to transform data prior to its analysis. Use of such operations is called prewhitening of the series.

5. For using Sims' filter, all variables used in the region were measured as natural logs and prefiltered using the filter $(1-0.75L)^2$ or $(1-1.5L + 0.5625L^2)$ i.e., each logged variable X_t is replaced by $X_t-1.5X_{t-1}+0.5625X_{t-2}$. By doing so, the regression residuals would be nearly white noise.

6. Nerlove used several filters of the type $(1-KL)^P$ for K=3/4 and P=1, 2 and 3. The data can again be prefiltered using the above filter.

7. Box and Jenkins suggested that the non-stationarity of time series could be removed by using filter $(1-L)^P$ with P taking the value 1 or 2.

8. The covariance between an observation Z_t and its value Z_{t+k}, separated by k intervals of time, is called the autocovariance at lag k and is defined by

$$\gamma_k = cor\,[Z_t, Z_{t+k}] = E\,[(Z_t-\mu)\,(Z_{t+k}-\mu)]$$

where μ is the mean. Similarly, the autocorrelation at lag k is

$$\rho_k = \frac{E[(Z_t-\mu)\,(Z_{t+K}-\mu)]}{\sqrt{E[(Z_t-\mu)^2]\,E[(Z_{t+k}-\mu)_2}}$$

$$= \frac{E[(Z_t-\mu)\,(Z_{t+k}-\mu)]}{\sigma_z^2}$$

$$\rho_k = \frac{\gamma_k}{\sigma_z^2}$$

The plot of autocovariance coefficient γ_k versus the lag k, is called the autocovariance function $\{\gamma_k\}$ of the stochastic process. Similarly, the plot of autocorrelation coefficient ρ_k as a function of lag k, is called the autocorrelation function $\{\rho_k\}$ of the process. The autocorrelation function is also called the correlogram.

9. The trend stationary (TS) process is one which can be expressed as a function of time called a trend, plus a stationary stochastic process with mean zero. If a time series is found to be a trend stationary process, then it can be converted to stationarity by transformation to natural logs on the assumption that trends are linear in the transformed data.

10. The second class of non-stationary processes are those for which the first or higher order difference is a stationary process. These are called the difference stationary (DS) processes.

11. $Z_t = \varphi_1 Z_{t-1} + \ldots + \varphi_p Z_{t-p} + a_t$ is a p^{th} order autoregressive (AR) process, where $\varphi_1, \varphi_2, \ldots, \varphi_p$ are the set of adjustable parameters.

12. The p^{th} order AR process above can be written as,

    $(1-\varphi_1 B - \ldots - \varphi_p B^P)\ Z_t = \varphi(B) Z_t = a_t$

    Similarly first-order AR process can be written as,

    $(1-\varphi_1 B) Z_t = a_t$

    In the first-order AR process written above, $1-\varphi_1 B = 0$ is called the characteristic equation.

13. Let $Y_t = B_0 + B_1 X_t + U_t$ ——> (a)

    be a two variable regression model. If the above equation holds true at time t, it also holds true at time t-1. Hence,

$Y_{t-1} = B_0 + B_1 X_{t-1} + U_t \longrightarrow$ (b)

Subtracting (b) from (a) gives

$$Y_t - Y_{t-1} = B_1 (X_t - X_{t-1}) + U_t - U_{t-1}$$

$$= B_1 (X_t - X_{t-1}) + \epsilon_t$$

or $\Delta Y_t = B_1 \Delta X_t + \epsilon_t$ is called the first difference equation.