Chapter 4

ERROR IDENTIFICATION AND DIAGNOSIS

ERROR IDENTIFICATION AND DIAGNOSIS

4.1 INTRODUCTION

One cannot start diagnosis without any basis regarding the information about the errors committed by the students. Diagnosis starts with the interest of identifying the learning difficulties faced by the students resulting in the observed errors in the responses of the students. The present chapter is a detailed description about the error identification and diagnosis. The entire process from error identification to the diagnosis of the learning difficulties in geometry is included. This chapter includes the detailed item-wise analysis of the responses of the students on achievement test. Also, the observations of the investigator and the experienced teachers on the common errors committed by the students in geometry are enunciated here. Further this was followed by the process of diagnosis, administration of the diagnostic test and the analysis of the responses of the students on the diagnostic test. Finally the identification of the learning difficulties in geometry is discussed.

4.2 PROCESS OF DIAGNOSIS

The investigator followed the process of Diagnosis and Remediation of Learning difficulties particularized in four steps according to Cooney, Davis and Henderson (1975) as below:

1. To discover which students are having learning difficulties. It is these to whom the teacher gives attention.

2. To find out what kind of errors a student or a group of students are making.

3. To infer the causes that explains the errors. Errors are observable but why an error is made i.e. its cause is not observed and can only be inferred.

4. To accept the cause of a difficulty and provide remedial teaching.

In the present study the process of diagnosis and remediation was done in the following manner:

i)   Construction and Administration of the Achievement Test

ii)   Scoring and analysis of the responses of the students on Achievement test

iii)   Identification of Error

iv)   Construction and Administration of Diagnostic Test

v)   Analysis of the responses of the students on the diagnostic test

vi)   Identification of the Learning Difficulties

vii)   Conducting Remedial Programme

viii)   Construction and Administration of the Parallel test for Achievement

ix)   Studying the impact of the remedial measures

4.3 ADMINISTRATION OF THE ACHIEVEMENT TEST

Achievement test was prepared by the investigator. The details of the construction of the Achievement test are given in the previous chapter. There were six major questions and hundred items in the achievement test. It was of hundred marks and each item was of one mark. The achievement test was administered by the investigator on all the two hundred and fifty-eight students of the two selected schools. The time given to the students for the test was two hours.

4.4 ANALYSIS OF THE ACHIEVEMENT TEST

The performance of the students for each item in the achievement was scored and studied using the error identification table. The error identification tables were used for each of the five divisions which are displayed in appendix section (Appendix- E).

4.4.1 Score-wise Analysis

From the error identification tables of all the five division in total, of both the schools, the analysis was done in terms of the scores obtained by the students. The table below represents the frequency distribution of the scores obtained by the students on the achievement test.

Table – 14

Frequency Distribution of Scores obtained by the Students on Achievement Test

Class	Frequency (*f*)
0-19	22
20-39	67
40-59	112
60-79	47
80-99	10
Total	**258**

From the above table it's clear that only fifty-seven students out of 258 i.e. 22.1 percent students could achieve more than sixty percent. Also, the mean score was 45.5 which is less than fifty percent. So in general the achievement of the students in geometry was not good.

4.4.2 Item-wise Analysis

Also, the item-wise analysis was carried out by the investigator in terms of the number of incorrect responses given by the students, and errors were identified by relating the percentage of incorrect responses with the instructional objective associated with each item as follows:

Table – 15

Item-wise Analysis of Achievement Test

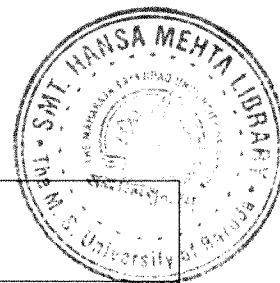| Item No.	Item	Number of Correct Responses	Percentage of Correct Responses	Number of Incorrect Responses	Percentage of Incorrect Responses	Error (Students committed error in...)
			Q. I			
	Fill in the blanks by selecting the correct alternative from the options given on the right.					
1	Every line has atleast _____ distinct points. (0,1,2)	202	78.3	56	21.7	Stating the postulate that "Every line has atleast two distinct points."
2	If $\overrightarrow{XY} = \overrightarrow{YZ}$ then Y_____$\overrightarrow{XZ}$. ($\subset, \in, \notin$)	179	69.4	79	30.6	Identifying the condition for the two equal lines
3	Exactly _____ lines can be determined by three distinct non-collinear points. (1,2,3)	143	55.4	115	44.6	Stating that three distinct non-collinear points determine three lines
4	If $P \in \overrightarrow{QR}$ then P, Q, R are _____. (equal, collinear, non-collinear)	197	76.4	61	23.6	Recognizing that the given points are collinear points
5	Two distinct points	150	58.1	108	41.9	Interpreting the

	P,Q are on both the lines ℓ_1 and ℓ_2. So ℓ_1 ___ ℓ_2. (=, ≠, ∈)					postulate that "Two distinct points belong to simultaneously one and only one line."
6	If P∈ℓ & Q∉ℓ, then $\overrightarrow{PQ}$ ___ ℓ. (=,≠,⊂)	162	62.8	96	37.2	Identifying that the given lines are distinct lines
7	If two distinct lines intersect in exactly one point, then they are _____ lines. (parallel, non-parallel, same)	104	40.3	154	59.7	Stating that the intersecting lines are non-parallel lines
8	If R∈$\overrightarrow{PQ}$ & S∉$\overrightarrow{PQ}$, then $\overrightarrow{PQ} \cap \overrightarrow{RS}$ = _____. ({S}, {P}, {R})	97	37.6	161	62.4	Locating the point of intersection for the given two lines
9	For points P, Q, R , PQ+QR _____ PR. (=, ≤, ≥)	29	11.2	229	88.8	Stating the property "PQ+QR ≥ PR" for three distinct points P,Q, and R
10	For distinct collinear points P, Q & R if RP+PQ=QR, then _____. (P-Q-R, R-P-Q, R-Q-P)	145	56.2	113	43.8	Identifying the betweenness for the given three collinear points
11	A & B are the end points of _____. ($\overrightarrow{AB}$, $\overrightarrow{AB}$, $\overrightarrow{AB}$)	152	58.9	106	41.1	Naming the end-points of the given line-segment

12	Line-segments having equal lengths are called _____ line-segments. (congruent, parallel, same)	176	68.2	82	31.8	Recalling the definition of congruent line-segments
13	If P-Q-R & _____ then Q is a midpoint of a line-segment $\overline{PR}$. (PQ=PR, PR=QR, PQ=QR)	187	72.5	71	27.5	Identifying the condition for the point to be the mid-point of the given line-segment
14	Every line-segment has _____ mid-point. (0, 1, 2)	191	74.0	67	26.0	Recollecting that the line-segment has only one mid-point
15	For D-E-F-G, $\overline{DF} \cap \overline{EG} =$ _____. ($\overline{DE}$, $\overline{FG}$, $\overline{EF}$)	150	58.1	108	41.9	Indicating the intersection of two line-segments in a given situation
16	For $\overrightarrow{AB}$, _____ is called the initial point. (A, {A}, {B})	181	70.2	77	29.8	Naming the initial point of the given ray
17	For $\overrightarrow{PQ}$, the ray extends infinitely towards _____. (P, Q, nowhere)	170	65.9	88	34.1	Recognizing the point towards which the ray is extended infinitely
18	$\overline{AB}$ _____ $\overrightarrow{AB}$. ($\in$, =, $\subset$)	162	62.8	96	37.2	Expressing the relation as line-segment is a subset of a line in a given situation

19	$\overline{AB} \cup \{P/A\text{-}B\text{-}P\}$ = _____ . ($\overrightarrow{PB}$, $\overrightarrow{BP}$, $\overrightarrow{AB}$)	157	60.9	101	39.1	Recalling the set representation of a ray
20	For E-D-F, _____ will be the opposite rays. ($\overrightarrow{ED}$ & $\overrightarrow{DF}$, $\overrightarrow{FD}$ & $\overrightarrow{DE}$, $\overrightarrow{DE}$ & $\overrightarrow{DF}$)	91	35.3	167	64.7	Locating the two opposite rays for the given betweenness of the three points
21	A _____ has a bisector. (line, line-segment, ray)	85	32.9	173	67.1	Recollecting that a line-segment has a bisector
22	If MN=PQ, then $\overline{MN}$ ___ $\overline{PQ}$. (=, ⊂, ≅)	165	64.0	93	36.0	Indicating that the given line-segments with same length are congruent
23	Three non-collinear points determine _____ plane. (more than two, two, one and only one)	200	77.5	58	22.5	Recalling the postulate that three non-collinear points determine one and only one plane
24	A plane α contains atleast _____ non-collinear points. (1, 2, 3)	159	61.6	99	38.4	Recognizing that given plane contains atleast three non-collinear points
25	If P & Q are points of plane α then $\overleftrightarrow{PQ}$ ___ α. (∈, =, ⊂)	87	33.7	171	66.3	Recollecting the postulate that a line passing through two distinct points of a plane is a subset of that plane
26	The intersection of two intersecting	131	50.8	127	49.2	Stating that the intersection of two

	distinct planes is a _____. (line, ray, plane)					intersecting distinct planes is a line
27	Let X_1 & X_2 be two half planes formed by line ℓ & plane α, then $X_1 \cap X_2 = $ _____. (X_1, ℓ, ϕ)	101	39.1	157	60.9	Inferring that the intersection of two half planes formed by a line is a null set
28	If A & B are in the same half plane made by ℓ then $\overline{AB} \cap \ell = $ ____. (A, ϕ, ℓ)	130	50.4	128	49.6	Interpreting that the intersection of the line and the line-segment formed by two distinct points in the same half plane formed by the line is an empty set
29	For points P & Q and a plane X, P≠Q & P, Q ∈ X; $X \cap \overrightarrow{PQ} = $ ____. ({P,Q}, Plane X, $\overrightarrow{PQ}$)	28	10.9	230	89.1	Indicating that the intersection of a line in a plane with the same plane is a line itself
30	If three non-collinear points A, B, C are in plane X as well as in plane Y, then _____. (X=Y, X≠Y, X⊂Y)	163	63.2	95	36.8	Recognizing that the two planes containing the same three non-collinear points are equal
31	For an angle $\angle ABC$ if A-O-C, then O is in the _____ of $\angle ABC$. (exterior, cross,	159	61.6	99	38.4	Identifying the point in the interior of an angle for the given situation

|						
	interior)					
32	If D is in the _____ of an angle $\angle ABC$, then $\overrightarrow{BD}$ intersects $\overline{AC}$. (interior, exterior, intersection)	118	45.7	140	54.3	Stating the cross-bar theorem " If D is in the interior of an angle $\angle ABC$, then $\overrightarrow{BD}$ intersects $\overline{AC}$."
33	An angle has _____ measure between 0 & 180. (exactly one, more than one, no)	129	50.0	129	50.0	Expressing that an angle has exactly one measure between 0 & 180
34	For point D in the interior of $\angle BAC$, $m\angle BAD +$_____ $=$ $m\angle BAC$. ($m\angle ACD$, $m\angle DAC$, $m\angle ADC$)	166	64.3	92	35.7	Recalling the postulate that "If a point D is in the interior of $\angle$ BAC, then $m\angle$ BAD + $m\angle$ DAC $= m\angle$ BAC."
35	An angle has _____ bisector. (one, two, no)	181	70.2	77	29.8	Stating that an angle has one bisector
36	If two congruent angles are supplementary, then each of them is a _____ angle. (right, acute, obtuse)	158	61.2	100	38.8	Inferring that the two congruent supplementary angles are right angles
37	If one angle of a pair of supplementary angles is acute then the other is _____	162	62.8	96	37.2	Identifying that the supplementary angle to the obtuse angle has to be an acute angle

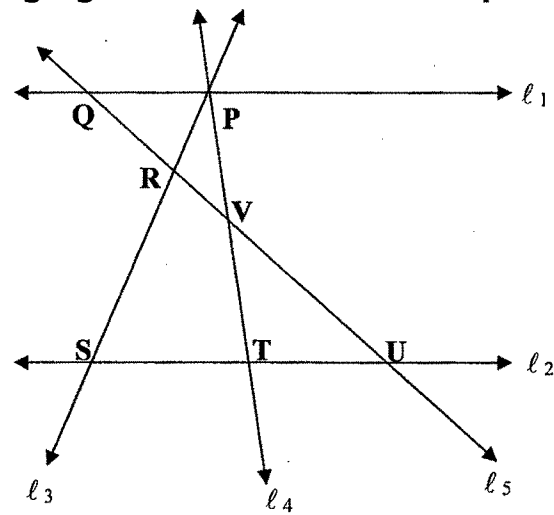|	angle. (acute, right, obtuse)				
38	The bisector of $\angle MON$ is $\overrightarrow{OP}$. If m$\angle MOP$ = 45°, then $\angle MOP$ is _____ to $\angle PON$. (supplementary, obtuse, complementary)	129	50.0	129	50.0	Discovering that for $\angle MON$ if $\overrightarrow{OP}$ is a bisector and m$\angle MOP$ = 45°, then $\angle MOP$ and $\angle PON$ will be a complementary pair of angles.
39	If S is in the interior of $\angle PQR$ then point P is in the exterior of _____. ($\angle PQS$, $\angle SQR$, $\angle SPQ$)	126	48.8	132	51.2	Extending that if S is in the interior of $\angle PQR$ then point P is in the exterior of $\angle SQR$
40	Each of the angles from a pair of complementary angles is _____. (obtuse, right, acute)	105	40.7	153	59.3	Recognizing that each of the angles from a pair of complementary angles is acute angle

Q.II [A]
Look at the following figure of the number-line and answer the respective questions:
Figure – 1
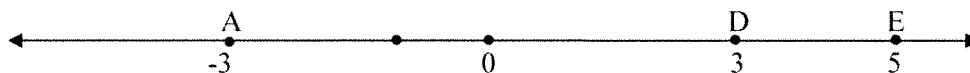
1	Is $\overline{AC} \cong \overline{CD}$? Ans. _____	137	53.1	121	46.9	Recognizing the two congruent line-segments on a number-line

101

#	Question					Description
2	What will be the number corresponding to the mid-point of $\overline{BD}$? Ans. _____	55	21.3	203	78.7	Finding the number corresponding to the mid-point of the line-segment on the number-line
3	What will be AE? Ans. _____	65	25.2	193	74.8	Calculating the length of the line segment on the number-line
4	For the point F, if A-F-E and AF=4, then what will be the number corresponding to F? Ans. _____	31	12.0	227	88.0	Computing the number corresponding to F for A-F-E on the number-line where AF and the numbers corresponding to A and E are given.
5	For C-O-P-E, if OP=2=PE, then what will be CO? Ans. _____	19	7.4	239	92.6	Computing the value of CO for C-O-P-E on the number line where OP=2=PE and the numbers corresponding to C and E are given.

Q.II [B]
**Look at the following figure and answer the respective questions:**
Figure – 2

1	What is $\ell_1 \cap \ell_2$? Ans. _____	110	42.6	148	57.4	Inferring based on the figure that the intersection of two parallel lines is a null set ($^\phi$)
2	Which are the points on line ℓ_4? Ans. _____	178	69.0	80	31.0	Recognizing in the figure all the points lying on the given line
3	Which four points are collinear? Ans. _____	133	51.6	125	48.4	Choosing the four collinear points in the given figure
4	Which are the lines that intersect in P? Ans. _____	141	54.7	117	45.3	Grouping the lines intersecting in a given point in the figure.
5	If PT=9.5 & PV=3, then what is VT? Ans. _____	137	53.1	121	46.9	Applying the property of distance " for P-V-T, PV+VT = PT" to find the value of VT based on the given figure
6	If T is a mid-point of $\overline{SU}$ & numbers corresponding to S & U on ℓ_2 are -6 & 7 respectively, then what is the number corresponding to T? Ans. _____	52	20.2	206	79.8	Computing the number corresponding to the mid-point of the line-segment in the figure based on the given numbers corresponding to the end-points of the line-segment
7	Are lines $\overleftrightarrow{QV}$ & $\overleftrightarrow{ST}$ intersecting? Ans. _____	57	22.1	201	77.9	Identifying based on the figure that the given lines are intersecting
8	What is the	121	46.9	137	53.1	Locating the point of

	intersection of $\overline{RV}$ & $\overline{UV}$? Ans. _____					intersection of two line-segments based on the given figure
9	What is $\overline{QV} \cap \overline{RU}$? Ans. _____	74	28.7	184	71.3	Identifying the line-segment which is the intersection of two given line-segments
10	What is $\overline{PV} \cap \overline{TU}$? Ans. _____	106	41.1	152	58.9	Recognizing based on the figure that the given line segments are not intersecting lines and the intersection is a null set (ϕ)
11	What is the intersection of $\overrightarrow{UQ}$ & $\overrightarrow{TS}$? Ans. _____	92	35.7	166	64.3	Inferring based on the figure that the intersection of given two rays is an empty set (ϕ)
12	What is the intersection of $\overrightarrow{UV}$ & $\overrightarrow{PQ}$? Ans. _____	22	8.5	236	91.5	Identifying the point of intersection of two rays in the given figure
13	Are $\overrightarrow{RQ}$ & $\overrightarrow{VU}$ opposite rays? Ans. _____	83	32.2	175	67.8	Recognizing based on the given figure that the given rays are opposite or not
14	What is the intersection of line ℓ_1 and $\overrightarrow{QP}$? Ans. _____	26	10.1	232	89.9	Inferring from the figure that the given lines are same and their intersection is the line itself

| 15 | What is $\overline{RS} \cap \ell_3$? Ans. _____ | 21 | 8.1 | 237 | 91.9 | Indicating based on the figure that the intersection of a line and a line-segment which is its subset is a line-segment itself. |

Q.II [C]

Look at the following figure and answer the respective questions:

Here α & β are parallel planes.

