

❖ Appendix A

➤ Reflectance of Multilayer Dielectric Thin-Films

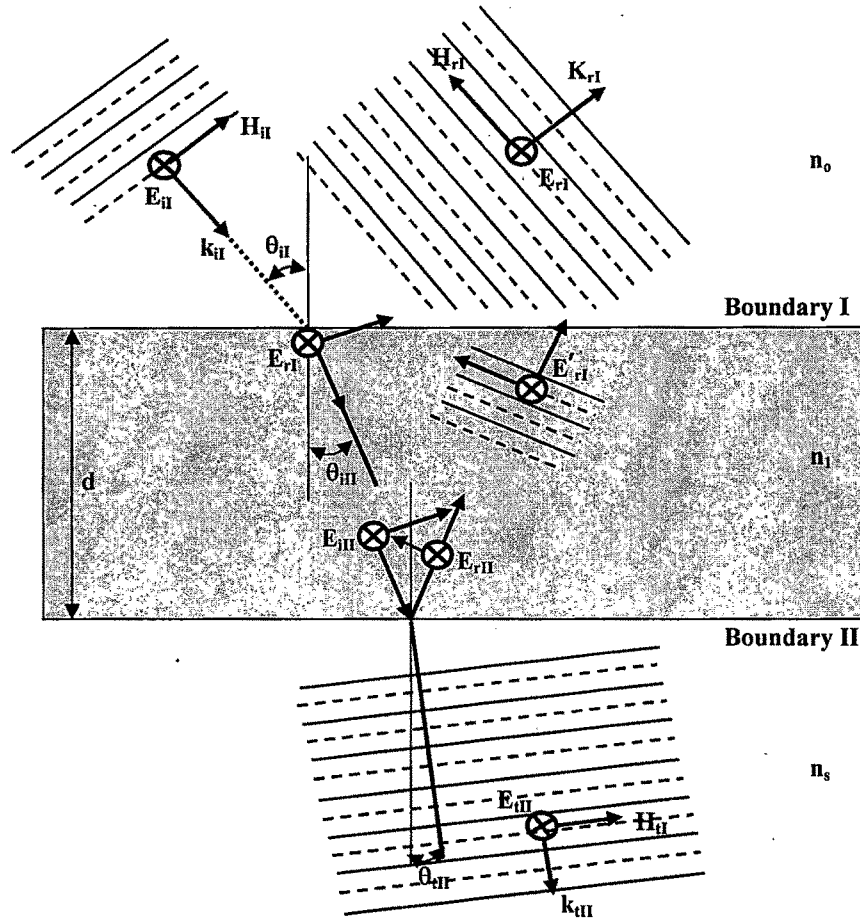


Fig A.1: Reflection from thin film boundaries.

Consider a linearly polarized wave incident on a thin dielectric film between two semi-infinite transparent media, which are semiconductor active layer material and air in the case of laser diode as shown in figure A.1. The refractive indices of air, thin-film and semiconductor material are n_0 , n_1 , and n_s respectively. Each wave E_{rI} , E'_{rII} , E_{tII} , and so forth, represents the resultant of all possible waves traveling in that direction, at that point in the medium. The summation process is therefore built in.

The boundary conditions require that the tangential components of both the electric field (\vec{E}) and magnetic field ($\vec{H} = \vec{B}/\mu$) be continuous across the boundaries.

At boundary I:

$$E_I = E_{iI} + E_{rI} = E_{tI} + E'_{rII} \quad (A.1)$$

$$H_I = \sqrt{\frac{\epsilon_0}{\mu_0}} (E_{iI} - E_{rI}) n_0 \cos \theta_{iI} = \sqrt{\frac{\epsilon_0}{\mu_0}} (E_{tI} - E'_{rII}) n_1 \cos \theta_{iI} \quad (A.2)$$

Here, \vec{E} and \vec{H} in nonmagnetic media are related through refractive index and the unit propagation vector:

$$\vec{H} = \sqrt{\frac{\epsilon_0}{\mu_0}} n \hat{k} \times \vec{E} \quad (A.3)$$