Figure – 3
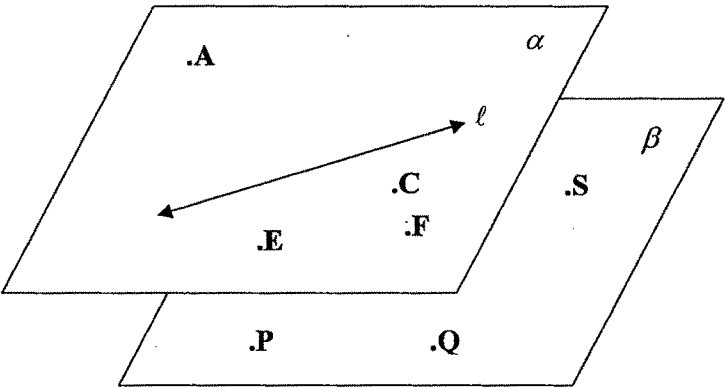
	Questions					Description
1	Which all points are in the plane α ? Ans. _____	169	65.5	89	34.5	Listing the points lying in the plane from the given figure
2	What is the intersection of the line ℓ and $\overline{PS}$? Ans. _____	134	51.9	124	48.1	Recognizing from the figure that the given lines are skew lines and their intersection is a null set
3	Of which plane is $\overleftrightarrow{PS}$ subset? Ans. _____	187	72.5	71	27.5	Naming the plane in the figure of which the given line is a subset.

No.	Question					Objective
4	Are $\overleftrightarrow{AF}$ and ℓ intersecting lines? Ans. _____	123	47.7	135	52.3	Identifying from the figure that the given lines are intersecting each other
5	$\overleftrightarrow{AC}\,\&\,\overrightarrow{PQ}$, are they coplanar or skew lines? Ans. _____	153	59.3	105	40.7	Identifying based on the figure whether the given two lines are coplanar or skew lines
6	Mention the points of α lying in the same half planes? Ans. _____	99	38.4	159	61.6	Locating from the figure the points lying in the same half-planes
7	Are $\overleftrightarrow{AE}$ & $\overrightarrow{QS}$ intersecting each other? Ans. _____	177	68.6	81	31.4	Identifying from the figure that the given lines are not intersecting each other
8	What is the intersection of planes α & β? Ans. _____	117	45.3	141	54.7	Inferring from the figure that the given two planes are not intersecting and their intersection is a null set
9	Is ℓ a subset of closed half plane of α? Ans. _____	161	62.4	97	37.6	Recognizing that the line ℓ is a subset of the closed half plane of α formed by the line ℓ
10	Are P, Q & S coplanar points? Ans. _____	192	74.4	66	25.6	Recognizing based on the figure that the given points are coplanar

Q.II [D]
Look at the following figure and answer the respective questions: **Figure – 4**
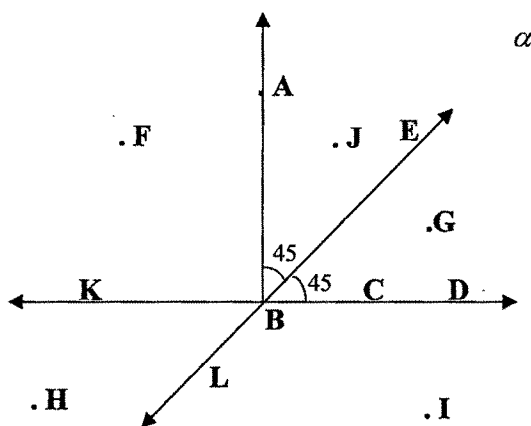
	Name the arms of					Naming the arms of the
1	∠ABC? Ans. _____	55	21.3	203	78.7	given angle from the figure
2	List the points in the exterior of ∠EBC? Ans. _____	80	31.0	178	69.0	Listing the points in the exterior of an angle from the figure
3	List the points in the interior of ∠ABD? Ans. _____	80	31.0	178	69.0	Listing the points in the interior of an angle from the figure
4	List the points on the ∠ABD? Ans. _____	114	44.2	144	55.8	Listing the points on the angle in the given figure
5	Which rays will intersect $\overline{AD}$? Ans. _____	48	18.6	210	81.4	Applying the cross-bar theorem to identify the rays intersecting the given line-segment in the figure
6	Are ∠ABC & ∠BCA same? Ans. _____	150	58.1	108	41.9	Identifying that the given angles are not equal

7	Are ∠ABE & ∠EBA same? Ans. _____	194	75.2	64	24.8	Identifying that the given angles are equal
8	Are ∠ABK & ∠KBL adjacent angles? Ans. _____	100	38.8	158	61.2	Recognizing that the given angles from the figure are adjacent angles
9	Which is the bisector of ∠ABD? Ans. _____	76	29.5	182	70.5	Locating the bisector of the given angle from the figure
10	Which is the complementary angle to an ∠ABE? Ans. _____	63	24.4	195	75.6	Finding the complementary angle to the given angle from the figure
11	Are ∠ABL & ∠ABE supplementary angles? Ans. _____	118	45.7	140	54.3	Identifying that the given angles in the figure are supplementary angles
12	Is ∠ABD & ∠ABL a linear pair of angles? Ans. _____	126	48.8	132	51.2	Recognizing that the given pair of angles in the figure are not forming linear pair of angles
13	Are ∠EBD & ∠KBL congruent angles? Ans. _____	176	68.2	82	31.8	Applying the property of vertically opposite angles that they are always congruent
14	Are ∠EBA & ∠LBD vertically opposite angles? Ans. _____	103	39.9	155	60.1	Identifying from the figure that the given angles are not vertically opposite angles
15	∠ABL is which type	121	46.9	137	53.1	Recognizing that the

	of angle? Ans. _____					given angle is an obtuse angle

Q.III: Represent each of the following by a figure:

1	$P \notin \overleftrightarrow{AB}$ but $Q \in \overrightarrow{BP}$.	107	41.5	151	58.5	Representing "$P \notin \overleftrightarrow{AB}$ but $Q \in \overrightarrow{BP}$" by a figure
2	$\overleftrightarrow{AB} = \overrightarrow{PQ}$, but $\overleftrightarrow{AB} \neq \overrightarrow{PR}$ & $S \in \overrightarrow{QR}$ & R-Q-S.	52	20.2	206	79.8	Representing "$\overleftrightarrow{AB} = \overrightarrow{PQ}$, but $\overleftrightarrow{AB} \neq \overrightarrow{PR}$ & $S \in \overrightarrow{QR}$ & R-Q-S" by a figure
3	A-B-C, C-D-E, A-F-E, D-G-A.	49	19.0	209	81.0	Representing "A-B-C, C-D-E, A-F-E, D-G-A" by a figure
4	$R \in \overrightarrow{PQ}$ & $S \notin \overrightarrow{PQ}$, $\overleftrightarrow{PQ} \cap \overleftrightarrow{RS} = \{R\}$.	93	36.0	165	64.0	Representing "$R \in \overrightarrow{PQ}$ & $S \notin \overrightarrow{PQ}$, $\overleftrightarrow{PQ} \cap \overleftrightarrow{RS} = \{R\}$" by a figure
5	For distinct lines ℓ_1, ℓ_2, ℓ_3; $\ell_1 \cap \ell_2 = \phi$ and $\ell_1 \cap \ell_3 = \{X\}$.	84	32.6	174	67.4	Representing "For distinct lines ℓ_1, ℓ_2, ℓ_3; $\ell_1 \cap \ell_2 = \phi$ and $\ell_1 \cap \ell_3 = \{X\}$" by a figure
6	$\overleftrightarrow{AB} = \overleftrightarrow{CD} \neq \overleftrightarrow{CE}$.	100	38.8	158	61.2	Representing "$\overleftrightarrow{AB} = \overleftrightarrow{CD} \neq \overleftrightarrow{CE}$" by a figure
7	X,Y and Z are collinear, ℓ is a line, $X \notin \ell$, $Y \in \ell$, $Z \notin \ell$.	116	45.0	142	55.0	Representing "X,Y and Z are collinear, ℓ is a line, $X \notin \ell$, $Y \in \ell$, $Z \notin \ell$" by a figure
8	P,Q,R & P,S,T are non-collinear triplets; but P,Q,S & P,R,T are	49	19.0	209	81.0	Representing "P,Q,R & P,S,T are non-collinear triplets; but P,Q,S &

	collinear points.					P,R,T are collinear points" by a figure
9	$\overrightarrow{PQ} \subset \overrightarrow{AB} \neq \overrightarrow{PR}$.	78	30.2	180	69.8	Representing "$\overrightarrow{PQ} \subset \overrightarrow{AB} \neq \overrightarrow{PR}$" by a figure
10	A, O, B are 3 non-collinear points and $\overrightarrow{AO} \cap \overrightarrow{OB} = \{O\}$.	18	7.0	240	93.0	Representing "A, O, B are 3 non-collinear points and $\overrightarrow{AO} \cap \overrightarrow{OB} = \{O\}$" by a figure
11	ℓ_1, ℓ_2 and ℓ_3 are three distinct lines and $\ell_1 \cap \ell_2 \cap \ell_3 = \{P\}$.	144	55.8	114	44.2	Representing "ℓ_1, ℓ_2 and ℓ_3 are three distinct lines and $\ell_1 \cap \ell_2 \cap \ell_3 = \{P\}$" by a figure
12	$A \notin \overrightarrow{PQ}$ but $B \in \overrightarrow{AQ}$.	92	35.7	166	64.3	Representing "$A \notin \overrightarrow{PQ}$ but $B \in \overrightarrow{AQ}$" by a figure
13	X,Y and Z are non-collinear points and ℓ is a line, $X \in \ell$, $Y \in \ell$ and $Z \notin \ell$.	126	48.8	132	51.2	Representing "X,Y and Z are non-collinear points and ℓ is a line, $X \in \ell$, $Y \in \ell$ and $Z \notin \ell$" by a figure
14	$\overrightarrow{RQ} \subset \ell_1$ and $S \in \ell_1$, S-R-Q.	101	39.1	157	60.9	Representing "$\overrightarrow{RQ} \subset \ell_1$ and $S \in \ell_1$, S-R-Q" by a figure
15	$\overrightarrow{PQ} \cap \ell_1 \cap \ell_2 = \{P\}$; $Q \notin \ell_1$, $Q \notin \ell_2$.	56	21.7	202	78.3	Representing "$\overrightarrow{PQ} \cap \ell_1 \cap \ell_2 = \{P\}$; $Q \notin \ell_1$, $Q \notin \ell_2$" by a figure

4.5 <u>IDENTIFICATION OF ERROR</u>

The commonly occurring errors by the students in geometry were located and identified with the help of corroborative references at three different stages viz. the error identification table, observations based on the note-books and answer sheets of the students and informal talk with experienced teachers about their observations. The details regarding all are described as follows:

4.5.1 Based on Error Identification Table

The error identification table was studied by the investigator and the points regarding the identification of errors are summarized as below.

Out of hundred items in only ten items it was observed that more than seventy percentages of students responded correctly. All the items were either of knowledge or comprehension level. It was inferred that the students performed well in the following areas:

- The postulate of line "Every line has atleast two distinct points."
- Collinear points
- Mid-point of the line-segment
- Initial point of the ray
- Postulate of plane "Three non-collinear points determine one and only one plane"
- Bisector of an angle
- Line is a subset of plane
- Coplanar points
- Equality of angles

Out of hundred items in the achievement test in ninety items the performance of the students was not good. The performance on the items was categorized as satisfactory (between sixty to seventy percent), moderate (between fifty to sixty percent) and poor (below fifty percent). The description about the categorization of the performance of students based on the errors committed by the students in particular area of geometry is given below.

I. Students performed satisfactorily but still quite a few students committed errors in the following areas:

Equality of lines, Relationship between point and line, Collinear and non-collinear points, Congruent line-segments, Concept of ray, $\overline{AB} \subset \overrightarrow{AB}$, Equality of planes, Measure of an angle, Interior of an angle, pair of supplementary angles, coplanar points, Closed half plane, Vertically opposite angles are congruent.

II. Students performed moderately and many students committed errors in the following areas:

Concept of line, Betweeenness of points, End-points of line-segment, Intersection of two planes, two congruent line-segments on a number line, four collinear points, three lines intersecting in one point, property of distance " for P-V-T, PV+VT = PT", Skew lines, Not equal angles.

III. Students performed poorly and most of the students committed errors in the following areas:

Intersection of two lines, the property "PQ+QR $\geq$ PR" for three distinct points P,Q, and R, Opposite rays, Bisector of a line-segment, Line is a subset of plane, Intersection of two half planes, Cross-bar theorem, Supplementary and Complementary pair of angles, Partitions of plane by an angle, Concept of distance, parallel lines, Intersection of line-segments, Intersection of a line and a line-segment, Parallel planes, Concept of an angle, Interior and exterior of an angle, Vertically opposite angles, linear pair of angles, Types of angles, Relationship between point and line, Intersection of rays, Relationship between point, line, line-segment and ray, Intersection of ray and line.

The above listed are the weak spots identified where the students commonly commit errors in geometry.

Following table shows the average performance of the students for each question.

Table – 16

Question-wise Average Performance

Question No.	Title of the question	Topics	Average performance (%)	Average Error (%)
I	Fill in the blanks – Multiple Choice	Miscellaneous	55.3	44.7
II - A	Refer the figure and answer the questions	Number-line, distance, and betweenness	23.8	76.2
II - B	Refer the figure (based on) and answer the questions	Line, Line-segment and Ray	35.0	65.0
II - C	Refer the figure and answer the questions	Plane	58.6	41.4
II - D	Refer the figure and answer the questions	Angle	41.4	58.6
III	Represent the geometrical statements by appropriate figure	Miscellaneous	32.7	67.3

From the above table it was observed that the students committed maximum errors in the items based on the geometrical figure of the number-line, distance and betweenness. Also, it was seen that the students committed more errors in case of representing the given geometrical statements with the help of a figure as it involves all the fundamental concepts collectively.

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The average performance of the students at three different levels of objectives was as follows:

Table – 17
Average Performance at Three Different Levels of Objectives

Level of Objectives	Average performance (%)	Average Error (%)
Knowledge	55.3	44.7
Comprehension	46.4	53.6
Application	35.9	64.1

From the above table it is very clear that students committed maximum errors in case of the items related to application level and minimum errors in case of items related to the knowledge level. The distribution of the errors committed by the students is represented in bar chart as follows:

Figure -1

Average performance and Average error at three levels of Objectives

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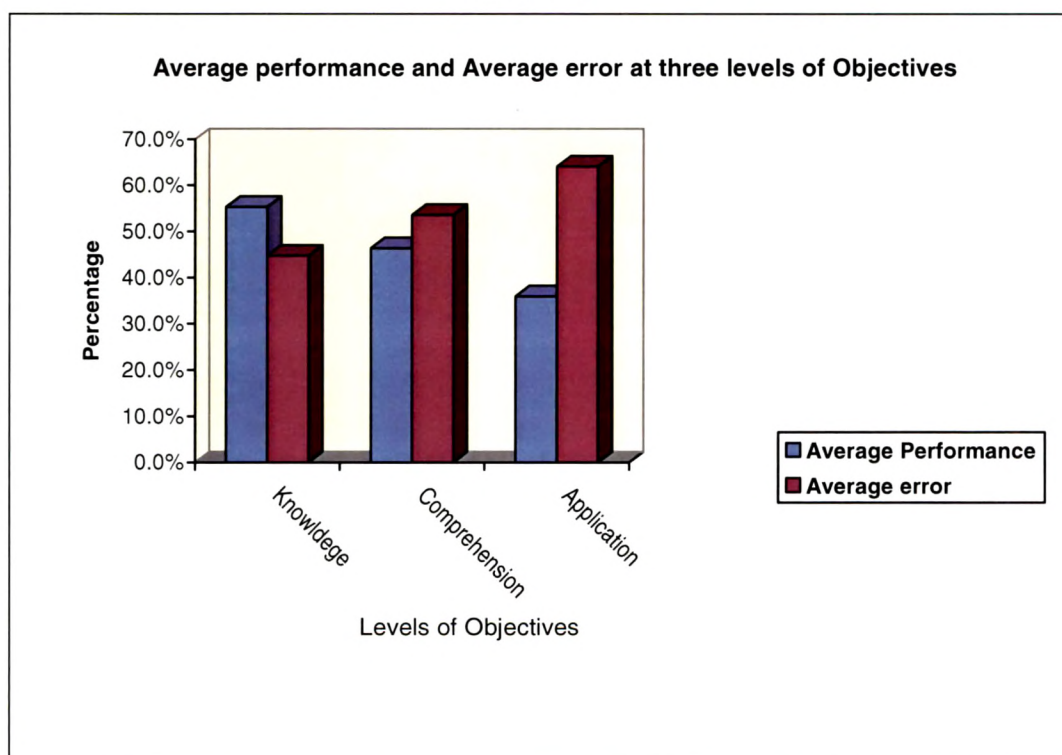
4.5.2 Based on Observations of the Note-books and Answer-sheets

The note-books of the standard VIII students and their answer sheets were observed and the observations were made by the investigator regarding the common errors committed by the students in geometry. The common errors committed by the students in geometry were as follows:

- Intersection of two parallel lines was represented as $\{\phi\}$
- The inappropriate use of the symbols $\in$, $\notin$, $\subset$, $=$, $\neq$ for the relationship between point, line, line-segment, ray, plane e.g. $X \subset \ell$, $\overline{AB} \in \overleftrightarrow{AB}$, $\ell \in \alpha$, $\overline{AB} \neq \overrightarrow{AB}$
- Naming more than one points in the plane by the same alphabet e.g. two points in the plane were named as 'S'
- Any two lines are coplanar lines
- Intersection of line and line-segment is a point (singleton set) e.g. $\overline{AB} \cap \overrightarrow{AB} = \{A\}$
- Length of line-segment was not found correctly e.g. for B=5 and D=9, BD=14
- The intersection of line segment based on the betweenness relation of the points was not correct e.g. For D-E-F-G-H, $\overline{DG} \cap \overline{GH} = \overline{DH}$, $\overline{DF} \cap \overline{EG} = \{F\}$
- For angle $\angle$ XYZ the arms are $\overrightarrow{XY}$ and $\overrightarrow{YZ}$
- For ray $\overrightarrow{AB}$ B is the initial point and A is the end point
- $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are same
- Opposite rays are the rays starting from the same point
- Ray has a mid-point
- Intersection of two rays i.e. $\overrightarrow{AB} \cap \overrightarrow{BD} = \phi$
- Consider the figure to the right –
  For this figure intersection of two parallel planes is all the points in the plane i.e. $X \cap Y = \{A,B,C,D\}$
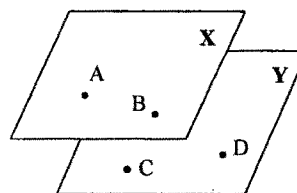
- In the above figure $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are coplanar lines
- In the above figure A,B,C,D are coplanar points
- In the above figure $\overrightarrow{CD} \in Y, B \subset X$
- Points on the angles are in the interior of an angle
- $\angle$ ABC and $\angle$ CBA are not equal
- Vertically opposite angles are not congruent
- In supplementary pair of angles both are acute angles
- The angles whose sum of the measures is 90 are supplementary angles

4.5.3 Based on Informal talk with the teacher

Investigator met two (Appendix J) experienced teachers of mathematics and conducted an informal talk with them regarding their observations on the common errors committed by the students in geometry of standard VIII. The list of their observations on the commonly observed errors is as below. Students committed errors in

- Stating point and line are undefined terms
- Identifying relationship between point and line, line-segment and line, line-segment and ray, ray and line
- Identifying parallel lines
- Finding intersection of lines, line-segments, rays in any combination
- Finding the length of line-segment
- Representing line, line-segment and ray in a set form
- Defining collinear points
- Identifying the opposite rays
- Using the appropriate geometrical symbols
- Interpreting the geometrical statements and drawing an appropriate figures
- Differentiating line, line-segment and ray
- Defining coplanar points and coplanar lines
- Identifying the half planes formed by the line
- Distinguishing between half plane and closed half plane

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- Stating the partitions of plane formed by an angle
- Listing the points in the interior and exterior of an angle
- Identifying and naming the arms of an angle
- Identifying the types of angles viz. Acute angle, Obtuse angle and Right angle
- Differentiating between Complementary pairs of angles and Supplementary angles
- Identifying the interrelation between different types of pair of angles viz. Adjacent angles, Linear angles, Congruent angles, Vertically opposite angles, Complementary angles and supplementary angles (Linear pair of angles are always supplementary angles, Vertically opposite angles are always congruent angles, Linear pair of angles are always adjacent angles, Supplementary angles are not always linear pair of angles, etc.)

Based on the identified errors as above the diagnosis was carried out by the investigator.

4.6 DIAGNOSIS OF LEARNING DIFFICULTIES

The above analysis provided the basis for the Diagnosis of the learning difficulties. Diagnosis is a process concerned with the students' persistent or recurring learning difficulties that are left unresolved during the classroom teaching. The purpose of diagnosis is to find the cause of student's learning difficulties so that the subsequent remediation can be directed at removing the cause.

4.6.1 Administration of Diagnostic Test

By this stage the investigator carried out the analysis of the responses of the students on the Achievement test and identified the commonly occurring errors in geometry. It was at this stage where the diagnostic test was constructed keeping in mind the errors identified. Achievement test's scores and performance provided the cues which must be followed up by further

study and observation. It provided the direction for framing the items in the diagnostic test for the further diagnosis of the learning difficulties.

The details about the construction of the diagnostic test are described in the previous chapter. In brief it included eleven major questions and one hundred and sixty-two items. The scoring scheme was not intended as it was designed with the purpose of critically confirming the errors committed by the students in geometry. It revealed in a comprehensive way, the probable causes for the errors committed by the students and the precise nature of the difficulties faced by the students in attempting the item correctly.

The diagnostic test was administered on the experimental group identified with the lowest mean score on the Achievement test. The diagnostic test was administered by the investigator on thirty-five students as two students were absent and did not appear for the diagnostic test. There was no time limit kept for the students to complete the test as the focus was not on measuring their performance but to locate the learning difficulties. It took two and half hours for the students to complete the diagnostic test.

4.6.2 Item-wise Analysis of the Diagnostic Test

The responses obtained from the students of experimental group on the diagnostic test were analyzed and studied in detail by the investigator. The analysis was carried out with the help of error analysis sheet for each item of the diagnostic test with the purpose of identifying the learning difficulties. The error analysis sheet included the following parameters the item, correct response, incorrect responses and the probable causes for the incorrect responses. Each item was studied with respect to these parameters and is represented in a tabular form followed by the description wherever required as follows:

Q.I. Group the following as defined & undefined terms:

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Point, Line, Line-Segment, Ray

Terms	Incorrect Answers		Correct Answers
	Defined	Unanswered	Undefined
Line	16	04	15
Point	14	05	16
	Undefined	Unanswered	Defined
Line-Segment	16	03	16
Ray	17	04	14

Probable Causes for the errors committed by the students in the above question were as follows:

Students were not clear about

- Line is an Undefined term
- Point is an Undefined term
- Line-Segment is a defined term
- Ray is a defined term
- "Defined term " & "Undefined term"

Q.II. Define the following terms:

1. Collinear Points

Correct Answers		No. of
- Points lying on the same line are called Collinear Points.		Students
- If there is a line passing through the points simultaneously then the points are collinear.		11
Incorrect Answer	Probable Cause	
- The points which lie on a line are collinear points.	- Each point lies on some line but mention of "same line" is important for collinear points was not understood	07
- The points which belongs from the same line are collinear	- The use of appropriate words (belongs from & belongs to) was not clear	08

points.		
- Collinear points are the points which are in same straight line shape.	- The difference between line & straight line shape was not clear	05
- Unanswered		04

2. Non-Collinear Points

Correct Answers		No. of
- Points not lying on the same line are called non-collinear points. - Points that are not collinear are called non-collinear points. - There does not exist any line passing through the points simultaneously then the points are non-collinear.		Students
		15
Incorrect Answer	Probable Cause	
- If there does not exists given points on the same line then they are non-collinear points.	- Definition of non-collienar points was memorized/crammed without understanding	06
- The points not lying in the same plane are non-collinear points.	- The difference between "non-collinear" & "non-coplanar" was not clear	04
- The points which are non-collinear.	- The concept of non-collinear points was not understood	05
- Unanswered		05

3. Co-planar Points

Correct Answer	No. of
- Points lying in the same plane are called co-planar	Students

points.		06
Incorrect Answers	Probable Cause	
- If there exists given points on same line then they are co-planar points.	- Difference between collinear points & co-planar points was not clear	06
- Points which are in Plane are coplanar points.	- All points are in some plane but they should be in "same plane" was missing	07
- Points have same line & same initial point are coplanar.	- There was a confusion between the definition of ray & co-planar points	05
- If there exists one & only one point or line then the points are coplanar.	- There was confusion between Point, line, Plane & their relation	05
- The points which are in one box are called coplanar points.	- The representation of plane was not related with the understanding of the concept of coplanar points	03
- Unanswered		03

4. Non-coplanar points

Correct Answers		No. of Students
- Points that are not co-planar are non-coplanar points.		
- Points that are not lying on the same plane are non-coplanar points.		10
Incorrect Answers	Probable Cause	
- The points which are non-collinear are non-coplanar points	- Difference between non-collinear & non-coplanar points was not understood	07
- If there exists points	- The concept of plane was not	08

not in any plane are non-coplanar points	understood	
- If there does not exist one & only one plane passing through the points then they are non-coplanar points	- The postulates of plane were not understood	06
- Unanswered		04

5. Opposite Rays

Correct Answers		No. of
- Rays having same initial point lying in the same line & extended in the opposite direction.		Students
		01
- For A-O-B, $\overrightarrow{OA}$ & $\overrightarrow{OB}$ are opposite rays.		
Incorrect Answers	Probable Cause	
- The ray which is opposite to the given ray is called opposite ray	- Both the rays are opposite to each other was not clear	07
- The rays which are different & in different direction are opposite rays	- Difference between "Distinct rays" & "Opposite rays" was not understood	07
- The rays lying in same line & same initial point are called opposite rays	- The condition of "Opposite direction" for the opposite rays was not understood	05
- The rays lying in same line but are in opposite direction are	- The condition of "having same initial point" for the opposite rays was not clear	06

opposite rays		
- The rays which have same initial point & end points are called opposite rays	- Rays do not have end point was not understood	04
- Unanswered	.	05

6. Define Angle

Correct Answer		No. of
- Union of two rays, with the common initial point & which are not on the same line is called an Angle.		Students
		05
Incorrect Answers	Probable Cause	
- It is an intersecting lines	- Angle is made up of rays & not lines was not understood	05
- Angle is union of two rays not lying in same line	- Two rays should have same initial point was not considered	07
- Union of two rays having same initial point is an angle	- Two rays should not lie on same line was not understood	08
- Union of two lines meeting at the same point	- Angle is made up of rays & not lines was not understood	06
Unanswered		04

Q.III Observe the figure & select the appropriate symbol to make the statement correct.
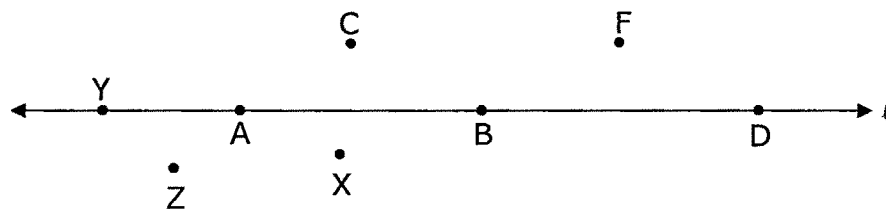
Sr. no.	Correct Answer – No. of students	Incorrect Answer – No. of Students				Unanswered
1.	$X \notin \ell$ - 19	$\in$ - 6	$\subset$ - 2	= - 3	$\neq$ - 4	1
2.	$Y \in \ell$ - 0	$\notin$ - 2	$\subset$ - 4	= - 2	$\neq$ - 5	2
3.	$A \in \ell$ - 6	$\notin$ - 5	$\subset$ - 6	= - 4	$\neq$ - 2	2
4.	$F \notin \ell$ - 5	$\in$ - 2	$\subset$ - 5	= - 3	$\neq$ - 6	4
5.	$\overline{YB} \subset \ell$ - 10	$\in$ - 7	$\notin$ - 2	= - 8	$\neq$ - 4	4
6.	$\overline{AB} \subset \ell$ - 11	$\in$ - 9	$\notin$ - 3	= - 7	$\neq$ - 5	0
7.	$\overleftrightarrow{BD} = \ell$ - 7	$\in$ - 10	$\notin$ - 5	$\subset$ - 8	$\neq$ - 0	5
8.	$\overleftrightarrow{AB} = \ell$ - 4	$\in$ - 6	$\notin$ - 8	$\subset$ - 7	$\neq$ - 5	5
9.	$\overrightarrow{BD} \subset \overrightarrow{AB}$ - 4	$\in$ - 3	$\notin$ - 4	= - 8	$\neq$ - 11	5
10.	$B \in \overrightarrow{YA}$ - 6	$\notin$ - 8	$\subset$ - 3	= - 10	$\neq$ - 3	5
11.	$\overline{AB} \subset \overrightarrow{AB}$ - 8	$\in$ - 0	$\notin$ - 3	= - 11	$\neq$ - 8	5
12.	$\overleftrightarrow{XZ} \neq \ell$ - 6	$\in$ - 3	$\notin$ - 17	$\subset$ - 3	= - 3	4
13.	$\overleftrightarrow{YB} = \ell$ - 6	$\in$ - 11	$\notin$ - 4	$\subset$ - 7	$\neq$ - 3	4
14.	$C \notin \overleftrightarrow{AB}$ - 14	$\in$ - 2	$\subset$ - 3	= - 1	$\neq$ - 10	5
15.	$A \in \overleftrightarrow{AD}$ - 8	$\in$ - 8	$\subset$ - 6	= - 4	$\neq$ - 4	5
16.	$X \in \overrightarrow{ZX}$ - 7	$\notin$ - 7	$\subset$ - 4	= - 5	$\neq$ - 8	4
17.	$Z \neq C$ - 7	$\in$ - 2	$\notin$ - 10	$\subset$ - 4	= - 7	5
18.	$\overline{BD} \subset \ell$ - 8	$\in$ - 10	$\notin$ - 2	= - 7	$\neq$ - 3	5

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No.	Statement					
19.	$\overline{BD} \subset \overline{BD}$ - 10	$\in$ - 3	$\notin$ - 4	= - 9	$\neq$ - 5	4
20.	$Z \notin \overrightarrow{YD}$ - 12	$\in$ - 4	$\subset$ - 3	= - 3	$\neq$ - 8	5
21.	$\overrightarrow{AB} \subset \ell$ - 9	$\in$ - 9	$\notin$ - 7	= - 2	$\neq$ - 4	4
22.	$\overrightarrow{AB} = \overrightarrow{AD}$ - 9	$\in$ - 5	$\notin$ - 4	$\subset$ - 6	$\neq$ - 5	6
23.	$D \in \overrightarrow{AB}$ - 4	$\notin$ - 7	$\subset$ - 6	= - 4	$\neq$ - 7	7
24.	$A \in \overrightarrow{AD}$ - 9	$\notin$ - 2	$\subset$ - 7	= - 5	$\neq$ - 6	6
25.	$A \notin \overrightarrow{BD}$ - 7	$\in$ - 8	$\subset$ - 4	= - 6	$\neq$ - 5	5
26.	$\overrightarrow{AB} \neq \overrightarrow{BA}$ - 3	$\in$ - 4	$\notin$ - 5	$\subset$ - 5	= - 13	5
27.	$\overrightarrow{AB} \subset \overrightarrow{AB}$ - 9	$\in$ - 4	$\notin$ - 2	= - 7	$\neq$ - 9	4
28.	$\overline{BD} \subset \overline{AB}$ - 4	$\in$ - 3	$\notin$ - 7	= - 6	$\neq$ - 9	6
29.	$\overrightarrow{AD} \neq \overrightarrow{BD}$ - 6	$\in$ - 7	$\notin$ - 4	$\subset$ - 5	= - 8	5
30.	$\overrightarrow{BA} = \overrightarrow{BY}$ - 4	$\in$ - 2	$\notin$ - 8	$\subset$ - 9	$\neq$ - 8	4
31.	$\overrightarrow{DA} \neq \overrightarrow{BA}$ - 6	$\in$ - 4	$\notin$ - 5	$\subset$ - 8	= - 7	5
32.	$\overline{BD} \subset \overrightarrow{AD}$ - 7	$\in$ - 8	$\notin$ - 3	= - 5	$\neq$ - 6	6

Probable Causes for the above errors committed by the students are as follows:

Students had
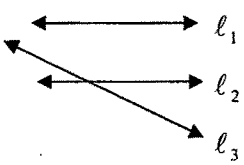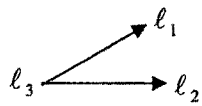
- No clarity about the correct use of $\in$, $\notin$, =, $\neq$, $\subset$
- Not understood the difference between $\notin$ & $\neq$
- Not understood the meaning of $\in$, $\notin$, =, $\neq$, $\subset$ symbols
- Confusion between the symbols $\in$, =, $\subset$
- Not understood that line-segment cannot be equal to line
- Not understood the equality of 2 rays
- Not understood the equality of 2 lines was not understood
- Line is extended infinitely on both the sides was not clear
- Ray is extended infinitely was not understood
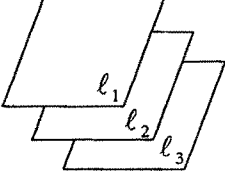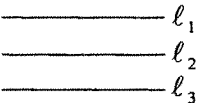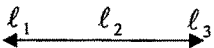
Also some of the probable causes of the above errors were found in the form of the misconceptions held by the students as follows:

- When point is on the line, then point was considered as a subset of line
- When point is not on line, then point was considered as not equal to line
- Rays with same initial point towards two distinct points in the same direction from the initial point are considered to be not equal rays
- Rays with different initial points but extended towards the same point were considered as same rays
- For two points A & B on the line ℓ, $\overline{AB}$ was considered as $\overline{AB} \in \ell$
- For two points A & B on the line ℓ, $\overline{AB}$ was considered as $\subset$ of ℓ
- For two points A and B, $\overrightarrow{AB} = \overrightarrow{BA}$ was considered
- For B-A-Y, $\overline{BA} \neq \overline{BY}$ was considered
- For B-A-Y, $\overline{BA} \subset \overline{BY}$ was considered
- For D-B-A, $\overrightarrow{DA} = \overrightarrow{BA}$ was considered
- For Y-A-B, $B \notin \overrightarrow{YA}$ was considered
- For A-B-D, $D \notin \overrightarrow{AB}$ was considered
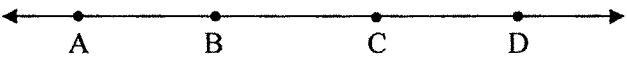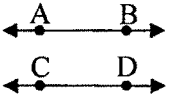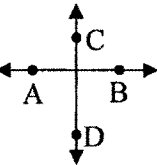
Q.IV Draw a figure representing the following situations.