At boundary II:

$$E_{II} = E_{iII} + E_{rII} = E_{tII} \quad (A.4)$$

$$H_{II} = \sqrt{\frac{\epsilon_0}{\mu_0}} (E_{iII} - E_{rII}) n_1 \cos \theta_{iII} = \sqrt{\frac{\epsilon_0}{\mu_0}} E_{tII} n_s \cos \theta_{iII} \quad (A.5)$$

where, n_s is the refractive index of the substrate. A wave that traverses the film once undergoes a shift in-phase of $k_0(2n_1d \cos \theta_{iII})/2$, which will be denoted by k_0h , so that

$$E_{iII} = E_{te} e^{(-ik_0h)} \quad (A.6)$$

$$E_{rII} = E'_{rII} e^{(+ik_0h)} \quad (A.7)$$

Equations (A.4) and (A.5) can now be written as

$$E_{II} = E_{te} e^{(-ik_0h)} + E'_{rII} e^{(+ik_0h)} \quad (A.8)$$

and

$$H_{II} = [E_{te} e^{(-ik_0h)} - E'_{rII} e^{(+ik_0h)}] \sqrt{\frac{\epsilon_0}{\mu_0}} n_1 \cos \theta_{iII} \quad (A.9)$$

These last two equations can be solved for E_{II} and E'_{III} , which when substituted into equations (A.1) and (A.2) yield

$$E_I = E_{II} \cos k_0 h + H_{II} (i \sin k_0 h) / Y_I \quad (\text{A.10})$$

and

$$H_I = E_{II} Y_I i \sin k_0 h + H_{II} (\cos k_0 h) \quad (\text{A.11})$$

where,

$$Y_I = \sqrt{\frac{\epsilon_0}{\mu_0}} n_1 \cos \theta_{II} \quad (\text{A.12})$$

When \vec{E} is in the plane-of-incidence above calculations result in similar equations, provided that now

$$Y_I = \frac{\sqrt{\frac{\epsilon_0}{\mu_0}} n_1}{\cos \theta_{II}} \quad (\text{A.13})$$

In matrix notation, the above linear relations take the form

$$\begin{bmatrix} E_I \\ H_I \end{bmatrix} = \begin{bmatrix} \cos k_0 h & (i \sin k_0 h) / Y_I \\ Y_I i \sin k_0 h & \cos k_0 h \end{bmatrix} \begin{bmatrix} E_{II} \\ H_{II} \end{bmatrix} \quad (\text{A.14})$$

or

$$\begin{bmatrix} E_I \\ H_I \end{bmatrix} = M_I \begin{bmatrix} E_{II} \\ H_{II} \end{bmatrix} \quad (\text{A.15})$$

The *characteristic matrix* M_I relates the fields at the two adjacent boundaries. It follows, therefore, that if two overlaying films are deposited on the substrate, there will be three boundaries or interfaces, and now

$$\begin{bmatrix} E_{II} \\ H_{II} \end{bmatrix} = M_{II} \begin{bmatrix} E_{III} \\ H_{III} \end{bmatrix} \quad (\text{A.16})$$

Multiplying both sides of this expression by M_I , we obtain

$$\begin{bmatrix} E_I \\ H_I \end{bmatrix} = M_I M_{II} \begin{bmatrix} E_{III} \\ H_{III} \end{bmatrix} \quad (\text{A.17})$$

In general, if p is the number of layers, each with a particular value of n and h , then the first and the last boundaries are related by

$$\begin{bmatrix} E_I \\ H_I \end{bmatrix} = M_I M_{II} \dots M_p \begin{bmatrix} E_{(p+1)} \\ H_{(p+1)} \end{bmatrix} \quad (\text{A.18})$$

The characteristic matrix of the entire system is the resultant of the product (in proper sequence) of the individual 2×2 matrices, that is,

$$M = M_1 M_2 \dots M_p \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (\text{A.19})$$