1. Three distinct lines ℓ_1, ℓ_2 & ℓ_3

Correct Answer		No. of Students
		13
Incorrect Answers	Probable Cause	
	- Representation of line was not clear	04
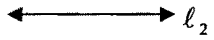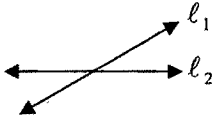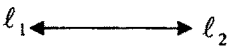
	- Difference between line & plane was not clear	03
	- Difference between line and line-segment was not understood	05
	- Concept of distinct lines was not clear	06
Unanswered		04

2. $\overrightarrow{AB} = \overleftarrow{CD}$
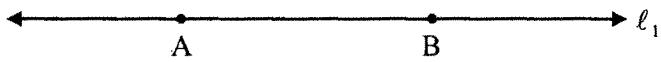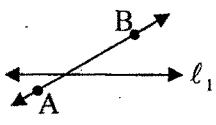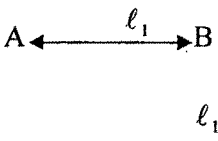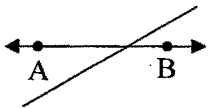
Correct Answer	No. of Students
	12

Incorrect Answers	Probable Cause	
	- Difference between parallel lines and equal lines was not clear	05
	- Representation of line & its figure was not understood	07
	- Difference between distinct intersecting lines & equal lines was not clear	05
Unanswered		04

3. $\ell_1 \cap \ell_2 = \phi$
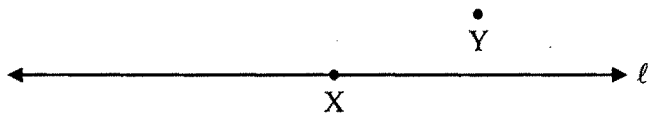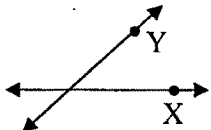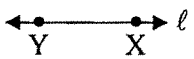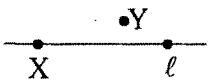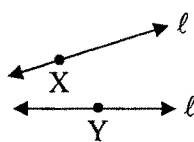
Correct Answer	No. of Students
$\longleftrightarrow \ell_1$	10

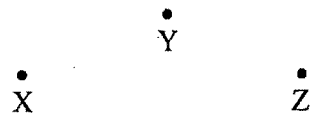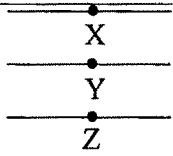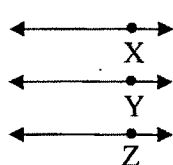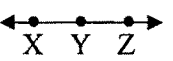|		
← —————— → ℓ_2		
Incorrect Answers	**Probable Cause**	
ℓ_1 ℓ_2	- The concept of parallel lines wais not clear - The meaning of ϕ was not understood	11
ℓ_1 ←————→ ℓ_2	- Difference between parallel / not intersecting lines & equal lines was not clear - The intersection of two equal lines is not empty set was not understood	09
Unanswered		05

4. $\ell_1 = \overleftrightarrow{AB}$

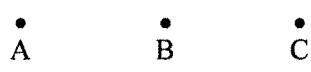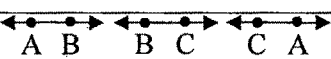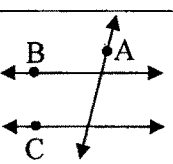| Correct Answer		No. of Students
←——•————•———→ ℓ_1 A B		15
Incorrect Answers	**Probable Cause**	
B, A ℓ_1	- Difference between equal lines and intersecting lines was not clear	04
A ←—ℓ_1—→ B ℓ_1	- In $\overleftrightarrow{AB}$, A & B are points on line & its representation in the figure was not understood	07
A B	- "ℓ_1" represents line & not line-segment was not understood - Difference between equal lines & intersecting lines was not clear	05
Unanswered		05

5. X∈ ℓ & Y∉ ℓ
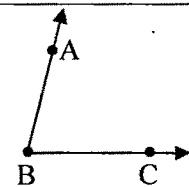
Correct Answer		No. of Students
		10
Incorrect Answers	Probable Cause	
	- The line ℓ needs to mentioned in the geometrical representation was not clear	04
	- Difference between "∈ - belongs to" & "∉ - does not belong to" was not understood	05
	-'ℓ' represents a line was not understood	05
	- There can be only one line represented as 'ℓ' was not clear	06
Unanswered		05

6. X,Y,Z are distinct non-collinear points

Correct Answer		No. of Students
		13
Incorrect Answers	Probable Cause	
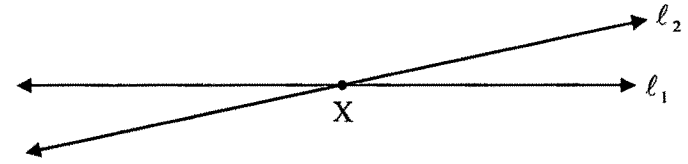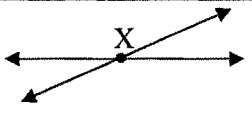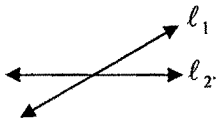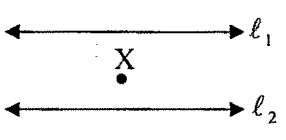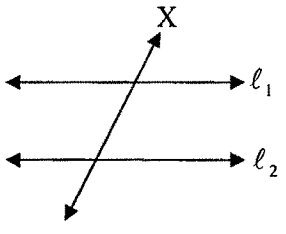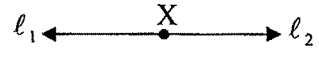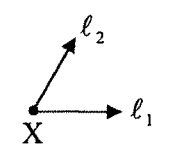
	- Three distinct points lying on three distinct lines may be collinear was not understood	04
		06
	- Difference between collinear points and non-collinear points was not understood	06
Unanswered		06

7. A,B,C are three distinct collinear points

Correct Answer		No. of Students
		15
Incorrect Answers	Probable Cause	
	- There is only one point represented by 'A', 'B' , 'C' in a plane or on a line was not clear	05
	- Three points lying on three distinct lines are collinear was the misconception	06
	- Three distinct points were focused but the concept of collinear points was not understood	04
Unanswered		05
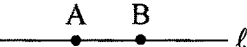
8. $\ell_2 \cap \ell_1 = \{X\}$

Correct Answer	No. of Students
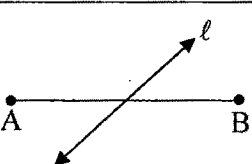
		09
Incorrect Answers	Probable Cause	
	- Naming the lines was not considered significant & was not clear	03
	- Point of intersection was not represented so "{X}" was not interpreted as point of intersection	02
	- Point lying between two parallel lines is the point of intersection was the misconception - Two parallel lines do not intersect was not clear	07
	- 'X' represents point and not line was not known - Intersection of two lines is either a point or an empty set was not understood	06
	- Two lines interesting in one point have to be distinct and not same was not understood	04
	- Difference between ray & line was not clear, "ℓ_1" represents line & not ray was not known	03
Unanswered		01

9. $\overline{AB} \subset \ell$

Correct Answer	No. of Students
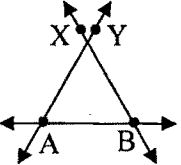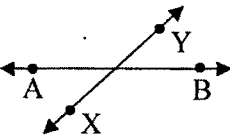
		12
←———————•———————•————————→ ℓ A · · · B		
Incorrect Answers	Probable Cause	
———•———•——— ℓ A B	- 'ℓ' represents line & not line-segment was not clear	06
•————————• ℓ A B	- Meaning of '⊂' & 'ℓ' was not understood	08
←————————→ ℓ A B	- The representation of points on the line & in $\overline{AB}$, A & B are points on line was not clear	05
Unanswered		04

10. $\overline{AB} \cap \ell = \overline{AB}$
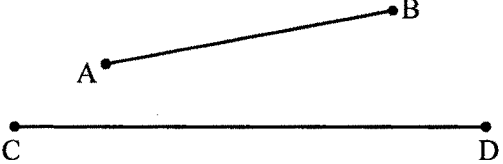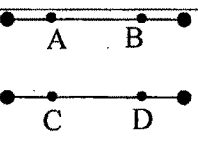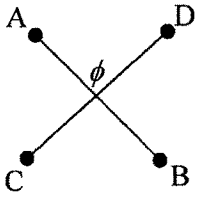
Correct Answer		No. of Students
←———————•———————•————————→ ℓ A · · · B		05
Incorrect Answers	Probable Cause	
———•———•——— ℓ A B	- 'ℓ' is a line & not line-segment was not clear	08
•————————• ℓ A B (ℓ crossing)	- If the intersection is a line-segment then '$\overline{AB}$' has to be subset of line 'ℓ' was not understood	09
•————————•ℓ A B	- Could not conclude that $\overline{AB} \subset \ell$ & 'ℓ' is a line. $\overline{AB}$ & ℓ are not same was not clear	07
Unanswered		06
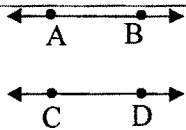
11. $\overleftrightarrow{AB} \cap \overleftrightarrow{XY} = \overrightarrow{AB}$

Correct Answer	No. of Students
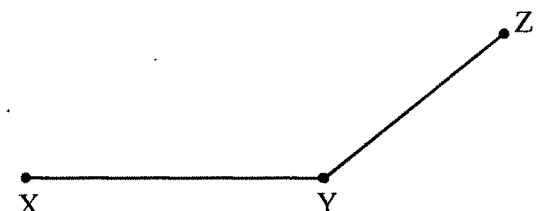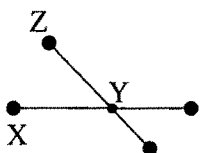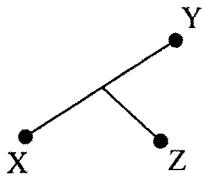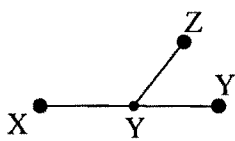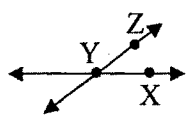
		06
A X Y B		
Incorrect Answers	Probable Cause	
	- $\overleftrightarrow{XY}$ represent the same line having points X and Y was not clear - The intersection is part in between two lines was a misconception	10
	- If intersection of two lines is a line, then relative points are on the same line was not understood	14
Unanswered		05

12. $\overline{AB} \cap \overline{CD} = \phi$
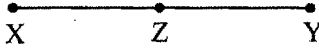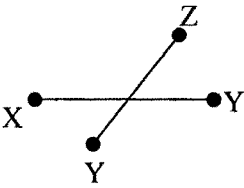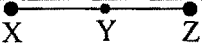
Correct Answer		No. of Students
		08
Incorrect Answers	Probable Cause	
	- In $\overline{AB}$, A & B are end-points of the line-segment was not clear	07
	- 'ϕ' cannot be the point of intersection but it represents empty set i.e. The line-segments are not intersecting was not understood	08

A B C D	- $\overline{AB}$ & $\overline{CD}$ are a line-segments & not lines was not clear	06
Unanswered		06

13. $\overline{XY} \cap \overline{YZ} = \{Y\}$
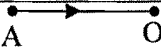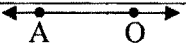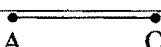
Correct Answer		No. of Students
		06
Incorrect Answers	Probable Cause	
	- No clarity about representation of line-segments. - In '$\overline{XY}$', X & Y are end-points of the line-segment was not clear.	05
	- Y is the point of intersection was not understood	04
	- 'Y' is only one point in a plane & is common to both the line-segments was not understood	06
	- Difference between the representation of line and line-segment was not clear i.e. $\overline{XY}$ represents line-segment & not line was not clear	08
Unanswered		06

14. $\overline{XY} \cap \overline{YZ} = \overline{YZ}$
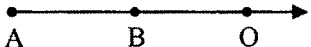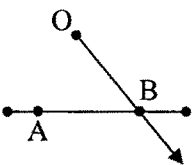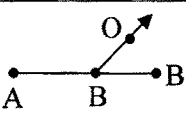
Correct Answer		No. of Students
X——•——Z——•——Y		00
Incorrect Answers	**Probable Cause**	
(diagram: X, Y, Z crossing segments)	- There can be only one point named as 'Y' in a plane was not clear - Intersection of two line-segments is a line-segment was not understood and in that case all points are collinear was not clear	11
X——Y——Z	- Here {Y} is the intersection & not $\overline{YZ}$ was not understood	11
Y——X——Z	- Here $\overline{XY}$ is the intersection & not $\overline{YZ}$; this difference was not clear	08
Unanswered		05

15. $\overrightarrow{AO}$
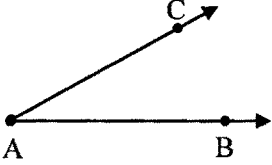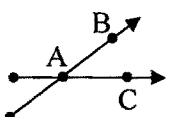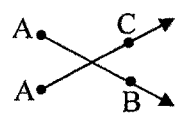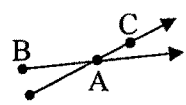
Correct Answer		No. of Students
•———•———▶ A O		10
Incorrect Answers	**Probable Cause**	
•———•——▶ A O	- In $\overrightarrow{AO}$, A is an initial point /	05

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	end point was not understood	
A O	- The geometrical figure of a ray was not clear	05
AO	- In $\overrightarrow{AO}$, A & O are points on the ray was not understood	04
A O	- Difference between ray & line was not clear	04
A O	- Difference between line-segment & line was not clear	05
Unanswered		02

16. $\overline{AB} \cap \overrightarrow{BO} = \{B\}$
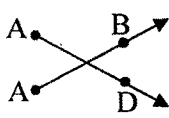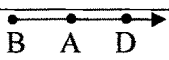
Correct Answer		No. of Students
A B O		07
Incorrect Answers	Probable Cause	
	- Concept of ray & the initial point of ray was not understood	06
	- There is only one point 'B' in the plane was not clear	07
	- $\overrightarrow{BO}$ is a ray & not line-segment was not clear	03
	- $\overline{AB}$ is a line-segment & not a line, the difference was not understood	06
Unanswered		06

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17. $\overrightarrow{AB} \cap \overrightarrow{AC} = \{A\}$
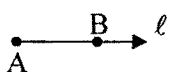
Correct Answer		No. of Students
A, B, C ray diagram with C up-right, A-B horizontal		05
Incorrect Answers	Probable Cause	
A B C on one ray	- The ray as a set of points is not understood. The concept of intersection of two rays is not clear	09
B, A, C diagram	- The figure representing ray is not understood where the concept of initial point of ray is not clear	05
crossing rays A, C, A, B	- There is only one point 'A' in the plane is not clear. Also, the concept of intersection of two sets (rays) is not understood	05
B, C, A diagram	- The difference between $\overrightarrow{AB}$ & $\overrightarrow{BA}$ is not understood	06
Unanswered		05

18. $\overrightarrow{AB} \cap \overrightarrow{AD} = \overrightarrow{AB}$

Correct Answer		No. of Students
A B D ray	A D B ray	03
Incorrect Answers	Probable Cause	
A, B, D diagram	- Intersection of two rays can be other than a point was not understood (other possibilities of	07
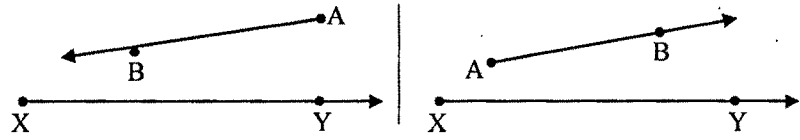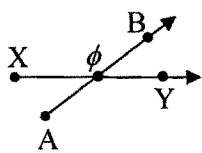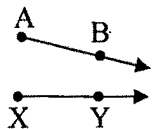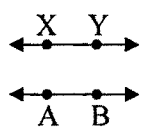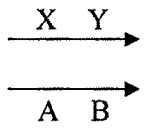
	∩ of two rays)	
B A D (rays)	-If intersection of two rays with the same initial point is a ray then they are same rays & are in same direction, was not clear	05
A, B, A, D (crossing rays)	- There is only one point 'A' in the plane & the intersection of rays is not always a point was not understood	07
B A D (ray)	- Difference between $\overrightarrow{AB}$ & $\overrightarrow{BA}$ was not clear	06
Unanswered		06

19. $\overrightarrow{AB} \subset \ell$

Correct Answer		No. of Students
◄────── A ────── B ──────► ℓ		09
Incorrect Answers	Probable Cause	
A ────── B ℓ	- Difference between line-segment & ray; 'ℓ' represents line & not line-segment; meaning of '⊂' was not understood	06
A ──B──► ℓ	- 'ℓ' represents line was not clear	15
Unanswered		05
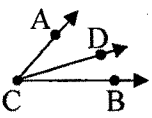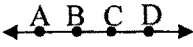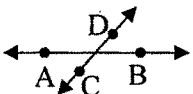
20. $\overrightarrow{XY} \cap \overrightarrow{AB} = \phi$

Correct Answer	No. of Students
	06

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Incorrect Answers	Probable Cause	
	- 'ϕ' represents empty set and not a pointy & hence two rays does not intersect was not understood	09
	- The concept of ray (its extended further on one side) was not clear	08
	- There was a confusion between concept of parallel lines & not intersecting rays. - Difference between figure of ray & line was not clear	04
	- Naming of points on ray was not clear (X & A are initial points)	04
Unanswered		04

21. A-C-D-B

Correct Answer	No. of Students
	12
Incorrect Answers	Probable Cause	

	- There was a misconception that in the betweenness relation of the points the in between point is initial point & others are rays w.r.t. that point	07
	- They are collinear but the betweeness of points was not clear	05
	- The betweenness relation represents intersecting lines was a misconception	04
Unanswered		06

Q.V Answer the following questions based on the figure below?
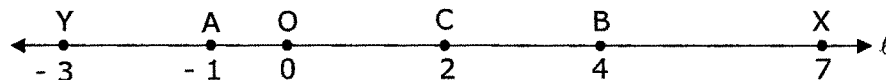
1. What is AB?