To see how all this fits together, we will derive expressions for the amplitude coefficients of reflection and transmission using the above scheme. By reformulating equation (A.19) in terms of the boundary conditions (A.1), (A.2) and (A.4) and setting

$$Y_0 = \sqrt{\frac{\epsilon_0}{\mu_0}} n_0 \cos \theta_{0i} \quad (\text{A.20})$$

and

$$Y_s = \sqrt{\frac{\epsilon_0}{\mu_0}} n_s \cos \theta_{0t} \quad (\text{A.21})$$

we obtain,

$$\begin{bmatrix} (E_{0i} + E_{0r}) \\ (E_{0i} - E_{0r}) Y_0 \end{bmatrix} = M_i \begin{bmatrix} E_{0t} \\ E_{0t} Y_s \end{bmatrix} \quad (\text{A.22})$$

When the matrices are expanded, the last relation becomes

$$1 + r = m_{11} t + m_{12} Y_s t \quad (\text{A.23})$$

and

$$(1 - r) Y_0 = m_{21} t + m_{22} Y_s t \quad (\text{A.24})$$

In as much as

$$r = \frac{E_{0r}}{E_{0i}} \text{ and } t = \frac{E_{0t}}{E_{0i}}$$

Consequently,

$$r = \frac{Y_0 m_{11} + Y_0 Y_s m_{12} - m_{21} - Y_s m_{22}}{Y_0 m_{11} + Y_0 Y_s m_{12} + m_{21} + Y_s m_{22}} \quad (\text{A.25})$$

and

$$t = \frac{2 Y_0}{Y_0 m_{11} + Y_0 Y_s m_{12} + m_{21} + Y_s m_{22}} \quad (\text{A.26})$$

To find either r or t for any configuration of films, we need only to compute the characteristic matrices for each film, multiply them and substitute the resulting matrix elements in the above equations.

Now further simplifying the above equations we can write,

$$r = \frac{n_0 m_{11} + n_0 n_s m_{12} - m_{21} - n_s m_{22}}{n_0 m_{11} + n_0 n_s m_{12} + m_{21} + n_s m_{22}} \quad (\text{A.27})$$

and

$$t = \frac{2n_0}{n_0 m_{11} + n_0 n_s m_{12} + m_{21} + n_s m_{22}} \quad (\text{A.28})$$

Also, without any trouble one can assume the matrix of the p^{th} layer to be

$$M_p = \begin{bmatrix} \cos k_0 h & (i \sin k_0 h) / n_p \\ n_p i \sin k_0 h & \cos k_0 h \end{bmatrix} \quad (\text{A.29})$$

where,

$n_p = n \cos k_0 h$ for perpendicular polarization

$n_p = n / \cos k_0 h$ for parallel polarization

$n_p = n$ for normal light incidence

n = refractive index of the p^{th} layer.

Now the layer matrix is a complex matrix. The elements m_{12} and m_{21} are purely complex while the other two elements are purely real. This can be seen from equation (A.29). So while making calculations using a computer program, this fact helps to make the program simpler while calculating the characteristic matrix. Here both r as well as t is imaginary.

So, Reflectance,

$$R = r r^*$$

And Transmittance,

$$T = t t^*$$

Which gives,

$$R = \frac{(n_0 m_{11} - n_s m_{22})(n_0 m_{11} - n_s m_{22}) + (n_0 n_s m_{12} - m_{21})(n_0 n_s m_{12} - m_{21})}{(n_0 m_{11} + n_s m_{22})(n_0 m_{11} + n_s m_{22}) + (n_0 n_s m_{12} + m_{21})(n_0 n_s m_{12} + m_{21})} \quad (\text{A.30})$$

and

$$T = \frac{4n_0^2}{(n_0 m_{11} + n_s m_{22})(n_0 m_{11} + n_s m_{22}) + (n_0 n_s m_{12} + m_{21})(n_0 n_s m_{12} + m_{21})} \quad (\text{A.31})$$

Equations (A.30) and (A.31) determine the reflection and transmission coefficients of the dielectric thin-films. These equations are used in the computer simulation.