Correct Answer		No. of Students
5		
		07
Incorrect Answers	Probable Cause	No. of Students
-5	- AB represents Distance & is always +ve was not clear (-1-4 = -5)	08
3	- Calculated (4)-1 instead of 4-(-1) i.e. 3 instead of 5	09
- Line-Segment	- AB represents length of the line-segment	04
- Collinear	& its value is a number is not understood	02
Unanswered		04

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2. What is YC?

Correct Answer		No. of
5		Students
		09
Incorrect Answers	Probable Cause	
-1	- Calculated YC=C-Y=2-3; instead of |2-3| - No clarity that length is always +ve	09
- Line-Segment - Ray	- Not understood that CY represents the length of the line-segment & its value is a number	06 06
Unanswered		05

3. What is AX?

Correct Answer		No. of
8		Students
		06
Incorrect Answers	Probable Cause	
6	-Calculated (7-1) instead of 7-(-1)	09
-6	- Calculated (1-7) instead of 7-(-1) & do not know that length is non-negative number	08
- Line-Segment - Ray	- Not understood that AX represents the length of the line-segment & is a number	05 03
Unanswered		04

4. What is CX?

Correct Answer		No. of
5		Students
		08

Incorrect Answers	Probable Cause	
9	- Added the value of X & C i.e. 7+2; instead of 7-2	09
- Line-Segment	- Not understood that CX represents the	07
- Ray	length of the line-segment & is a number	06
Unanswered		05

5. What is AY?

Correct Answer		No. of
2		Students
		02
Incorrect Answers	Probable Cause	
-4	- Added the value of Y & A i.e. (-3)+(-1) instead of 7-2	07
4	-Considered the positive value after adding the value of Y & A instead of subtracting it i.e. $|(-3)+(-1)|$, instead of $|(-3)-(-1)|$	08
-2	- Not taken the modulus & just subtracted the value of A from Y i.e. (-3)- (-1) =-2. No clarity that the value (length) is always non-negative	06
- Line-Segment	- Not understood that AY represents the length of the line-segment & is a number	06
Unanswered		05

6. Which are the points in the positive direction of line ℓ?

Correct Answer	No. of
C,B,X	Students

	10
Incorrect Answers	Probable Cause	
O,C,B,X	- O is called the origin point & is not in any direction was not clear	11
2,4,7	- They are values associated to the points in positive direction was not understood	04
Y,A	- They are the points in the negative direction was not understood	02
C,B	- 'X' is also a point in the positive direction & is to be considered was not clear	02
Unanswered		06

7. Which are the points in the negative direction of line ℓ ?

Correct Answer	No. of
A,Y	Students
	09
Incorrect Answers	Probable Cause	
Y,A,O	- O is called the origin point & is not in any direction was not clear	10
-3,-1	- They are values associated to the points in negative direction was not understood	05
-3,-1,0	- 0 is a value associated to the origin point & is not in any direction is not clear - They are values associated to the points in negative direction was not understood	04
Unanswered		06

8. Which is the origin of line ℓ ?

Correct Answer	No. of

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O		Students
		11
Incorrect Answers	Probable Cause	
Y	- The left most point on the line was considered	06
C	- The point which tentatively seemed to be at the centre of the line was considered. The point whose value is '0' is called origin was not clear	08
X	- The right most point on the line was considered	05
Unanswered		05

9. Which is the mid-point of $\overline{OB}$?

Correct Answer C		No. of Students
		16
Incorrect Answers	Probable Cause	
B	- The difference between mid-point & end-point was not clear	04
O	- The difference between mid-point & end-point is not clear	05
2	- It's not the mid-point but a number associated to the mid-point of $\overline{OB}$	04
Unanswered		06

10. What will be the number corresponding to the mid-point of $\overline{CX}$?

Correct Answer	No. of

4.5		Students
		04
Incorrect Answers	Probable Cause	
4	- It's the value associated to the point given in between C & X but is not a mid-point. The mid-point was to be found was not understood	11
B	- It's the point in between C & X but the mid-point was to be found was not clear	08
5	- It was the value estimated for the mid-point based on the figure (i.e. point after B(4) but was not exactly calculated	04
Unanswered		06

11. What will be the no. corresponding to the mid-point of $\overline{CY}$?

Correct Answer		No. of
-0.5		Students
		02
Incorrect Answers	Probable Cause	
-1	- It's the value associated to the point given in between C & Y but is not mid-point. The mid-point was to be found was not understood	06
0	- It's the value associated to the point given in between C & Y but is not mid-point. The mid-point was to be found was not understood	08
A,O	- It's the point in between C & Y but the mid-point was to be found was not clear	06
3.5/2.5/5	- Not aware that the mid-point has to be in between C & Y which divides the line-	04

	segment in two equal parts (equal length)	
-1,0	- Not clear that there can be only one mid-point for the given line-segment	03
Unanswered		06

12. Which are the congruent line-segments to $\overline{YA}$?

Correct Answer $\overline{OC}$, $\overline{CB}$		No. of Students
		02
Incorrect Answers	Probable Cause	
$\overline{CB}$	- $\overline{OC}$ is also congruent which was not considered	08
$\overline{BX}$	- It seemed to be of equal length (estimated) but was actually of 3 units & not 2 units ($\overline{YA}$)	09
OC,CB	- It represents lengths of $\overline{OC}$ & $\overline{CB}$ respectively but are not line-segments was not understood. The difference between OC & $\overline{OC}$ is not clear	09
Unanswered		07

13. Which is the congruent line-segment to $\overline{AC}$?

Correct Answer $\overline{YO}$ / $\overline{BX}$		No. of Students
		03
Incorrect Answers	Probable Cause	
$\overline{YA}$	- Not clear about finding the length & comparing it with the length of $\overline{AC}$ (equal or not)	12
$\overline{CB}$		12
Unanswered		08

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14. Which point is equidistant from X & Y?

Correct Answer		No. of
C		Students
		04
Incorrect Answers	Probable Cause	
O	- Origin was considered to be equidistant point from the extreme points to the left & right	14
O,C	- There is only one point equidistant from the given two points was not understood	11
Unanswered		06

Q.VI Answer the following questions:

1. Represent $\overline{AB}$ in a set form?

Correct Answer		No. of
{A,B} $\cup$ {P$\in$ $\overrightarrow{AB}$ /A-P-B}		Students
		02
Incorrect Answers	Probable Cause	
{A,B}	- Not clear that line-segment is not just end-points but a set of all the points in between A & B including A & B	17
{A,B} $\cap$ {A-P-B}	- This will be null set was not understood. Its union of {A,B} & {A-P-B} and not intersection was not clear.	10
Unanswered		06

2. Represent $\overrightarrow{AB}$ in a set form?

Correct Answer	No. of

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$\overline{AB} \cup \{P \in \overrightarrow{AB}/A\text{-}B\text{-}P\}$		Students
		01
Incorrect Answers	Probable Cause	
{A,B}	- Not clear that ray is not just two points on the ray but a set of all the points in from A towards and beyond B including point A	14
{A,B} ∪ {A-P-B}	- The difference between the set representation of line-segment & ray was not understood	07
{ $\overrightarrow{AB}$ }	- The relation of points forming ray as a set was not clear	06
Unanswered		07

3. Line-segment has how many end-points?

Correct Answer		No. of
Two		Students
		17
Incorrect Answers	Probable Cause	
No end-points	- Not clear about end-points - Not understood the difference between line & line-segment	06
One end-point	- The difference between ray & line-segment was not clear	10
Unanswered		03

4. Line has how many end-points?

Correct Answer		No. of
Zero / No		Students
		12
Incorrect Answers	Probable Cause	
Two end-points	- Not clear about end-points & the difference	12

	between line & line-segment	
One end-point	- The difference between ray & line was not clear	07
Unanswered		04

5. Ray has how many end-points?

Correct Answer		No. of
One		Students
		22
Incorrect Answers	Probable Cause	
No end-points	- Difference between line & ray was not clear	05
Two end-points	- Difference between line-segment & ray was not clear	05
Unanswered		03

6. How many planes pass through one point?

Correct Answer		No. of
Infinitely many		Students
		08
Incorrect Answers	Probable Cause	
1	- Not understood that infinite planes pass through one point as the plane is	07
2	determined by 3 non-collinear points	07
3		04
4		04
Unanswered		05

7. How many lines pass through two distinct points?

Correct Answer		No. of
One & Only one		Students
		06
Incorrect Answers	Probable Cause	
2	- One & only one line passes through two	03
	distinct points as line is determined by two	
3	distinct points was not understood	07
4		08
Infinite		07
Unanswered		04

8. How many lines pass through one point?

Correct Answer		No. of
Infinitely many		Students
		04
Incorrect Answers	Probable Cause	
No	- Two distinct points determine a line &	01
	hence there are infinitely many lines passing	
2	through one point was not clear	12
3		03
4		02
1		05
8		03
Unanswered		05

9. How many planes pass through two distinct points?
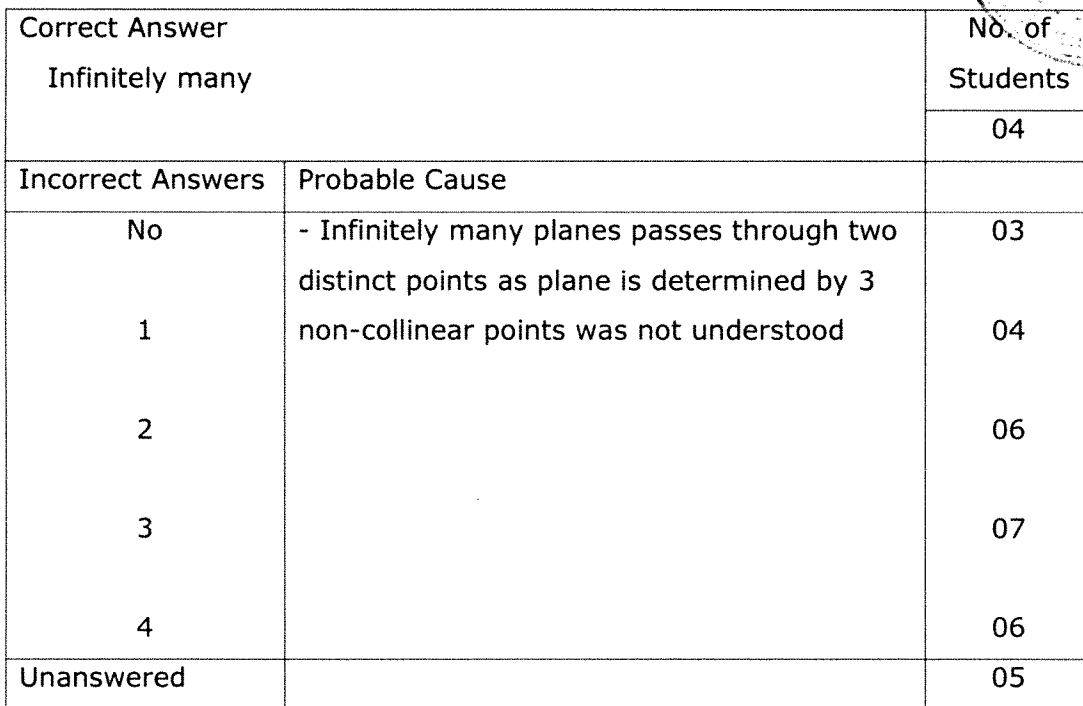
Correct Answer		No. of
Infinitely many		Students
		04
Incorrect Answers	Probable Cause	
No	- Infinitely many planes passes through two distinct points as plane is determined by 3 non-collinear points was not understood	03
1		04
2		06
3		07
4		06
Unanswered		05

10. How many planes pass through three distinct non-collinear points?

Correct Answer		No. of
One & Only one		Students
		03
Incorrect Answers	Probable Cause	
2	- One & only one plane pass through three distinct collinear points was not clear	04
3		06
4		05
6		09
Infinite		04
Unanswered		04

11. How many distinct points determine a line?

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Correct Answer		No. of
Two		Students
		10
Incorrect Answers	Probable Cause	
1	- Two distinct points determine a line was not understood	06
5		02
3	- Two distinct points determine a line was not understood - The difference between 2 distinct points determine a line & 3 distinct non-collinear points determine a plane was not clear & there was a confusion	11
Unanswered		06

12. How many distinct non-collinear points determine a plane?

Correct Answer		No. of
Three		Students
		03
Incorrect Answers	Probable Cause	
1	- Three distinct non-collinear points determine a plane was not understood	07
4		05
2	- Three distinct non-collinear points determine a plane was not understood - The difference between 2 distinct points determine a line & 3 distinct non-collinear points determine a plane was not clear & there was a confusion	12
Unanswered		08

13. Into how many parts does a line divide the plane?

Correct Answer		No. of
Three		Students
		03
Incorrect Answers	Probable Cause	
2	- Line itself is the third part along with two half planes was not clear	21
4	- line divides the plane in three parts was not understood	06
Unanswered		05

14. What is the intersection of two distinct intersecting lines?

Correct Answer		No. of
Point		Students
		06
Incorrect Answers	Probable Cause	
Line	- The meaning of 'distinct lines' and its intersection was not understood	06
Line-segment	- Intersection of two lines can never be a line-segment was not understood	05
Plane	- Intersection of two lines can never be a plane was not understood	07
Mid-point	- The point of intersection is not a mid-point & line does not have a mid-point was not understood	05
Unanswered		06

15. Does line have a bisector?

Correct Answer		No. of
No		Students
		12
Incorrect Answers	Probable Cause	

Yes	- Line cannot have a bisector as it is extended infinitely was not understood	17
Unanswered		06

16. A line-segment has how many mid-points?

Correct Answer One		No. of Students
		18
Incorrect Answers	Probable Cause	
2	- There was confusion between the end-points & a mid-point. Also, there is only one mid-point of a line-segment was not clear	07
0	- Line has a no mid-point but a line-segment has a mid-point was not understood	05
Unanswered		05

17. How many distinct lines determine a plane?

Correct Answer Two		No. of Students
		10
Incorrect Answers	Probable Cause	
1	- There are infinitely many planes passing through one line was not understood	09
3	- More than two lines can determine more than one plane was not clear	07
4		05
Unanswered		04

18. What is the intersection of two distinct planes?

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Correct Answer		No. of
Line		Students
		09
Incorrect Answers	Probable Cause	
Point	- Difference between the intersection of two distinct lines and two distinct planes was not clear	07
Line-segment	- Line-segment cannot be intersection of plane ever was not understood	08
Rectangle	- Rectangle can never be the intersection of the plane was not understood	06
Unanswered		04

19. When will two rays be opposite to each other?

Correct Answer		No. of
- Two rays having same initial point & lying on the same line in opposite direction will be opposite rays		Students
- For A-O-B, $\overrightarrow{OA}$ & $\overrightarrow{OB}$ will be opposite rays		01
Incorrect Answers	Probable Cause	
Has same initial point	- The condition that the two rays should lie on one line & in opposite direction for being opposite rays was not understood	06
Are in opposite direction	- The condition that the opposite rays should have same initial point & lie on one line was not considered	05
Lie on same line & has same initial point	- The condition that the two rays should be in opposite direction for opposite rays was not clear	06
Lie on same line	- For opposite rays the two rays should have same initial point & lie in opposite direction was not understood	05

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They intersect each other	- Difference between intersecting rays & opposite rays was not clear	07
Unanswered		05

20. How many arms does an angle have?

Correct Answer		No. of
Two		Students
		20
Incorrect Answers	Probable Cause	
1	- Difference between angle & ray was not clear	05
3	- It has only two arms was not understood	02
Infinite		03
Unanswered		05

21. How many vertices does an angle have?

Correct Answer		No. of
One		Students
		23
Incorrect Answers	Probable Cause	
3	- An angle has only one vertex was not understood	04
2		02
Infinite		02
Unanswered		03

22. How many bisectors does an angle have?

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Correct Answer		No. of
One		Students
		14
Incorrect Answers	Probable Cause	
2	- An angle has only one bisector was not understood	08
3		05
Infinite		04
Unanswered		04

23. Are supplementary angles congruent?

Correct Answer		No. of
Not necessary / Not always		Students
		09
Incorrect Answers	Probable Cause	
Yes	- Supplementary angles are congruent was a misconception	26
Unanswered		00

24. Are vertically opposite angles congruent?

Correct Answer		No. of
Yes		Students
		12
Incorrect Answers	Probable Cause	
No	- Vertically opposite angles are always congruent was not understood	22
Unanswered		01

25. Do adjacent angles always form a linear pair of angles?

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Correct Answer		No. of
No		Students
		22
Incorrect Answers	Probable Cause	
- Yes	- There are adjacent angles which do not form a linear pair of angles was not understood	10
Unanswered		03

26. Is linear pair of angles adjacent?

Correct Answer		No. of
Yes		Students
		30
Incorrect Answers	Probable Cause	
- No	- All linear pair of angles are adjacent was not clear	04
Unanswered		01

27. Are complementary angles adjacent?

Correct Answer		No. of
Yes		Students
		18
Incorrect Answers	Probable Cause	
- No	- Complementary angles are always adjacent was not understood	16
Unanswered		01

28. Is linear pair of angles congruent?

Correct Answer	No. of
Not necessary / Not always	Students
	15

Incorrect Answers	Probable Cause	
- Yes	- Linear pair of angles are not always congruent was not understood	17
Unanswered		03

Q.VII Answer the following questions based on the figure below:

1. $\overleftrightarrow{AD} \cap \overleftrightarrow{AB}$

Correct Answer	No. of
{A}	Students
	14

Incorrect Answers	Probable Cause	
ϕ	- Doesn't know the difference between parallel & intersecting lines	04
$\overleftrightarrow{BD}$	- There was a misconception that the line passing through both the lines is the intersection	06
$\angle ABD$	- Intersection of two lines is either 'ϕ' or a point was not understood	02

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$\overrightarrow{AB}$	- Intersection of two lines is a line, if they are the same line was not clear	02
A	- The point of intersection should be represented as a set was not clear	04
Unanswered		03

2. $\overleftrightarrow{BC} \cap \overrightarrow{DA}$

Correct Answer {A}		No. of Students
		04
Incorrect Answers	Probable Cause	
ϕ	- There is no clarity about the line being extended infinitely on both the sides - $\overleftrightarrow{BC}$ was perceived as $\overline{BC}$, so the difference between the line and line-segment was not understood	10
ℓ_2	- The line intersecting (passing through) both the lines was assumed as intersection of the two lines	04
Line-segment	- Intersection of two lines can never be a line-segment was not clear	03
Unanswered		04

3. $\ell_2 \cap \ell_4$

Correct Answer ϕ		No. of Students
		04
Incorrect Answers	Probable Cause	
{C,B}	- There was a misconception that the line which intersects the two lines, the set of those intersecting points is the intersection	05

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	- The concept of parallel lines was not understood	
ℓ_1	- The transversal is the intersection of the two parallel lines was a misconception	08
{F,B,D,C,E}	Set of all the points named either on ℓ_2 or ℓ_4 is intersection, was a misconception. Intersection of lines is a point was not clear	06
$\{\phi\}$	- The difference between ϕ & $\{\phi\}$ was not clear	07
Unanswered		05

4. $\overline{CB} \cap \overline{CA}$

Correct Answer $\overline{CB}$		No. of Students
		02
Incorrect Answers	Probable Cause	
{C,A,B}	- Set of all the points named either on $\overline{CB}$ or $\overline{CA}$ is the intersection was a misconception - Intersection of two line-segments can be either ϕ or singleton set or a line-segment was not clear	06
$\overline{CA}$	- The relation of $\overline{CB}$ & $\overline{CA}$ in terms of sets was not clear and the intersection will be a subset & not superset was not understood.	04
{B}	- There was a misconception that the point in between C & A i.e. B is considered as intersection	14
$\overrightarrow{CA}$	- Intersection of two line-segments can never be a line was not clear	03

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Unanswered		06

5. $\overline{CB} \cap \overline{DA}$

Correct Answer ϕ		No. of Students
		05
Incorrect Answers	Probable Cause	
$\ell_1 \& \ell_3$	- The two lines of which the two line-segments are subsets is the intersection of the line-segments was a misconception	07
Line-segment/$\overline{AB}$	- Not aware that intersection of two line-segments can be an empty set	05
		04
$\{\phi\}$	- The difference between ϕ & $\{\phi\}$ was not clear	09
Unanswered		05

6. $\ell_2 \cap \ell_3$

Correct Answer $\{D\}$		No. of Students
		07
Incorrect Answers	Probable Cause	
$\{B,A\}$	- There was a misconception that the intersection of two lines is the point of intersection of another line passing through both the lines, $\ell_2 \& \ell_3$	07
ℓ_1	- The intersection of two lines is the line passing through both the lines, $\ell_2 \& \ell_3$ was a misconception	09
ϕ	- The difference between parallel lines &	06

	intersecting lines was not clear	
Unanswered		06

7. $\overrightarrow{BC} \cap \overrightarrow{DA}$

Correct Answer ϕ		No. of Students
		06
Incorrect Answers	Probable Cause	
$\{\phi\}$	- The difference between ϕ & $\{\phi\}$ was not clear	04
$\{A\}$	- The difference between $\overrightarrow{BC}$ & $\overrightarrow{CB}$ was not understood & are perceived to be same	07
ℓ_2	- A line passing through both the rays was considered as intersection of the rays	08
$\{B,D\}$	- There was a misconception that the end-points of both the rays is a set of intersection (as they were lying on one given line)	05
Unanswered		05

8. $\overrightarrow{AG} \cap \overrightarrow{CB}$

Correct Answer $\overrightarrow{AG}$		No. of Students
		02
Incorrect Answers	Probable Cause	
$\{A,G,C,B\}$	- All the points named on the line ℓ_1 of which $\overrightarrow{AG}$ & $\overrightarrow{CB}$ are subsets was considered as a set of intersection	07
ϕ	- The concept of ray being extended on one-side was not clear & were perceived as line-segments	12

ℓ_1	- The line on which both the rays lie wais considered as intersection	08
Unanswered		06

9. $\overline{BD} \cap \overrightarrow{FD}$

Correct Answer $\overline{BD}$		No. of Students
		01
Incorrect Answers	Probable Cause	
{B,D}	- Instead of the line-segment the endpoints of the line-segment were considered as intersection	07
ℓ_2	- The line on which both line-segment & ray lies was considered as intersection	07
$\overrightarrow{BD}$	- The intersection of line-segment & ray, can never be a ray was not clear	05
{D}	- The points named and which were common to both $\overline{BD}$ & $\overrightarrow{FD}$ were considered as an intersection	08
Unanswered		07

10. $\overrightarrow{AG} \cap \overrightarrow{BC}$

Correct Answer ϕ		No. of Students
		04
Incorrect Answers	Probable Cause	
$\overline{AB}$	- The line-segment lying between the end-points (initial points) of two rays was considered	06
ℓ_1	- The line on which both the rays lie was considered as intersection	07

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$\{\phi\}$	- The difference between ϕ & $\{\phi\}$ is not clear	06
$\{C,B,A,G\}$	- Points named and which are on either of the ray were taken as an intersection	05
Unanswered		07

11. $\overrightarrow{CE} \cap \ell_4$

Correct Answer $\overrightarrow{CE}$		No. of Students
		02
Incorrect Answers	Probable Cause	
ℓ_4	- The intersection was considered as a superset of the two given sets instead of the subset. - Not clear that intersection of ray & line can never be a line	12
$\{C,E\}$	- The points named & are common to both ray & line were considered	14
Unanswered		07

12. $\overrightarrow{CB} \cap \overrightarrow{BA}$

Correct Answer $\{B\}$		No. of Students
		06
Incorrect Answers	Probable Cause	
ϕ	- Not clear that the end-point is common to both the line-segments is the intersection as it is a part of both the line-segments	09
$\{\phi\}$	- Not clear that the end-point is common to both the line-segments is the intersection as it is a part of both the line-segments - The difference between ϕ & $\{\phi\}$ is not clear	05

$\overline{CA}$	- The union of two line-segments was considered instead of the intersection of two line-segments	04
ℓ_1	- The line of which both the line-segments are subsets was considered as intersection	04
B	- The intersection should be represented in a set form was not clear	02
Unanswered		05

13. $\overline{FB} \cap \overrightarrow{BA}$

Correct Answer $\{B\}$		No. of Students
		07
Incorrect Answers	Probable Cause	
ϕ	- The end-point of line-segment & the initial point of the ray is common to both so it is the intersection was not understood	08
$\{\phi\}$	- The end-point of line-segment & the initial point of the ray is common to both so it is the intersection was not understood - The difference between ϕ & $\{\phi\}$ is not understood	04
$\{F,B,A\}$	- The points named & lying either on line-segment or ray were considered as intersection	07
ℓ_2, ℓ_1	- The lines on which line-segment & ray lie were considered as intersection	05
Unanswered		04

14. $\overline{FB} \cap \ell_2$

Correct Answer $\overline{FB}$		No. of Students
		04
Incorrect Answers	Probable Cause	
ℓ_2	- Intersection was considered as a line of which line-segment is a subset	11
{F,B}	- The points which were named & common to both $\overline{FB}$ & ℓ_2 were considered as intersection	12
Unanswered		08

15. $\overrightarrow{BD} \cap \overrightarrow{FD}$

Correct Answer $\overrightarrow{BD}$		No. of Students
		03
Incorrect Answers	Probable Cause	
{B,D,F}	- The points which were named & common to both ray & line were considered as intersection	12
{D}	- The point which was common in the symbolic representation of the rays $\overrightarrow{BD}$ & $\overrightarrow{FD}$ was considered as intersection	14
Unanswered		06

16. $\overrightarrow{BD} \cap \overrightarrow{BF}$

Correct Answer {B}		No. of Students
		05
Incorrect Answers	Probable Cause	
$\overrightarrow{BD}$	- The $\overrightarrow{BF}$ is perceived as $\overrightarrow{FB}$ & the difference was not understood	06

ϕ	- The end-point is included & is part of ray was not clear; & is common to both the rays was not considered	07
$\{\phi\}$	- The end-point is included & is part of ray was not clear; & is common to both the rays was not considered - Difference between ϕ & $\{\phi\}$ was not clear	05
$\overrightarrow{FD}$	- Irrespective of the direction of both the rays looking to the position of points F-B-D on line ℓ_2, the ray from left to right $\overrightarrow{FD}$ was considered as a intersection	07
Unanswered		05

17. $\overrightarrow{CE} \cap \overrightarrow{BC}$

Correct Answer		No. of
$\{C\}$		Students
		06
Incorrect Answers	Probable Cause	
ϕ	- The initial point 'C' was not included as a part of ray and was not considered as common point to both the rays	07
$\{\phi\}$	- The initial point 'C' was not included as a part of ray and was not considered as common point to both the rays - Difference between ϕ & $\{\phi\}$ was not clear	05
ℓ_2	- The line which intersect both the rays was considered as intersection	05
ℓ_3		07
Unanswered		05

18. $\overrightarrow{DA} \cap \overrightarrow{CE}$

Correct Answer		No. of
ϕ		Students
		04
Incorrect Answers	Probable Cause	
$\{\phi\}$	- Difference between ϕ & $\{\phi\}$ was not clear	07
ℓ_3, ℓ_4	- The lines which include the rays $\overrightarrow{DA}$ & $\overrightarrow{CE}$ were considered as intersection	09
ℓ_1	- The line which intersects both the rays was considered as intersection	09
Unanswered		06

Q.VIII Answer the following questions based on the figure below:

In the figure X, Y & Z are 3 parallel planes.
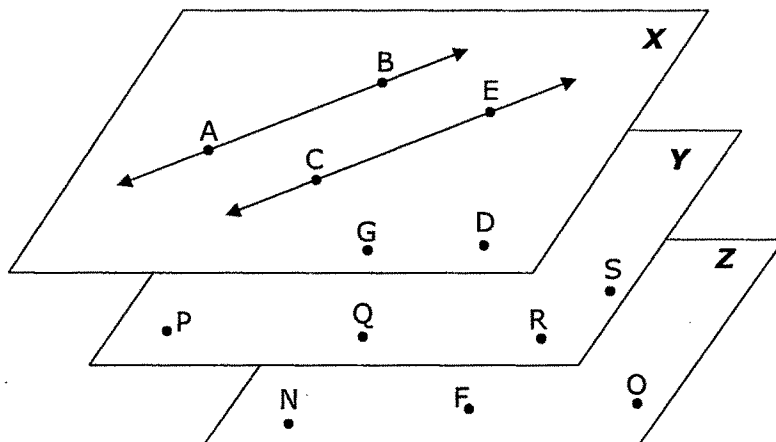
1. Which all points are coplanar with respect to plane X?

Correct Answer		No. of
A,B,C,D,E,G		Students
		09
Incorrect Answers	Probable Cause	
G,D	- Points not lying on the line were considered as co-planar points	09

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A,B,C,E	- Points lying on the line in the plane were considered as co-planar points	06
$\overrightarrow{AB}$, $\overrightarrow{CE}$	- The difference in the terms co-planar points & co-planar lines was not clear	06
Unanswered		05

2. Which all points are coplanar with respect to plane Y?

Correct Answer P,Q,R,S		No. of Students
		23
Incorrect Answers	Probable Cause	
P,Q,R,S,Y	- 'Y' is a plane & not a point was not understood	06
Unanswered		06

3. Which points are coplanar with respect to plane Z?

Correct Answer N,F,O		No. of Students
		22
Incorrect Answers	Probable Cause	
N,F,O,Z	- 'Z' is a plane & not a point was not clear	08
Unanswered		05

4. List all the points that are coplanar to the point R?

Correct Answer P,Q,S		No. of Students
		05
Incorrect Answers	Probable Cause	
P,Q	- All the three points are coplanar i.e. P,Q,S was not clear, only two points were listed as coplanar points	05
Q,S		06

P,Q,R,S	- The other three points are coplanar to R so it should not include 'R' was not understood	07
P,Q,S,Y	- 'Y' is a plane where the points lie was not understood	08
Unanswered		04

5. Are $\overrightarrow{AB}$ & $\overleftarrow{CE}$ parallel?

Correct Answer Yes		No. of Students
		23
Incorrect Answers	Probable Cause	
No	- The figure of parallel lines was not comprehended	07
Unanswered		05

6. Are $\overrightarrow{AB}$ & $\overrightarrow{PQ}$ parallel?

Correct Answer No		No. of Students
		20
Incorrect Answers	Probable Cause	
Yes	- The two lines in parallel planes are not always parallel was not understood	09
Unanswered		06

7. Are $\overrightarrow{QS}$ & $\overleftarrow{FG}$ parallel?

Correct Answer No		No. of Students
		21
Incorrect Answers	Probable Cause	
Yes	- The two lines in parallel planes are not	09

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	always parallel was not understood	
Unanswered		05

8. Mention the points lying in the same half planes w.r.t. $\overleftrightarrow{CE}$?

Correct Answer		No. of
A,B and G,D		Students
		06
Incorrect Answers	Probable Cause	
A,B & G,D	- The points to be represented in set form was not clear	07
$\overrightarrow{AB}$ & $\overrightarrow{GD}$	- The lines were written instead of points, the difference between points & lines was not clear	08
A,B,C,D,E,G	- Points lying on the line are not in any half-plane was not clear and all the points of plane X in which the $\overleftrightarrow{CE}$ lies were written	09
Unanswered		05

9. What is the relation between $\overleftrightarrow{CE}$ & plane X?

Correct Answer		No. of
$\overleftrightarrow{CE} \subset$ plane X		Students
		07
Incorrect Answers	Probable Cause	06
$\overleftrightarrow{CE} \in X$	- The difference between '$\subset$' & '$\in$' was not understood	07
$\overleftrightarrow{CE}$ is coplanar to X	- Two lines can be coplanar and not the line and a plane was not understood	07
They lie in same plane	-'X' is a plane & not a point, & cannot lie on a plane itself was not clear	05
Unanswered		04

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10. Are lines $\overrightarrow{AB}$ & $\overrightarrow{GD}$ coplanar?

Correct Answer		No. of
Yes		Students
		14
Incorrect Answers	Probable Cause	
No	- As line $\overrightarrow{GD}$ was not drawn in the figure but points were given in the plane in which $\overrightarrow{AB}$ lies so the lines $\overrightarrow{AB}$ & $\overrightarrow{GD}$ were perceived as non-coplanar	16
Unanswered	-	05

11. Are lines $\overrightarrow{AB}$ & $\overrightarrow{QS}$ coplanar?

Correct Answer		No. of
No		Students
		23
Incorrect Answers	Probable Cause	
Yes	- The two lines not lying in the same plane are not coplanar but skew lines was not understood	08
Unanswered		04

Q.IX[A] Look at the following figure below & answer the following questions:

1. Name the arms of $\angle QRS$

Correct Answer		No. of
$\overrightarrow{RQ}$ & $\overrightarrow{RS}$ / $\overrightarrow{RP}$ & $\overrightarrow{RS}$		Students
		09
Incorrect Answers	Probable Cause	
$\overrightarrow{QR}$, $\overrightarrow{QS}$	- The difference between $\angle QRS$ & $\angle RQS$ was not understood. - The arms are two rays where the point in	10

	the centre in $\angle QRS$ is an initial point	
$\overrightarrow{QR}$, $\overrightarrow{RS}$	- In $\angle QRS$ the arms are considered as two rays starting from Q to R & R to S i.e. $\overrightarrow{QR}$ & $\overrightarrow{RS}$. R is the initial point for both the rays was not understood.	10
Unanswered		06

2. List the points lying in the interior of $\angle PRS$

Correct Answer A,B		No. of Students
		16
Incorrect Answers	Probable Cause	
- A,B,Q,P,S	- The points which are on the angle are not in the interior was not clear	15
- D,C	- The difference between interior of an angle & exterior of an angle was not understood	02
Unanswered		02

3. List the points lying in the exterior of $\angle PRS$

Correct Answer D,C		No. of Students
		14
Incorrect Answers	Probable Cause	
- A,B,C,D	- There was confusion in the three partitions made by an angle	12
- D,P,Q,R,C,S	- The points which are on the angle are not in the exterior was not clear	04
- A,B	- The difference between interior of an angle & exterior of an angle was not understood	03
Unanswered		02

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4. List the points lying on the angle $\angle QRS$

Correct Answer P,Q,R & S		No. of Students
		04
Incorrect Answers	Probable Cause	
- P	- The other points on the angle were not considered	09
- P,Q		11
- A,B	- The difference between the points in the interior of an angle & points on the angle was not clear	06
Unanswered		05

5. Are angles $\angle QRS$ & $\angle PRS$ same?

Correct Answer Yes		No. of Students
		16
Incorrect Answers	Probable Cause	
No	- If the vertex & arms of both the angles are same then they are same angles was not understood	14
Unanswered		05

6. Are angles $\angle PRS$ & $\angle PRC$ same?

Correct Answer No		No. of Students
		20
Incorrect Answers	Probable Cause	
Yes	- If the arms of both the angles are not same then they are not same angles in spite of same vertex is not clear	11

Unanswered		04

7. How many partitions of the plane are made by $\angle QRS$?

Correct Answer Three		No. of Students
		10
Incorrect Answers	Probable Cause	
2	- The points on the angle is the third partition was not clear	14
1	- There are in total 3 partitions of plane made by an angle was not understood	05
Unanswered		06

8. Will the ray $\overrightarrow{RD}$ intersects $\overline{PS}$?

Correct Answer No		No. of Students
		22
Incorrect Answers	Probable Cause	
Yes	- If the point is in the exterior of an angle then the ray from vertex of an angle to that point will not intersect the line-segment joining the points on two arms of an angle was not understood.	09
Unanswered		04

9. Will the ray $\overrightarrow{RA}$ intersect $\overline{PS}$?

Correct Answer Yes		No. of Students
		23
Incorrect Answers	Probable Cause	
No	- In $\angle PRS$ if A is in the interior of an angle	09

	then $\overrightarrow{RA}$ intersects $\overline{PS}$ (Cross-bar theorem) was not understood	
Unanswered		03

10. Will the ray $\overrightarrow{RS}$ intersect $\overline{PS}$?

Correct Answer Yes		No. of Students
		14
Incorrect Answers	Probable Cause	
No	- If S is a point on $\angle PRS$ then the intersection of one arm $\overrightarrow{RS}$ & $\overline{PS}$ is a singleton set & not empty set was not clear. Also 'S' is the end-point & is a point of $\overline{PS}$ was not understood	15
Unanswered		06

Q.IX[B] Fill up the table below having the arms & vertices of the corresponding angles:

1. $\angle DEF$

Correct Answer Arms: $\overrightarrow{ED}$ & $\overrightarrow{EF}$ Vertex: E		No. of Students
Correct Answers	Both (Arms and Vertex)	13
	Only Arms	0
	Only Vertex	11
Incorrect Answers	Probable Cause	
Arms: $\overrightarrow{DE}$, $\overrightarrow{DF}$	- $\overrightarrow{DE}$ & $\overrightarrow{DF}$ are arms was a misconception	12
Arms: $\overline{ED}$, $\overline{EF}$	- For $\angle DEF$, $\overline{ED}$ & $\overline{EF}$ are line-segments & not arms of an angle; arms of an angle are	10

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	always rays was not clear	
Vertex: D	- D is a vertex is a misconception	11

From the above table it is observed that there were 13 (13+0) correct responses for arms of the given angle and 24 (13+11) correct responses for the vertex of the given angle.

2. $\overrightarrow{PQ}, \overrightarrow{PR}$

Correct Answer Angle: $\angle QPR$ / $\angle RPQ$ Vertex: P		No. of Students
Correct Answer	Both (Angle and Vertex)	12
	Only Angle	0
	Only Vertex	16
Incorrect Answers	Probable Cause	
Angles: $\angle PQR$	- $\overrightarrow{PQ}$ & $\overrightarrow{PR}$, the angle formed is $\angle PQR$ was a misconception	23
Vertex: $\angle P$	- Difference between Vertex P & $\angle P$; was not clear	03
Vertex: Q	- Q or R is not a vertex, the common initial point of both the arms in an angle is a vertex was not understood	02
Vertex: R	- Q or R is not a vertex, the common initial point of both the arms in an angle is a vertex was not understood	02

From the above table it is observed that there were 12 (12+0) correct responses for Angle of the given arms and 28 (12+16) correct responses for the vertex of the given angle.

Q.X Refer the figure below & select appropriate option(s) for the given pair of angles (Put a mark '✓' in the table against the selected options):
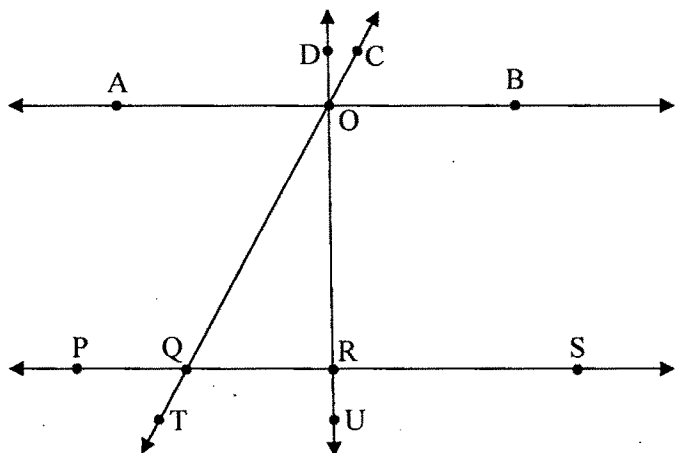
1. $\angle DOA$, $\angle DOC$

Correct Answer Adjacent Angles		No. of Students
		03
Incorrect Answers	Probable Cause	
Complementary & Adjacent	- Complementary angles were not understood	09
Supplementary & Adjacent	- Supplementary angles were not understood	09
Vertically Opposite & Adjacent	- Vertically Opposite angles & Adjacent angles were not clear. - Vertically Opposite angles & Adjacent angles are not possible together was not understood	09
Unanswered		05

2. $\angle PQT$, $\angle TQR$

Correct Answer Supplementary Angles, Adjacent Angles, Linear Pair of Angles	No. of Students
	02

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Incorrect Answers	Probable Cause	
Linear Pair	- Linear pair of angles is always Supplementary was not clear - Linear pair of angles is always adjacent was not clear	08
Linear Pair, Supplementary	- Linear pair of angles is always adjacent was not clear	09
Linear pair, Complementary	- Difference between Supplementary & Complementary angles was not clear	05
Adjacent	- Adjacent angles were not understood	07
Unanswered		04

3. $\angle DOC$, $\angle COB$

Correct Answer Complementary Angles, Adjacent Angles		No. of Students
		01
Incorrect Answers	Probable Cause	
Complementary	- Adjacent angles were not clear	07
Adjacent	- Complementary angles were not clear	09
Adjacent, Linear & Vertically Opposite	- Vertically opposite angles can never be Linear & Adjacent was not clear	08
Supplementary, Adjacent	- Difference between Complementary angles & Supplementary angles was not clear	05
Unanswered		05

4. $\angle PQT$, $\angle OQR$

Correct Answer Vertically Opposite Angles	No. of Students
	06

Incorrect Answers	Probable Cause	
Vertically Opposite & Adjacent	- Vertically opposite angles & Adjacent angles are not possible together was not clear	09
Vertically Opposite & Linear	- Vertically opposite angles & Linear pair of angles are not possible together was not clear	07
Complementary & Supplementary	- Complementary angles & Supplementary angles were not clear & they cannot be possible together was not understood	09
Unanswered		04

5. $\angle ORS$, $\angle OQR$

Correct Answer		No. of
Supplementary Angles, Adjacent Angles, Linear Pair of Angles		Students
		00
Incorrect Answers	Probable Cause	
Linear, Adjacent	- Linear pair of angles is always adjacent was not clear	07
Linear, Supplementary	- Supplementary angles was not clear & Linear pair of angles is always Supplementary was not understood	06
Complementary, Vertically Opposite	- Complementary angles and vertically opposite angles were not clear	07
Supplementary, Vertically Opposite	- Supplementary angles and vertically opposite angles were not clear	09
Unanswered		05

6. $\angle URP$, $\angle URS$

Correct Answer		No. of
Supplementary Angles, Adjacent Angles, Linear Pair of Angles		Students
		01
Incorrect Answers	Probable Cause	
Adjacent	- Supplementary angles were not clear & Linear pair of angles was not clear	09
Supplementary	- Adjacent not clear & Linear not clear	08
Supplementary, Linear	- Linear pair of angles is always Adjacent was not clear	05
Complementary, Adjacent	- Difference between Supplementary angles & Complementary angles was not clear	07
Unanswered		05

7. $\angle DOC$, $\angle QOR$

Correct Answer		No. of
Vertically Opposite Angles		Students
		05
Incorrect Answers	Probable Cause	
Linear	- Vertically opposite angles was not understood and can never be Linear was not clear	08
Linear, Supplementary	- Vertically opposite angles was not understood and can never be Linear was not clear - Supplementary angles was not clear	09
Adjacent	- Vertically opposite angles can never be adjacent was not understood	08
Unanswered		05

8. $\angle OQR$, $\angle OQP$

Correct Answer		No. of
Supplementary Angles, Adjacent Angles, Linear Pair of Angles		Students
		03
Incorrect Answers	Probable Cause	
Linear, Supplementary	- Linear pair of angles and supplementary angles are always adjacent was not clear	06
Supplementary	- Adjacent angles & Linear pair of angles were not clear	05
Adjacent	- Linear pair of angles & Supplementary angles were not clear	07
Supplementary, Complementary	- Supplementary angles and complementary angles are not possible together was not understood	04
Linear, Adjacent, Complementary	- Linear pair of angles can never be complementary was not clear & difference between Complementary angles and Supplementary angles was not clear	04
Unanswered		06

9. $\angle QOR$, $\angle ROB$

Correct Answer		No. of
Adjacent Angles		Students
		04
Incorrect Answers	Probable Cause	
Complementary	- Complementary angles were not clear	09
Adjacent, Linear	- Adjacent angles are not always Linear was not clear	09
Supplementary	- Supplementary angles were not clear	08
Unanswered		05

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10. $\angle QOR$, $\angle COB$

Correct Answer		No. of
No Relation		Students
		03
Incorrect Answers	Probable Cause	
Vertically Opposite	- Vertically Opposite angles were not clear	10
Vertically Opposite, Adjacent	- Vertically opposite angles and adjacent angles are not possible together was not understood	09
Supplementary	- Supplementary angles were not clear	09
Unanswered		04

11. $\angle ORS$, $\angle PRU$

Correct Answer		No. of
Supplementary Angles, Vertically Opposite Angles		Students
		02
Incorrect Answers	Probable Cause	
Vertically Opposite	- Supplementary angles were not clear	06
Vertically Opposite & Linear	- Vertically opposite angles and Linear pair of angles are not possible together was not understood - Difference between Linear pair of angles & Supplementary angles was not understood	08
Supplementary & Linear	- Supplementary angles are not always Linear was not understood; but Linear pair of angles is always Supplementary	07
Vertically Opposite &	- Difference between Supplementary angles & Complementary angles was not	06

Complementary	clear	
Vertically Opposite & Adjacent	- Vertically opposite angles and adjacent angles are not possible together was not clear	03
Unanswered		03

Q.XI Answer the following questions based on the same figure above (in last question):
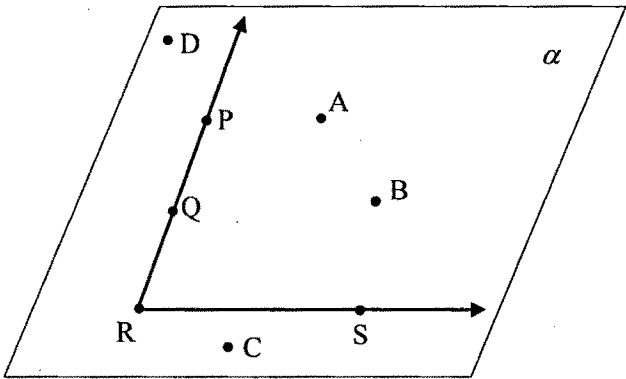
1. Which type of angle is $\angle COB$?

Correct Answer	No. of
Acute angle	Students
	15
Incorrect Answers	Probable Cause	
Right	- Difference between Acute angle & Right angle was not clear	04
Complementary	- An angle cannot be Complementary or Supplementary; it always refers to pair of	07
Supplementary	angles was not understood	06
Unanswered		03

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2. Which type of angle is $\angle OQP$?

Correct Answer Obtuse angle		No. of Students
		13
Incorrect Answers	Probable Cause	
Obtuse	- Difference between Acute angle & Obtuse angle was not clear	11
Complementary	- An angle cannot be Complementary or Supplementary; it always refers to pair of	05
Supplementary	angles was not understood	04
Unanswered		02

3. Which type of angle is $\angle OQR$?

Correct Answer Acute angle		No. of Students
		16
Incorrect Answers	Probable Cause	
Obtuse	- Difference between Acute angle & Obtuse angle was not clear	05
Right	- Difference between Acute angle & Right angle was not clear	03
Complementary	- An angle cannot be Complementary or Supplementary; it always refers to pair of	03
Supplementary	angles was not understood	04
Unanswered		04

4. Which type of angle is $\angle ORS$?

Correct Answer Right angle		No. of Students
		14
Incorrect Answers	Probable Cause	

Acute	- Difference between Acute angle & Right angle was not clear	05
Obtuse	- Difference between Obtuse angle & Right angle was not clear	06
Complementary	- An angle cannot be Complementary or Supplementary; it always refers to pair of	05
Supplementary	angles was not understood	02
Unanswered		03

5. Which type of angle is $\angle ROA$?

Correct Answer		No. of
Right angle		Students
		12
Incorrect Answers	Probable Cause	
Acute	- Difference between Acute angle & Right angle was not clear	06
Obtuse	- Difference between Obtuse angle & Right angle was not clear	08
Complementary	- An angle cannot be Complementary or Supplementary; it always refers to pair of	04
Supplementary	angles was not understood	03
Unanswered		02

6. Which type of angle is $\angle COA$?

Correct Answer		No. of
Obtuse angle		Students
		11
Incorrect Answers	Probable Cause	
Acute	- Difference between Acute angle & Obtuse angle was not clear	08
Right	- Difference between Obtuse angle & Right	08

	angle was not clear	
Complementary	- An angle cannot be Complementary or	02
	Supplementary; it always refers to pair of	
Supplementary	angles was not understood	04
Unanswered		02

7. Which type of angle is $\angle QOA$?

Correct Answer		No. of
Acute angle		Students
		09
Incorrect Answers	Probable Cause	
Obtuse	- Difference between Obtuse angle & Right angle was not clear	08
Right	- Difference between Acute angle & Right angle was not clear	06
Complementary	- An angle cannot be Complementary or Supplementary; it always refers to pair of	05
Supplementary	angles was not understood	04
Unanswered		03

4.6.3 Learning Difficulties

From the above item-wise analysis of the responses of the students on Diagnostic Test the learning difficulties in geometry were identified with respect to each topic and subtopic by inferring the probable cause of the incorrect answer. The probable causes for the errors were based on the lack of understanding related to the previous and basic concepts, the misconceptions held by the students, and no clarity about the concept addressed. Learning difficulties are referred in terms of understanding the content. It is seen as difficulty in:

- acquiring the knowledge of different geometrical symbols and statements
- understanding about different geometrical terms and concepts.
- misconceptions related to different geometrical concepts

The learning difficulties related to the very basic understanding about the geometrical concepts were identified broadly which resulted in other learning difficulties. They were as listed below.

- No clarity about defined and undefined terms
- The basic concept of different geometrical terms was not understood. There was confusion in the different geometrical terms viz. Point, line, line-segment, ray, Plane and Angle.
- Difference between the geometrical representation and · the geometrical figure for line, line-segment and ray was not understood
- The concept of the geometrical figures in terms of set of points was not understood and the intersection of any two sets was not understood
- The geometrical representation and the symbols used for all the geometrical concepts viz. point, line, line-segment, ray, Plane and Angle was not clear
- The relation of all the geometrical terms in terms of their set formation and subset of each other was not understood. Thus the intersection of line, line-segment and ray in different combinations was a difficulty.
- For the geometrical terms (line, line-segment and ray) only the points which are named are considered as a part of it, apart from these named points there are infinitely many points on them was not understood
- The relation of point, line and plane was not understood
- The difference between line and plane was not understood
- Types of pair of angles and types of angles were not understood

Further, the learning difficulties were analyzed in detail and were categorized topic-wise as below:

4.6.3.1 Point and Line

- Line is an undefined term was not clear
- Point is an undefined term was not clear
- Line is always straight was not understood

- Equality of two lines was not understood
- Each point lie on some line was not understood
- Definition of collinear points was crammed or memorized without understanding
- The concept of line extended infinitely on both the sides was not clear
- Geometrical figure and representation of line was not clear
- Meaning of distinct lines was not understood
- Difference between parallel lines and equal lines was not clear
- Difference between parallel and intersecting lines was not understood
- Difference between distinct lines & equal lines was not clear
- The figure of parallel lines was not comprehended
- In $\overleftarrow{AB}$, A & B are points on line
- Line does not have a bisector and a mid-point was not clear
- "ℓ_1" represents line & not line-segment was not understood
- There can be only one line represented as 'ℓ' was not clear
- Three distinct points lying on three distinct lines may be collinear was not understood
- Difference between collinear points and non-collinear points was not understood
- Two parallel lines do not intersect was not clear
- In betweenness relation of the points the points are collinear and the order of the points on the line was not understood
- One & only one line passes through two distinct points as line is determined by two distinct points was not understood
- There are infinitely many lines passing through one point was not clear

4.6.3.2 Point, Line and Plane

- Difference between line and plane was not understood
- Differences between the collinear points and coplanar points, non-collinear and non-coplanar points were not understood and how the context of point with respect to plane and line is different was not clear.

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- The importance of "same line" in case of collinear points and "same plane" in case of coplanar points was not clear
- The difference between 2 distinct points determine a line & 3 distinct non-collinear points determine a plane was not clear & there was a confusion
- The difference in the terms co-planar points & co-planar lines was not clear
- Line divides the plane in three parts was not clear
- Line itself is the third partition of the plane along with two half planes was not understood
- Points lying on the line are not in any half-plane was not clear and all the points of plane were considered as the points in the half-planes
- Points not lying on the line were considered as co-planar points
- The two lines in parallel planes are not always parallel was not understood
- Difference between parallel lines, intersecting lines and skew lines was not clear
- Two lines can be coplanar and not the line and a plane was not understood
- For line $\overleftrightarrow{GD}$ which was not drawn in the figure but points G and D were given in the plane in which $\overleftrightarrow{AB}$ lies so the lines $\overleftrightarrow{AB}$ & $\overleftrightarrow{GD}$ were perceived as non-coplanar

4.6.3.3 Line-segment

- Line-Segment is a defined term was not clear
- Line-segment can never be equal to a line was not understood
- For A, B ∈ ℓ, $\overline{AB} \subset \ell$ but $\overline{AB}$ was considered to be equal to $\overleftrightarrow{AB}$. So, the difference between the symbolic representation of line and line-segment was not clear
- Difference between line and line-segment was not understood
- In $\overline{AB}$, A & B are points on line-segment was not clear

- In $\overline{AB}$, A & B are end-points of the line-segment was not clear
- The difference between mid-point & end-point of the line-segment was not clear
- Not aware that the mid-point has to be in between the two points and which divides the line-segment in two equal parts (equal length)
- Mid-point of the line-segment is the point in between the end-points of the line-segment was not understood
- There is only one mid-point for the given line-segment was not clear
- On the number-line, the mid-point of the line-segment is to be computed was not clear and the value associated to the point in between on the line-segment was assumed as the mid-point of the line-segment i.e. the value was estimated for the mid-point based on the figure but was not exactly calculated
- The two line-segments seemed to be of equal length were considered (estimated) to be the congruent line-segments, actually the length of the line-segments is to be calculated and compared was not clear
- Not clear that line-segment is not just end-points but is a set of all the points in between A & B including A & B.
- The set representation of the line-segment was not understood. Its union of {A,B}& {A-P-B} and not intersection was not clear.

4..6.3.4 Ray
- Ray is a defined term was not clear
- Ray do not have both initial point and end point was not understood
- The concept of ray that its extended infinitely on one side was not clear
- The end-point / initial point is included & is part of ray was not clear
- In $\overrightarrow{AO}$, A is an initial point / end point was not understood
- The geometrical figure of a ray was not clear
- In $\overrightarrow{AO}$, A & O are points on the ray was not understood
- The difference between $\overrightarrow{AB}$ & $\overrightarrow{BA}$ is not understood

- Difference between ray, line and line-segment in terms of the number of end-points they have was not clear

- For A-B-D, $D \notin \overrightarrow{AB}$ was considered, so the concept of ray extended infinitely was not understood

- For two opposite rays both the rays are opposite to each other was not clear

- Difference between "Distinct rays" & "Opposite rays" was not understood

- Difference between intersecting rays & opposite rays was not clear.

- The significance of all the three conditions to be satisfied simultaneously for the opposite rays was not understood viz. both the rays lying on the same line, having same initial point and are in opposite direction.

- Equality of two rays was not understood

- For B-A-Y, $\overrightarrow{BA} = \overrightarrow{BY}$ was not clear

- There was confusion between concept of parallel lines & not intersecting rays.

- The relation of points forming ray as a set was not clear and ray is not just two points on the ray but a set of all the points in from A towards and beyond B including point A

- The difference between the set representation of line-segment & ray was not understood

4.6.3.5 Intersection

- Intersection of two lines is either a point or an empty set was not understood

- Two lines intersecting in one point have to be distinct and not same was not understood

- Intersection of two lines can never be a line-segment or plane was not understood

- If the intersection is a line-segment then '$\overline{AB}$' has to be subset of line 'ℓ' was not understood

- The point of intersection of two lines is not a mid-point & line does not have a mid-point was not understood

- The intersection of two equal lines is not empty set was not understood

- If intersection of two lines is a line, then they are the same line and the relative points are on the same line was not understood

- The point of intersection is common to both the lines / line-segments / rays was not understood

- Intersection of two line-segments can be a line-segment was not understood and in that case all points are collinear was not clear

- The concept of intersection of two rays is not clear

- Intersection of two rays can be other than a point was not understood (other possibilities of $\cap$ of two rays were not clear)

- If intersection of two rays with the same initial point is a ray then they are same rays & are in same direction, was not clear

- Difference between the intersection of two distinct lines and two distinct planes was not clear i.e. intersection of two lines is a point and that of two planes is a line

- Line-segment cannot ever be the intersection of two planes was not understood

- Intersection of two line-segments can be either ϕ or singleton set or a line-segment was not clear

- In case of the intersection of the two sets where one is a subset of the other, the intersection will be a subset & not superset was not understood

- Intersection of two line-segments can never be a line was not clear

- Instead of the line-segment the endpoints of the line-segment were considered as intersection

- The intersection of line-segment & ray can never be a ray was not clear

- Points named and which are on either of the ray were taken as an intersection of the two rays

- Intersection of ray & line can never be a line was not clear
- The end-point which is common to both the line-segments is the intersection as it is a part of both the line-segments was not understood
- The point which was common in the symbolic representation of the rays $\overrightarrow{BD}$ & $\overrightarrow{FD}$ was considered as intersection i.e {D}
- Intersection of two rays can never be a line was not understood

4.6.3.6 Distance
- Distance / Length of the line segment is always positive was not clear.
- The modulus value for finding the distance was not considered
- AB represents length of $\overline{AB}$ & its value is a non-negative number was not understood
- For finding the length of the line-segment the values of two points is to be subtracted & the modulus is to be considered was not understood
- An algebraic rule that a-(-b)=a+b; a,b > 0 was not known
- An algebraic rule that (-a)-(-b)=-a+b (& not (-a)-b; a,b > 0) was not known
- Positive direction and negative direction on the number-line was not clear
- 0 is a value associated to the origin point O & is not in any direction is not clear
- The difference between the points on the number-line and the values associated to the points on the number line was not clear i.e. Alphabets represents points and the numbers are the values associated to the points
- The point which tentatively seemed to be at the centre of the line was considered as origin. The point whose value is '0' is called origin was not clear
- Origin was considered to be equidistant point from the extreme points to the left & right

- There is only one point equidistant from the given two points was not understood

4.6.3.7 Plane

- The concept of plane and the postulates related to plane were not understood. i.e. three non-collinear points determine a plane then the significance of non-colinearity was not understood, two distinct lines determine a plane was not understood
- Infinite planes pass through one point as the plane is determined by 3 non-collinear points was not understood
- All points are in some plane was not understood
- The representation of a plane with the help of a figure was misunderstood as a rectangular box and was not related with the understanding of the concept of plane and hence the coplanar points
- Infinitely many planes passes through two distinct points as plane is determined by 3 non-collinear points was not understood
- There are infinitely many planes passing through one line was not understood
- More than two lines can determine more than one plane was not clear

4.6.3.8 Angle

- An angle is made up of rays & not lines was not understood
- An angle has only two arms and one vertex was not understood
- For an angle two rays should have same initial point was not considered
- For an angle two rays should not lie on same line was not understood
- The difference between $\angle QRS$ & $\angle RQS$ was not understood.
- If the vertex & arms of both the angles are same then they are same angles was not understood
- An angle has only one bisector was not understood
- For an angle the vertex point (i.e. for $\angle QRS$, R the in between point) is the initial point of both the arms was not clear

- Difference between angle & ray was not clear. Also, ray is a subset of an angle was not clear

- The points which are on the angle are neither in the interior nor in the exterior of an angle was not clear.

- There are in total 3 partitions of plane made by an angle was not understood There was confusion in the three partitions of plane made by an angle

- In $\angle PRS$ if A is in the interior of an angle then $\overrightarrow{RA}$ intersects $\overline{PS}$ (Cross-bar theorem) was not understood

- If the point is in the exterior of an angle then the ray from vertex of an angle to that point will not intersect the line-segment joining the points on two arms of an angle was not understood

- For $\angle PRS$ the intersection of one arm $\overrightarrow{RS}$ & $\overline{PS}$ is a singleton set & not empty set was not clear.

- The difference between interior of an angle & exterior of an angle was not understood

- For $\angle DEF$, $\overline{ED}$ & $\overline{EF}$ are line-segments & not arms of an angle; arms of an angle are always rays ($\overrightarrow{ED}$ & $\overrightarrow{EF}$) was not clear

- Difference between Vertex P & $\angle P$ was not clear

- For $\overrightarrow{PQ}, \overrightarrow{PR}$, Q or R is not a vertex, the common initial point of both the arms in an angle is a vertex was not understood

- There was confusion in identification of types of pair or angles and types of angles from the given figure

- Difference between Supplementary & Complementary angles was not clear

- Complementary angles & Supplementary angles were not clear & they cannot be possible together was not understood

- Vertically opposite angles are always congruent was not understood

- All linear pair of angles is adjacent but adjacent angles do not always form a linear pair of angles was not understood

- Complementary angles are always adjacent was not understood

- Linear pair of angles are not always congruent was not understood
- Vertically Opposite angles can never be Adjacent and Linear was not understood
- Linear pair of angles is always Supplementary was not clear
- Linear pair of angles is always adjacent was not clear
- Linear pair of angles can never be complementary was not clear
- Supplementary angles are not always Linear was not understood
- An angle cannot be Complementary or Supplementary; it always refers to pair of angles was not understood
- Difference between Acute angle, Right angle, and Obtuse angle was not clear

4.6.3.9 Symbols and Geometrical figures

- The appropriate use of symbols to express the relation of point and line, line and line-segment, line and plane, ray and line, ray and line-segment was missing, i.e. No clarity about the correct use of $\in$, $\notin$, $=,\neq,\subset$
- The meaning and difference between the symbols $\in$, $\notin$, $=,\neq,\subset$ was not understood
- The difference between $\notin$ & $\neq$ was not understood
- There was a confusion between the symbols $\in$, $=,\subset$
- 'ϕ' cannot be the point of intersection but it represents empty set i.e. The line-segments are not intersecting was not understood. The meaning of ϕ was not understood
- Difference between "$\in$ - belongs to" & "$\notin$ - does not belong to" was not understood
- Naming the lines was not considered significant in the geometrical representation & was not clear
- Naming of points on the line / line-segment / ray was not understood
- There is only one point represented by 'A', 'B' , 'C' in a plane or on a line was not clear

- Point of intersection was not represented so "{X}" was not interpreted as point of intersection in the geometrical statement "$\ell_2 \cap \ell_1 = \{X\}$"
- 'X' represents point and not line was not known
- "ℓ_1" represents line & not ray or line-segment was not clear
- The representation of the points on the line or line-segment or ray was not clear
- $\overline{XY}$ represents the same line having points X and Y both was not clear
- AB represents distance between points A and B or length of the line segment $\overline{AB}$ & is always +ve was not understood
- On a number-line, O is called the origin point & is not in any direction was not clear
- The point of intersection should be represented as a set was not clear. In general the points to be represented in set form was not clear
- The difference between ϕ & $\{\phi\}$ was not clear
- 'Y' is a plane & not a point was not understood

4.6.3.10 Misconceptions

Also there were many misconceptions held by the students related to the fundamentals of geometry which are listed below.

- When point is on the line, then point was considered as a subset of line
- When point is not on line, then point was considered as not equal to line
- Rays with same initial point towards two distinct points in the same direction from the initial point were considered to be not equal rays
- Rays with different initial points but extended towards the same point are considered as same rays
- For two points A & B on the line ℓ, $\overline{AB}$ was considered as $\overline{AB} \in \ell$
- For two points A & B on the line ℓ, $\overrightarrow{AB}$ was considered as $\subset$ of ℓ
- For two points A and B, $\overrightarrow{AB} = \overrightarrow{BA}$ was considered
- For B-A-Y, $\overrightarrow{BA} \neq \overrightarrow{BY}$ was considered

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- For B-A-Y, $\overline{BA} \subset \overline{BY}$ was considered
- For D-B-A, $\overrightarrow{DA} = \overrightarrow{BA}$ was considered
- Three points lying on three distinct lines are collinear
- Point lying between two parallel lines is the point of intersection
- The intersection is part in between two lines
- In the betweenness relation of the points the in between point is an initial point & other points are forming rays with respect to that point
- The right most point on the line was considered as the origin of the number-line
- Rectangle is the intersection of two planes
- Supplementary angles are congruent
- the line passing through both the lines is the intersection of the two lines i.e. The transversal is the intersection of the two parallel lines
- The line which intersects the two lines, the set of those intersecting points is the intersection of the two lines
- Set of all the points named either on the line of intersection was considered as intersection of the two lines
- Set of all the points named either on $\overline{CB}$ or $\overline{CA}$ is the intersection of the two line segments $\overline{CB}$ and $\overline{CA}$
- For C-B-A, $\overline{CB} \cap \overline{CA}$ is the point in between C & A i.e. B is considered as intersection
- The two lines of which the two line-segments are subsets is the intersection of the line-segments
- A line passing through both the rays was considered as intersection of the rays
- The line on which both the rays lie was considered as intersection of two rays
- The left most point on the line was considered as the origin of the line
- The right most point on the line was considered
- Points lying on any line in the plane were considered as co-planar points

- For $\angle DEF$, $\overline{DE}$ & $\overline{DF}$ are arms of an angle
- For $\angle DEF$, D is a vertex of an angle
- For $\overrightarrow{PQ}$ & $\overrightarrow{PR}$, the angle formed is $\angle PQR$

The above listed learning difficulties were identified in the geometry by the investigator. This was followed by a remedial programme which is detailed out in the next chapter.

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